Project 1 - Ramon Iglesias

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1 Method

1.1 Algorithm

I implemented a variation of the K2 Search algorithm shown in the book with some minor changes mostly inspired from [1].

The main change is that, during the main loop, when considering which parents to add to the current node I only consider the preceding nodes as feasible parents, i.e. the initial ordering of nodes is treated as a topological sort and respected as such. This gives the advantage of knowing that the graph will continue to be acyclic with every parental addition. Thus, the initial ordering plays an important role. To be able to explore different initial orderings on each trial, the initial search begins by randomizing the node ordering.

The second change is that I included the option to bound the number of parents. The most evident benefit in this case is the reduction in computation time. But it also helps in guiding the search, moving it away from too complex structures (this turn out to be beneficial in the School Grades dataset).

The third modification involves computing only the Bayesian Score for the node under consideration. This comes as a realization (and pointed out in [1]) that the score is separable. Therefore, at each step we avoid computing the Bayesian Score for the whole graph, and use only individual scores for comparison. While not implemented in this specific instance, this allows the parallel computation of the best parental arrangement for each node. It is worth noting that this is only possible due to the algorithm treating the original ordering as a topological sort (removes constraint dependencies between nodes).

1.2 Implementation

The code was written in Python and makes use of NetworkX (graph operations), Numpy (linear algebra and numerical operaitons), Scipy (special scientific functions) and Pandas (data frames).

1.3 Results

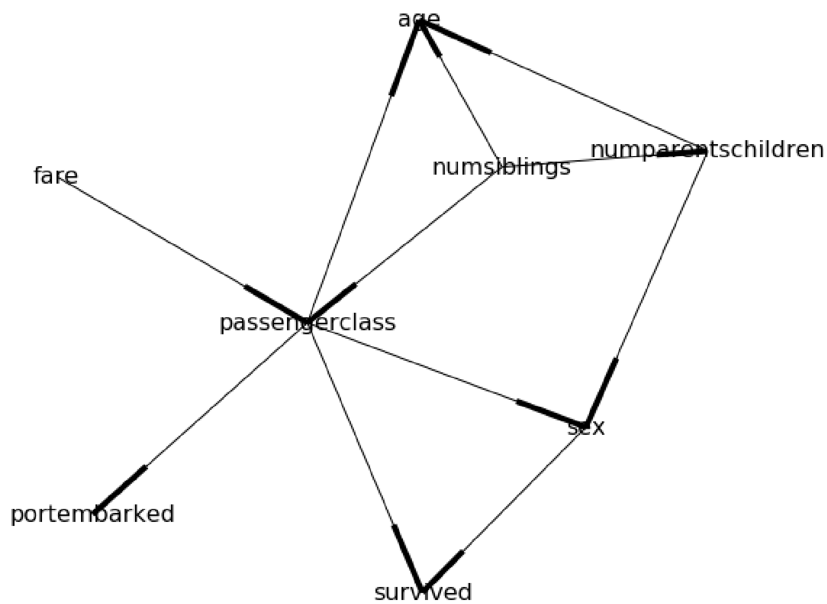
1.3.1 Titanic

Score: -3802.96

Time: 0.336s

Intution: the resulting graph shows that, as is repeatedly discussed, passenger class and gender play a big role in who survived (rich people and women). Other intuitive relations: fare and class, and siblings/children and age.

```
In [339]: pos=nx.graphviz_layout(G)
          nx.draw(G,pos=pos,with_labels=True, node_size=0, edge_size = 0.1, font_size=16)
```



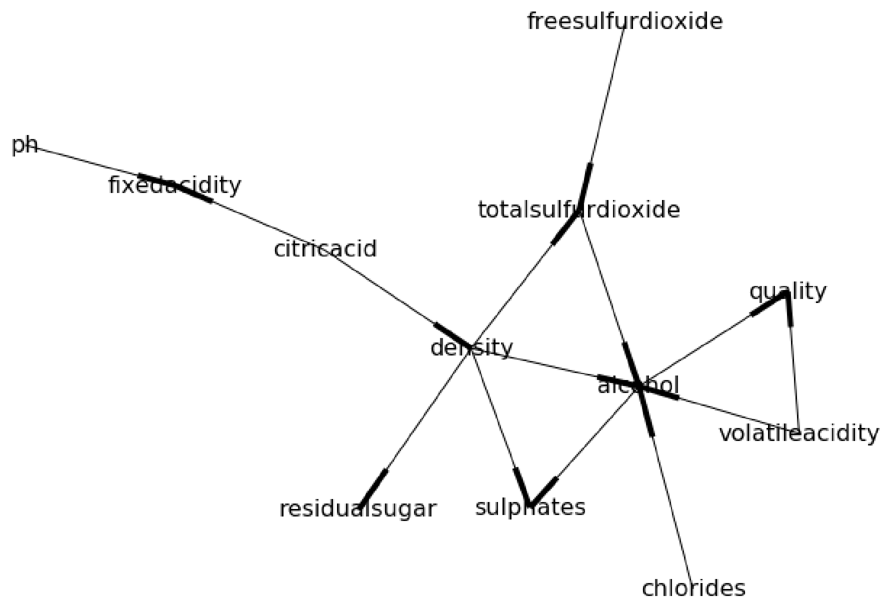
1.3.2 White Wine

Score: -42084.37

Time: 0.814

Intuition: Main intution is that the quality is dependent on the level of alcohol. This I can expect and agree with. Additionally, pH is a parent to fixed acidity, which makes sense. The rest of the relationships are difficult for me to interpret since I am far from a wine connoisseur.

```
In [340]: pos=nx.graphviz_layout(wine_dag)
          nx.draw(wine_dag,pos=pos,with_labels=True, node_size=0,font_size=16)
```



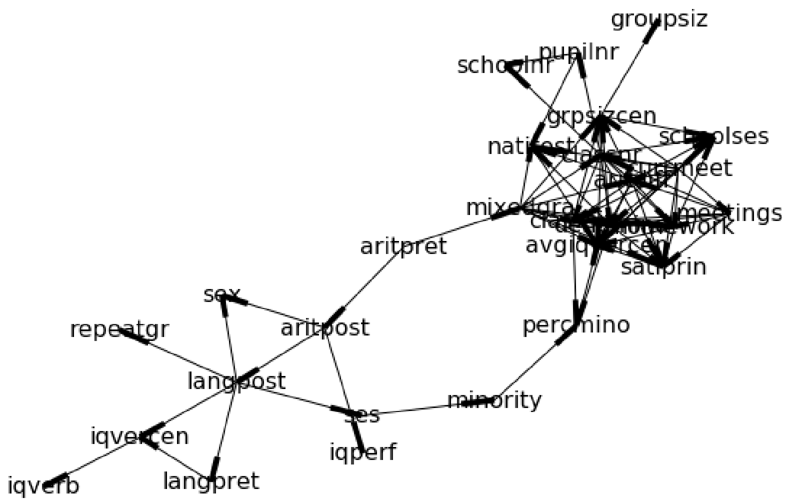
1.3.3 School Grades

Score: -44179.81

Time: 10.069

Intuition: I do not have much to say in this result. The information on the dataset is rather vague, so I can't draw any relevant conclusions.

```
In [342]: pos=nx.graphviz_layout(school_dag)
          nx.draw(school_dag,pos=pos,with_labels=True, node_size=0, font_size=16)
```



References

[1] Teyssier, Marc, and Daphne Koller. *Ordering-based search: A simple and effective algorithm for learning Bayesian networks.*. arXiv preprint arXiv:1207.1429 (2012).

1.4 Appendix: Code

```
In [331]: import numpy as np
          import pandas as pd
          import networkx as nx
          import itertools
          from random import shuffle
          from scipy.special import gammaln
          from collections import OrderedDict
          import matplotlib.pyplot as plt
          import time
          %matplotlib inline
          from matplotlib import rcParams
          rcParams['figure.figsize'] = (10, 7) #Size of figure
          rcParams['figure.dpi'] = 125
```

```
In [177]: # compute score from list of node paramters
          def bayesianScore(L):
              '''
              L is a list where each entry i is
              aa numpy array of shape (q_i, r_i)
```

```
        we assume that alpha_ijk = 1 for all i,j,k

        and P(G) can be disregarded
        '''
        score = 0
        for i,m in enumerate(L):
            alphas = np.ones(m.shape)
            #print m.sum(axis=0)
            A = (gammaln(alphas.sum(axis=1)) - gammaln(alphas.sum(axis=1) + m.sum(axis=1))).sum()
            B = (gammaln(alphas + m) - gammaln(alphas)).sum(axis=0).sum()
            #print A, B
            score += (A + B).sum()
        return score
```

```
In [124]: # compute mijk from given node and parents
    def compute_counts(node, parents, dataset):
        '''
        returns a (q_i, r_i) numpy array with counts

        node is a string, representing the node/column

        parents is a list of strings

        dataset is a pandas dataframe
        '''

        # for each ijk get count of ocurrences
        if parents:
            m = pd.pivot_table(dataset[[node]+parents],
                               index=parents, columns=node, aggfunc=len).fillna(0).as_matrix()
        else:
            vals = dataset[node].value_counts().as_matrix()
            m = np.zeros((1,len(vals)))
            m[0,:] = vals

        return m
```

```
In [217]: def k2Search(dataset, max_parents=5):
        '''
        implementation of k2 search

        returns networkx graph

        dataset is a pandas dataframe

        '''
        dag = nx.DiGraph()

        # get random ordering of nodes
        nodes = list(dataset.columns)
        shuffle(nodes)

        #add nodes
        dag.add_nodes_from(nodes)
```

```
        #keep track of total score
        total_score = 0

        #keep track of parents
        for i,node in enumerate(nodes):
            feasible_parents = nodes[:i]
            current_parents = []
            current_counts = compute_counts(node, current_parents, dataset)

            #note that we only need to compare the score for the current score
            current_score = bayesianScore([current_counts])
            while feasible_parents:
                scores = []
                for j,parent in enumerate(feasible_parents):
                    new_counts = compute_counts(node, current_parents + [parent], dataset)
                    new_score = bayesianScore([new_counts])
                    #print i,j, new_score
                    scores.append(new_score)
                best_score = np.max(scores)

                if best_score > current_score:
                    new_parent = feasible_parents.pop(np.argmax(scores))
                    current_parents.append(new_parent)
                    dag.add_edge(new_parent, node)
                    current_score = best_score
                    if len(current_parents) >= max_parents:
                        # reached max parenthood
                        break
                else:
                    #  basically no new parent would improve the score
                    break

            #update the total score
            total_score += current_score
        return total_score, dag
```

In [275]:
```
def compute_score(dag, dataset):
    nodes = dag.nodes()
    score = 0
    for i, node in enumerate(nodes):
        parents = dag.predecessors(node)
        counts = compute_counts(node, parents, dataset)
        score += bayesianScore([counts])
    return score
```

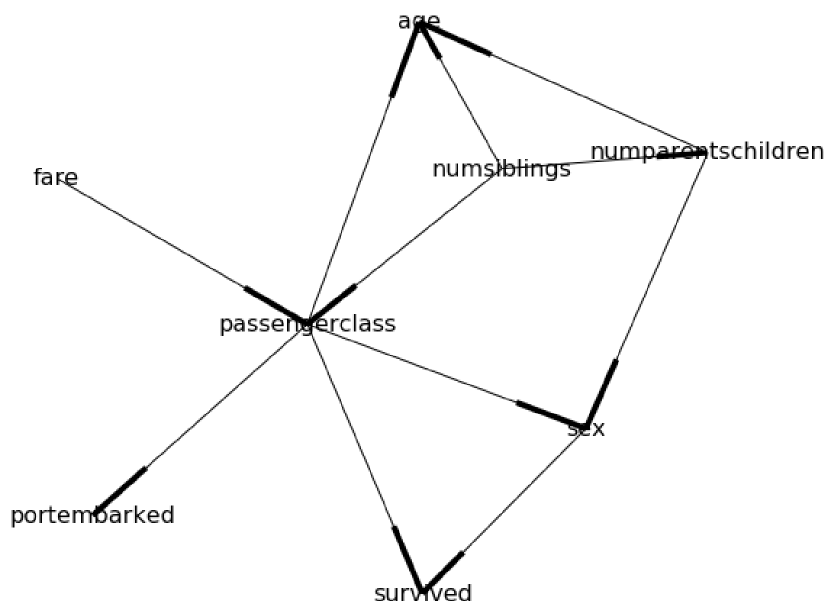
In [272]:
```
titanicdf = pd.read_csv('assets/titanic.csv')
whitewinedf = pd.read_csv('assets/whitewine.csv')
schoolgradesdf = pd.read_csv('assets/schoolgrades.csv')
```

In [265]:
```
t0 = time.time()
score, G = k2Search(titanicdf, max_parents=100)
dt = time.time() - t0
print compute_score(G, titanicdf), dt
```

```
-3802.96021241 0.336395978928
```

```
In [266]: with open('assets/titanic.gph','w') as f:
              text = ''
              for e in G.edges():
                  text += '\n' + e[0] + ',' + e[1]
              f.write(text)
```

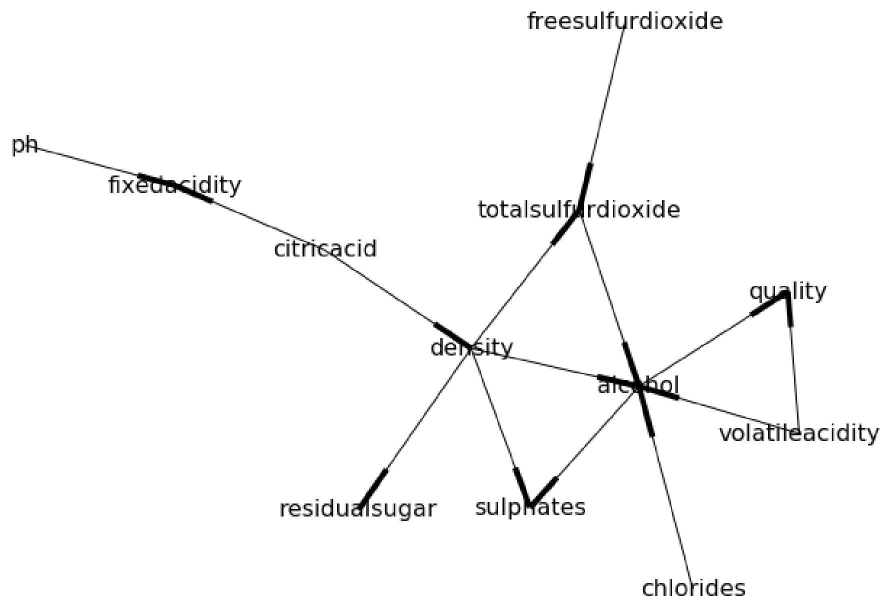
```
In [337]: pos=nx.graphviz_layout(G)
          nx.draw(G,pos=pos,with_labels=True, node_size=0, edge_size = 0.1, font_size=16)
```




```
In [303]: t0 = time.time()
          wine_score, wine_dag = k2Search(whitewinedf, max_parents=100)
          dt_wine = time.time() - t0
          print compute_score(wine_dag, whitewinedf), dt_wine
```

```
-42084.3787949 0.814061880112
```

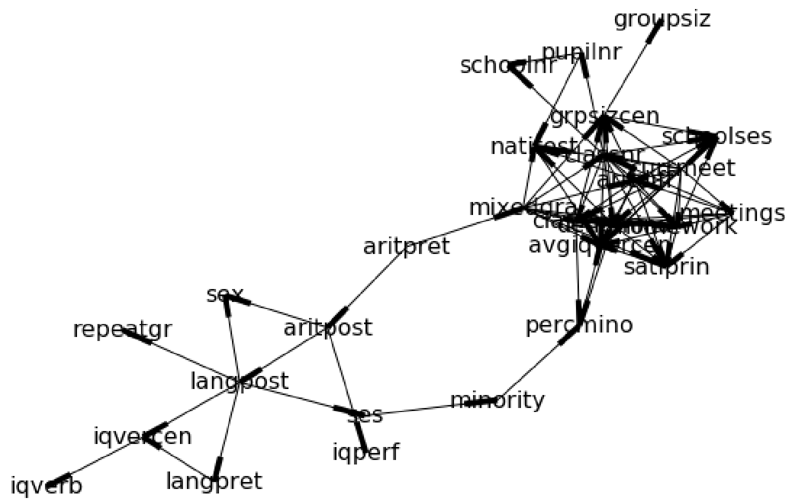
```
In [336]: pos=nx.graphviz_layout(wine_dag)
          nx.draw(wine_dag,pos=pos,with_labels=True, node_size=0,font_size=16)
```




```
In [305]: t0 = time.time()
          school_score, school_dag = k2Search(schoolgradesdf, max_parents=10)
          dt_school = time.time() - t0
          print compute_score(school_dag, schoolgradesdf), dt_school
```

```
-44179.8184466 10.069065094
```

```
In [338]: pos=nx.graphviz_layout(school_dag)
          nx.draw(school_dag,pos=pos,with_labels=True, node_size=0, font_size=16)
```




```
In [307]: with open('assets/whitewine.gph','w') as f:
              text = ''
              for e in wine_dag.edges():
                  text += '\n' + e[0] + ',' + e[1]
              f.write(text)

          with open('assets/schoolgrades.gph','w') as f:
              text = ''
              for e in school_dag.edges():
                  text += '\n' + e[0] + ',' + e[1]
              f.write(text)
```