

A Variational Framework for Phase-Field Fracture Modeling with Applications to Fragmentation, Desiccation, Ductile Failure, and Spallation

Dissertation Defense

Tianchen (Gary) Hu

Department of Mechanical Engineering & Materials Science
Pratt School of Engineering
Duke University

Committee:

John Dolbow
Wilkins Aquino
Johann Guilleminot
Manolis Vlaveakis
Benjamin Spencer

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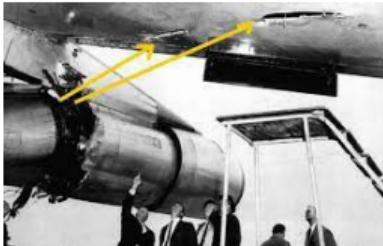
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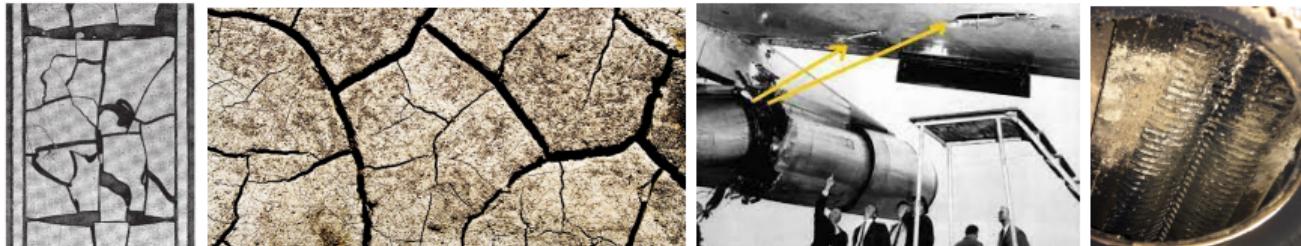
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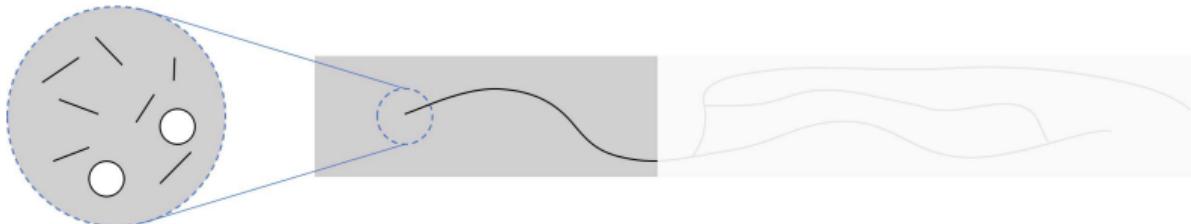


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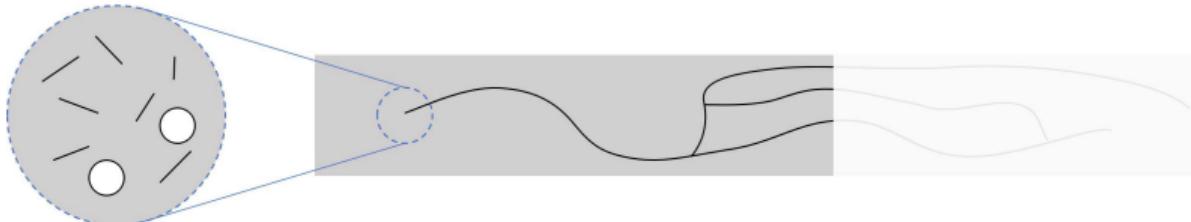
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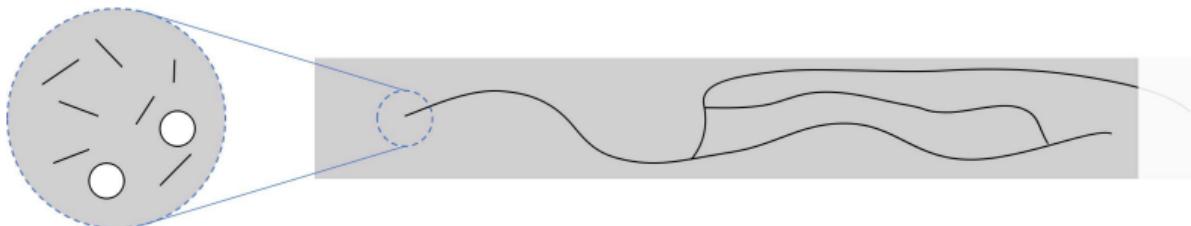
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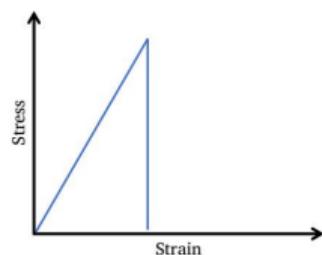


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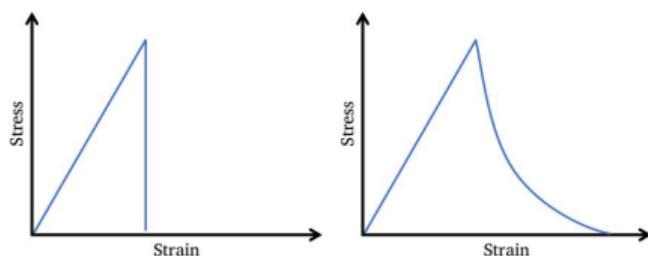


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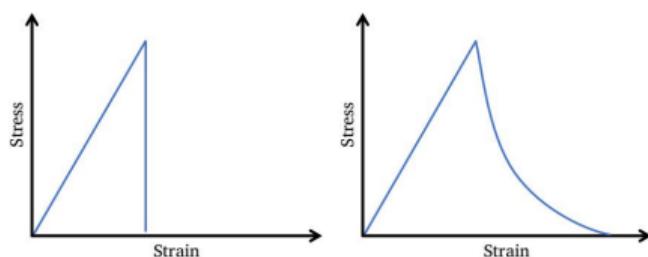
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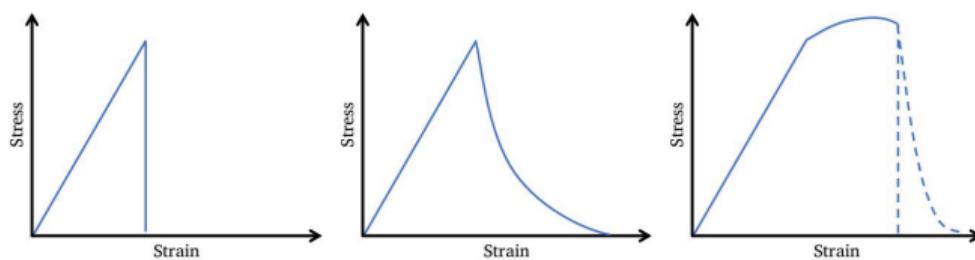
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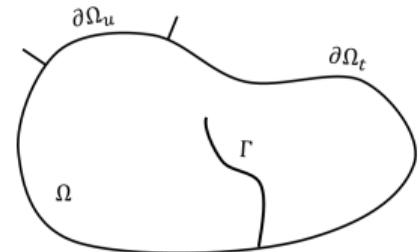
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To date, fracture is still one of the most challenging phenomena to model and predict.

The permanent crack set Γ and its associated fracture energy

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is approximated with

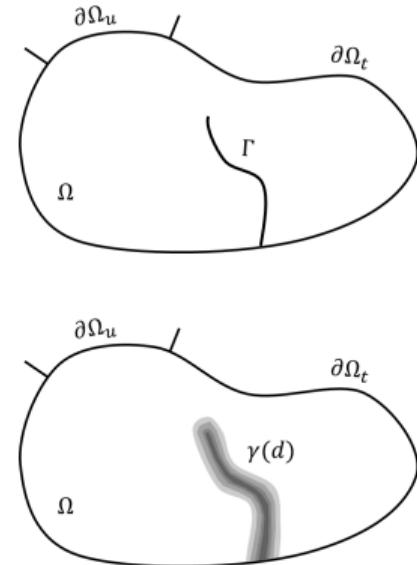
the crack surface density function $\gamma = \hat{\gamma}_l(d)$:

$$\Psi^f \approx \int_{\Omega} \mathcal{G}_c \gamma \, dV, \quad \gamma = \frac{1}{c_0 l} \left(\alpha + l^2 \nabla d \cdot \nabla d \right).$$

- $d \in [0, 1]$ is the phase field;
- $\alpha = \hat{\alpha}(d)$ is the crack geometric function, $\hat{\alpha}(0) = 0$, $\hat{\alpha}(1) = 1$;
- $g = \hat{g}(d)$ is the degradation function, $\hat{g}(0) = 1$, $\hat{g}(1) = 0$;
- c_0 is chosen such that

$$\lim_{l \rightarrow 0^+} \int_{\Omega} \mathcal{G}_c \gamma \, dV = \int_{\Gamma} \mathcal{G}_c \, dA.$$

See [1] for more details.



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$$\Phi, \quad \mathbf{F}^p, \quad \bar{\varepsilon}^p, \quad d, \quad T.$$

- Conservations and thermodynamic laws:

$$\dot{\rho}_0 = 0,$$

$$\rho_0 \mathbf{a} = \nabla \cdot \mathbf{P} + \rho_0 \mathbf{b},$$

$$\mathbf{P}\mathbf{F} = \mathbf{F}\mathbf{P}^T,$$

$$f - \nabla \cdot \boldsymbol{\xi} = 0,$$

$$\dot{u} + \dot{k} = \mathcal{P}^{\text{ext}} + \rho_0 q - \nabla \cdot \mathbf{h},$$

$$\dot{s}^{\text{int}} = \dot{s} - \frac{\rho_0 q}{T} + \nabla \cdot \frac{\mathbf{h}}{T} \geq 0.$$

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- Generalized forces are

$$\begin{aligned} \mathbf{P} &= \mathbf{P}^{\text{eq}} + \mathbf{P}^{\text{vis}}, \quad \mathbf{T} = \mathbf{T}^{\text{eq}} + \mathbf{T}^{\text{vis}}, \quad Y = Y^{\text{eq}} + Y^{\text{vis}}, \\ f &= f^{\text{eq}} + f^{\text{vis}}, \quad \boldsymbol{\xi} = \boldsymbol{\xi}^{\text{eq}} + \boldsymbol{\xi}^{\text{vis}}, \end{aligned}$$

- Following the Coleman-Noll procedure:

$$\begin{aligned} \mathbf{P}^{\text{eq}} &= \psi, \mathbf{F}, \quad \mathbf{T}^{\text{eq}} = \psi, \mathbf{F}^p, \quad Y^{\text{eq}} = \psi, \bar{\varepsilon}^p, \\ f^{\text{eq}} &= \psi, d, \quad \boldsymbol{\xi}^{\text{eq}} = \psi, \nabla d, \quad -s = \psi, T. \end{aligned}$$

- Viscous forces follow from the dual kinetic potential Δ^* :

$$\begin{aligned} \mathbf{P}^{\text{vis}} &= \Delta^*, \quad \mathbf{T}^{\text{vis}} = \Delta^*, \quad Y^{\text{vis}} = \Delta^*, \\ f^{\text{vis}} &= \Delta^*, \quad \boldsymbol{\xi}^{\text{vis}} = \Delta^*, \end{aligned}$$

- To satisfy the second law:

$$\delta = \mathbf{P}^{\text{vis}} : \dot{\mathbf{F}} + \mathbf{T}^{\text{vis}} : \dot{\mathbf{F}}^p + Y^{\text{vis}} \dot{\bar{\varepsilon}}^p + f^{\text{vis}} \dot{d} + \boldsymbol{\xi}^{\text{vis}} \cdot \nabla \dot{d} \geq 0.$$

With $\mathcal{V} = \{\dot{\phi}, \dot{\mathbf{F}}^p, \dot{\bar{\varepsilon}}^p, \dot{d}\}$:

$$(\mathcal{V}, \dot{s}, T) = \arg \left(\inf_{\mathcal{V}, \dot{s}} \sup_T L \right)$$

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Benefits:

- From a theoretical standpoint:
 - The direct method of **calculus of variations** informs conditions for the existence and uniqueness of solutions.
 - Localization effects can be studied within the framework of **free-discontinuity problems**.
- From a computational standpoint:
 - Discretization leads to a **symmetric operator**.
 - Discretization leads to robust and efficient variational constitutive update.
 - The total potential can assist **line search**.
 - The total potential can be directly used as an **error indicator** for adaptive mesh refinement.
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State-of-the-art: variational brittle fracture that concerns with

$$\Phi, \quad d,$$

while we are looking at

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The total potential L is constructed as

$$L = \int_{\Omega_0} \varphi \, dV - \mathcal{P}^{\text{ext}},$$

$$\varphi = \dot{k} + \dot{u} + \Delta^* - T\dot{s} - \chi,$$

The external power expenditure $\mathcal{P}^{\text{ext}}(\dot{\phi}, T)$ is defined as

$$\begin{aligned} \mathcal{P}^{\text{ext}} = & \underbrace{\int_{\Omega_0} \rho_0 \mathbf{b} \cdot \dot{\phi} \, dV}_{\text{body force}} + \underbrace{\int_{\partial_t \Omega_0} \mathbf{t} \cdot \dot{\phi} \, dA}_{\text{surface traction}} + \underbrace{\int_{\partial_h \Omega_0} \bar{h}_n \ln\left(\frac{T}{T_0}\right) \, dA}_{\text{external heat flux}} \\ & + \underbrace{\int_{\partial_r \Omega_0} h \left[T - T_0 \ln\left(\frac{T}{T_0}\right) \right] \, dA}_{\text{external heat convection}} - \underbrace{\int_{\Omega_0} \rho_0 q \ln\left(\frac{T}{T_0}\right) \, dV}_{\text{heat source}}, \end{aligned}$$

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- Mass balance and angular momentum balance are satisfied by construction.
- Linear momentum balance:

$$\rho_0 \mathbf{a} = \nabla \cdot \mathbf{P} + \mathbf{b}.$$

- Micro-macro force balance:

$$\nabla \cdot \boldsymbol{\xi} - f = 0.$$

- Plastic flow:

$$\left(\dot{\mathbf{F}}^p, \dot{\boldsymbol{\varepsilon}}^p \right) = \arg \inf_{\dot{\mathbf{F}}^p, \dot{\boldsymbol{\varepsilon}}^p} \left[\mathbf{T}^{\text{eq}} : \dot{\mathbf{F}}^p + Y^{\text{eq}} \dot{\boldsymbol{\varepsilon}}^p + \Delta^* \right]$$

subject to $\mathbf{L}(\mathbf{Z}) \dot{\mathbf{Z}} = \mathbf{0}$.

- Heat transfer:

$$T\dot{s} = \rho_0 q - \nabla \cdot \mathbf{h} + \delta.$$

- (Strict) dissipation inequality requires Δ^* to be convex in each rate.

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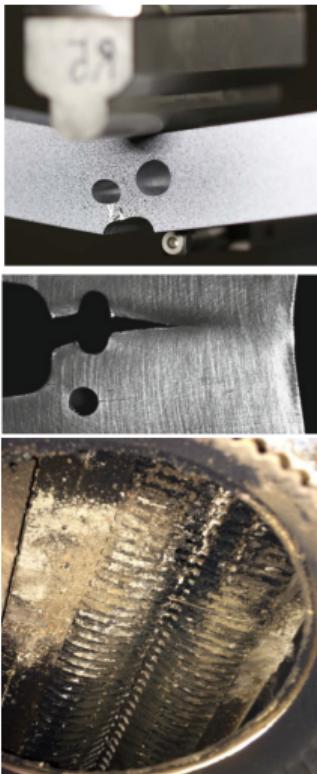
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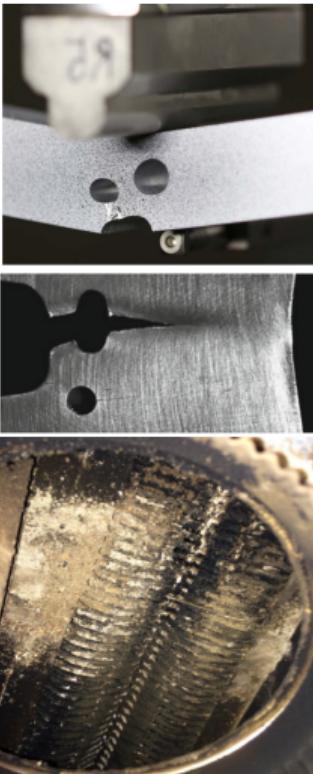
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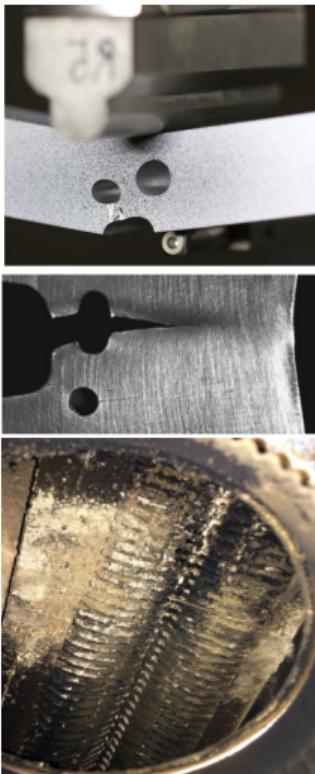


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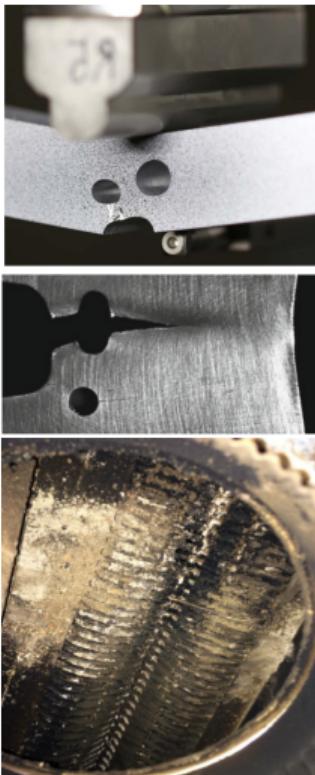
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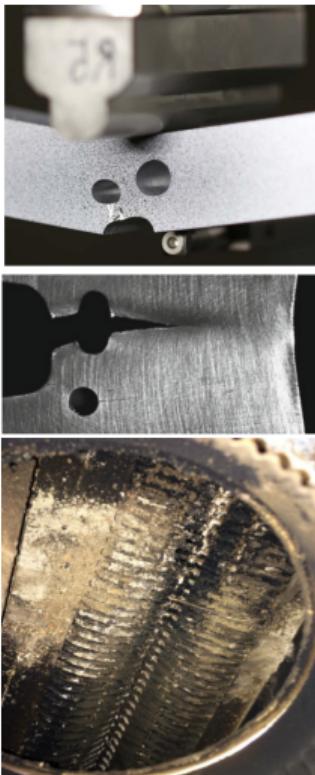
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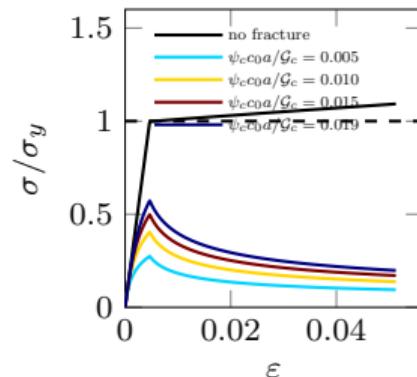
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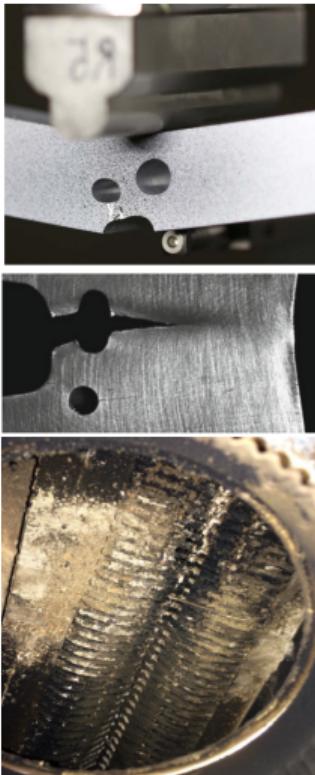


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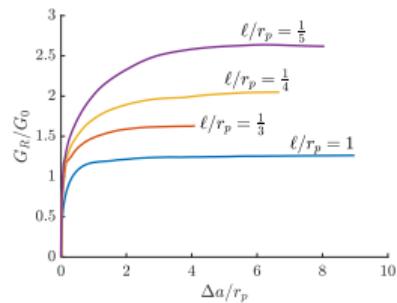


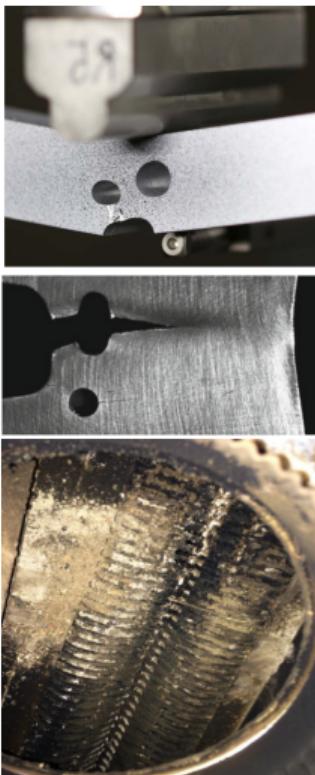


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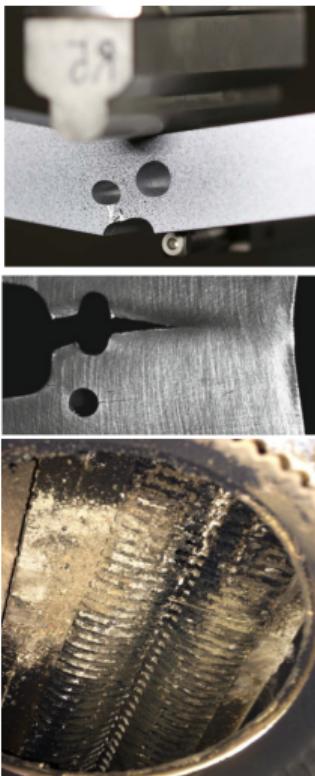




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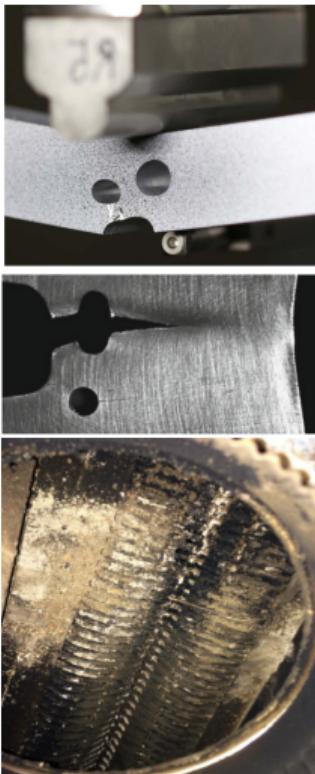
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All of the issues above can be addressed by the proposed framework
(with proper constitutive choices).

$$L = \int_{\Omega_0} \left[\dot{\psi}^e + \psi^{e*} + (1 - \mathcal{Q})\dot{\psi}^p + \mathcal{Q}\psi^{p*} + \dot{\psi}^f + \psi^{f*} - T\dot{s} - \chi \right] dV - \mathcal{P}^{\text{ext}},$$

subject to $\mathbf{L}(\mathbf{Z})\dot{\mathbf{Z}} = \mathbf{0}$.

Option 1 (Compressible Neo-Hookean):

$$\begin{aligned} \psi^e &= g^e \psi_{\langle A \rangle}^e + \psi_{\langle I \rangle}^e, \\ \psi_{\langle A \rangle}^e &= \mathbb{H}_1(J) \left\{ \frac{1}{2} K \left[\frac{1}{2} (J^2 - 1) - \ln(J) \right] \right\} \\ &\quad + \frac{1}{2} G (\bar{\mathbf{C}} : \mathbf{C}^{p-1} - 3), \\ \psi_{\langle I \rangle}^e &= (1 - \mathbb{H}_1(J)) \left\{ \frac{1}{2} K \left[\frac{1}{2} (J^2 - 1) - \ln(J) \right] \right\}. \end{aligned}$$

Option 2 (Hencky):

$$\begin{aligned} \psi^e &= g^e \psi_{\langle A \rangle}^e + \psi_{\langle I \rangle}^e, \\ \psi_{\langle A \rangle}^e &= \frac{1}{2} K \langle \text{tr}(\boldsymbol{\varepsilon}^e) \rangle_+^2 + G \text{dev } \boldsymbol{\varepsilon}^e : \text{dev } \boldsymbol{\varepsilon}^e, \\ \psi_{\langle I \rangle}^e &= \frac{1}{2} K \langle \text{tr}(\boldsymbol{\varepsilon}^e) \rangle_-^2, \\ \boldsymbol{\varepsilon}^e &= \frac{1}{2} \ln(\mathbf{C}^e). \end{aligned}$$

$$L = \int_{\Omega_0} \left[\dot{\psi}^e + \psi^{e*} + (1 - \mathcal{Q})\dot{\psi}^p + \mathcal{Q}\psi^{p*} + \dot{\psi}^f + \psi^{f*} - T\dot{s} - \chi \right] dV - \mathcal{P}^{\text{ext}},$$

subject to $\mathbf{L}(\mathbf{Z})\dot{\mathbf{Z}} = \mathbf{0}$.

Newtonian viscosity:

$$\begin{aligned}\psi^{e*} &= g^e J \left[\frac{1}{2} \zeta \text{tr}(\mathbf{d})^2 + \eta \mathbf{d} : \mathbf{d} \right], \\ \mathbf{d} &= \text{sym} \left(\dot{\mathbf{F}} \mathbf{F}^{-1} \right).\end{aligned}$$

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subject to $\mathbf{L}(\mathbf{Z})\dot{\mathbf{Z}} = \mathbf{0}$.

Flow rule constraints:

$$\text{tr} \left(\dot{\mathbf{F}}^p \mathbf{F}^{p-1} \right) = 0,$$

$$\left\| \dot{\mathbf{F}}^p \mathbf{F}^{p-1} \right\|^2 - \frac{3}{2} |\dot{\bar{\varepsilon}}^p|^2 = 0.$$

Remark (Flow rule)

It recovers the Prandtl-Reuss flow rule:

$$\dot{\mathbf{F}}^p \mathbf{F}^{p-1} = \dot{\bar{\varepsilon}}^p \mathbf{N}^p, \quad \mathbf{N}^p = \sqrt{\frac{3}{2}} \frac{\text{dev}(\mathbf{M})}{\|\text{dev}(\mathbf{M})\|},$$

and the loading/unloading conditions:

$$\phi^p \leqslant 0, \quad \dot{\bar{\varepsilon}}^p \geqslant 0, \quad \phi^p \dot{\bar{\varepsilon}}^p = 0,$$

$$\phi^p = \|\text{dev}(\mathbf{M})\| - \sqrt{\frac{2}{3}} \left(Y^{\text{eq}} + Y^{\text{vis}} \right).$$

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Option 1 (Linear hardening):

$$\begin{aligned}\psi^p &= g^p \left(\sigma_y \bar{\varepsilon}^p + \frac{1}{2} H \bar{\varepsilon}^p {}^2 \right), \\ \psi^{p*} &= g^p (\sigma_y + H \bar{\varepsilon}^p) \dot{\bar{\varepsilon}}^p.\end{aligned}$$

Option 2 (Power-law hardening):

$$\begin{aligned}\psi^p &= g^p \frac{n}{n+1} \sigma_y \epsilon_0 \left[\left(1 + \frac{\bar{\varepsilon}^p}{\epsilon_0} \right)^{(n+1)/n} - 1 \right], \\ \psi^{p*} &= g^p \sigma_y \left(1 + \frac{\bar{\varepsilon}^p}{\epsilon_0} \right)^{1/n} \dot{\bar{\varepsilon}}^p.\end{aligned}$$

Option 3 (Perfect plasticity with thermal softening):

$$\begin{aligned}\psi^p &= g^p \sigma_y^T \bar{\varepsilon}^p, & \psi^{p*} &= g^p \sigma_y^T \dot{\bar{\varepsilon}}^p, \\ \sigma_y^T &= \frac{\sigma_0}{\exp\left(-\frac{Q}{RT}\right)}\end{aligned}$$

Remark (The Taylor-Quinney factor)

Due to thermal softening (option 3), to get an increase in temperature from plastic dissipation:

$$\frac{Q}{Q + RT} \leq \mathcal{Q} \leq 1.$$

$$L = \int_{\Omega_0} \left[\dot{\psi}^e + \psi^{e*} + (1 - \mathcal{Q})\dot{\psi}^p + \mathcal{Q}\psi^{p*} + \dot{\psi}^f + \psi^{f*} - T\dot{s} - \chi \right] dV - \mathcal{P}^{\text{ext}},$$

subject to $\mathbf{L}(\mathbf{Z})\dot{\mathbf{Z}} = \mathbf{0}$.

Fracture energy density:

$$\psi^f = \frac{\mathcal{G}_c}{c_0 l} (C\alpha + l^2 \nabla d \cdot \nabla d).$$

Viscous regularization and coalescence dissipation:

$$\begin{aligned} \psi^{f*} &= \frac{1}{2} v d^2 + (1 - C) \frac{\mathcal{G}_c}{c_0 l} \alpha_{,d} d \\ &\quad - (1 - \beta) \frac{\mathcal{G}_c}{c_0 l} \alpha_{,d} \left(1 - e^{-\bar{\varepsilon}^p/\varepsilon_0}\right) \dot{d}. \end{aligned}$$

Irreversibility constraint:

$$\dot{d} \geq 0.$$

Remark

The contributions from plasticity is clear in the fracture envelope:

$$vd = \nabla \cdot \frac{2\mathcal{G}_c l}{c_0} \nabla d - \left(\frac{\widehat{\mathcal{G}}_c}{c_0 l} \alpha_{,d} + \psi^d \right),$$

$$\psi^d = \psi_{,d}^e + (1 - \mathcal{Q})\psi_{,d}^p,$$

$$\widehat{\mathcal{G}}_c = g^c \mathcal{G}_c, \quad g^c = 1 - (1 - \beta) \left(1 - e^{-\bar{\varepsilon}^p/\varepsilon_0}\right),$$

Remark

To satisfy the second law: $0 < C \leq \beta$.

$$L = \int_{\Omega_0} \left[\dot{\psi}^e + \psi^{e*} + (1 - \mathcal{Q})\dot{\psi}^p + \mathcal{Q}\psi^{p*} + \dot{\psi}^f + \psi^{f*} - T\dot{s} - \textcolor{red}{x} \right] dV - \mathcal{P}^{\text{ext}},$$

subject to $\mathbf{L}(\mathbf{Z})\dot{\mathbf{Z}} = \mathbf{0}$.

Fourier potential:

$$\begin{aligned}\chi &= \frac{1}{2}\kappa\mathbf{g} \cdot \mathbf{g}, \\ \mathbf{g} &= -\nabla T/T.\end{aligned}$$

Remark

The heat conduction equation can be written as

$$\begin{aligned}\rho_0 c_v \dot{T} &= \rho_0 q + \nabla \cdot \kappa \nabla T + \delta + \delta_T, \\ \delta &= \mathbf{P}^{\text{vis}} : \dot{\mathbf{F}} + Y^{\text{vis}} \dot{\bar{\varepsilon}}^p + f^{\text{vis}} \dot{d}, \\ \delta_T &= -g^p(1 - \mathcal{Q}) \frac{Q}{RT} \sigma_y^T \dot{\bar{\varepsilon}}^p.\end{aligned}$$

How to best design the coupling between plasticity and fracture remains an open question. Some challenges/issues:

- Fracture tends to propagate along the plastic zone.
- Most existing models
 - are not variational,
 - require re-calibration of material parameters,
 - result in regularization-dependent response.
- How much plastic work contributes to fracture evolution?
- How much plastic work converts to heat generation?

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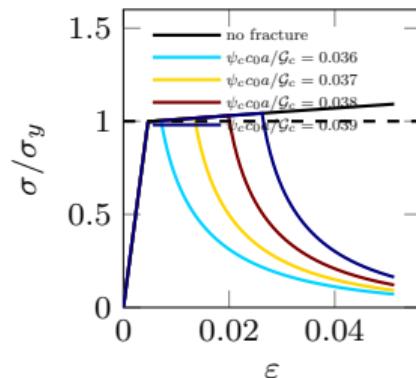
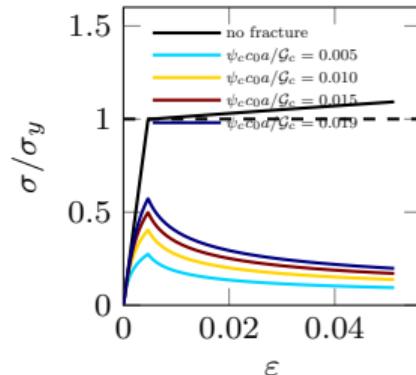
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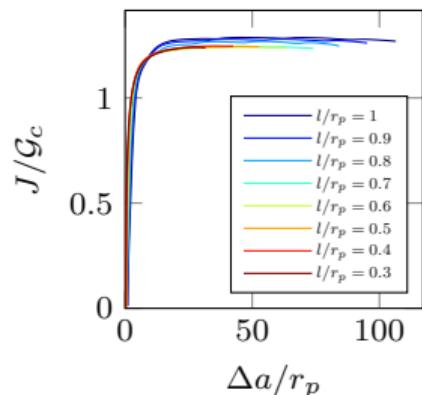
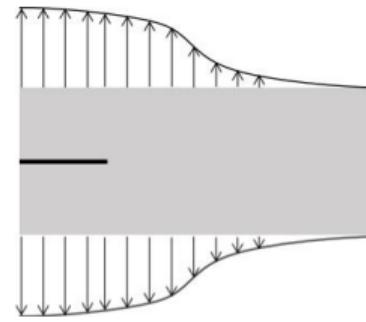


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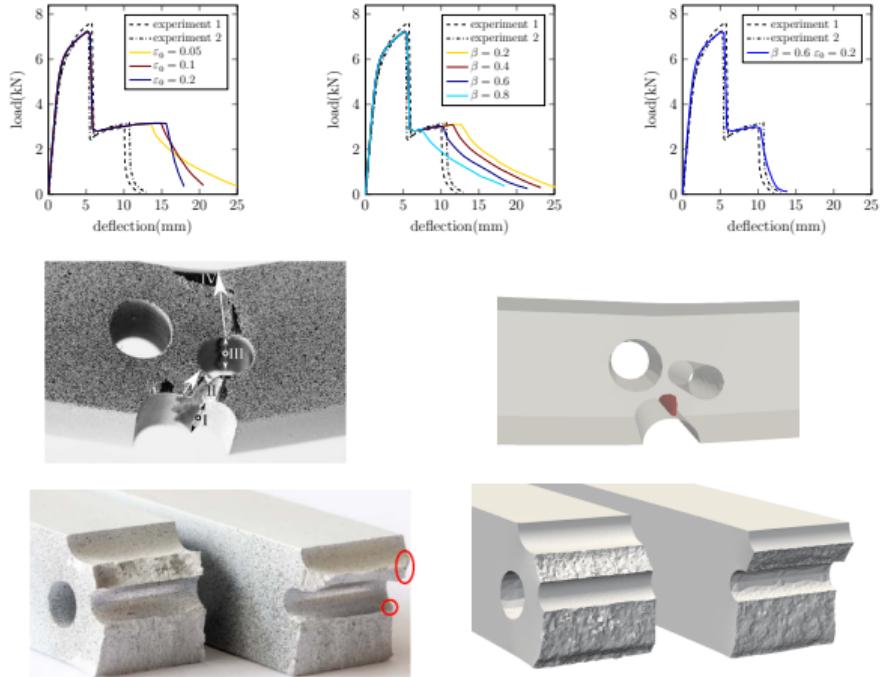


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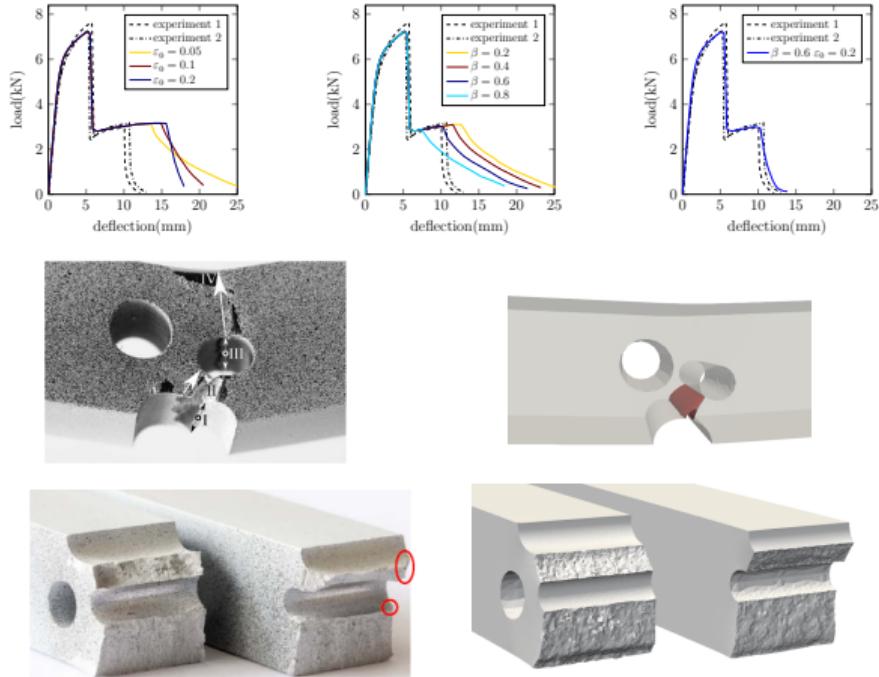
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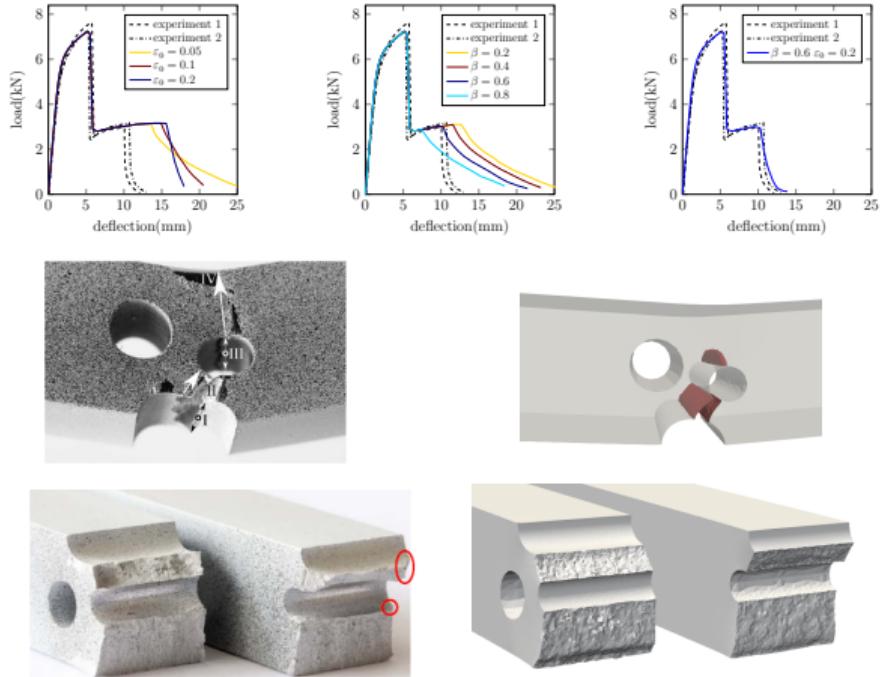
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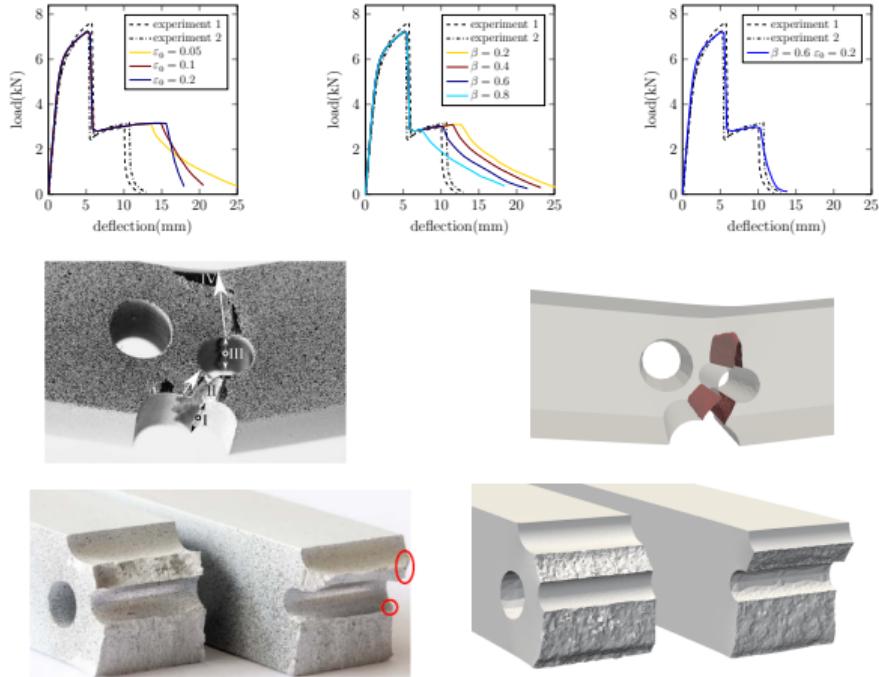
- A **three-point bending** experiment is simulated.
- The **aluminum** specimen is modeled as a **compressible Neo-Hookean** material, with **linear hardening**, $\mathcal{Q} = 1$.
- **Coalescence dissipation** is included. The effects of β and ε_0 are investigated in a 2D setting.
- Parameters are calibrated based on a **tensile tension test**.
- “Shear lips” are not captured by numerical simulations.
- Crack paths and load deflection curves have **excellent agreement** with the experiment.



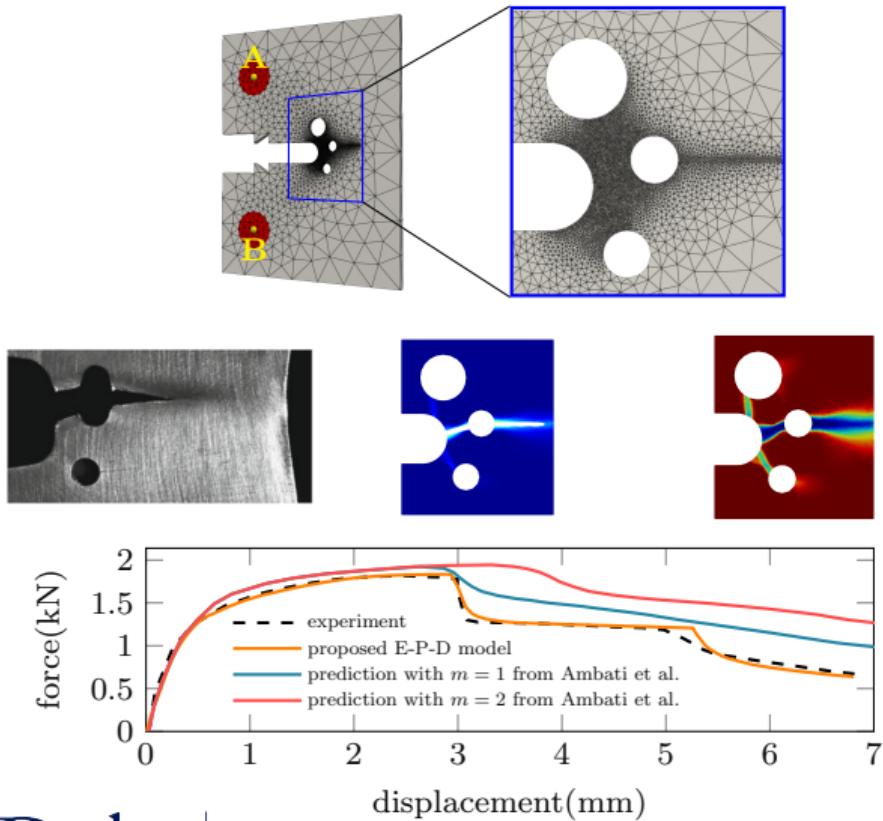
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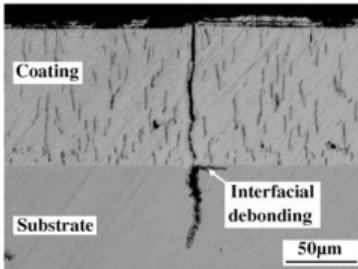
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- A recent [Sandia Fracture Challenge](#).
- Material properties are calibrated using provided [tensile tension test](#) data.
- Loading pins A and B are modeled as purely elastic materials with the same constants as the specimen.
- The predicted [force-displacement curve](#) is compared with the experimental data and predictions by other existing phase-field models of ductile fracture.
- The agreement between the experiment and our simulation is [remarkable](#), both in terms of the crack path and the force-displacement curve.



Background:

- High temperature heat exchangers are key components of many power conversion systems, including advanced nuclear power generation systems.
- They operate in the inlet temperature range of 750-1100 °C and are subject to unique operating challenges including oxidation, corrosion, creep and fracture.

Model:

- To model energy release associated with debonding:

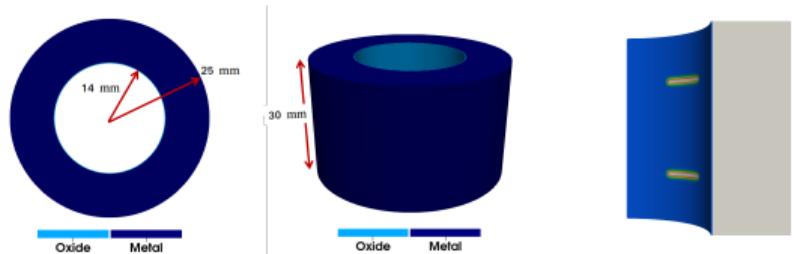
$$\psi^f = \frac{\mathcal{G}_c}{c_0 l} (C\alpha(d) + l^2 \nabla d \cdot \nabla d) + \frac{1}{\tau} \mathcal{G}\omega(c).$$

- A lower dimensional representation of the oxide layer:

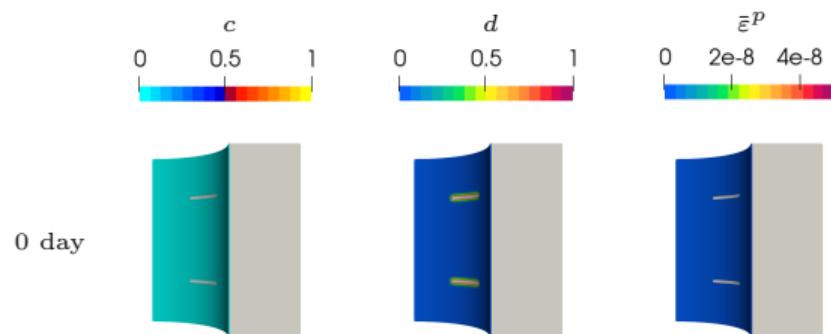
$$\psi^e = g_{ip}^e \psi_{ip,\langle A \rangle}^e + \psi_{ip,\langle I \rangle}^e + g_{op}^e \psi_{op,\langle A \rangle}^e + \psi_{ip,\langle I \rangle}^e.$$

- The perfect plasticity model with thermal softening is approximated using a power-law creep model:

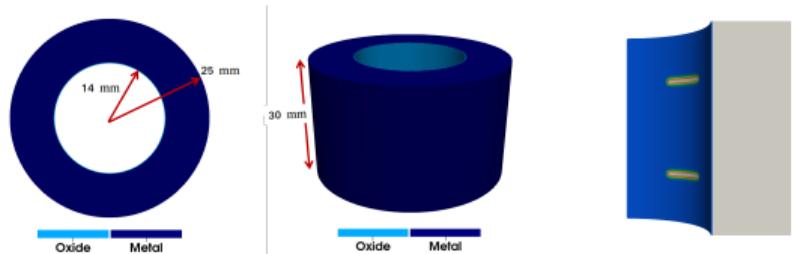
$$\dot{\varepsilon}^p = A \left(\frac{\bar{\sigma}}{g^p \sigma_0} \right)^n \exp \left(-\frac{nQ}{RT} \right).$$



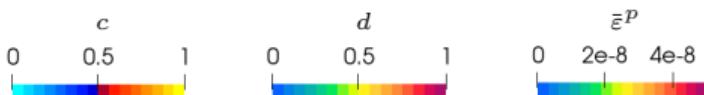
Case I: Initial transverse cracks



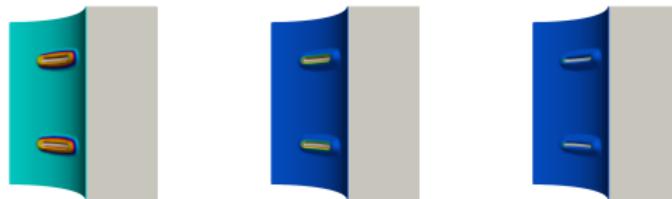
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- The HTHX is surround by **high temperature pressurized** fluids during normal operation. The temperature and the pressure of the fluids drop after shutting down.
- Most model parameters are adopted from [10].
- Creep deformation accumulates under normal operating conditions. The effective creep strain localizes around the cracks.
- Debonding occurs at weaker locations in the vicinity of the crack surfaces.
- Transverse cracks nucleate and propagate while shutting down.



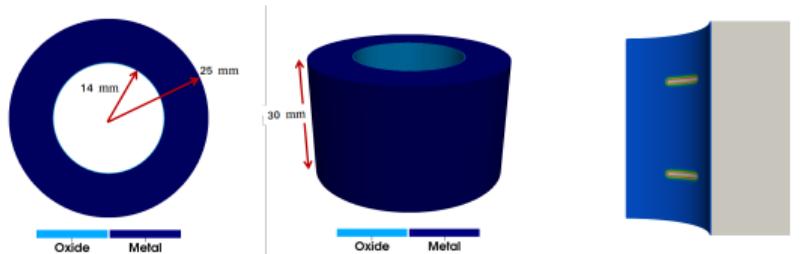
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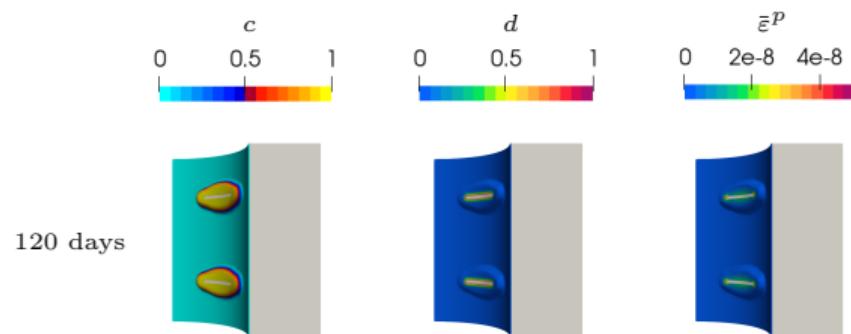
60 days



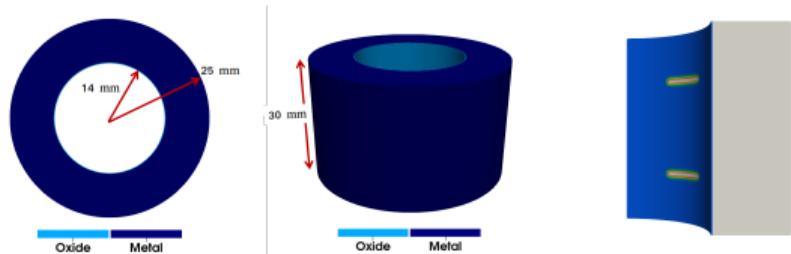
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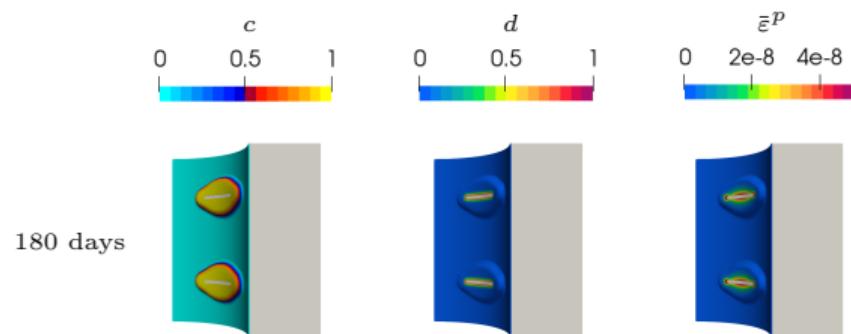
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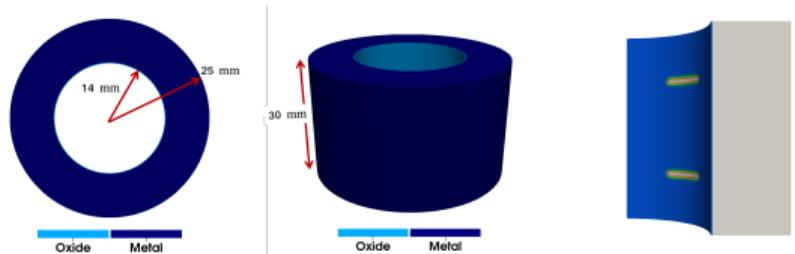
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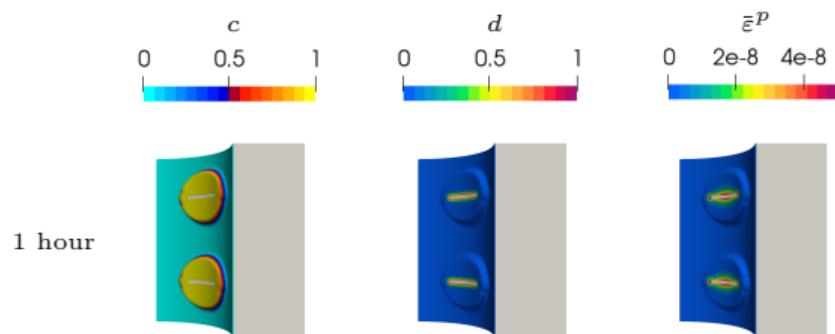
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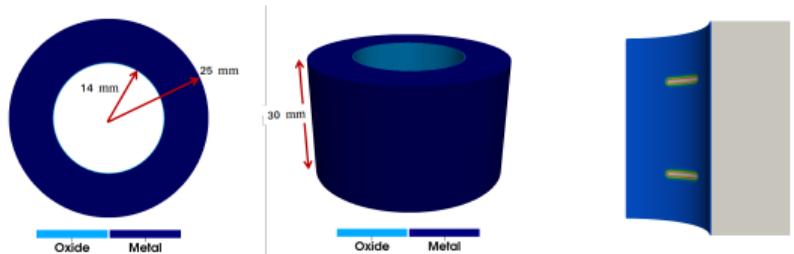
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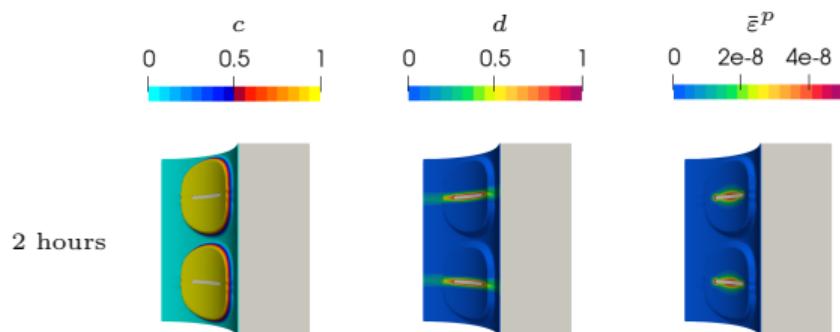
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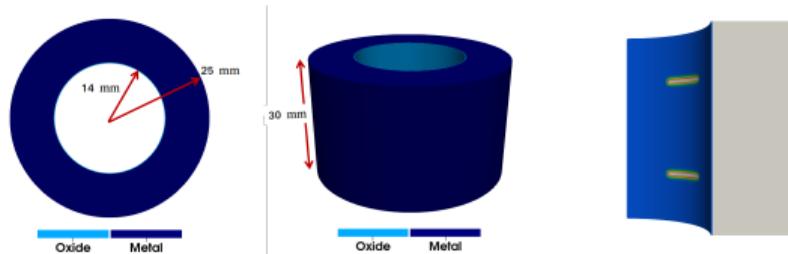
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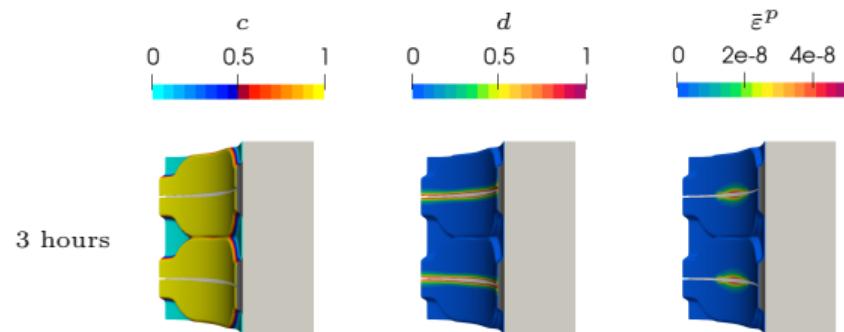
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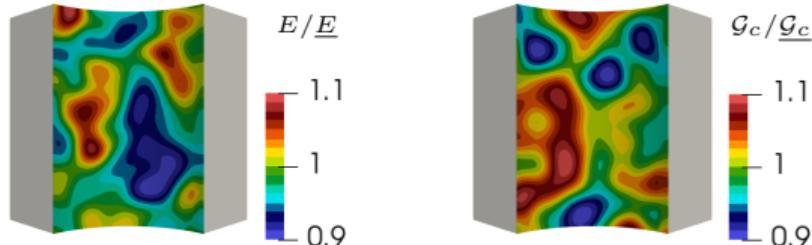
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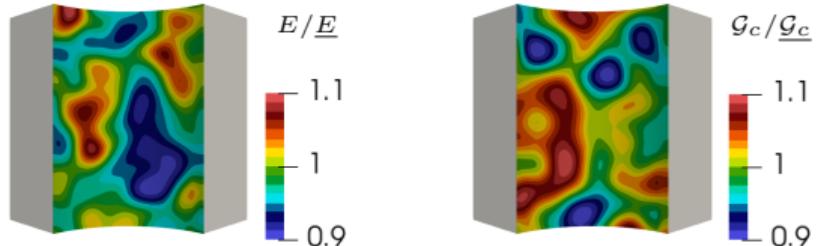
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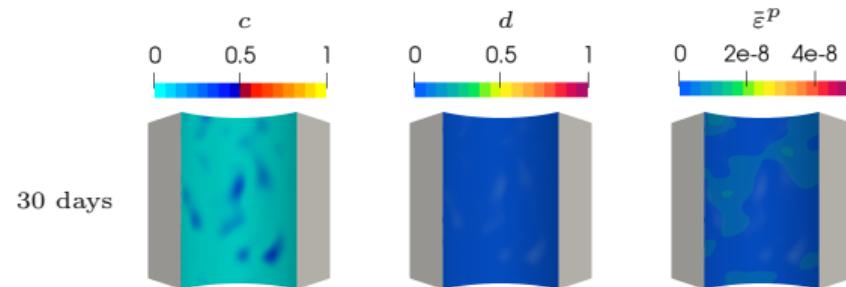
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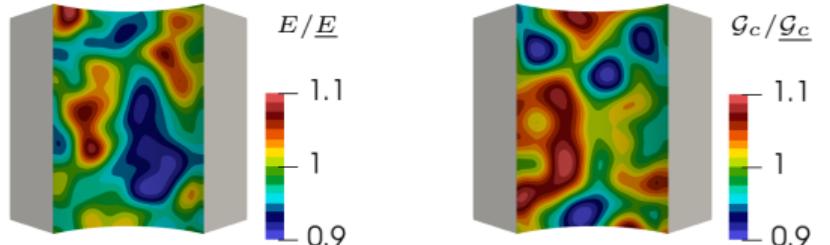
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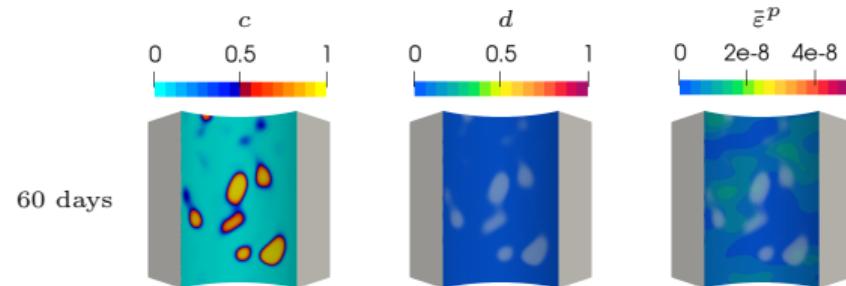
Case II: Inhomogeneous Young's modulus and Fracture toughness



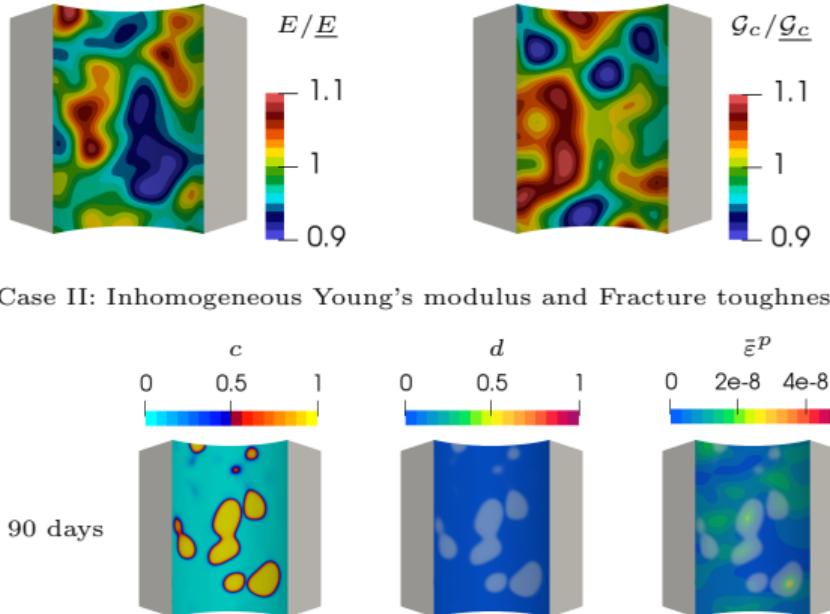
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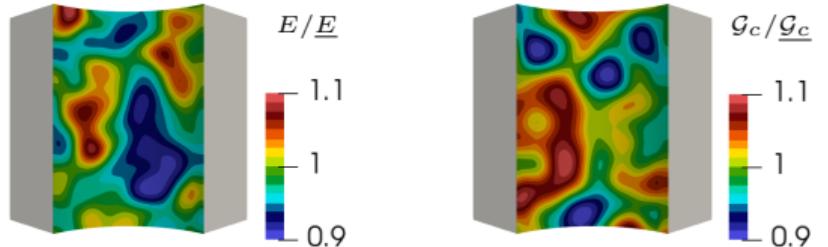
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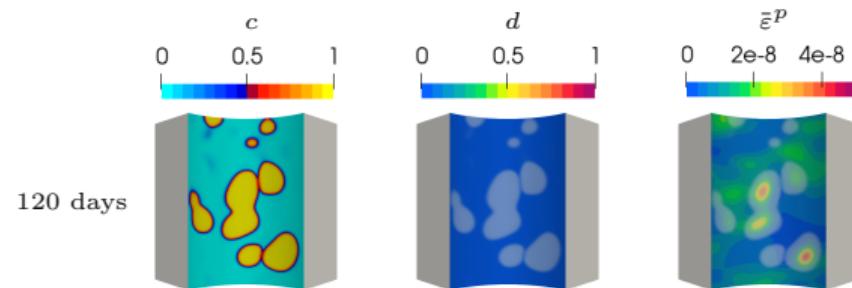
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- Transverse cracks nucleate and propagate while shutting down.



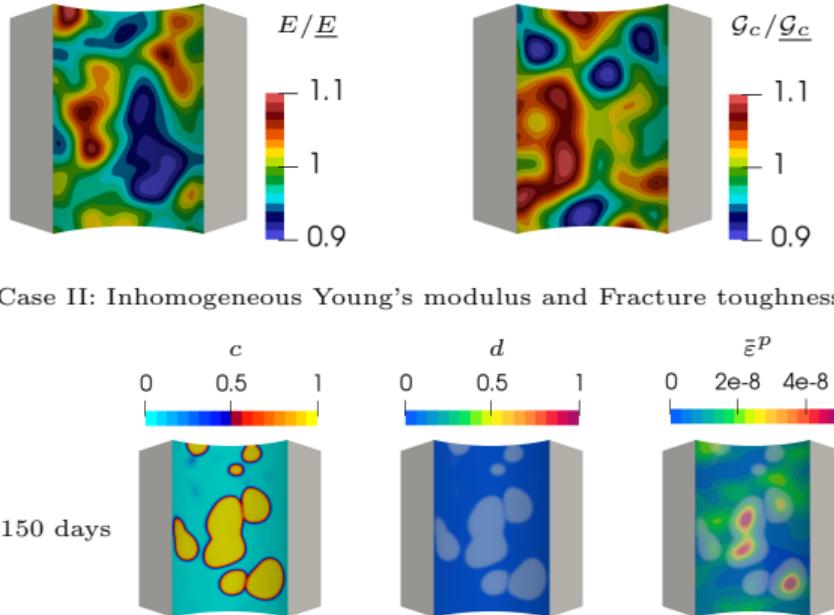
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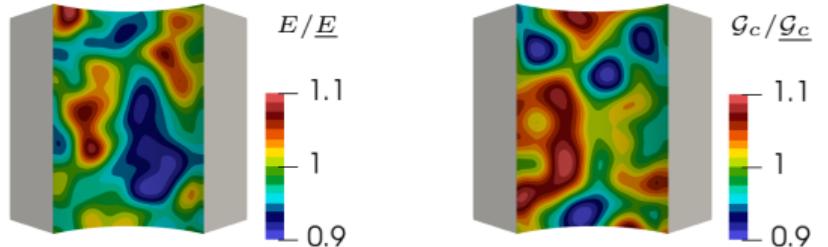
Case II: Inhomogeneous Young's modulus and Fracture toughness



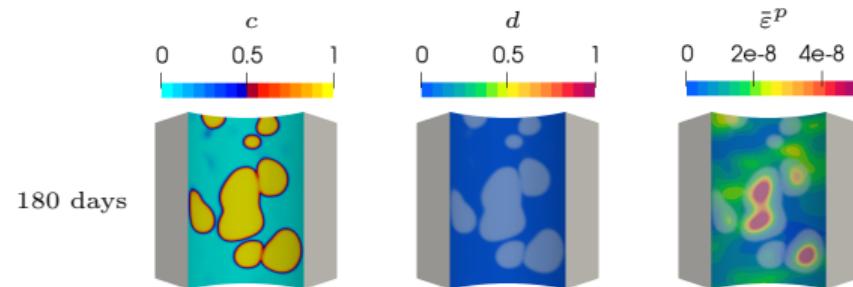
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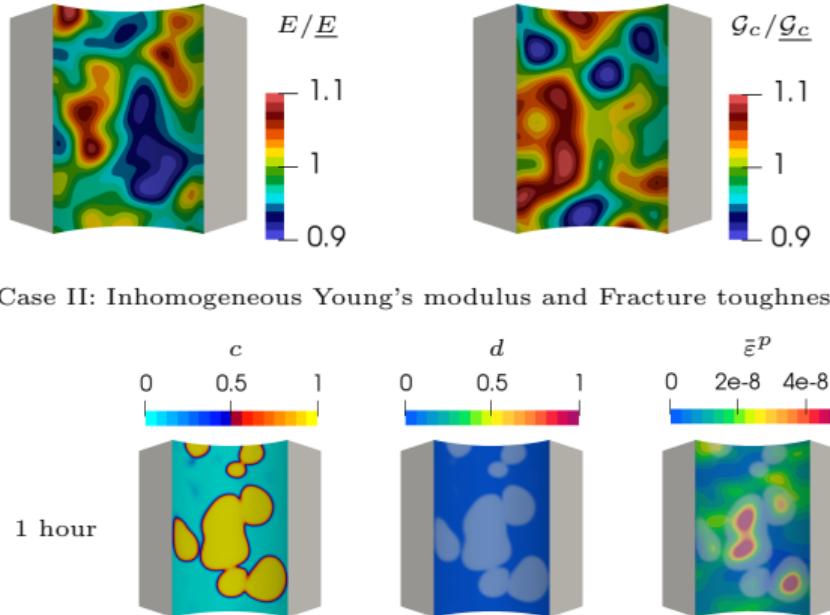
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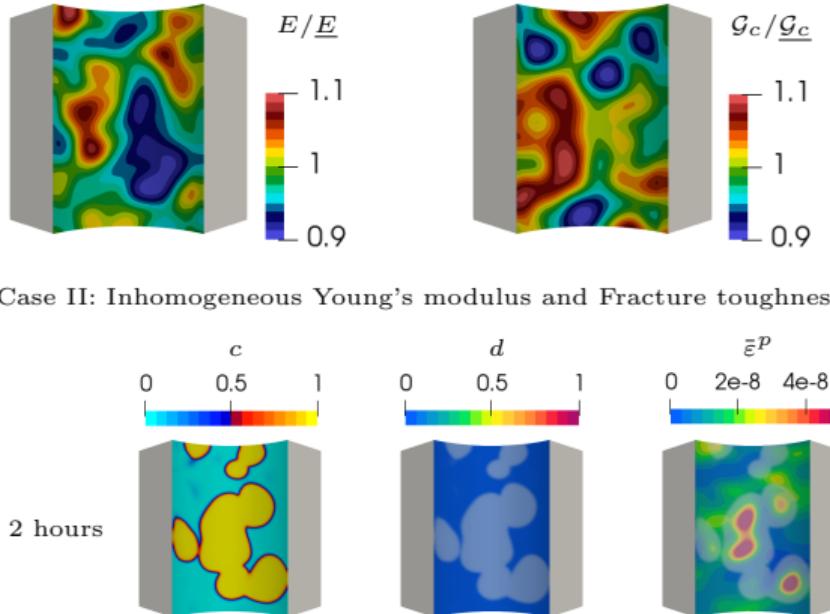
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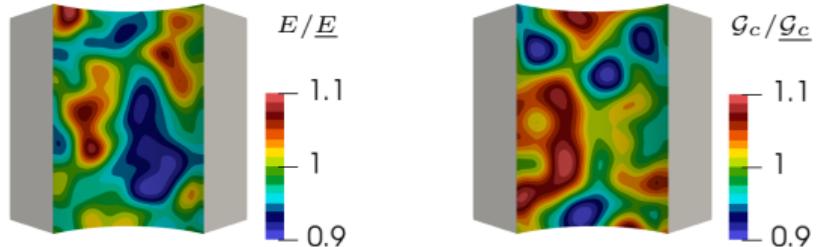
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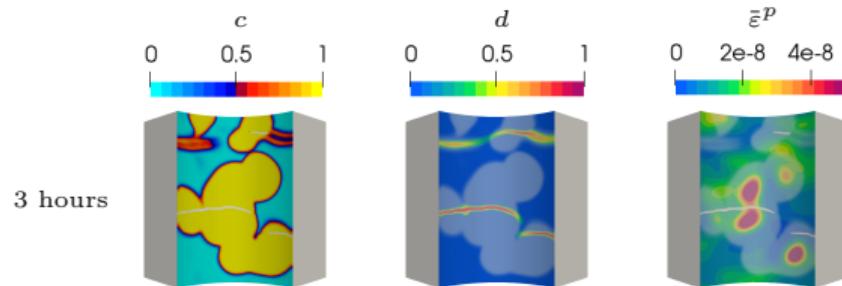
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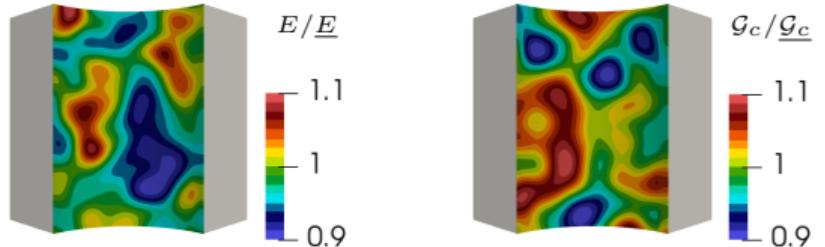
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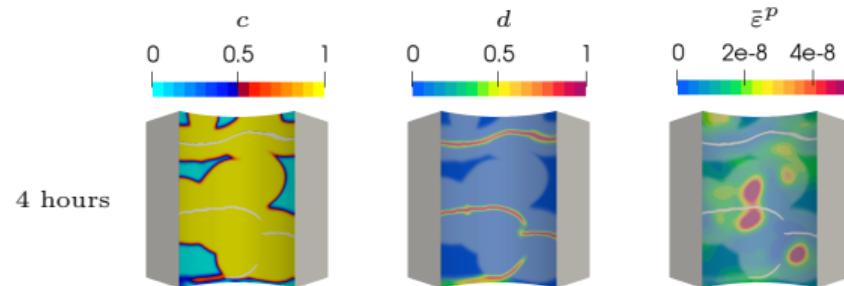
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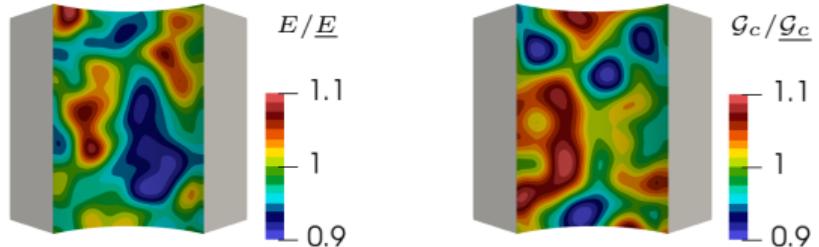
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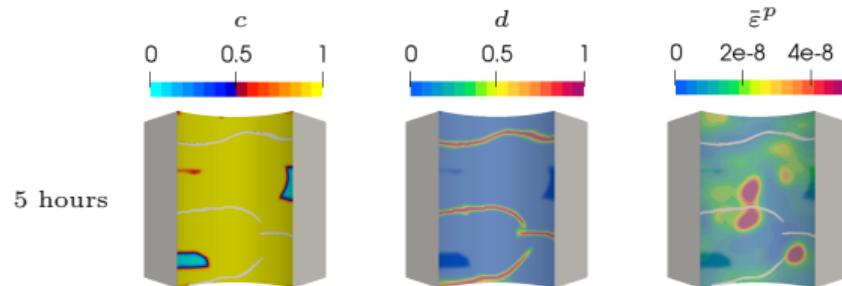
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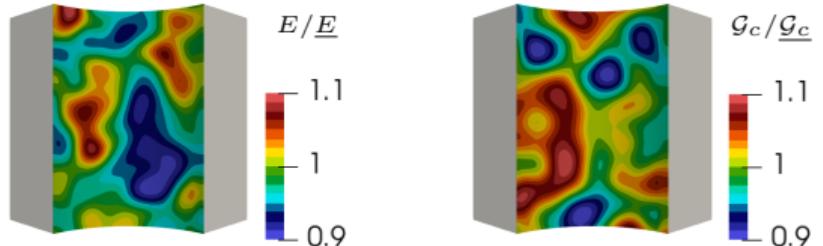
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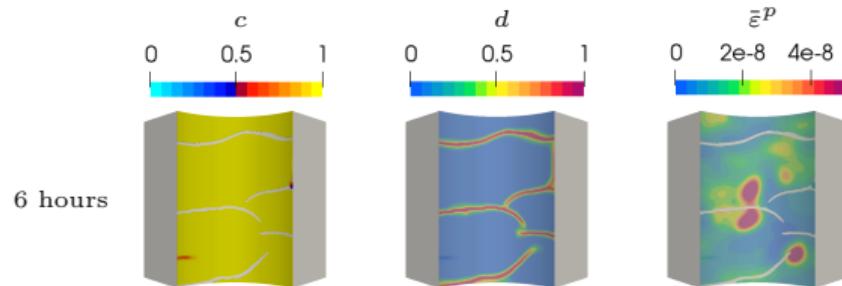
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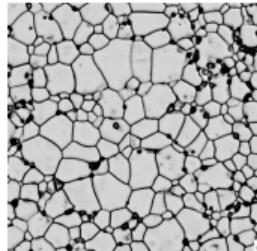
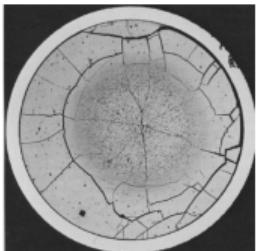
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Background:

- Fission of UO₂ produces a variety of fission products.
- Properties of UO₂ are strongly influenced by fracture.
- Gas bubbles, grains, and grain boundaries alter fracture properties.
- Existing 2D models over-simplifies the microstructure and results in inaccurate strength-porosity relations.

Model:

- Helmholtz free energy density:

$$\psi = \psi^e + \psi^f.$$

- Strain energy density:

$$\psi^e = g\psi_{\langle A \rangle}^e + \psi_{\langle I \rangle}^e,$$

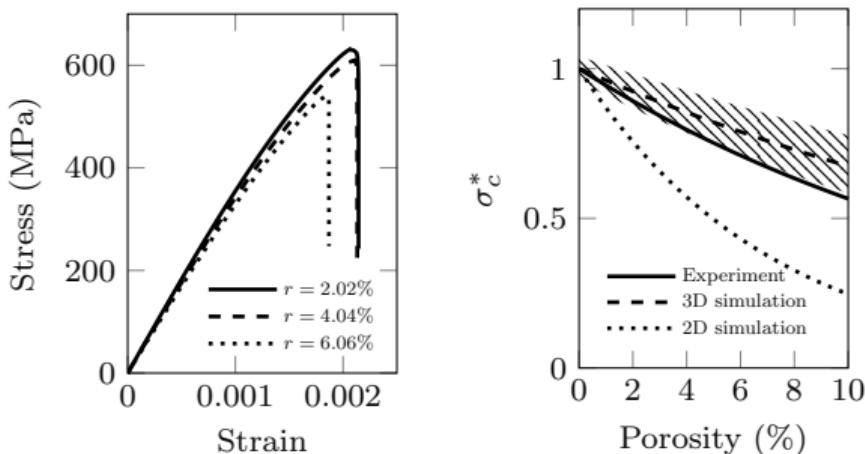
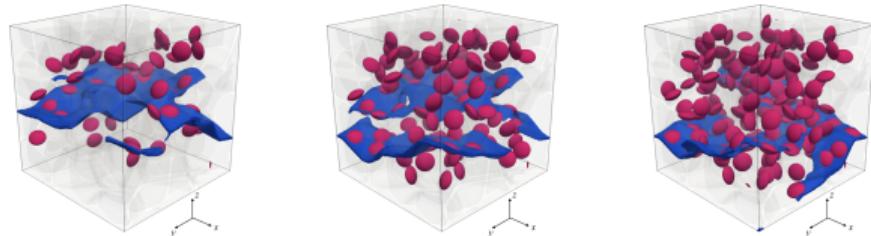
$$\psi_{\langle A \rangle}^e = \frac{1}{2}\lambda \langle \text{tr}(\boldsymbol{\varepsilon}) \rangle_+^2 + G\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+,$$

$$\psi_{\langle I \rangle}^e = \frac{1}{2}\lambda \langle \text{tr}(\boldsymbol{\varepsilon}) \rangle_-^2 + G\boldsymbol{\varepsilon}^- : \boldsymbol{\varepsilon}^-,$$

- Fracture energy density:

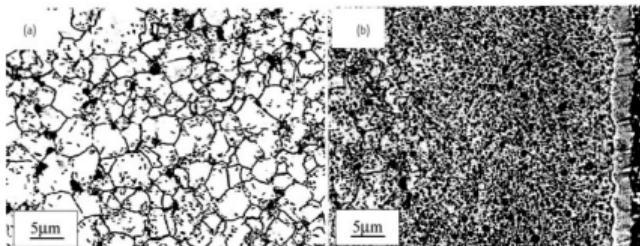
$$\psi^f = \frac{\mathcal{G}_c}{c_0 l} (\alpha + l^2 \nabla d \cdot \nabla d),$$

$$\alpha = d^2, \quad g = (1 - d)^2.$$



- A set of random close-packing voronoi structures are realized by **maximal Poisson-disk sampling**.
- The microstructure is generated using a **phase-field grain growth model** [11].
- Grain boundaries have an arbitrarily high fracture toughness to facilitate intergranular fracture.
- Numerical studies are performed to investigate the effects of **bubble geometry**, **loading conditions**, and **porosity** on the critical fracture strength.
- Results of 15 realizations of 3 porosity levels are fitted using the relation suggested by experiments [12]:

$$\frac{\sigma_c}{\sigma_0} = \exp(-ar).$$

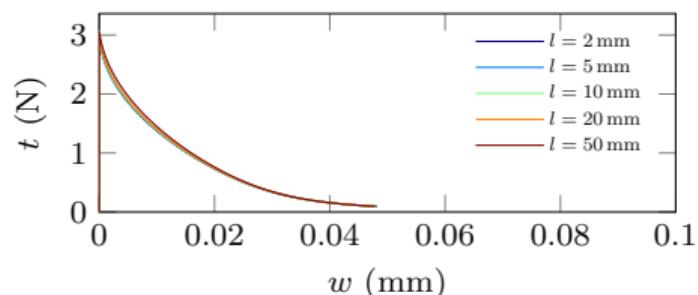
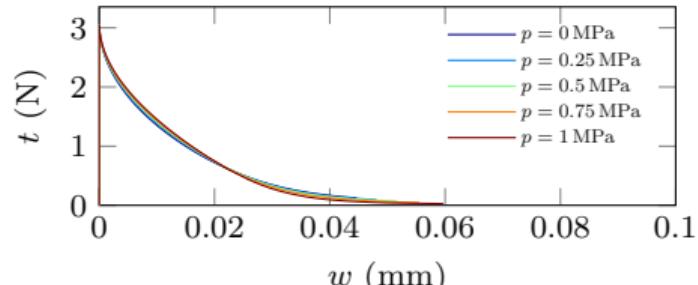


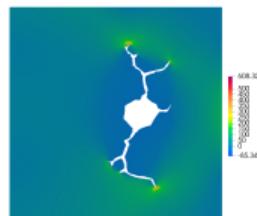
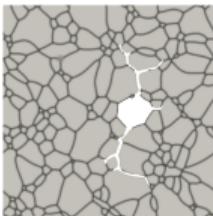
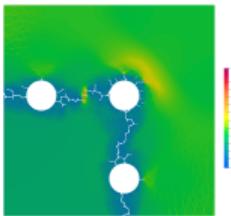
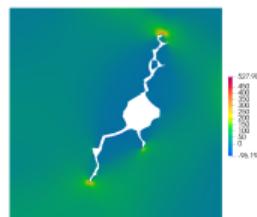
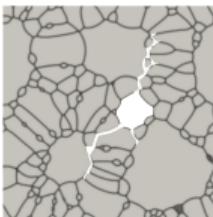
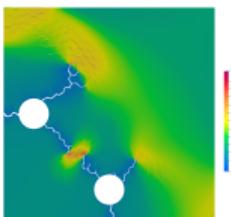
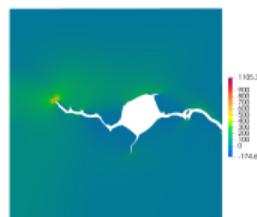
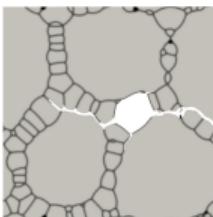
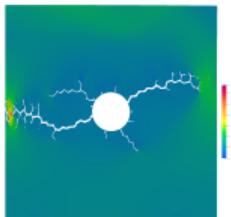
Background:

- Utilities are seeking to increase the allowable **burnup limit** for UO₂ fuel.
- The risk of **fragmentation** during a loss-of-coolant accident (LOCA) is a major limitation.
- **Over-pressurization** of fission gas bubbles results in fine fragmentation of high burnup structures.
- A model for predicting the onset of fragmentation is essential.

External pressure power:

$$\mathcal{P}^{\text{ext}} = - \int_{\Omega_0} \bar{p} \nabla d \cdot \dot{\phi} I_{d,d} \, dV.$$





- A 2D REV is considered. Plane strain conditions are assumed to hold.
- LOCA pressure transients:
 - The temperature as a function of time at the edge of a representative pellet for each rod is obtained from simulations.
 - The temperature transient is used as an input to a Kim-Kim-Suzuki (KKS) phase-field model [13] to determine the pressure transient.
 - The pressure transient is treated as a known in the fracture model.
- Effects of **bubble size**, **bubble pressure**, **surrounding pressure**, and **multi-bubble interaction** are investigated.
- **Defect evolution** and **recrystallization** can be incorporated into the fracture model.

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Background:

- **Film-substrate systems** widely exist in nature and in engineering applications.
- Fracture of thin films has been studied using model-based simulations based on a wide range of methodologies.
- Many soil materials are “**cohesive**” in nature. It calls for a phase-field model for cohesive fracture.
- Many film-substrate systems are symmetric or axisymmetric. It is important to incorporate **stochastic models** for material properties.

Model:

- Enforcing traction-free boundary conditions:

$$\psi^e = g\psi_{\langle A \rangle}^e + \psi_{\langle I \rangle}^e,$$

$$\psi_{\langle A \rangle}^e = \frac{1}{2}\boldsymbol{\sigma}_{\langle A \rangle} : \boldsymbol{\varepsilon}, \quad \psi_{\langle I \rangle}^e = \frac{1}{2}\boldsymbol{\sigma}_{\langle I \rangle} : \boldsymbol{\varepsilon},$$

$$\boldsymbol{\sigma}_{\langle A \rangle} = \boldsymbol{\sigma}_n^+ + \boldsymbol{\sigma}_t, \quad \boldsymbol{\sigma}_{\langle I \rangle} = \boldsymbol{\sigma}_n^-,$$

$$\boldsymbol{\sigma}_n^\pm = \langle -t_N \rangle_\pm \tilde{\mathbf{n}} \otimes \tilde{\mathbf{n}}, \quad \boldsymbol{\sigma}_t = \boldsymbol{\sigma} - \boldsymbol{\sigma}_n^+ - \boldsymbol{\sigma}_n^-.$$

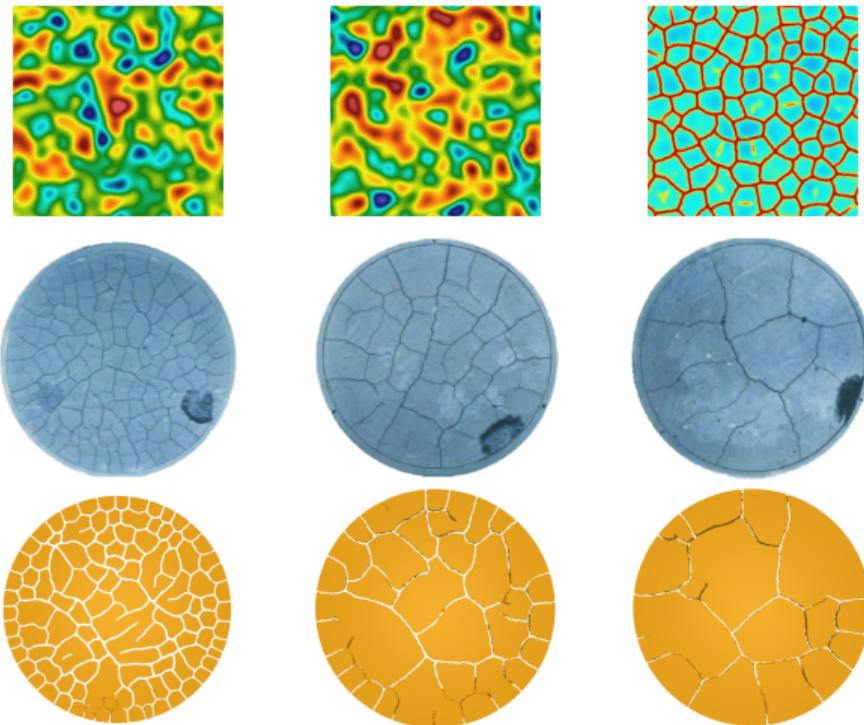
- Cohesive fracture:

$$\alpha = \xi d - (1 - \xi)d^2, \quad g = \frac{1}{1 + \phi},$$

$$\phi = \frac{a_1 d + a_1 a_2 d^2 + a_1 a_2 a_3 d^3}{(1 - d)^p}.$$

- A nonlinear softening law:

$$\xi = 1, \quad p = 2, \quad a_1 = \frac{\mathcal{G}_c}{c_0 l \psi_c}, \quad a_2 = 1, \quad a_3 = 0.$$



- Only “channeling” cracks in the thin film are considered.
- Thermal effects are neglected. Dehydration is modeled as pre-stress (or equivalent eigenstrains).
- The fracture model is verified with analytical solutions in a periodic quasi-1D context.
- Pervasive fracture is studied with a 2D simplification.
- Material property inhomogeneity is represented by two pointwise correlated random fields $\{\mathcal{G}_c(\mathbf{X}), \mathbf{X} \in \Omega\}$ and $\{\psi_c(\mathbf{X}), \mathbf{X} \in \Omega\}$.
- The versatility offered by the probabilistic framework is highlighted by solving a 3D problem based on physical experiments.

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Conclusions:

- A variational framework is proposed to model **general dissipative solids with fracture**.
- Several models are constructed within the framework to study **practical engineering problems**:
 - Intergranular fracture: brittle fracture, quasi-brittle fracture, fragmentation, pressurized cracks;
 - Soil desiccation: cohesive fracture, traction-free BCs, random fracture properties;
 - Ductile failure: no re-calibration, regularization-independent response (J -resistance curves), thermal effects, three-point bending, the Sandia Fracture Challenge, oxide spallation in HTHX.

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Future work:

- Fracture **nucleation with arbitrary strength surface**.
- Ductile failure with **impact loading**, where dynamic effects, abrupt thermal softening, and heat generation are important.
- **Fatigue** effects are important in structures subject to cyclic loading. Existing fatigue models do not fit into the framework as is. The interplay between fatigue and plasticity could be interesting.

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