Parameter Noise Wave Extraction

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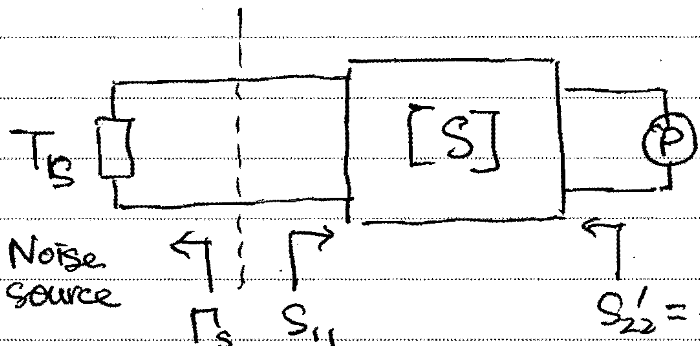
Consider a network connected to a matched power measurement device.

* This network is characterized by a scattering matrix [S]
* The network is reasonably well matched at output (say with attenuators)
* The network has $S_{12} \approx 0$



$$S'_{22} = S_{22} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{11}\Gamma_L}$$

Available gain of network

$$G_{av} = \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2} \cdot \frac{1-|\Gamma_S|^2}{1-|S'_{22}|^2} = |S_{21}|^2 \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}$$

Noise Temp. of network

$$T_n = T_{min} + T_0 \frac{R_N}{G_S} |Y_S - Y_{opt}|^2$$

T_0 ambient temp. (K)

where T_{min}, R_N and Y_{opt} are the 4 noise parameters

$Y_S = G_S + jB_S$ is the source admittance

Power measured at output

$$P = k B G_{av} (T_S + T_n)$$

k: Boltzmann constant

B: Noise equivalent BW

$$P = kB \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2}\left[(1-|\Gamma_S|^2)(T_S + T_{min}) + T_0 R_N \frac{1-|\Gamma_S|^2}{G_S}|Y_S - Y_{opt}|^2\right]$$

We note that

$$1-|\Gamma_S|^2 = \frac{4 G_S Y_0}{|Y_0 + Y_S|^2} \qquad \text{where } Y_0 = \frac{1}{50\Omega}$$

$$P = kB \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2}\left[\frac{4G_S Y_0}{|Y_0+Y_S|^2}(T_S + T_{min}) + \frac{4 Y_0 R_N}{|Y_0+Y_S|^2} T_0 |Y_S - Y_{opt}|^2\right]$$

$$= \underbrace{kB \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2}\cdot\frac{4Y_0}{|Y_0+Y_S|^2}}_{G_{net}}\left[G_S(T_S + T_{min}) + R_N T_0 |Y_S - Y_{opt}|^2\right]$$

The network gain G_{net} can be found using a hot/cold measurement with a matched calibrated noise source (no impedance diff ON/OFF)

$$P_{hot} = G_{net}\left[G_S(T_{hot} + T_{min}) + R_N T_0 |Y_S - Y_{opt}|^2\right]$$

$$P_{cold} = G_{net}\left[G_S(T_{cold} + T_{min}) + R_N T_0 |Y_S - Y_{opt}|^2\right]$$

$$\boxed{P_{hot} - P_{cold} = G_{net} G_S (T_{hot} - T_{cold})}$$

$$P = G_{net}\left[G_S(T_S + T_{min}) + R_N T_0 \left((G_S - G_{opt})^2 + (B_S - B_{opt})^2\right)\right.$$

$$= G_{net}\left[G_S T_S + G_S(T_{min} - 2R_N G_{opt} T_0) + R_N(G_{opt}^2 + B_{opt}^2)T_0\right.$$

$$\left. + R_N(G_S^2 + B_S^2)T_0 - 2R_N B_S B_{opt} T_0\right]$$

There are 4 independent unknowns

$$C_1 = \frac{T_{min}}{T_0} - 2R_N G_{opt} \qquad C_2 = R_N$$

$$C_3 = (G_{opt}^2 + B_{opt}^2)R_N \qquad C_4 = -R_N B_{opt}$$

$$R = G_{net}\cancel{\left[G_S T_S + G_S C_1 + (G_S^2+B_S^2) C_2 T_0 +\right.}$$

$$P = G_{net} T_0 \left[G_S \frac{T_S}{T_0} + G_S C_1 + (G_S^2+B_S^2) C_2 + C_3 + 2 B_S C_4 \right]$$

If we make 4 power measurements with different Γ_S or $G_S + jB_S$, the above equations can be solved for C_1, C_2, C_3 and C_4

from which T_{min}, R_N and $Y_{opt} = G_{opt} + jB_{opt}$ can be solved.

Note: For passive terminations, $T_S = T_0$

If more than 4 measurements are made, then one arrive at an over-determined system, which can be tackled by error minimization

Reference: Juan M O'Callaghan, and Jyoti P Mondal. "A vector approach for noise parameter fitting and selection of source admittances," IEEE MTT-39, pp. 1376-1382, Aug 1991