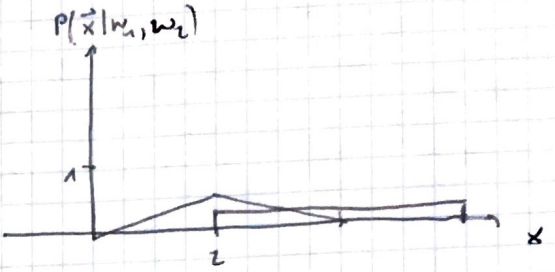
1. $p(\vec{x}|\omega_1) = \begin{cases} \frac{1}{4}x & x \in (0,2) \\ -\frac{1}{4}x + 1 & x \in [2,4) \end{cases}$

$p(\vec{x}|\omega_2) = \frac{1}{4} \quad x \in [2,6]$



i) $P_e = \int_T^\infty p(x|\omega_1) \cdot p(\omega_1)\, dx + \int_{-\infty}^{T} p(x|\omega_2)\, p(\omega_2)\, dx$

$= \int_T^4 \left(-\frac{1}{4}x + 1\right) \cdot \frac{5}{9}\, dx + \int_{-\infty}^{T} \frac{1}{4} \cdot \frac{4}{9}\, dx$

$= \frac{5}{9}\left(-\frac{1}{8}x^2 + x\Big|_T^4\right) + \frac{1}{9}x\Big|_2^T$

$= \frac{8}{9} + \frac{5}{72}T^2 - \frac{4}{9}T$

$\Rightarrow \frac{dP_e}{dT} = \frac{10}{72}T - \frac{4}{9} \stackrel{!}{=} 0 \quad \Rightarrow T = \frac{288}{90} = \underline{3.2}$

$\rightarrow$ $R_1: 2 < x < 3.2$

$R_2: 3.2 \leq x < 6$

$P_e = \frac{8}{45} = \underline{17.8\,\%}$