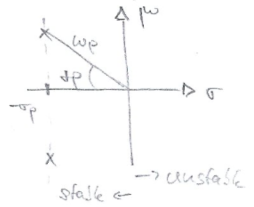
ASF Summary 2021 - Noah Hütter

L1: Intro

Poles & Zeros

$F(s) = \frac{N(s)}{D(s)} = \frac{\cdots}{(s^2+\frac{\omega_p}{q_p}s+\omega_p^2)\cdot\ldots}$

Any poly can be factorized into terms of order 1 & 2 w/ real coefficients



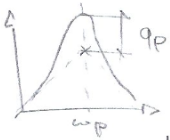
$q_p = \frac{\omega_p}{2\sigma_p}$ $\quad 2q_p = \frac{\omega_p}{\sigma_p} = \frac{1}{\cos \varphi_p}$

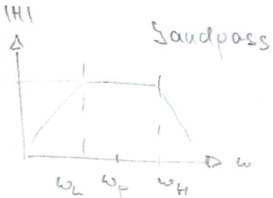
2nd order

$H(s) = \frac{N(s)}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}$ $\quad N(s) = \begin{cases} k\,\omega_0 & \text{LP} \\ k\frac{\omega_0}{q_p}s & \text{BP} \\ k(\omega_p - s^2) & \text{BR (notch)} \\ ks^2 & \text{HP} \end{cases}$

1st order

$H(s) = \frac{k_1 s + k_0}{s+\omega_0}$ $\quad \begin{cases} k_0 = 0 & \text{HP} \\ k_1 = 0 & \text{LP} \end{cases}$

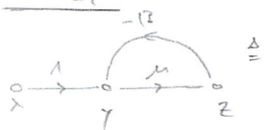




Bandpass

$H(s) = \frac{k' 2D\omega_p s}{s^2+2D\omega_p s+\omega_p^2}$ $\quad B = 2Df_p = f_H - f_L$ $\quad q_p = \frac{1}{2D}$

SFG



$\Rightarrow \begin{array}{l} y = x - Bz \\ z = \mu y \end{array}$

$\boxed{G = \frac{z}{x} = \frac{\sum_i \Delta_i p_i}{\Delta}}$

$\Delta = 1$ − sum of all loops + sum of prod. of all comb. of two loop that have no nodes in common − sum ... three ... + ...

p_i = forward paths

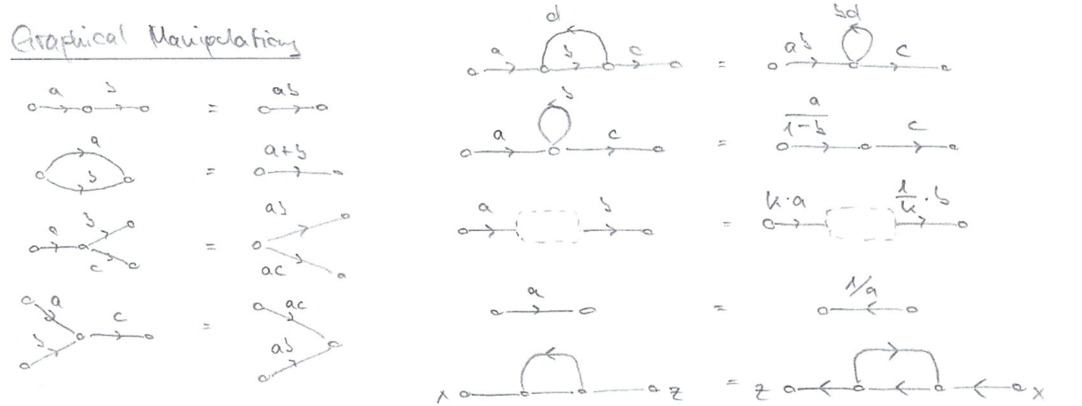
$\Delta_i = \Delta$ with terms rem. that touch p_i

An SFG establishes relationship between variables. Causality comes only from source superposition (eg. z is controlled by y)

DPSFG

1. Add magic sources where no source exists already
2. Draw driving-point impedances
3. Draw inp. & op.
4. Draw active devices
5. Draw passive devices
6. Add currents where required
7. Mason!

Graphical Manipulations



→ check units (I → V, ..)

SFG Impedances

Input: - Write I_{in} node equ - Solve $\frac{1}{Z_{in}} = \frac{I_{in}}{V_{in}}$

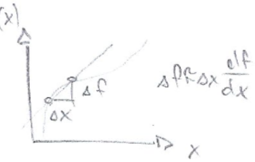
Output - Inject test current I_{test} - Solve for $Z_{out} = \frac{V_{out}}{I_{test}}$

Sensitivity

$\boxed{S_x^f = \frac{df}{dx}\cdot\frac{x}{f}}$

S_x^f: sensitivity of $f(x)$ to changes of x

$\%f \approx \underbrace{\%x}_{\text{change of x in percent}} \cdot \frac{df}{dx}\cdot\frac{x}{f} = \%x \cdot S_x^f$



$\Delta f \approx \Delta x \frac{df}{dx}$

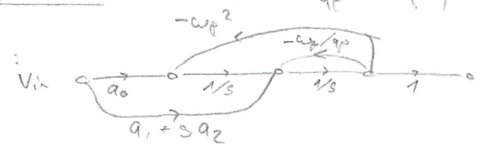
L2: Gm-C Filters

$\boxed{v_c(t) = \frac{1}{C}\int^t i_c(\tau)d\tau}$

Integrator-Loop Filters

Need $D(s) = s^2 + \frac{\omega_p}{q_p}s + \omega_p^2$, with integrator: $\Delta = \frac{D(s)}{s^2} = 1 + \underbrace{\frac{\omega_p}{q_p}\frac{1}{s}}_{-L_1} + \underbrace{\omega_p^2\frac{1}{s^2}}_{-L_2}$

For LP/BP/HP:



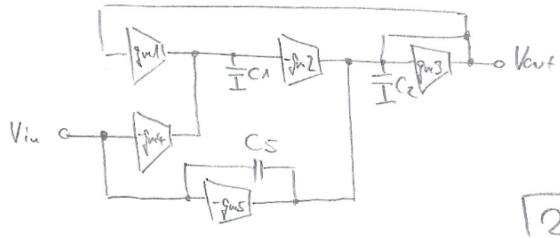
$= \boxed{\frac{a_2 s^2 + a_1 s + a_0}{s^2 + \frac{\omega_p}{q_p}s + \omega_p^2}}$

1st problem: $\circ \xrightarrow{1/s} \circ$ can't implement → scale to

2nd problem: $\underset{I_1}{\circ} \xrightarrow{?} \underset{V_1}{\circ}$ convert I to V → OTA

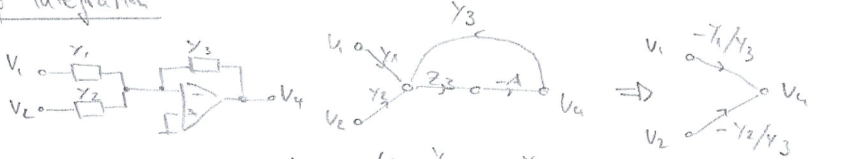
$I = g_m \cdot V$ Operational Transcond. Amplifier

Gm-C Biquad



OpAmp Integration

Basic form

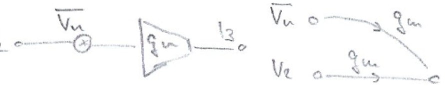


$$V_4 = -\left(V_1 \frac{Y_1}{Y_3} + V_2 \frac{Y_2}{Y_3}\right)$$

Gm-C noise

! Physically, noise is generated at output !

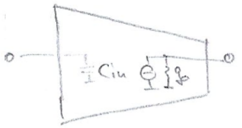
Model:



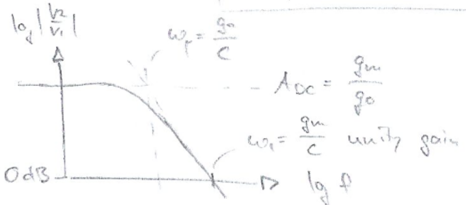
! noise is uncorrelated → sum squares !

$$V_{n,out}^2 = \sum \overline{V_{ni}^2}\, |T_{ni}(s)|^2$$

Non-ideal Gm-C

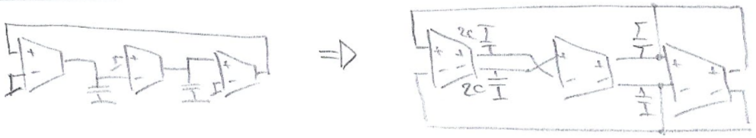


$$\frac{V_2}{V_1} = -\frac{g_m}{g_o + sC}$$



- C_{in} parallel to signal cap C → $C \geq k \cdot C_{in}$, $k \gg 1$
- g_o sets ω_p and gain. All ω_p must be lower than freq. of interest
- $g_m(s)$ have a phase lag → stability problems!

Balanced Circuit

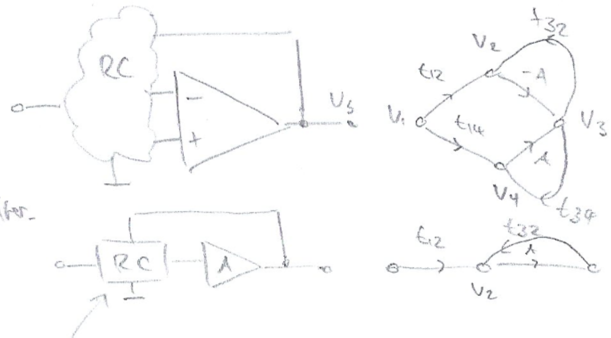


better power supply noise rejection

L3: OpAmp RC-Filters

SAB (Single Amp Biquad)

Gm-C difficult to realize linear.
High-gain opamp are better.
q_p less sensitive on 3-amp filter.



$$T = \frac{A(t_{14} - t_{12})}{1 + At_{32} - At_{34}} \overset{A\to\infty}{=} \frac{t_{14} - t_{12}}{t_{32} - t_{34}}$$

If fwd & fb path go through same net: $t_{12} = \frac{n_{12}}{\hat{d}}$ $t_{32} = \frac{n_{32}}{\hat{d}}$ → $\hat{d} = a_2 s^2 + a_1 s + a_0$

$\hat{d}$ can't have complex poles → require feedback

$$\to T = \frac{A n_{12}}{\hat{d} - A n_{32}}$$

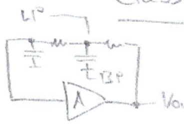
poles of $\hat{d}$ ⟹ $s = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$, $a_1^2 \geq 4a_2a_0$

and $\hat{d} - An_{32}$ should have complex poles:

I) $\uparrow a_0 \to A<0$, $n_{32} = b_0$	LP	neg. gain amp	
II) $\uparrow a_2 \to A<0$, $n_{32} = b_2 s^2$	HP		
III) $\uparrow a_0 \uparrow a_2 \to A<0$, $n_{32} = b_2 s^2 + b_0$	BR	only one works with $A \to -\infty$	
IV) $\downarrow a_1 \to A>0$, $n_{32} = b_1 s$	BP	pos. gain amp	

Class I

LP in FB

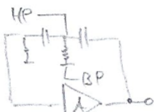


✓ simple
× HP not possible
× $\omega p/qp$ depend on $|A|$

suitable for low-qp

Class II

HP in FB



✓ 2nd-H2 HP possible
× LP not possible
× for $A \to -\infty$, $\omega_p \to 0$

Class III

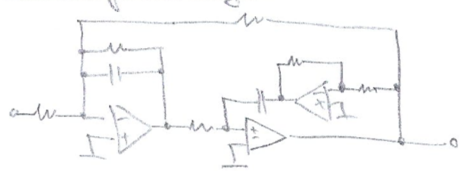
BR in FB

✓ versatile
✓ $A \to -\infty$ → high slew
✓ zeros close to $j\omega$ ax → good high-q_p
✓ linear

Class IV

✓ versatile (LP, BP, HP)
✓ ω_p constant in A
× unstable for large A
× q_p sensitive to components

Ackerberg-Mossberg



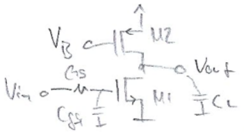
more robust to A than Tow-Thomas.
Suitable for high ω_p, q_p

L4: SFG Videos

Already covered in L1: Intro

L5: Current Mirror OTA

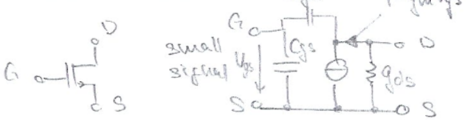
Common-Source Amp $A_{DC} = -\frac{g_{m1}}{g_{ds1}+g_{ds2}}$ $A(s) = -\frac{g_{m1}}{g_{ds1}+g_{ds2}+sC_L}$



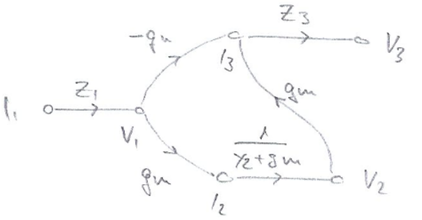
$C_{out} \rightarrow \boxed{\omega_{p2} = \frac{g_{ds1}+g_{ds2}}{C_L}}$ $C_{gs1} \rightarrow \boxed{\omega_{p1} = \frac{G_S}{C_{gs1}}}$

! Every driving-point impedance of a node with resistive and capacitive components connected to it gives one pole! !

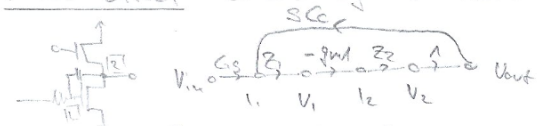
MOS transistor



$g_m \approx 100 \cdot g_{ds}$

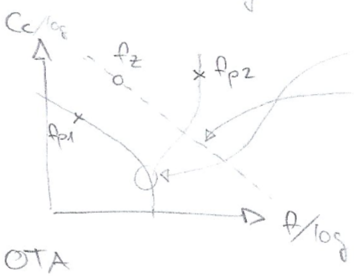


Miller Effect
caused by ⊣⊢ from gate to output (in to out)



$T = -\frac{-G_S(C_CS - g_{m1})}{C_CS(C_CS - g_{m1}) - Y_1Y_2}$

$C_C = C_{gd} + \text{all } C \text{ from } G \text{ to } D$

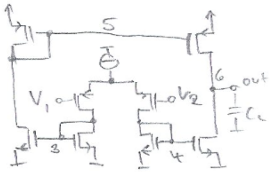


pole splitting: increases stability → phase margin

RHP-zero

$\boxed{\omega_{p1} \approx -\frac{G_1}{C_C} \cdot \frac{G_2}{g_m}}$ → effect of C_C is multiplied by gain & its load!

OTA



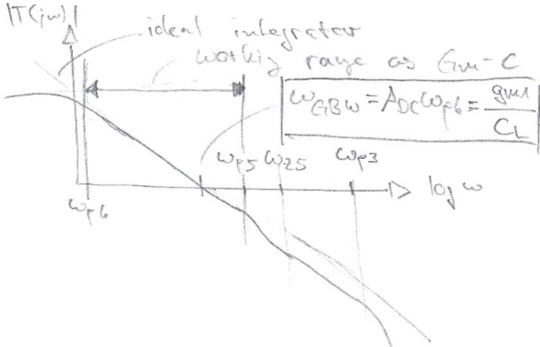
$\omega_{p3} \approx \frac{g_{m2}}{2C_{gs2}}$

$\omega_{p5} \approx \frac{g_{m3}}{2C_{gs3}}$

$\omega_{p6} \approx \frac{g_{ds2}+g_{ds3}}{C_L}$

$\omega_{Z5} \approx 2\omega_{p5}$

1 for each DP-impedance ($Y_3 = Y_4$)



ideal integrator

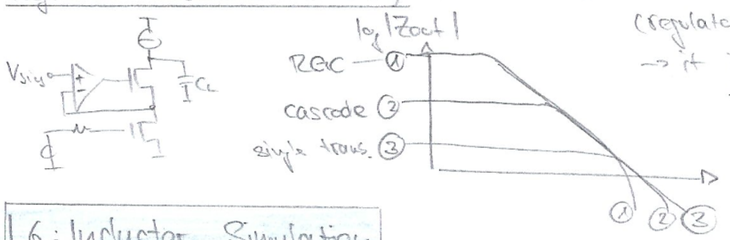
working range as G_m-C

$\boxed{\omega_{GBW} = A_{DC}\omega_{p6} = \frac{g_{m1}}{C_L}}$

Reducing ω_{p6} ("gain enhancement") [not treated in class]

to reduce ω_{p6} must decrease $g_{ds2}+g_{ds3}$ → cascode transistors (transistors in series)

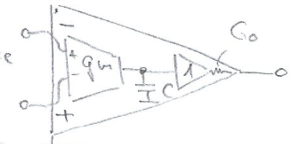
Regulated cascode (RGC) [not treated]



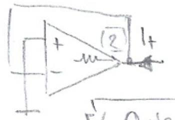
(regulated) cascode more DC gain → it is "more slower" not faster.

L6: Inductor Simulation

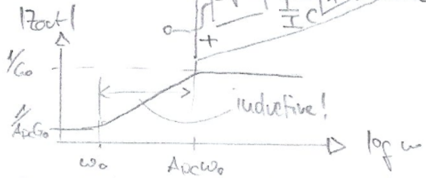
OpAmp with feedback generally opamps are built like



$V_0 = A(V_1 - V_2)$ $A = \frac{A_{DC}\omega_0}{s+\omega_0} = \frac{GBWP}{s+\omega_0}$



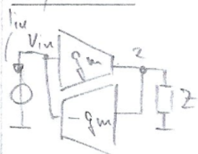
$Z_{out} = \frac{1}{Y_2 + AG_0} \overset{A_{DC} \gg 1}{\approx} \frac{1}{G_0}\frac{s+\omega_0}{s + A_{DC}\omega_0}$



! Output of any opamp with FB is inductive over wide range !

3 approximations for A: I) $A \to \infty$ II) $A \approx \frac{\omega_1}{s}$ III) $A \approx \frac{\omega_1}{s+\omega_0}$ $\omega_1 = GBWP$

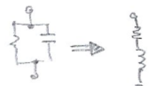
Gyrator



$\boxed{Y_{in} = g_m^2 Z = g_m^2 \frac{1}{Y}}$ with $\frac{1}{\bot} = \frac{1}{\bot}$ $Y_{in} = \frac{g_m^2}{sC} \triangleq \frac{1}{sL_{eq}}$ ← simulated inductor

std. gyrator symbol

$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & 1/g_m \\ g_m & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

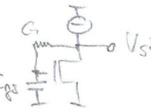


$Re\{P_1\} + Re\{P_2\} = 0$ ← gyrator is a passive component

floating

$L = \frac{C}{g_m^2}$

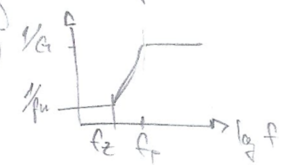
Single transistor [not treated]



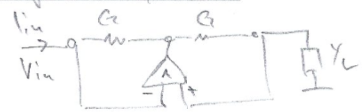
$Z_{sig} = \frac{sC_{gs} + G}{sC_{gs}(G+g_{ds}) + G(g_{ds}+g_m)}$

pole: $2\pi f_p = \frac{G(g_{ds}+g_m)}{C_{gs}(G+g_{ds})}$

zero: $2\pi f_z = \frac{G}{C_{gs}}$

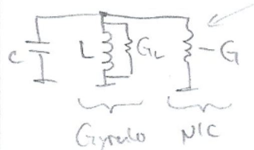


Negative Impedance NIC



$Z_{in} \overset{A \gg 1}{\approx} \frac{G}{G(Y_3 - Y_1)} = -Z_L$

oscillator



L7: Current Conveyors

Universal active element (UAE) can build all ckts with this element. OpAmp is one implementation of it

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

→ I no inp. current $I_1 = 0$

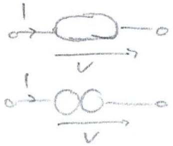
→ II inp. voltage diff is zero. $V_1 = 0$

4-terminal:

→ external ckt forces same current on inp

Nullors

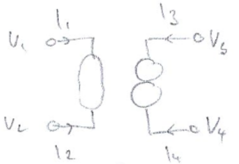
UAE forces 2 eq but two-port nets have 3 → unlimited possibilities!
2 term nets have 1 equ.



Nullator: $V=0$ $\quad I=0$ $\quad$ 2 equ.

Norator: V and I arbitrary $\quad \varnothing$ equ.

must come in same number in ckt.



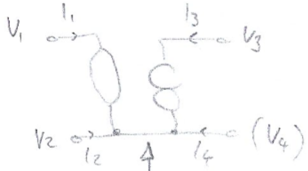
$V_1 - V_2 = 0$, $I_1 = 0$ } Two-port equ

$I_3 = -I_4$ } "inverted"

$I_1 = -I_2$ } Kirchhoff

$\{ V_1 - V_2 = 0,\ I_1 = 0 \}$

$\{ V_2 = V_4,\ I_2 = -I_3 \}$

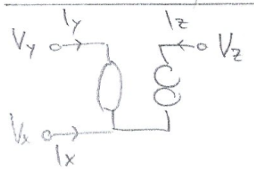


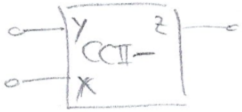
golden opamp rule

something new!

Current Conveyors

implementation of "≤" causal equality



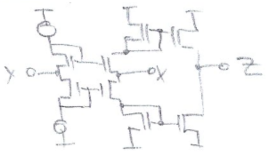


$$\boxed{\begin{aligned} I_Y &= \varnothing \\ V_X &\stackrel{\leftarrow}{=} V_Y \\ I_Z &\stackrel{\leftarrow}{=} -I_X \end{aligned}}$$

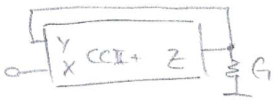
CCII−	CCII+	CCI±	CCIII±
$I_Y=0$	$I_Y=0$	$I_Y=0$	$I_Y=0$
$V_X \stackrel{\leftarrow}{=} V_Y$	$V_X \stackrel{\leftarrow}{=} V_Y$	$V_X \stackrel{\leftarrow}{=} V_Y$	$V_X \stackrel{\leftarrow}{=} V_Y$
$I_Z \stackrel{\leftarrow}{=} -I_X$	$I_Z \stackrel{\leftarrow}{=} I_X$	$I_Y \stackrel{\leftarrow}{=} I_X$	$I_Y \stackrel{\leftarrow}{=} -I_X$

Application

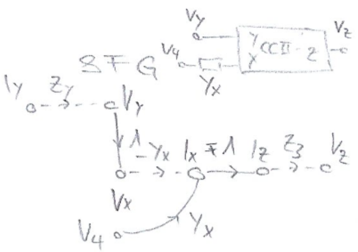
CCII+ in CMOS



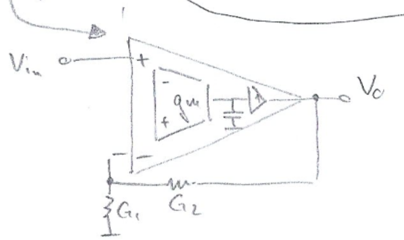
NIC

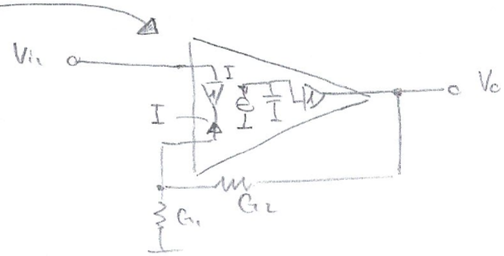


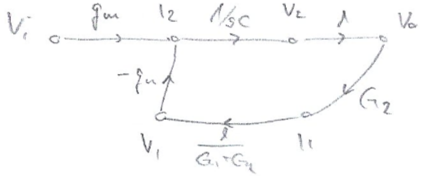
SFG

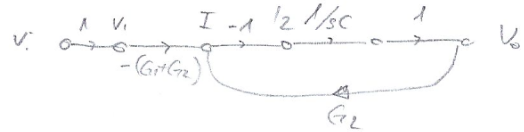


Voltage vs. Current Feedback







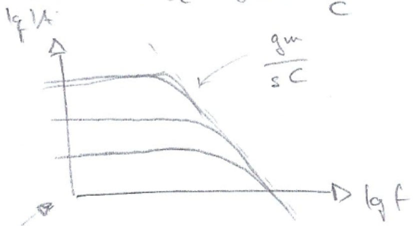


$A_{DC} = \frac{G_1+G_2}{G_2} \qquad BW = \frac{G_1}{G_1+G_2} \cdot \frac{g_m}{C}$

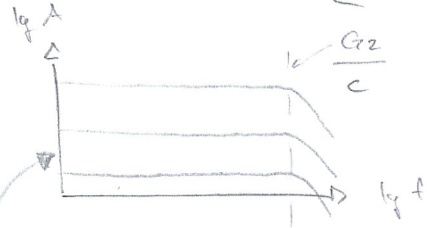
$A_{DC} = \frac{G_1+G_2}{G_2} \qquad BW = \frac{G_2}{C}$

$GBWP = A_{DC} \cdot BW = \frac{g_m}{C}$

$GBWP = A_{DC} \cdot BW = \frac{G_1+G_2}{C}$



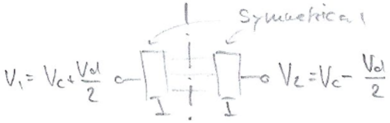
BW decreases as A_{DC} increases



change gain at const. BW

L8: Quality Measures

Common & Differential Mode



Common: V_c  V_c
$V_d = 0$

Differential: $V_c = 0$ 

Parasitic Gains & Rejection

A_{xx}: freq. dep. small signal gains

$V_o = A_d V_d + A_c V_c + A_{dd} V_{dd} + A_{ss} V_{ss}$

$CMR = \frac{1}{A_c}$ $PSR_+ = \frac{1}{A_{dd}}$ $PSR_- = \frac{1}{A_{ss}}$ | $CMRR = \frac{A_d}{A_c}$ $PSRR_+ = \frac{A_d}{A_{dd}}$ $PSRR_- = \frac{A_d}{A_{ss}}$

$\frac{1}{CMR} + \frac{1}{PSR_+} + \frac{1}{PSR_-} = 1$ $\frac{1}{CMRR} + \frac{1}{PSRR_+} + \frac{1}{PSRR_-} = \frac{1}{A_d}$

If all need to be good, add V_{ref} and set $A_{ref} = 1$ $\Rightarrow A_c + A_{dd} + A_{ss} = 0$

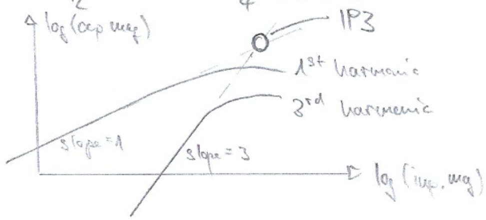
Harmonic Distortion (HD)

Any signal through non-linearity $g(x)$ will give HD

$g(x) = a_1 x + a_2 x^2 + \ldots \rightarrow g(A\cos\omega t) = \ldots = a_1 A\cos\omega t + \frac{a_2 A^2}{2}\cos 2\omega t + \frac{a_3 A^3}{4}\cos 3\omega t + \ldots$

$\boxed{HD2 \approx \frac{\frac{a_2 A^2}{2}}{a_1 A} = \frac{A}{2}\cdot\frac{a_2}{a_1}}$ $\boxed{HD3 \approx \frac{A^2}{4}\frac{a_3}{a_1}}$

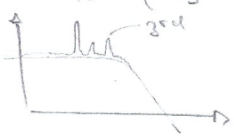
often small in diff. largest



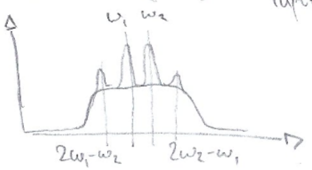
IP3: third order intercept point. Complete info about weak-non-linearity of a circuit.

Measuring HD

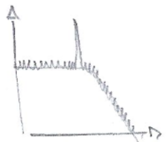
low pass LP after HD3



band pass input sum of sin



SNR, dynamic range

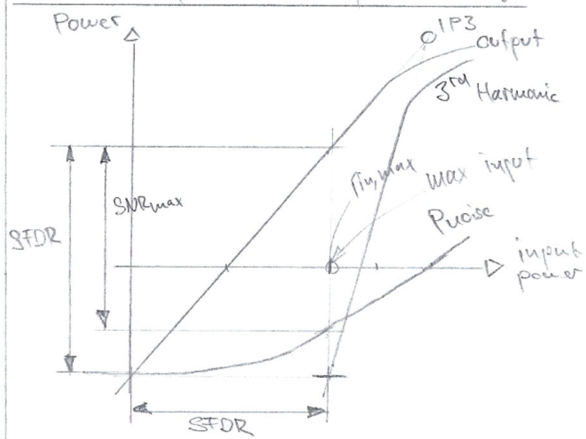


$P_{sig} = \frac{A^2}{2}$ $P_{noise} = \int_{f_{min}}^{f_{max}} S_{noise}\, df$ $SNR = \frac{P_{sig}}{P_{noise}}$ has no meaning if f_{min}, f_{max} not fixed!

$SNR_{max} = \frac{P_{sig,max}}{P_{noise} \text{ when } P_{sig,max} \text{ applied}}$ $DR = \frac{P_{sig,max}}{P_{sig,min}}$

dep. on application

SFDR Spurious-free dynamic range

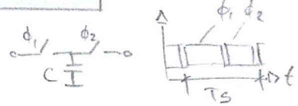


$\boxed{SFDR = \frac{P_{in,max}}{P_{in,min}}}$

$P_{in,min} = P_{noise}$ @ no signal

$P_{in,max}$ = where $P_{3rd} = P_{noise}$ @ no signal

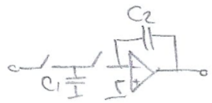
L9: SC Intro

Resistor 

$\phi_1: Q_{c,1} = CV_1$
$\phi_2: Q_{c,2} = CV_2$
$\Delta Q = C(V_1 - V_2)$

$I = \frac{\Delta Q}{T_S} = f_S C(V_1 - V_2)$

$R_{eq} = \frac{1}{f_S C}$ $\boxed{G_{eq} = f_S C}$

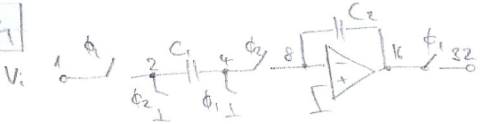
Integrator 

$\boxed{\tau = \frac{1}{f_S}\cdot\frac{C_1}{C_2}}$ this ratio in CMOS is accurate

Parasitic Insensitive problem  $\Rightarrow$ 

pos/neg - resistor
are periodically discharged

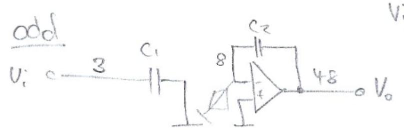
L10: SC SFG



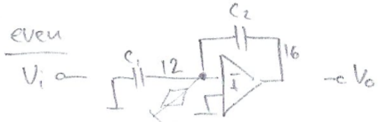
1. label nodes as powers of 2
2. Draw 2x, once odd closed, once else
3. Add node labels for closed switches
4. For e/o draw driving-point capacitances where required: $Q \xrightarrow{1/C} V$

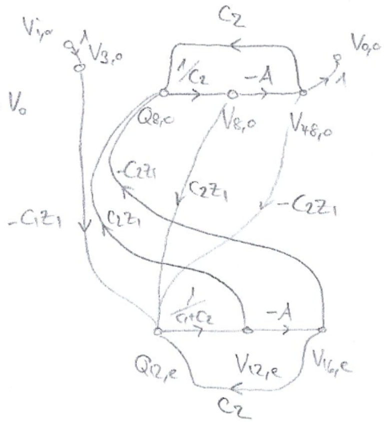
   label nones o/e
5. Add in & out
6. Add actives
7. Add passives

odd



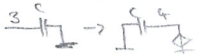
even





8. Add charge transfers e→o and o→e

e.g.



$V_{3,o} \xrightarrow{-C_2 z_1} Q_{4,e}$

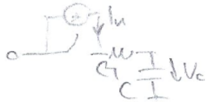
x) z_1 is ½ cycle

→ replace z^{-2} with z^{-1}

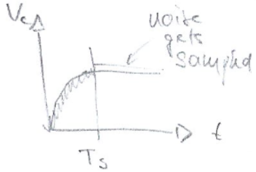
L11: SC offset & noise

OPAMP offset — node $V_{off} \xrightarrow{1}$ to opamp offset → $T_{off} = \frac{V_{out}}{V_{offs}}$

SC noise — C is not noisy but switch has R that contributes noise



$I_{n,c}^2 = 4kTG$ $\quad V_{n,c}^2 = \frac{4kT}{G}$ $\quad V_c = \frac{I_n}{G+sC} = V_n \frac{1}{1+\frac{C}{G}s}$



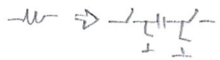
$V_{c,rms}^2 = \int_0^\infty V_n^2 |T| \, df \propto$

simpler analysis: $E_C = \frac{V_C^2}{2} C$ $\quad E_M = \frac{kT}{2}$ $\Rightarrow E_C = E_M \Rightarrow \boxed{V_{c,n}^2 = \frac{kT}{C}}$

higher C = less noise more power.

SC filter synthesis [not treated]

One way: take RC circuit & replace R with SC

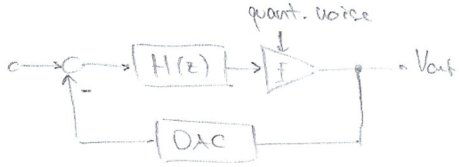


→ |T| becomes periodic in f_s,

→ only accurate in low frequencies

L12 Sigma-Delta Modulation

ΣΔ Main idea: ADC are subject to quantization noise. ΣΔ introduces feed-back, such that a different TF is applied to the input and the quant. noise



G: quant. gain

x_n: quant. noise

$H_{sig} = \frac{X_{out}}{X_{in}} = \frac{H(z)}{1+H(z)}$

$H_{noise} = \frac{X_{out}}{X_n} = \frac{1}{1+H(z)}$

for $H(z)$ integrator $\frac{z^{-1}}{1-z^{-1}}$

$H_{sig} = z^{-1}$ ← delay → LP

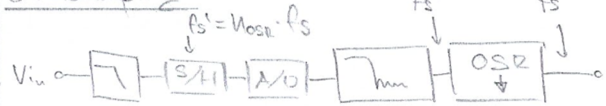
$H_{noise} = 1 - z^{-1}$ ← differentiator → HP

Quantization

N-bit ADC, the signal to quant noise ratio

Signal: sine $V_{rms} = \frac{V_{ref}}{2} \cdot \frac{1}{\sqrt{2}}$ $\quad$ Noise: $V_{q,rms} = \frac{V_{ref}}{2^N} \cdot \frac{1}{\sqrt{12}}$ $\Rightarrow \boxed{SQNR = N \cdot 6.02\,dB + 1.76\,dB}$

Oversampling



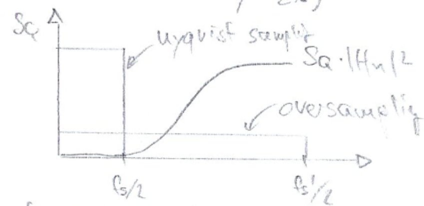
⊕ AA filter needs not to be steep

⊕ $V_{Q,rms}^2$ noise power is now spread over larger range

$\boxed{SNR = N \cdot 6.02\,dB + 1.76\,dB + 10 \log(n_{OSR})}$ → win ½ per octave of oversampling

Q-noise shaping (what is done by ΣΔ)

$|H_n(f)| \approx 2\pi \frac{f}{f_s'}$



$P_Q \approx \frac{V_{LSB}^2}{12} \frac{\pi^2}{3} n_{OSR}^{-3}$ $\quad n_{OSR} \gg 1$

$\boxed{SNR = N \cdot 6.02\,dB + 1.76\,dB - 5.17\,dB + 30 \cdot \log n_{OSR}}$

→ noise shaping costs ~1 bit but 1.5 bits/octave OSR gained.

for k-th order ΣΔ (= noise shaping)

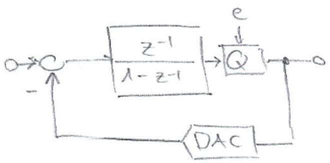
→ gain $(k+\frac{1}{2})$ bits/octave OSR!

$\boxed{SNR = N \cdot 6.02 + 1.76 + 10\log(2k+1) - 20k \log \pi + 10(2k+1) \log n_{OSR}}$

(for $k \geq 5$ get unstable)

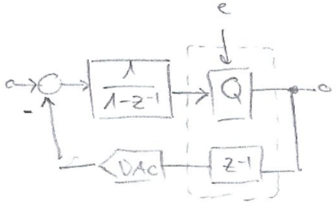
L13: Higher Order ΣΔ

1st to 2nd order



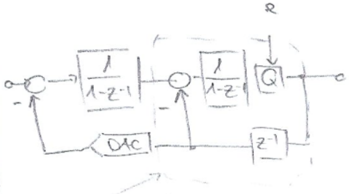
can move z^{-1} to other part of loop, still stable

$T_{sig} = z^{-1}$ $T_n = 1 - z^{-1}$



any circuit that goes from high to low res, e.g. ΣΔ!

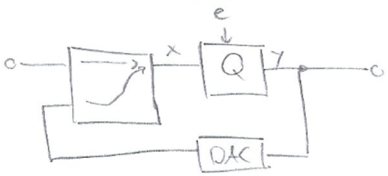
$T_{sig} = 1$ $T_{noise} = 1 - z^{-1}$



2nd order noise shaping

$T_{sig} = 1$ $T_{noise} = (1-z^{-1})^2$

Multibit Quantizer



$$T_S = \frac{H_0}{1 - H_1 G_0}$$

$$T_n = \frac{1}{1 - H_1 G_0}$$

$$\boxed{\begin{aligned} SQNR = {} & N \cdot 6.02\,dB \\ & + 1.76\,dB \\ & + 10\log(2k+1) - 20k\lg\pi \\ & + 10(2k+1)\log n_{OSR} \end{aligned}}$$

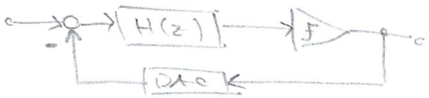
G_0: quantizer gain

N: quantizer bits → 6dB per bit

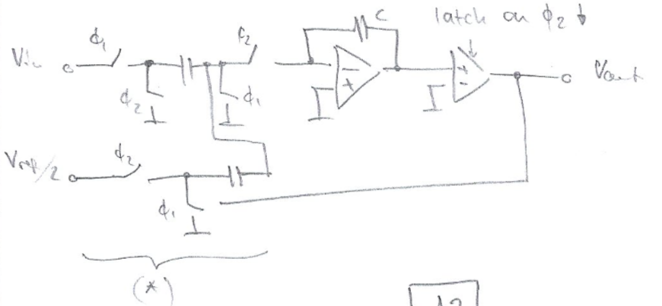
k: ΣΔ order → $k + \frac{1}{2}$ bits/n_{OSR} octave

n_{OSR}: Oversampling ratio → 1.5 bits/n_{OSR} octave

ΣΔ with switched capacitor

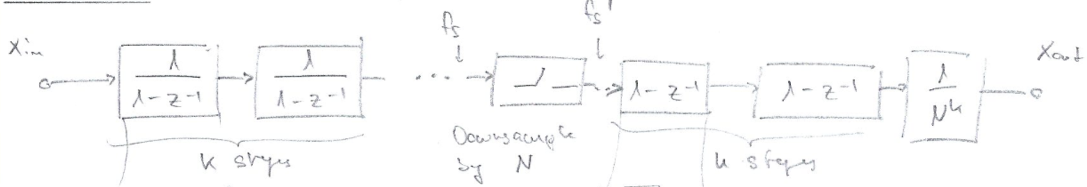


(*) resistor positive / negative controlled by Vout → DAC

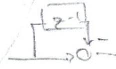


L14: In and out of ΣΔ

CIC filter cascaded integrator comb (or Hogenauer filter)



integrator



comb

$$H(z) = \frac{1}{N^k} \cdot \left(\frac{1 - z^{-N}}{1 - z^{-1}}\right)^k$$

- same as k moving average
- low pass linear phase