A Variational Framework for Phase-Field Fracture Modeling with Applications to Fragmentation, Desiccation, Ductile Failure, and Spallation

Dissertation Defense

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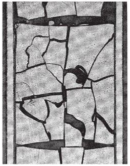

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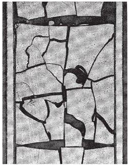


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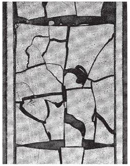

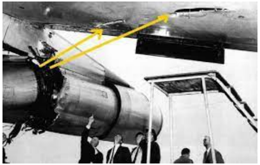

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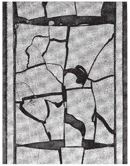

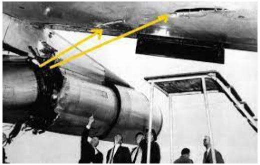


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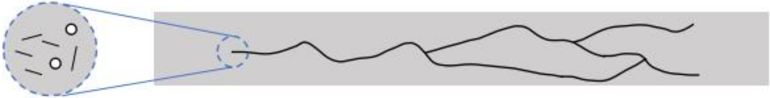

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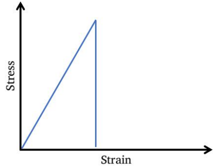

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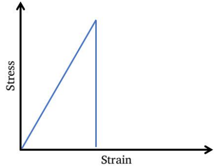

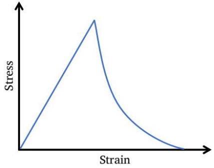

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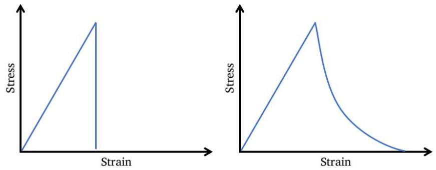

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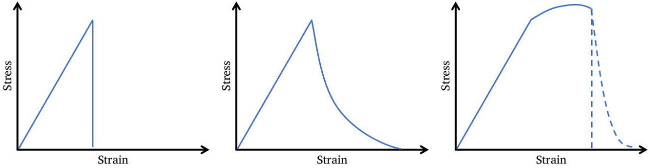

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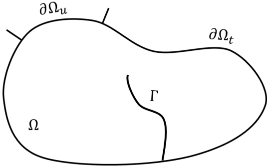
To date, fracture is still one of the most challenging phenomena to model and predict.

The permanent crack set Γ and its associated fracture energy

$$\Psi^f = \int_\Gamma \mathcal{G}_c \, \mathrm{d}A$$



The permanent crack set Γ and its associated fracture energy

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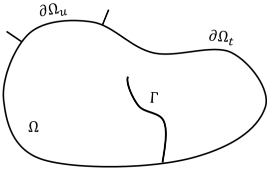
is approximated with

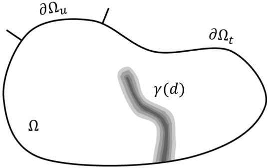
the crack surface density function $\gamma = \widehat{\gamma}_l(d)$:

$$\Psi^f \approx \int_\Omega \mathcal{G}_c \gamma \, \mathrm{d}V, \quad \gamma = \frac{1}{c_0 l}\left(\alpha + l^2 \boldsymbol{\nabla} d \cdot \boldsymbol{\nabla} d\right).$$

- $d \in [0, 1]$ is the phase field;
- $\alpha = \widehat{\alpha}(d)$ is the crack geometric function, $\widehat{\alpha}(0) = 0$, $\widehat{\alpha}(1) = 1$;
- $g = \widehat{g}(d)$ is the degradation function, $\widehat{g}(0) = 1$, $\widehat{g}(1) = 0$;
- c_0 is chosen such that

$$\lim_{l \to 0^+} \int_\Omega \mathcal{G}_c \gamma \, \mathrm{d}V = \int_\Gamma \mathcal{G}_c \, \mathrm{d}A.$$





For more details: [1].

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[1] Blaise Bourdin, Gilles A Francfort, and Jean-Jacques Marigo.
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