Coding the Matrix — Written Questions

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Here we will have the written answers to selected problems that do not require code or drawings to solve.

1 The Field

1.6 Review

1. The complex numbers, the reals, and the integers.
2. z.real − z.imag, and the formula for the absolute value of a complex number is $z * z_c$
3. Adding the real and imaginary components seperately.
4. Putting them in an equation and using distributive property.
5. Adding two complex numbers together.
6. Multiplying a real number by a complex number.
7. Multiplying by -1.
8. Multiplying by $e^{\frac{\pi i}{2}}$.
9. Adding the two bits and then applying modulo 2.
10. Setting the result to 0 if one of the bits is 0 and 1 otherwise.

1.7 Problems

10. (a) $5 + 3i$
    (b) i
    (c) $-1 + 0.001i$
    (d) $0.001 + 9i$
11. (a) e^{3i}
    (b) $e^{(\frac{11\pi}{12})i}$

(c) $e^{(\frac{5\pi}{12})i}$

12. (a) $a = (2)(e^{(\frac{\pi}{2})i})$, $b = 1 + 1i$

    (b) Not possible to scale the real part by two and imaginary part by three in only one multiplication.

13. (a) $1 + 1 + 1 + 0 = (1 + 1) + (1 + 0) = (0) + (1) = 1$

    (b) 0

    (c) 0

2 The Vector

2.6 Combining Vector Addition and Scalar Multiplication

1. $[3, 4]$, and the translation vector is $[2, 3]$.

2. $\{\alpha[5, -1] + [1, 4] : \alpha \in \mathbb{R}, 0 \leq \alpha \leq 1\}$

6. Same approach as in 2.5.5. For each element k of the domain D, entry k of $(\alpha + \beta)u$ is $(\alpha + \beta)u[k]$. By the distributive law for fields, this is equal to $\alpha u[k] + \beta u[k]$, which is the kth element of $\alpha u + \beta u$.

2.8 Vectors over $GF(2)$

3. Have two n-bit keys chosen randomly and uniformly over $GF(2)$ this time, v_a and v_b, given to Alice and Bob. Give a third TA the key $v_c := v - v_a - v_b$.

2.9 Dot Product

23. Suppose $u = [u_1, u_2 \ldots u_n]$ and $v = [v_1, v_2 \ldots v_n]$.
    $(\alpha u) * v$
    $= (\alpha u_1)(v_1) + (\alpha u_2)(v_2) \ldots (\alpha u_n)(v_n)$
    $= \alpha(u_1)(v_1) + \alpha(u_2)(v_2) \ldots \alpha(u_n)(v_n)$
    $= \alpha((u_1)(v_1) + (u_2)(v_2) \ldots (u_n)(v_n))$
    $= \alpha(u * v)$

24. $\alpha = 2, u = [2, 2], v = [2, 2]$
    $(2 * [2, 2]) * (2 * [2, 2]) = [4, 4] * [4, 4] = 32$
    $2 * ([2, 2] * [2, 2]) = 2 * (4 + 4) = 2 * 8 = 16$

26. $u, v, w, x = [2, 2]$
    $(u + v) * (w + x) = [4, 4] * [4, 4] = 32$
    $(u * v) + (w * x) = 8 + 8 = 16$

29. For the first number, sum the last three challenges, $101010 + 111011 + 001100 = 011101$.
    The sum of their responses is 0.

For the second number, sum the first, third, and last challenge, $110011 + 111011 + 001100 = 000100$.
The sum of their responses is also 0.

2.11 Solving a triangular system of linear equations

3. $x_3 = -\frac{6}{5}$
$x_2 = \frac{4-4x_3}{2} = 2 - 2x_3 = 2 - 2(-\frac{6}{5}) = 2 + \frac{12}{5} = \frac{22}{5}$
$x_1 = 7 + 3x_2 + 2x_3 = 7 + 3(-\frac{6}{5}) + 2(\frac{22}{5}) = 7 - \frac{18}{5} + \frac{44}{5}$
$= 7 + \frac{26}{5} = \frac{61}{5}$

2.13 Review Questions

- Vector addition is, as the name suggests, the summation of two or more vectors. If you consider, as the book does, vectors to be special types of functions, then vector addition is the act of creating a new function equal to the sum of two "vector functions".
- Points in an X-D space can be modeled as arrows from the origin in that X-D space. Similarly, the sum of vectors that can be represented as points can be interpreted as "summing" the arrows, or laying their displacements from the origin on top of each other.
- Scalar-Vector multiplication is the act of multiplying a "scalar", that's within a field, and a vector together, in the manner that the above definition of vector addition would imply.
- $(\alpha + \beta)u = \alpha u + \beta u$
- $\alpha(u + v) = \alpha u + \alpha v$
- The set of all possible solutions to the equation $\alpha[x, y]$, which is analogous to the line connecting and moving beyond the points $(0, 0)$ and (x, y).
- The similar set $\{\alpha[w, x] + \beta[y, z] : \alpha, \beta \in \mathbb{F}, \alpha + \beta = 1\}$, where $\mathbb{F}$ is your field of choice.
- The sum of the product of the corresponding entries of two vectors. We defined vectors as functions earlier and so I don't know if this is a sane way to go about talking about dot products, but here is the book's definition. Two vectors must have the same domain for this dot product definition to make sense.
- $(\alpha u) * v = \alpha(u * v)$
- $(u + v) * w = u * w + v * w$
- An equation of the form $a * x = \beta$, where a is a static vector, b is a vector variable, and β is a scalar.

- A set of linear equations.
- A linear system in upper-triangular form.
- Solving for the variable at the bottom, and progressing orderly to the top, solving for one variable each equation, until you have reached the top of the triangle and solved for all x_k.

3 Problems

1. $[-1, 7]$,
   $[-1, -1]$, and
   $[-3, 1]$.
2. $[1, 0, 6]$,
   $[3, -2, -4]$,
   $[5, -3, 9]$,
   $[0, 1, 7]$
3. $[one, 0, 0]$
   $[0, one, one]$
4. (a) $c + d + e$
   (b) $b + c + d + e$
5. (a) $c + d$
   (b) Could not find one.
6. - 1011
   - 1101
   - 1000
   - 1011 + 1111 = 0100
     0100 * 1100 = 0 + 1 + 0 + 0 = 1
     1101 + 1111 = 0010
     1010 + 0010 = 0 + 0 + 1 + 0 = 1
     1000 + 1111 = 0111
     1111 * 0111 = 0 + 1 + 1 + 1 = 1 + 0 = 1
7. (a) $v_1 = [2, 3, 4, 3]$
   (b) $v_2 = [1, -5, 2, 0]$
   (c) $v_3 = [4, 1, -1, -1]$