Fourier Transform

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1 Fourier transform formula

We will write the signal (in the spatial domain) as a lower case letter and the Fourier transform (in the spatial frequency domain) as a capital letter. The double harpoon inbetween means that $F(s)$ is the Fourier transform of $f(x)$ and that $f(x)$ is the inverse Fourier Transform of $F(s)$.

$$f(x) \rightleftharpoons F(s)$$

Note that $F(s)$ contains all the information from $f(x)$, so we can go back and forth from the spatial domain to the spatial frequency domain with no loss of information.

The formula for the Fourier transform is

$$F(s) = \int\limits_{-\infty}^{\infty} f(x)\ e^{-2\pi isx}\ dx$$

where s is the spatial frequency. The range of s is referred to as the spectrum. We will use i to denote complex numbers.

$$i = \sqrt{-1}$$

2 Inverse Fourier transform formula

The formula for the inverse Fourier Transform is

$$f(x) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} F(s)\ e^{2\pi isx}\ ds$$

In both formulas we have and exponential term. The Fourier transform has a negative exponential and the inverse formula as positive exponential. The general form is $e^{i\theta}$. With Euler's formula we see that

$$e^{i\theta} = \cos\theta + i\sin\theta$$

So $e^{i\theta}$ is an oscillator that varies between 1 and -1. This is the sinusoid nature of the frequency domain.