# Predictable Variation in Subjective Risk Premia and Currency Returns

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#### Abstract

Using survey data on exchange rate and interest rate expectations, this paper investigates the source of predictable currency returns by interest rate differentials. With a present value decomposition of exchange rates, I highlight that the predictable innovation in subjective currency risk premia plays a crucial role in explaining the ex-post predictability of exchange rate forecast errors, which results in predictable currency returns widely-documented in the literature. This is a novel channel of predictability not emphasized in exchange rate models featuring rational expectations or assuming investors make predictable errors for interest rates. As an illustration, I propose a reduced-form model with time-varying subjective perceptions of risk that generates predictable innovation in subjective risk premia. I further present empirical evidence that is consistent with a key property of the model—a positive relationship between subjective perceptions of risk and subjective return expectations.

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### 1 Introduction

The forward premium puzzle—interest rate differentials today predict currency excess returns tomorrow—is one of the most widely studied stylized facts in international finance that reflects the failure of uncovered interest parity (UIP) condition. Explanations of this puzzle generally fall into one of the following two strands of literature. One argues that the timeseries predictability is due to time-varying risk premia, captured by interest rate differentials<sup>1</sup>. A common assumption made in this literature is that agents have rational expectations (i.e. they know the data-generating process (DGP) underlying the economy). An alternative and complementary explanation relaxes the rational expectations assumption. Studies in this literature measure investor expectations directly with survey data and view the puzzle as a result of predictable forecast errors in exchange rates by interest rate differentials ex-post. For instance, in Gourinchas and Tornell (2004), agents have distorted beliefs about the interest rate process and their underreaction to interest rate changes gives rise to predictable forecast errors in exchange rates, which leads to the observed forward premium puzzle. Most models generating predictable forecast errors in this strand of literature focus on subjective beliefs about first moments—investors having beliefs of the interest rate process that deviate from the underlying DGP.

This paper relates to the second strand of literature and provides evidence on an understudied mechanism—predictable innovation in subjective risk premia—that generates predictable forecast errors in exchange rates by interest rate differentials. In classical rational expectations models, while subjective return expectations vary over time, the changes are unpredictable. In models generating predictable forecast errors that focus on first moments, dynamics of investors' subjective return expectations (i.e. subjective perceptions of second and higher moments) are often shut down.

To explore predictable innovation in subjective risk premia as a source of predictable forecast errors in exchange rates, I employ a well-known present value decomposition of exchange rates and decompose exchange rate forecast errors into three components:

$$\begin{split} s_{t+1} - \mathbb{E}^s_t[s_{t+1}] \\ &= \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}^s_{t+1} - \mathbb{E}^s_t)[i^*_{t+k} - i_{t+k}]}_{\text{revision in expected interest rate diff.}} - \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}^s_{t+1} - \mathbb{E}^s_t)[rx^{FX}_{t+k+1}]}_{\text{the innovation in subj. r.p.}} - \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}^s_{t+1} - \mathbb{E}^s_t)[\pi^*_{t+k} - \pi_{t+k}]}_{\text{change in expected relative inflation}} \end{split}$$

<sup>1.</sup> See, for instance, Verdelhan (2010), Bansal and Shaliastovich (2013), Colacito and Croce (2013) and Farhi and Gabaix (2016)

That is, the revision in expected interest rate differentials, the innovation in subjective currency risk premia and the change in expected relative inflation. I measure each component with survey data on exchange rates, interest rates and inflation rates from *Consensus Economics*. While my measure of each component is an approximation due to data limitations, I show that they capture variations in exchange rate forecast errors reasonably well.

After decomposing exchange rate forecast errors into three components, I then evaluate the contribution of each component to the predictability of exchange rate forecast errors by interest rate differentials. I find that it is through the innovation in subjective risk premia component, instead of interest rate forecast errors, that interest rate differentials predict exchange rate forecast errors. The result suggests not only are investors' subjective return expectations time-varying, their ex-post predictability leads to predictable realized currency excess returns. One economic reason for this may be that investors have subjective risk perceptions that are time-varying and are often "too" high from the econometrician's point of view when interest rate differentials are high.

To illustrate how investors' subjective return expectations can vary in a predictable manner that gives rise to predictable exchange rate forecast errors and realized currency returns, I write down a simple reduced-form SDF where investors have sticky risk perceptions. I assume dynamics of investors' expectations on volatility follows a process from the macroeconomic literature on information rigidity<sup>2</sup>. With this reduced-form SDF, I show that investors have time-varying subjective return expectations and their innovations are predictable by the econometrician.

Lastly, I present empirical evidence that is consistent with a key property of the reducedform model. I show that subjective currency return expectations are positively correlated
with subjective risk perceptions. I measure investors' subjective risk perceptions with the survey data on the perceived variance of US real GDP growth, which is in line with consumptionbased asset pricing models. The empirical pattern also supports the economic interpretation
of the paper's main result that investors overestimate risk from the econometrician's point
of view when interest rate differentials are high, which leads to predictable realized currency
returns.

The paper's first contribution lies in highlighting a source of currency return predictability—predictable innovation in subjective currency risk premia—that has not yet been documented

<sup>2.</sup> See for instance, Mankiw and Reis (2002), Reis (2006), Coibion and Gorodnichenko (2012), Coibion and Gorodnichenko (2015)

empirically<sup>3</sup> and not emphasized in most exchange rate models, which either feature rational expectations or assume investors make predictable forecast errors for interest rates<sup>4</sup>. My result suggests not only are investors' subjective return expectations time-varying, they change in a way that is predictable by interest rate differentials today. In other words, investors tend to have return expectations that are "too" high in the ex-post sense when interest rate differentials are high.

The paper also contributes to the literature examining dynamics of subjective currency return expectations and the literature studying risk-return relationships empirically. I document that subjective currency return expectations are positively correlated with subjective risk perceptions, measured by perceived variance of US real GDP. This result complements the findings of Kalemli-Özcan and Varela (2021) that subjective currency return expectations are correlated with VIX among currencies of developed countries. The findings imply not only are subjective currency return expectations correlated with risk-neutral measures of risk (i.e. VIX), they are also correlated with investors' "subjective" risk perceptions. The connection between subjective risk premia and subjective perceptions of risk in currency markets adds to the findings of Nagel and Xu (2022) where they show perceived stock market risks and subjective excess return expectations are positively correlated.

The paper is organized as follows. Section 2 describes data used in this paper. Section 3 reviews earlier findings on the predictability of exchange rate forecast errors with the sample in this paper. Section 4 outlines the decomposition of exchange rate forecast errors into three components. Section 5 empirically examines the contribution of each component to the predictability of exchange rate forecast errors. Section 6 presents a simple reduced-form model that generates results consistent with the main findings. Section 7 documents dynamics of subjective currency return expectations that are consistent with the model and the economic interpretation of the paper's main result. Section 8 concludes.

<sup>3.</sup> Empirical works studying the UIP puzzle and return predictability using survey data predominantly look at the predictability of exchange rate forecast errors by interest rate differentials and how it changes across regimes or markets. See for instance, Froot and Frankel (1989), Bacchetta, Mertens, and Van Wincoop (2009), Bussiere et al. (2018) Kalemli-Özcan and Varela (2021), Candian and De Leo (2021). Some others focus on FX forecast errors due to forecast errors in interest rate differentials (Valente, Vasudevan, and Wu (2021) Granziera and Sihvonen (2020)).

<sup>4.</sup> In these models, innovations in subjective risk premia play little to no role in explaining predictable variation in currency returns. In rational expectation models, innovations in subjective risk premia are unpredictable. In models with investors having distorted beliefs on interest rates, subjective risk premia generally do not vary over time.

### 2 Data

In this section, I describe data used in this paper. More detailed descriptions (e.g. summary statistics) are provided in Appendix A. Throughout the paper, I consider a panel of 8 advanced economies—Australia, Canada, Eurozone, Japan, New Zealand, Sweden, Switzerland, UK. Before Jan-1990, I use Germany to replace the Eurozone. My sample period ranges from November 1997 to May 2019.

#### 2.1 Exchange rates, interest rates and inflation rates data

I obtain monthly observations of spot exchange rates, short-term interest rates for each country in the sample from Bloomberg. Monthly CPI data is from the International Financial Statistics (IFS) of IMF. For Australia and New Zealand, since the CPI data is available only at quarterly frequency, I linearly interpolate missing values to obtain monthly observations. Inflation rate is calculated as year-on-year change in log CPI.

### 2.2 Survey data on exchange rates, interest rates and inflation rates

To measure investor expectations directly, I obtain consensus (i.e. average) forecasts<sup>5</sup> of exchange rates, short-term interest rates and inflation from *Consensus Economics*. This dataset is commonly used in the literature (for instance, Stavrakeva and Tang (2020a), Stavrakeva and Tang (2020b), Kalemli-Özcan and Varela (2021), De Marco, Macchiavelli, and Valchev (2020)).

For data from *Consensus Economics*, exchange rate forecasts are available at 3-month, 12-month and 24-month horizons. Short-term interest rate forecasts are available at 3-month, 12-month horizons. Inflation rate forecasts are available at 12-month horizon.

#### 2.3 Measure of perceived risk by US investors

To measure investors' perceived risk, I use "quarterly" survey on perceived variance of US real GDP growth from Survey of Professional Forecasters (SPF). Survey respondents are asked to assign a probability that the real GDP growth next year falls into a certain range. An example of the bins is, 6+, 5 to 5.9, 4 to 4.9, 3 to 3.9, 2 to 2.9,  $\cdots$ , < -2. To obtain the perceived variance of real GDP growth, I assign the mean value to each bin (i.e. 2.5 to the bin 2 to 2.9) and compute variance according to the assigned probability. For edge bins (i.e. 6+ and < -2), I assume they represent 6.5 and -2.5 respectively.

<sup>5.</sup> That is, the mean of survey respondents' forecasts on exchange rates, interest rates and inflation rates. The survey respondents are typically global banks or active participants in FX market.

## 3 Predictable currency excess returns and forecast errors

In this section, I review earlier findings on the predictability of *realized* currency excess returns and exchange rate forecast errors to make sure earlier results hold in the sample used by this paper.

### 3.1 Review of currency excess return predictability

The predictability of currency excess returns by interest rate differentials (or equivalently forward premium puzzle) is robustly documented in the literature (e.g. Fama (1984)). In other words, the regression coefficient ( $\beta$ ) in the following panel regression is often found to be significantly positive and larger than 1:

$$rx_{c,t+1}^{FX} = \alpha_c + \beta(i^* - i)_{c,t} + \varepsilon_{c,t+1}$$

$$\tag{1}$$

where  $\alpha_c$  denotes country fixed effect,  $rx_{c,t+1}^{FX} = s_{c,t+1} - s_{c,t} + (i^* - i)_{c,t}$  is 12-month excess return of currency c,  $s_t$  is log spot exchange rate (quoted in USD per foreign currency),  $(i^* - i)_{c,t}$  is the interest rate differential in log between country c and US. Under the rational expectations assumption<sup>6</sup>, the objective expected excess returns observed by an econometrician with the statistical model in 1 is equivalent to investors' subjective expectation up to some random noise. In other words, the wedge between econometrician's  $(\mathbb{E}_t[\cdot])$  and investors'  $(\mathbb{E}_t^s[\cdot])$  expectation should be unpredictable by information today:

$$\mathbb{E}_{t}[rx_{c,t+1}^{FX}] = \underbrace{\left(\mathbb{E}_{t}[rx_{c,t+1}^{FX}] - \mathbb{E}_{t}^{s}[rx_{c,t+1}^{FX}]\right)}_{\text{wedge in expectation}} + \underbrace{\mathbb{E}_{t}^{s}[rx_{c,t+1}^{FX}]}_{\text{subjective risk premia}}$$
(2)

Hence, under the rational expectations assumption, the predictability of currency excess returns by interest rate differentials is thought of as a violation of uncovered interest parity<sup>7</sup> and evidence of the existence of time-varying currency risk premia which correlate with contemporaneous interest rate differentials in the time series.

Alternatively, instead of assuming rational expectations, we can measure investor expectations directly with survey data<sup>8</sup>. With survey forecasts of exchange rates, we can decompose

<sup>6.</sup> The term, rational expectations, refers to "rational expectations methodology" mentioned in Chinn and Frankel (2019) that says "ex ante expectations can be inferred from ex post outcomes up to an expectational error term that is statistically uncorrelated with information available today"

<sup>7.</sup> Under uncovered interest parity,  $E_t^s[s_{c,t+1} - s_{c,t}] = -(i^* - i)_{c,t}$ . Hence, if investors have rational expectations, we would expect  $\beta$  in Equation 1 to be 0, which is counterfactual.

<sup>8.</sup> One of the forerunners of this approach is Frankel and Froot (1987) studying UIP condition.

realized currency excess returns into expected and forecast error components, i.e.

$$rx_{c,t+1}^{FX} = (rx_{c,t+1}^{FX} - \mathbb{E}_t^s[rx_{c,t+1}^{FX}]) + \mathbb{E}_t^s[rx_{c,t+1}^{FX}]$$

$$= \underbrace{(s_{c,t+1} - \mathbb{E}_t^s[s_{c,t+1}])}_{\text{FX forecast error}} + \underbrace{\mathbb{E}_t^s[rx_{c,t+1}^{FX}]}_{\text{expected return}}$$
(3)

Note  $\mathbb{E}_t^s[rx_{t+1}^{FX}] = \mathbb{E}_t^s[s_{c,t+1}] - s_{c,t} + (i^* - i)_{c,t}$  is subjective expected currency excess returns and  $\mathbb{E}_t^s[s_{c,t+1}]$  is the consensus (average) survey forecast of 12-month ahead spot exchange rate of currency c. Then, we can regress each component on interest rate differentials, i.e.

$$\mathbb{E}_t^s[rx_{c,t+1}^{FX}] = \alpha_c^{subj} + \beta^{subj}(i^* - i)_{c,t} + \varepsilon_{c,t+1}^{subj} \tag{4}$$

$$s_{c,t+1} - \mathbb{E}_t^s[s_{c,t+1}] = \alpha_c^{fe} + \beta^{fe}(i^* - i)_{c,t} + \varepsilon_{c,t+1}^{fe}$$
(5)

If the survey correctly measures investor expectations and that investors have rational expectations, we would expect  $\beta^{subj}$  to be close to the regression coefficient in Equation 1. However, the literature measuring exchange rate expectations using survey data has instead found  $\beta^{subj}$  to be indistinguishable from zero statistically and  $\beta^{fe}$  to be positive and large. This evidence suggests interest rate differentials' correlation with exchange rate forecast errors, instead of subjective currency risk premia, is the source of predictability in Equation 1.

To confirm results in earlier studies hold in the sample used in this paper, I estimate Equation 1, 4 and 5. Results are presented in Table I. Column (1) in Table I is the estimation result of Equation 1. It suggests interest rate differentials positively and significantly predict realized excess currency returns in the time series (which is the famous result in Fama (1984)). Next, I consider a decomposition as in Equation 3, measured with survey data. Column (2) is the estimation result of Equation 4 and column (3) is the estimation result of Equation 5. Put together, the results indicate that subjective currency risk premia (i.e. subjective expected currency excess returns) are not significantly correlated with interest rate differentials in the time series, which is documented in Frankel and Froot (1987) and many subsequent papers 10. Moreover, the results support the idea that the main source of predictable realized currency excess returns in the time series by interest rate differentials is predictable exchange rate

<sup>9.</sup> Note, coefficients in column (2) and column (3) should sum up to the coefficient in column (1)

<sup>10.</sup> See, for instance, Bussiere et al. (2018), Chinn and Frankel (1994), Chinn and Frankel (2019), Kalemli-Özcan and Varela (2021)

forecast errors.

Table I: Predictable excess currency returns and forecast errors

	$\mathbb{E}_t[rx_{c,t+1}^{FX}]$	$\mathbb{E}_t^s[rx_{c,t+1}^{FX}]$	$(3)$ $s_{c,t+1} - \mathbb{E}_t^s[s_{c,t+1}]$
$(i^*-i)_{c,t}$	1.731***	-0.237	1.967**
	(0.481)	(0.357)	(0.566)
Observations Adjusted $\mathbb{R}^2$	2072	2072	2072
	0.068	0.091	0.089
N(Countries) N(Months) Country FE Twoway Clustered	8	8	8
	259	259	259
	✓	✓	✓

Note: This table presents estimates of the panel regression of realized 12-month excess currency returns (Column (1)), subjective expected excess currency returns (Column (2)) and exchange rate forecast errors (Column (3)) on interest rate differentials  $((i^*-i)_{c,t})$  with country fixed effects. c denotes currency (country). Each period is 12 months and the regressions are estimated at monthly frequency. Sample includes 8 advanced economies from Nov. 1997 to May 2019. Standard errors are two-way-clustered at country and month level. \*\*\*, \*\*, and \* denote statistical significance at the 1%, 5%, and 10% level, respectively.

# 4 Decomposition of exchange rate forecast errors

To disentangle the sources of predictable FX forecast errors we found in the previous section and the literature, I introduce the decomposition of exchange rate forecast errors into three components—the revision in expected interest rate differentials, the innovation in subjective currency risk premia and the change in expected long-term nominal exchange rates.

The decomposition of exchange rate forecast errors is based on an accounting identity widely used in the literature<sup>11</sup>. By definition, the currency excess return is the carry trade return where one takes a long position in one-period, risk-free bonds in foreign currency that is financed by shorting one-period, risk-free bonds in home currency. In our case, the home currency is US dollar. In other words, the excess currency return for each foreign currency

<sup>11.</sup> See, for instance, Froot and Ramadorai (2005), Engel and West (2004), Engel and West (2005), Engel et al. (2007), Engel and West (2010), Evans (2012), Engel (2014), Engel (2016), Stavrakeva and Tang (2020a).

can be written as:

$$rx_{t+1}^{FX} = \Delta s_{t+1} + (i_t^* - i_t) \tag{6}$$

where  $\Delta s_{t+1}$  is the one-period (i.e. 12-month in this paper) change in log spot exchange rates (quoted in USD) and  $i_t^*$ ,  $i_t$  are log short-term interest rates of foreign and home country respectively. Rearranging Equation 6, we get  $s_t = (i_t^* - i_t) - rx_{t+1}^{FX} - s_{t+1}$ . Then, iterate the rearranged equation forward and take subjective expectation, we get

$$s_t = \sum_{k=0}^{\infty} \mathbb{E}_t^s \left[ i_{t+k}^* - i_{t+k} \right] - \sum_{k=0}^{\infty} \mathbb{E}_t^s \left[ r x_{t+k+1}^{FX} \right] + \mathbb{E}_t^s \left[ \lim_{t \to \infty} s_t \right]$$
 (7)

Using the exchange rate identity, Equation 7, we can write exchange rate forecast errors as

$$s_{t+1} - \mathbb{E}_{t}^{s}[s_{t+1}]$$

$$= \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s})[i_{t+k}^{*} - i_{t+k}]}_{\text{revision in expected interest rate diff.}} - \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s})[rx_{t+k+1}^{FX}]}_{\text{innovation in subj. r.p.}} + \underbrace{(\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s})\lim_{K \to \infty} s_{t+K}}_{\text{change in expected long-term FX}}$$
(8)

Equation 8 shows that we can decompose exchange rate forecast errors into three components: (1) the revision in the expected path of interest rate differentials, (2) the innovation in subjective currency risk premia and (3) the change in expected long-term nominal exchange rates. To measure the change in expected long-term nominal exchange rates, I will assume that changes in real exchange rates (defined as  $\Delta q_{t+k} = \Delta s_{t+k} + (\pi_{t+k}^* - \pi_{t+k})^{12}$ ) is stationary. Under this assumption, the change in the long-run expectation of real exchange rates will be 0 and, thus, the change in expected long-run nominal exchange rates will simply reflect the change in the expected path of relative inflation between foreign and home country. In particular,

$$(\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s}) \lim_{K \to \infty} s_{t+K}$$

$$= \lim_{K \to \infty} \mathbb{E}_{t+1}^{s} \left( s_{t+K} - s_{t} \right) - \lim_{K \to \infty} \mathbb{E}_{t}^{s} \left( s_{t+K} - s_{t} \right)$$

$$= \lim_{K \to \infty} (\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s}) \sum_{k=1}^{K-1} \left( \Delta q_{t+k} - (\pi_{t+k}^{*} - \pi_{t+k}) \right)$$

$$= -\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s}) [\pi_{t+k}^{*} - \pi_{t+k}]$$
(9)

<sup>12.</sup>  $\pi^*, \pi$  are one-period log inflation of foreign and home country respectively.

Rewriting Equation 8, we have

$$s_{t+1} - \mathbb{E}_{t}^{s}[s_{t+1}] = \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s})[i_{t+k}^{*} - i_{t+k}]}_{\text{revision in expected interest rate diff.}} - \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s})[rx_{t+k+1}^{FX}]}_{\text{the innovation in subj. r.p.}} - \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^{s} - \mathbb{E}_{t}^{s})[\pi_{t+k}^{*} - \pi_{t+k}]}_{\text{change in expected relative inflation}}$$
(10)

### 4.1 Interpretation of the decomposition

We can interpret Equation 10 as follows. From the exchange rate identity, Equation 7, we know that next-period exchange rates  $(s_{t+1})$  depend on investors' expected future interest rate differentials  $\left(\sum_{k=1}^{\infty} \mathbb{E}_{t+1}^{s} \left[i_{t+k}^{*} - i_{t+k}\right]\right)$ , subjective currency risk premia  $\left(\sum_{k=1}^{\infty} \mathbb{E}_{t+1}^{s} \left[rx_{t+k+1}^{FX}\right]\right)$  and expected future long-term nominal exchange rates  $\left(\mathbb{E}_{t+1}^{s} \left[\lim_{t\to\infty}s_{t}\right]\right)$ . Hence, investors' expectation on next-period exchange rates today  $(E_{t}^{s}[s_{t+1}])$  would be their expectation on each of the components today. Equation 10 says that FX forecast errors  $(s_{t+1} - \mathbb{E}_{t}^{s}[s_{t+1}])$  will be positive for the following three reasons: (1) investors' expectations of future interest rate differentials are too low today  $\left(\text{ i.e. } \mathbb{E}_{t+1}^{s} \left[i_{t+k}^{*} - i_{t+k}\right] > \mathbb{E}_{t}^{s} \left[i_{t+k}^{*} - i_{t+k}\right]\right)$ ; (2) investors' subjective risk premia today are higher than that of next period  $\left(\mathbb{E}_{t+1}^{s} \left[rx_{t+k+1}^{FX}\right] < \mathbb{E}_{t}^{s} \left[rx_{t+k+1}^{FX}\right]\right)$ ; (3) investors' expected relative inflation is too high today  $\left(\mathbb{E}_{t+1}^{s} \left[\pi_{t+k}^{*} - \pi_{t+k}\right] < \mathbb{E}_{t}^{s} \left[\pi_{t+k}^{*} - \pi_{t+k}\right]\right)^{13}$ .

### 4.2 Why should we care

While it is well-documented that interest rate differentials predict FX forecast errors, little is known about the channel through which interest rate differentials derive the predictive power. Most exchange rate models in the literature focus on explaining the predictability of FX forecast errors through the revision in expected interest rate differentials channel in Equation 10. In general, those models assume investors have distorted beliefs about interest rate processes such that they have expectations of future interest rate differentials that are too low today (i.e. some sort of underreaction). However, it is also possible that exchange rate forecast errors are due to investors having imperfect expectations on subjective risk premia or relative inflation. Understanding FX forecast errors in the lens of the decomposition, Equation 10, allows us to disentangle different economic sources of predictable exchange rate forecast errors and evaluate their roles in explaining the forward premium puzzle.

<sup>13.</sup> A high expected relative inflation implies a low expected nominal exchange rate.

### 5 Source of predictable exchange rate forecast errors

In this section, I first discuss how I measure the decomposition of exchange rate forecast errors as in Equation 10 with survey data. Next, I empirically examine the contribution of each component to the predictability of exchange rate forecast errors by interest rate differentials.

### 5.1 Measuring the decomposition using survey data

Ideally, to assess through which channel interest rate differentials predict exchange rate forecast errors, I would like to measure each component of exchange rate forecast errors in Equation 10 with survey data that contains forecasts at every horizon in the future. However, the survey data available only allows us to measure the first term of each component. That is,

$$i_{t+1}^* - i_{t+1} - \mathbb{E}_t^s[i_{t+1}^* - i_{t+1}] \text{ instead of } \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^s - \mathbb{E}_t^s)[i_{t+k}^* - i_{t+k}]}_{\text{revision in expected interest rate diff.}}$$

$$\mathbb{E}_{t+1}^s[rx_{t+2}^{FX}] - \mathbb{E}_t^s[rx_{t+2}^{FX}] \text{ instead of } \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^s - \mathbb{E}_t^s)[rx_{t+k+1}^{FX}]}_{\text{innovation in subj. r.p.}}$$

$$\pi_{t+1}^* - \pi_{t+1} - \mathbb{E}_t^s[\pi_{t+1}^* - \pi_{t+1}] \text{ instead of } \underbrace{\sum_{k=1}^{\infty} (\mathbb{E}_{t+1}^s - \mathbb{E}_t^s)[\pi_{t+k}^* - \pi_{t+k}]}_{\text{change in expected relative inflation}}$$

where  $E_t^s[i_{t+1}^*]$ ,  $E_t^s[i_{t+1}]$  are 12-month ahead consensus forecasts for short-term interest rate of foreign country and US.  $E_t^s[\pi_{t+1}^*]$ ,  $E_t^s[\pi_{t+1}]$  are 12-month ahead consensus forecasts for inflation rate of foreign country and US.  $\mathbb{E}_t^s[rx_{t+2}^{FX}]$  is investors' consensus forecast of expected excess currency returns next period today.

$$\mathbb{E}_{t}^{s}[rx_{t+2}^{FX}] = \mathbb{E}_{t}^{s}[s_{t+2}] - \mathbb{E}_{t}^{s}[s_{t+1}] + \mathbb{E}_{t}^{s}[i_{t+1}^{*} - i_{t+1}]$$

where  $\mathbb{E}_t^s[s_{t+2}]$  is the 24-month ahead exchange rate consensus forecast today and  $\mathbb{E}_t^s[s_{t+1}]$  is the 12-month ahead exchange rate consensus forecast today. Lastly,  $\mathbb{E}_{t+1}^s[rx_{t+2}^{FX}]$  is the expected excess currency return next period, defined as

$$\mathbb{E}_{t+1}^{s}[rx_{t+2}^{FX}] = \mathbb{E}_{t+1}^{s}[s_{t+2}] - s_{t+1} + (i_{t+1}^* - i_{t+1})$$

If investor expectations follow an AR(1) process<sup>14</sup>, measuring the first term of each component will be sufficient. To see this, we can write the decomposition of exchange rate forecast errors

<sup>14.</sup> This assumption is not uncommon in the literature (see, for instance, De La O and Myers (2021), Cieslak (2018))

in Equation 10 as

$$s_{t+1} - \mathbb{E}_{t}^{s}[s_{t+1}] = \frac{1}{1 - \alpha_{i}} (i_{t+1}^{*} - i_{t+1} - \mathbb{E}_{t}^{s}[i_{t+1}^{*} - i_{t+1}]) - \frac{1}{1 - \alpha_{rx}} (\mathbb{E}_{t+1}^{s}[rx_{t+2}^{FX}] - \mathbb{E}_{t}^{s}[rx_{t+2}^{FX}]) - \frac{1}{1 - \alpha_{\pi}} (\pi_{t+1}^{*} - \pi_{t+1} - \mathbb{E}_{t}^{s}[\pi_{t+1}^{*} - \pi_{t+1}])$$

$$(11)$$

where  $0 < \alpha_i, \alpha_{rx}, \alpha_{\pi} < 1$  capture the degree of persistence in expectations of each component. Shown later in Section 5.2, regressing exchange rate forecast errors on the first term of each component as in Equation 11 not only produces a surprisingly high R-squared, but also coefficient estimates whose sign are consistent with mean-reverting investor expectations. This reflects, while the measurement of the decomposition outlined in Equation 10 with survey data is imperfect, it captures a significant amount of variations in FX forecast errors.

### 5.2 Empirical strategy

Section 5.1 discusses the data limitation on measuring the decomposition of exchange rate forecast errors indicated in Equation 10 and the sufficient condition for the measurement to be exact. In this section, I outline my empirical strategy to assess the source of predictable exchange rate forecast errors.

First, I will estimate the decomposition of exchange rate forecast errors as

$$s_{c,t+1} - \mathbb{E}_{t}^{s}[s_{c,t+1}] = \alpha_{c} + \beta_{1} \left( (i^{*} - i)_{c,t+1} - \mathbb{E}_{t}^{s}[(i^{*} - i)_{c,t+1}] \right) + \beta_{2} \left( \mathbb{E}_{t+1}^{s}[rx_{c,t+2}^{FX}] - \mathbb{E}_{t}^{s}[rx_{c,t+2}^{FX}] \right) + \beta_{3} \left( (\pi^{*} - \pi)_{c,t+1} - \mathbb{E}_{t}^{s}[(\pi^{*} - \pi)_{c,t+1}] \right) + u_{c,t+1}$$

$$(12)$$

where  $s_{c,t+1} - \mathbb{E}_t^s[s_{c,t+1}]$  is the exchange rate forecast error of currency c.  $(i^*-i)_{c,t+1} - \mathbb{E}_t^s[(i^*-i)_{c,t+1}]$  is the forecast error in interest rate differential between foreign currency c and US. This is meant to capture the revision in expected path of interest rate differentials in Equation 10.  $\mathbb{E}_{t+1}^s[rx_{t+2}^{FX}] - \mathbb{E}_t^s[rx_{t+2}^{FX}]$  is the innovation in subjective risk premia of currency c. It captures the path of the innovation in subjective risk premia.  $(\pi^* - \pi)_{c,t+1} - \mathbb{E}_t^s[(\pi^* - \pi)_{c,t+1}]$  is the forecast error in relative inflation between foreign country c and US. It captures the change in expected long-run nominal exchange rate of currency c under the stationary assumption. If investors have approximately AR(1) expectations, we would expect  $\beta_1 > 0$  and  $\beta_2, \beta_3 < 0$ .

Next, I will regress the fitted value of each right-hand-side variable in Equation 12 on

interest rate differentials. Namely,

$$(i^* - i)_{c,t+1} \widehat{-\mathbb{E}_t^s}[(i^* - i)_{c,t+1}] = \alpha_c^i + \beta^i (i^* - i)_{c,t} + \varepsilon_{c,t+1}^i$$

$$\mathbb{E}_{t+1}^s [rx_{c,t+2}^{FX}] - \mathbb{E}_t^s [rx_{c,t+2}^{FX}] = \alpha_c^{rx} + \beta^{rx} (i^* - i)_{c,t} + \varepsilon_{c,t+1}^{rx}$$

$$(\pi^* - \pi)_{c,t+1} - \widehat{\mathbb{E}_t^s}[(\pi^* - \pi)_{c,t+1}] = \alpha_c^\pi + \beta^\pi (i^* - i)_{c,t} + \varepsilon_{c,t+1}^\pi$$
(13)

where the LHS variables are the fitted value of each RHS variable in Equation 12.  $(i^* - i)_{c,t}$  is the interest rate differential between foreign currency c and US.  $\alpha_c$  is country-fixed effect. Since we know from Section 3.1 that interest rate differentials positively predict FX forecast errors, we would expect at least one of  $\beta^i$ ,  $\beta^{rx}$ ,  $\beta^{\pi}$  to be positive.

#### 5.3 Empirical results

Table II presents estimation results of Equation 12. Estimating Equation 12 allows us to understand how well the decomposition of exchange rate forecast errors is measured with survey data. Column (1) reports estimation with no fixed effect. Column (2) reports estimation including only country-fixed effect. Column (3) reports estimation including only month-fixed effect. Column (4) reports estimation including both country and time-fixed effect. As we can see from Table II, regardless of the fixed effect included, the estimated coefficients (i.e.  $\beta_1, \beta_2$  and  $\beta_3$ ) are pretty stable over time. This is in line with the decomposition relying solely on an accounting identity. Moreover, we see that  $\beta_1 > 0$  and  $\beta_2, \beta_3 < 0$ , consistent with what we would expect if investor expectations are mean-reverting. Importantly, the estimated panel regression with no fixed effect has an adjusted R-squared as high as 41%. This is impressive since only the first term of each component in the decomposition of exchange rate forecast errors (Equation 10) is measured. The result indicates our measure of each component of exchange rate forecast errors, while imperfect, captures a significant amount of variations in FX forecast errors. It also supports the idea that investor expectations do not deviate too much from an AR(1) model.

Table III presents estimation results of regressions in Equation 13. Estimating regressions in Equation 13 allows us to pin down which component of the FX forecast error interest rate differentials are predicting. As we can see in column (1) where we regress forecast errors in interest rate differentials on interest rate differentials today, the estimated coefficient is significantly negative. This is inconsistent with interest rate differentials positively predicting FX forecast errors. Similarly, the estimation results in column (3) suggest interest rate differentials do not significantly predict forecast errors in relative inflation with the right

Table II: Decomposition of exchange rate forecast errors

Dependent variable: $s_{c,t+1} - \mathbb{E}_t^s[s_{c,t+1}]$							
	(1)	(2)	(3)	(4)			
	No FE	Country FE	Month FE	Twoway FE			
$i_{t+1}^* - i_{t+1} - \mathbb{E}_t^s [i_{t+1}^* - i_{t+1}]$	1.736	1.698	2.938***	2.798***			
	(0.935)	(0.947)	(0.620)	(0.675)			
$\mathbb{E}_{t+1}^{s}[rx_{t+2}^{FX}] - \mathbb{E}_{t}^{s}[rx_{t+2}^{FX}]$	-1.285***	-1.257***	-1.162***	-1.097***			
.,	(0.115)	(0.112)	(0.126)	(0.131)			
$\pi_{t+1}^* - \pi_{t+1} - \mathbb{E}_t^s [\pi_{t+1}^* - \pi_{t+1}]$	-3.616***	-3.806***	-2.655***	-3.135***			
	(0.415)	(0.439)	(0.460)	(0.579)			
Observations	2053	2053	2053	2053			
Adjusted $R^2$	0.409	0.426	0.633	0.653			
N(Countries)	8	8	8	8			
N(Months)	259	259	259	259			
From	Nov 1997	Nov 1997	Nov 1997	Nov 1997			
То	May 2020	May 2020	May 2020	May 2020			
Country FE		$\checkmark$		$\checkmark$			
Month FE			$\checkmark$	$\checkmark$			
Twoway Clustered	$\checkmark$	$\checkmark$	$\checkmark$	✓			

Note: This table presents estimates of

$$\begin{split} s_{c,t+1} - \mathbb{E}_t^s[s_{c,t+1}] &= \alpha_c + \beta_1 \ \left( (i^* - i)_{c,t+1} - \mathbb{E}_t^s[(i^* - i)_{c,t+1}] \right) + \beta_2 \ \left( \mathbb{E}_{t+1}^s[rx_{c,t+2}^{FX}] - \mathbb{E}_t^s[rx_{c,t+2}^{FX}] \right) \\ &+ \beta_3 \left( (\pi^* - \pi)_{c,t+1} - \mathbb{E}_t^s[(\pi^* - \pi)_{c,t+1}] \right) + u_{c,t+1} \end{split}$$

where  $s_{c,t+1} - \mathbb{E}^s_t[s_{c,t+1}]$  is the exchange rate forecast error of currency c.  $(i^*-i)_{c,t+1} - \mathbb{E}^s_t[(i^*-i)_{c,t+1}]$  is the forecast error in interest rate differential between foreign currency c and US.  $\mathbb{E}^s_{t+1}[rx_{t+2}^{FX}] - \mathbb{E}^s_t[rx_{t+2}^{FX}]$  is the innovation in subjective risk premia of currency c.  $(\pi^* - \pi)_{c,t+1} - \mathbb{E}^s_t[\pi^* - \pi)_{c,t+1}$  is the forecast error in relative inflation between foreign country c and US. Each period is 12 months and the regressions are estimated at monthly frequency. Sample includes 8 advanced economies from Nov. 1997 to May 2019. Column (1) reports estimation with no fixed effect. Column (2) reports estimation including only country-fixed effect. Column (3) reports estimation including only month-fixed effect. Column (4) reports estimation including both country and time-fixed effect. Standard errors are two-way-clustered at country and month level. \*\*\*, \*\*, and \* denote statistical significance at the 1%, 5%, and 10% level, respectively.

sign. The only component of FX forecast errors that is significantly predicted by interest rate differentials today and with the right sign is the innovation in subjective risk premia. This is shown in column (2) in Table III.

Put together, results in Table III are evidence that it is through the predictable innovation in subjective risk premia that interest rate differentials predict exchange rate forecast errors. This implies interest rate differentials predict realized currency excess returns not by capturing contemporaneous currency risk premia or expectation errors in interest rates but by forecasting the revision in investors' required compensation for taking currency risk.

Table III: Sources of predictable exchange rate forecast errors

	(1)	(2)	(3)
	$(i^* - i)_{c,t+1} - \widehat{\mathbb{E}_t^s}[(i^* - i)_{c,t+1}]$	$\widehat{\mathbb{E}_{t+1}^s[rx_{c,t+2}^{FX}]} - \widehat{\mathbb{E}_{t}^s[rx_{c,t+2}^{FX}]}$	$(\pi^* - \pi)_{c,t+1} - \widehat{\mathbb{E}_t^s}[(\pi^* - \pi)_{c,t+1}]$
$(i^*-i)_{c,t}$	-0.295***	0.743**	-0.109
, , ,	(0.069)	(0.298)	(0.216)
Observations	2053	2053	2053
Adjusted $\mathbb{R}^2$	0.071	0.053	0.030
N(Countries)	8	8	8
N(Months)	259	259	259
Country FE	$\checkmark$	$\checkmark$	$\checkmark$
Twoway Clustered	$\checkmark$	$\checkmark$	$\checkmark$

Note: This table presents estimates of the panel regression of the forecast error in interest rate differential between foreign currency c and US (column (1)), the innovation in subjective risk premia of currency c (column (2)) and the forecast error in relative inflation between foreign country c and US (column (3)) on interest rate differentials  $((i^* - i)_{c,t})$  with country fixed effects. c denotes currency (country). All LHS variables are fitted values from regression in Equation 12. Each period is 12 months and the regressions are estimated at monthly frequency. Sample includes 8 advanced economies from Nov. 1997 to May 2019. Standard errors are two-way-clustered at country and month level. \*\*\*, \*\*, and \* denote statistical significance at the 1%, 5%, and 10% level, respectively

## 6 Model of predictable innovation in subjective risk premia

Section 5.3 demonstrates that interest rate differentials predict FX forecast errors through predictable innovation in subjective risk premia. Since this is a channel of predictability that most models in the exchange rate literature shut down, I propose a simple reduced-form affine SDF that can generate predictable innovation in subjective risk premia qualitatively. Note, as this model is meant to be an illustration, I remain agnostic on why the innovation in subjective risk premia can be predicted by interest rate differentials.

Assume representative investors in each country c have a stochastic discount factor (SDF) of the following form and denote investors' subjective expectations as  $E^s[\cdot]$ :

$$m_{t+1}^c = \alpha - \phi^c z_t - \sqrt{\gamma^c \sigma_{t+1}^2} u_{t+1} + \sqrt{\omega^c} \eta_{t+1}^s$$

where  $m_{t+1}$  is logged SDF,  $z_t$  denotes state variables,  $\sigma_{t+1}^2$  is time-varying conditional variance of the SDF.  $u_{t+1}$  denotes global shocks that are white noise and independent of  $\eta_{t+1}^s$ ,  $\sigma_{t+1}^2$ .  $\eta_{t+1}^s$  is defined as  $\eta_{t+1}^s := \sigma_{t+1}^2 - E_t^s[\sigma_{t+1}^2]$ .  $\eta_{t+1}^s$  ensures forecast errors of exchange rates are predictable ex-post<sup>15</sup>. Assume that  $\sigma_{t+1}^2$  follows an AR(1) process:

$$\sigma_{t+1}^2 = \rho \sigma_t^2 + \varepsilon_{t+1}$$

Investors know  $\varepsilon_{t+1} \stackrel{iid}{\sim} N(0,1)$  but their belief of the conditional variance  $\left(E_t^s[\sigma_{t+1}^2]\right)$  is

$$E_t^s[\sigma_{t+1}^2] = (1-\lambda)\mathbb{E}_t[\sigma_{t+1}^2] + \lambda E_{t-1}^s[\sigma_{t+1}^2]$$

where  $\mathbb{E}_t[\sigma_{t+1}^2]$  is the rational expectation of  $\sigma_{t+1}^2$  conditional on information sets at time t. This form of investor belief follows the literature on information rigidity. One interpretation of this belief formation process is that agents are inattentive and only update their belief with probability  $1 - \lambda$  each period (Mankiw and Reis 2002). When  $\lambda = 0$ , investors have rational expectations. When  $\lambda < 0$ , investors form expectations as if they overreact to news.

<sup>15.</sup> The included  $\eta_{t+1}^s$  term is not arbitrary. Lochstoer and Muir (2022) obtain a SDF of similar form where they try to model the empirical dynamics of investors' subjective stock market risk perceptions in a classical representative-agent asset pricing model.

Under complete market assumption, we can write expected currency excess returns as

$$E_t^s[rx_{c,t+1}^{FX}] = \frac{1}{2} \operatorname{Var}_t(m_{t+1}^{US}) - \frac{1}{2} \operatorname{Var}_t(m_{t+1}^c)$$

$$= \frac{1}{2} (\gamma^{US} - \gamma^c) E_t^s[\sigma_{t+1}^2] + (\omega^{US} - \omega^c)$$
(14)

From Equation 14, we see that investors' conditional expected currency excess returns are time-varying and correlated with perceived conditional variance,  $E_t^s[\sigma_{t+1}^2]$ .

Recall our goal is to show that the following term in Equation 10 is predictable ex-post:

$$(E_{t+1}^{s}[rx_{t+2}^{FX}] - E_{t}^{s}[rx_{t+2}^{FX}]) + (E_{t+1}^{s}[rx_{t+3}^{FX}] - E_{t}^{s}[rx_{t+3}^{FX}]) + \cdots$$
(15)

Let's first consider  $rx_{t+2}^{FX} = s_{t+2} - s_{t+1} + (i_{t+1}^* - i_{t+1}^{US})$ . Its innovation is  $E_{t+1}^s[rx_{t+2}^{FX}] - E_t^s[rx_{t+2}^{FX}]$ , which can be written as

$$E_{t+1}^{s}[rx_{i,t+2}^{FX}] - E_{t}^{s}[rx_{i,t+2}^{FX}] = \frac{1}{2}(\gamma^{US} - \gamma^{i}) \Big[ E_{t+1}^{s}[\sigma_{t+2}^{2}] - E_{t}^{s}[\sigma_{t+2}^{2}] \Big]$$
$$= \frac{1}{2}(\gamma^{US} - \gamma^{i})(1 - \lambda)\rho \Big[ \sigma_{t+1}^{2} - E_{t}^{s}[\sigma_{t+1}^{2}] \Big]$$

To see the ex-post predictability to econometrician, take objective expectation  $\mathbb{E}[\cdot]$  and we get

$$\mathbb{E}\left[E_{t+1}^{s}[rx_{i,t+2}^{FX}] - E_{t}^{s}[rx_{i,t+2}^{FX}]\right] \propto (1 - \lambda) \left(\mathbb{E}_{t}[\sigma_{t+1}^{2}] - E_{t}^{s}[\sigma_{t+1}^{2}]\right)$$

In general,  $E_t^s \neq \mathbb{E}$  and  $\lambda \neq 1$  so an econometrician can find predictable innovation in subjective currency risk premia ex-post. When  $\lambda = 0$ , investors have rational expectations so  $E_t^s = \mathbb{E}$ . This means the innovation in subjective risk premia is unpredictable. The entire Equation 15 can be written as

$$\sum_{j=0}^{\infty} E_{t+1}^{s} [rx_{t+2+j}^{FX}] - E_{t}^{s} [rx_{t+2+j}^{FX}] = \frac{1}{2} (\gamma^{US} - \gamma^{i}) \frac{\rho}{1-\rho} (1-\lambda) \left[ \sigma_{t+1} - E_{t}^{s} [\sigma_{t+1}] \right]$$

We can see this is also predictable by econometricians ex-post in general when we take  $\mathbb{E}[\cdot]$ .

The simple reduced-form model in this section is an affine SDF that generates predictable innovation in subjective risk premia by assuming that investors have sticky expectations on conditional variance. However, there are potentially many other mechanisms that can generate this result. In addition, I did not specify why interest rate differentials can predict the innovation in subjective risk premia as it is not the focus of the paper.

### 7 Empirical evidence on key properties of the reduced-form model

While the reduced-form model in Section 5.3 is meant to be stylized, I explore to what extent some key properties of the model hold in the data in this section. Specifically, I ask whether investors' subjective return expectations are correlated with their subjective risk perceptions.

I first explore whether there is a correlation between investors' subjective return expectations and their subjective risk perceptions. I measure investors' subjective risk perception with survey forecasts on variance of US real GDP growth. This measure captures investors' perceived conditional variance of consumption growth process that is consistent with standard consumption-based asset pricing models. The construction of this measure is detailed in Section 2. Table IV reports estimates of the following quarterly regression:

$$E_t^s[rx_{c,t+1}^{FX}] = \alpha_c + \beta \text{ Perceived } risk_t + \varepsilon_{t+1}$$
 (16)

where  $E_t^s[rx_{c,t+1}^{FX}]$  is the subjective excess return of currency c and Perceived risk can be survey forecasts on variance of US real GDP growth rate next period or logged VIX index. Note, the regression is quarterly since forecasts on variance are only available at quarterly frequency. As we can see in column (1), the measure of "subjective" risk perception is positively correlated with subjective currency excess returns. This confirms the relationship between subjective currency excess returns and risk perceptions in Equation 14. The positive correlation between risk perceptions and return expectations suggests there is indeed a risk-return trade-off predicted by standard asset pricing models. Results in column (3) imply the correlation between subjective risk perception and subjective excess return expectations is not absorbed by market-based measures of risk (i.e. VIX).

### 8 Conclusion

In this paper, I highlight a channel of currency return predictability—predictability of the innovation in subjective risk premia—that is under-explored when explaining the forward premium puzzle. In particular, I show that investors make predictable errors in forecasting the evolution of expected currency returns. Such errors contribute to the ex-post predictability of exchange rate forecast errors and, thus, realized currency returns. One economic interpretation is that investors tend to have risk perceptions that are "too" high from an econometrician's point of view when interest rate differentials are high. As most models in the international finance literature do not feature this channel of predictability, I propose a

Table IV: Subjective Excess Returns and Risk Perceptions

	$(1)$ $E_{t}^{s}[rx^{FX}]$	$(2)$ $E_t^s[rx_{c,t+1}^{FX}]$	$(3)$ $E_{c}^{s}[rx^{FX}]$
Percevied R.GDP growth risk	$\frac{\Delta_{t} \left[ \cos_{c,t+1} \right]}{0.019^{***}}$	$r \sim c, t+1$	$\frac{2_{t} \left[ v \approx c, t+1 \right]}{0.018^{**}}$
G	(0.005)		(0.005)
$\log VIX$	,	0.014*	0.013
		(0.007)	(0.007)
Observations	688	688	688
Adjusted $R^2$	0.137	0.109	0.166
N(Countries)	8	8	8
N(Quarters)	86	86	86
Country FE	$\checkmark$	$\checkmark$	$\checkmark$
Twoway Clustered	$\checkmark$	$\checkmark$	$\checkmark$

Note: This table presents estimates of the panel regression of the subjective currency excess returns of foreign currency c on measures of perceived risk with country fixed effects. The perceived risk measure is survey forecast variance of real GDP growth rate next period in column (1). The perceived risk measure is logged VIX index in column (2). All LHS variables are standardized to have mean 0 and standard deviation 1. Each period is 12 months and the regressions are estimated at quarterly frequency. Sample includes 8 advanced economies from Nov. 1997 to May 2019. Standard errors are two-way-clustered at country and month level. \*\*\*, \*\*, and \* denote statistical significance at the 1%, 5%, and 10% level, respectively

simple reduced-form affine SDF that can generate ex-post predictable innovation in subjective return expectations. Finally, I present empirical evidence that is consistent with a key property of the reduced-form SDF—a positive relationship between subjective perceptions of risk and subjective return expectations. The positive relationship also supports our economic interpretation of predictable innovation in subjective risk premia.

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### A Data Details

#### A.1 Variable construction

In this section, I describe the construction of main variables used in the paper.

- ▶ Interest rate differentials  $(i_t^* i_t)$ : take difference of short-term interest rate between foreign country  $(i_t^*)$  and US  $(i_t)$
- ▶ Realized excess currency returns  $(rx_{t+1}^{FX})$ :  $rx_{t+1}^{FX} = (s_{t+1} s_t) + i_t^* i_t$  (i.e.12-month difference in log exchange rates  $(s_{t+1} s_t)$  plus interest rate differentials between a foreign country and US  $(i_t^* i_t)$ ).
- ▶ Survey-based expected excess currency returns  $\left(\mathbb{E}_{t}^{s}\left[rx_{t+1}^{FX}\right]\right)$ :  $\mathbb{E}_{t}^{s}\left[rx_{t+1}^{FX}\right] = \left(\mathbb{E}_{t}^{s}[s_{t+1}] s_{t}\right) + i_{t}^{*} i_{t}$ , where  $E_{t}^{s}[s_{t+1}]$  is consensus forecasts of spot exchange rates in 12 months.
- ▶ Realized exchange rate forecast errors  $(s_{t+1} \mathbb{E}_t^s[s_{t+1}])$ : difference between spot exchange rate in 12-months  $(s_{t+1})$  and consensus forecasts of spot exchange rates in 12 months today $(\mathbb{E}_t^s[s_{t+1}])$ .
- ▶ Interest rate forecast errors  $(i_{t+1}^* i_{t+1} \mathbb{E}_t^s[i_{t+1}^* i_{t+1}])$ : difference between interest rate differentials in 12-months  $(i_{t+1}^* i_{t+1})$  and consensus forecasts of interest rate differentials in 12 months today( $\mathbb{E}_t^s[i_{t+1}^* i_{t+1}]$ ).
- ▶ The innovation in subjective risk premia  $(\mathbb{E}_{t+1}^s[rx_{t+2}^{FX}] \mathbb{E}_t^s[rx_{t+2}^{FX}])$ :

$$\mathbb{E}_{t}^{s}[rx_{t+2}^{FX}] = \mathbb{E}_{t}^{s}[s_{t+2}] - \mathbb{E}_{t}^{s}[s_{t+1}] + \mathbb{E}_{t}^{s}[i_{t+1}^{*} - i_{t+1}]$$

where  $\mathbb{E}_t^s[s_{t+2}]$  is 24-month ahead exchange rate consensus forecast today and  $\mathbb{E}_t^s[s_{t+1}]$  is 12-month ahead exchange rate consensus forecast today. Lastly,  $\mathbb{E}_{t+1}^s[rx_{t+2}^{FX}]$  is the expected excess currency returns next period, defined as

$$\mathbb{E}_{t+1}^{s}[rx_{t+2}^{FX}] = \mathbb{E}_{t+1}^{s}[s_{t+2}] - s_{t+1} + (i_{t+1}^{*} - i_{t+1})$$

- ▶ Foreign inflation rate  $(\pi_t^*)$  and US inflation rate  $(\pi_t)$ : 12-month difference in log CPI.
- ▶ Inflation rate forecast errors  $(\pi_{t+1}^* \pi_{t+1} \mathbb{E}_t^s[\pi_{t+1}^* \pi_{t+1}])$ : difference of relative inflation between foreign country and US in 12-months  $(\pi_{t+1}^* \pi_{t+1})$  and consensus forecasts of relative inflation in 12 months today $(\mathbb{E}_t^s[\pi_{t+1}^* \pi_{t+1}])$ .

▶ Subjective risk perception: I use "quarterly" survey on perceived variance of US real GDP growth from Survey of Professional Forecasters (SPF). Survey respondents are asked to assign a probability that the real GDP growth next year falls into a certain range. An example of the bins is, 6+, 5 to 5.9, 4 to 4.9, 3 to 3.9, 2 to 2.9, ···, < -2. To obtain the perceived variance of real GDP growth, I assign the mean value to each bin (i.e. 2.5 to the bin 2 to 2.9) and compute variance according to the assigned probability. For edge bins (i.e. 6+ and < -2), I assume they represent 6.5 and -2.5 respectively.

## A.2 Summary statistics

In this section, I report the summary statistics of main variables used in the analysis. For the definition of each variable, see Appendix A.1. Table A1 reports pooled summary statistics across countries. Table A2 reports summary statistics by country.

Table A1: Summary statistics of main variables

	Observation	Mean	Std	25th perc.	75th perc.
$i_t^* - i_t \text{ (decimal)}$	2072	0.0006	0.0206	-0.0106	0.0134
$rx_{t+1}^{FX}$ (decimal)	2072	0.0031	0.1097	-0.0655	0.0776
$\mathbb{E}_{t}^{s}\left[rx_{t+1}^{FX}\right]$ (decimal)	2072	0.0001	0.0757	-0.0333	0.0357
$s_{t+1} - \mathbb{E}_t^s \left[ s_{t+1} \right] $ (decimal)	2072	0.0030	0.1231	-0.0716	0.0799
$\mathbb{E}_t^s[i_{t+1}^* - i_{t+1}] \text{ (decimal)}$	2065	0.0015	0.0194	-0.0102	0.0133
$i_{t+1}^* - i_{t+1} - \mathbb{E}_t^s[i_{t+1}^* - i_{t+1}]$ (decimal)	2065	-0.0009	0.0103	-0.0072	0.0039
$\mathbb{E}_{t+1}^s[rx_{t+2}^{FX}] - \mathbb{E}_t^s[rx_{t+2}^{FX}] \text{ (decimal)}$	2072	-0.0014	0.0543	-0.0333	0.0299
$\pi_t^* - \pi_t$ (decimal)	2072	-0.0070	0.0132	-0.0155	0.0009
$\pi_{t+1}^* - \pi_{t+1} - \mathbb{E}_t^s [\pi_{t+1}^* - \pi_{t+1}]$ (decimal)	2053	-0.0025	0.0107	-0.0095	0.0037
Percevied R.GDP growth risk (decimal)	688	0.0212	0.0087	0.0151	0.0257

Note: This table presents summary statistics (observation, mean, standard deviation, min, max) of main variables used in this paper. For definition of each variable, see Appendix A.1.

Table A2: Summary statistics of main variables by country

	Australia	Canada	Eurozone	Japan	New Zealand	Sweden	Switzerland	UK
$i_t^* - i_t \text{ (log)}$	0.0187 (0.0167)	0.0019 (0.0077)	-0.0048 (0.0137)	-0.0212 (0.0195)	0.0223 (0.0156)	-0.0026 (0.0169)	-0.0162 (0.0137)	0.0064 (0.0122)
$rx_{t+1}^{FX}$ (log)	0.0212 $(0.1314)$	0.0062 $(0.0822)$	-0.0046 (0.1038)	-0.0118 (0.1013)	0.0294 $(0.1361)$	-0.0116 (0.1220)	0.0024 $(0.0901)$	-0.0063 (0.0916)
$\mathbb{E}_t^s \left[ r x_{t+1}^{FX} \right] \text{ (log)}$	0.0184 (0.0590)	0.0098 $(0.0327)$	-0.0329 (0.1561)	-0.0274 (0.0462)	0.0109 (0.0669)	0.0386 (0.0508)	-0.0187 (0.0503)	0.0023 (0.0337)
$s_{t+1} - \mathbb{E}_t^s \left[ s_{t+1} \right] \left( \log \right)$	0.0028 $(0.1323)$	-0.0036 (0.0807)	0.0283 $(0.1721)$	0.0156 $(0.1143)$	0.0185 $(0.1407)$	-0.0503 (0.1173)	0.0212 $(0.0956)$	-0.0086 (0.0887)
$\mathbb{E}_{t}^{s}[i_{t+1}^{*} - i_{t+1}] \text{ (log)}$	0.0188 $(0.0161)$	0.0022 $(0.0079)$	-0.0042 (0.0124)	-0.0209 (0.0175)	0.0230 $(0.0137)$	-0.0000 (0.0153)	-0.0136 (0.0122)	0.0064 $(0.0107)$
$i_{t+1}^* - i_{t+1} - \mathbb{E}_t^s [i_{t+1}^* - i_{t+1}] \text{ (log)}$	-0.0003 (0.0109)	-0.0001 (0.0069)	-0.0007 (0.0097)	$0.0011 \\ (0.0105)$	-0.0020 (0.0122)	-0.0027 (0.0124)	-0.0012 (0.0086)	-0.0013 (0.0092)
$\mathbb{E}_{t+1}^{s}[rx_{t+2}^{FX}] - \mathbb{E}_{t}^{s}[rx_{t+2}^{FX}] \text{ (log)}$	-0.0039 (0.0555)	0.0015 $(0.0319)$	-0.0023 (0.0753)	-0.0093 (0.0547)	-0.0110 (0.0577)	0.0217 $(0.0525)$	-0.0034 (0.0516)	-0.0044 (0.0378)
$\pi_t^* - \pi_t \text{ (log)}$	0.0033 $(0.0122)$	-0.0027 (0.0080)	-0.0073 (0.0085)	-0.0201 (0.0148)	-0.0016 (0.0120)	-0.0095 (0.0110)	-0.0163 (0.0075)	-0.0015 (0.0112)
$\pi_{t+1}^* - \pi_{t+1} - \mathbb{E}_t^s [\pi_{t+1}^* - \pi_{t+1}] $ (log)	-0.0004 (0.0119)	-0.0002 (0.0086)	-0.0017 (0.0089)	-0.0021 (0.0112)	-0.0013 (0.0117)	-0.0047 (0.0112)	-0.0038 (0.0092)	-0.0058 (0.0111)

Note: This table presents summary statistics (mean, standard deviation) of main variables used in this paper by country. Standard deviations are in parentheses. For definition of each variable, see Appendix A.1.