T.C.

SAMSUN ÜNİVERSİTESİ

MÜHENDİSLİK VE DOĞA BİLİMLERİ FAKÜLTESİ

YAZILIM MÜHENDİSLİĞİ BÖLÜMÜ

MAT301 NUMERİK YÖNTEMLER ARA SINAV CEVAP ANAHTARI

1) $f(x)=x^3-2x^2-5$ fonksiyonunu Newton-Raphson yöntemini ile $x_0=3$ başlangıç şartına bağlı olarak bağıl hata 0.01 oluncaya kadar yaklaşık kökünü hesaplayınız.

Cevap: Newton-Raphson formülü

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \quad f(x)=x^3-2x^2-5 \quad f'(x)=3x^2-4x$$

Bağıl Hata $B.H=\left|\frac{x_{n+1}-x_n}{x_{n+1}}\right|$

1.İterasyon

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$f(3)=3^3-2\times3^2-5=27-18-5=4$$
$$f'(3)=3\times3^2-4\times3=27-12=15$$
$$x_1=3-\frac{4}{15}=3-0.2667=2.7333$$

$$B.H=\left|\frac{x_1-x_0}{x_1}\right|=\left|\frac{2.7333-3}{2.7333}\right|=\frac{0.2667}{2.7333}\approx0.0976$$

2. İterasyon

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

$$f(2.7333)\approx(2.7333)^3-2\times(2.7333)^2-5=0.4788$$
$$f'(2.7333)\approx3\times(2.7333)^2-4\times(2.7333)=11.48$$
$$x_2=2.7333-\frac{0.4788}{11.48}=2.7333-0.0417=2.6916$$

$$B.H=\left|\frac{x_2-x_1}{x_2}\right|=\left|\frac{2.6916-2.7333}{2.6916}\right|=\frac{0.0417}{2.6916}\approx0.0155$$

3. İterasyon

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$f(2.6916) \approx (2.6916)^3 - 2\times(2.6916)^2 - 5 = 0.0107$$
$$f'(2.6916) \approx 3\times(2.6916)^2 - 4\times(2.6916) = 10.968$$
$$x_3 = 2.6916 - \frac{0.0107}{10.968} = 2.6916 - 0.001 = 2.6906$$

$$B.H = \left|\frac{x_3 - x_2}{x_3}\right| = \left|\frac{2.6906 - 2.6916}{2.6906}\right| = \frac{0.001}{2.6714} \approx 0.0036$$

2)
$$\begin{aligned} 4x - y + z &= 7 \\ -x + 3y - z &= -8 \\ x - y + 3z &= 4 \end{aligned}$$

lineer denklem sistemini Jacobi metodu ile 5 iterasyonda yaklaşık çözümünü bulunuz. Başlangıç değerleri $x^{(0)} = y^{(0)} = z^{(0)} = 0$.

Cevap: **Bir değişkeni diğerleri cinsinden ifade etmek için her denklemi yeniden düzenleyelim**

$$4x - y + z = 7 \Rightarrow x = \frac{7 + y - z}{4}$$
$$-x + 3y - z = -8 \Rightarrow y = \frac{-8 + x + z}{3}$$
$$x - y + 3z = 4 \Rightarrow z = \frac{4 - x + y}{3}$$

$$x^{(k+1)} = \frac{7 + y^{(k)} - z^{(k)}}{4}$$
$$y^{(k+1)} = \frac{-8 + x^{(k)} + z^{(k)}}{3}$$
$$z^{(k+1)} = \frac{4 - x^{(k)} + y^{(k)}}{3}$$

1. İterasyon

$$x^{(1)} = \frac{7 + 0 + 0}{4} = 1.75$$
$$y^{(1)} = \frac{-8 + 0 + 0}{3} = -2.6667$$
$$z^{(1)} = \frac{4 - 0 - 0}{3} = 1.3333$$

2. İterasyon

$$x^{(2)} = \frac{7+(-2.6667)-1.3333}{4} = 0.75$$

$$y^{(2)} = \frac{-8+1.75+1.3333}{3} = -1.6388$$

$$z^{(2)} = \frac{4-1.75+(-2.6667)}{3} = -0.1388$$

3. İterasyon

$$x^{(3)} = \frac{7+(-1.6388)-(-0.1388)}{4} = 1.375$$

$$y^{(3)} = \frac{-8+0.75-0.1388}{3} = -2.4629$$

$$z^{(3)} = \frac{4-0,75+(-1.6388)}{3} = 0.5371$$

4. İterasyon

$$x^{(4)} = \frac{7+(-2.4629)-0.5371}{4} = 1$$

$$y^{(4)} = \frac{-8+1.375+0.5371}{3} = -2.0293$$

$$z^{(4)} = \frac{4-1.375+(-2.4629)}{3} = 0.054$$

5. İterasyon

$$x^{(5)} = \frac{7+(-2.0293)-0.054}{4} = 1.2291$$

$$y^{(5)} = \frac{-8+1+0.054}{3} = -2.3153$$

$$z^{(5)} = \frac{4-1+(-2.0293)}{3} = 0.3255$$

Sonuç: $x = 1.2291, y = -2.3153, z = 0.3235$

3) $(1,6),(2,3)\,ve\,(3,2)$ noktalarından geçen polinom denklemini (lineer denklem sistemini oluşturarak) bulunuz.

Cevap: (1, 6), (2, 3) ve (3, 2) noktalarından geçen polinom denklemi 2. derecedendir.

$y(x) = ax^2 + bx + c$ *olmak üzere*

$(1,\ 6)$ *noktasında* $y(x) = a(1)^2 + b(1) + c = 6$

$=> a + b + c = 6$

$(2,\ 3)$ *noktasında* $y(x) = a(2)^2 + b(2) + c = 3$

$=> 4a + 2b + c = 3$

$(3, 2)$ *noktasında* $y(x) = a(3)^2 + b(3) + c = 2$

$=> 9a + 3b + c = 2$

Denklem sistemi 3 bilinmeyenli 3 denklemden oluşur ve matris formunda

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

olarak bellidir. Çözümü Gauss-Jordan ile gerçekleştirirsek,

$$\left[\begin{array}{ccccc} 1 & 1 & 1 & \vdots & 6 \\ 4 & 2 & 1 & \vdots & 3 \\ 9 & 3 & 1 & \vdots & 2 \end{array}\right] \xrightarrow{r_2 = r_2 - 4r_1} \left[\begin{array}{ccccc} 1 & 1 & 1 & \vdots & 6 \\ 0 & -2 & -3 & \vdots & -21 \\ 9 & 3 & 1 & \vdots & 2 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 1 & 1 & \vdots & 6 \\ 0 & -2 & -3 & \vdots & -21 \\ 9 & 3 & 1 & \vdots & 2 \end{array}\right] \xrightarrow{r_3 = r_3 - 9r_1} \left[\begin{array}{ccccc} 1 & 1 & 1 & \vdots & 6 \\ 0 & -2 & -3 & \vdots & -21 \\ 0 & -6 & -8 & \vdots & -52 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 1 & 1 & \vdots & 6 \\ 0 & -2 & -3 & \vdots & -21 \\ 0 & -6 & -8 & \vdots & -52 \end{array}\right] \xrightarrow{r_3 = r_3 - 3r_2} \left[\begin{array}{ccccc} 1 & 1 & 1 & \vdots & 6 \\ 0 & -2 & -3 & \vdots & -21 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 1 & 1 & \vdots & 6 \\ 0 & -2 & -3 & \vdots & -21 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right] \xrightarrow{r_2 = r_2 + 3r_3} \left[\begin{array}{ccccc} 1 & 1 & 0 & \vdots & 6 \\ 0 & -2 & 0 & \vdots & 12 \\ 0 & 0 & 1 & \vdots & 2 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 1 & 0 & \vdots & 6 \\ 0 & -2 & 0 & \vdots & 12 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right] \xrightarrow{r_1 = r_1 - r_2} \left[\begin{array}{ccccc} 1 & 1 & 0 & \vdots & -5 \\ 0 & -2 & 0 & \vdots & 12 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 1 & 0 & \vdots & -5 \\ 0 & -2 & 0 & \vdots & 12 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right] \xrightarrow{r_2 = -\frac{r_2}{2}} \left[\begin{array}{ccccc} 1 & 1 & 0 & \vdots & -5 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 1 & 0 & \vdots & -5 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right] \xrightarrow{r_1 = r_1 - r_2} \left[\begin{array}{ccccc} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 11 \end{array}\right]$$

$a = 1$

$b = -6$

$c = 11$

Böylece aranılan polinom denklemi $p_2(x) = x^2 - 6x + 11$ olarak bulunur.

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4) (1,4), (2,7), (4,19) ve (5,31) noktalarından geçen P(x) polinomunu Langrange interpolasyonu ile hesaplayınız ve buna göre P(3) değerini bulunuz.

Cevap: Langrange interpolasyon polinomu

$$P(x) = \sum_{i=1}^{n} y_i \times L_i(x)$$

$$L_i(x) = \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

Veri noktaları tanımlanır

$$(x_1, y_1) = (1,4), (x_2, y_2) = (2,7), (x_3, y_3) = (4,19), (x_4, y_4) = (5,31)$$

$$L_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-2)(x-4)(x-5)}{(1-2)(1-4)(1-5)}$$

$$L_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x-1)(x-4)(x-5)}{(2-1)(2-4)(2-5)}$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_1-x_4)} = \frac{(x-1)(x-2)(x-5)}{(4-1)(4-2)(4-5)}$$

$$L_4(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x-1)(x-2)(x-4)}{(5-1)(5-2)(5-4)}$$

$$P(x) = 4L_1(x) + 7L_2(x) + 19L_3(x) + 31L_4(x)$$

$$P(x) = -\frac{1}{4}x^3 + \frac{5}{2}x^2 - \frac{25}{4}x + 4$$

$$P(3) = 11.5$$