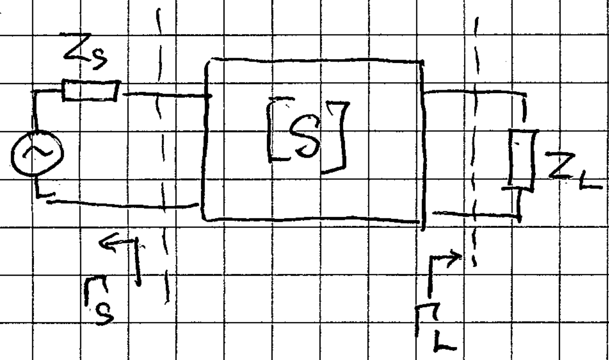


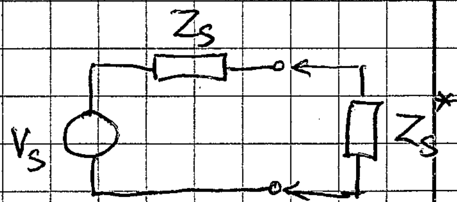

Input match $\quad \Gamma_{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$

Output match $\quad \Gamma_{out} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$

Available Gain $\quad G_{av} = \dfrac{|S_{21}|^2}{|1 - \Gamma_S S_{11}|^2} \cdot \dfrac{1 - |\Gamma_S|^2}{1 - |\Gamma_{out}|^2}$

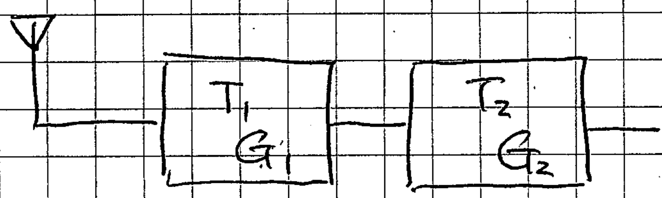
Definition of Available Gain

$$G_{av} = \frac{P_{out-av}}{P_{in-av}}$$




Maximum Available power from source

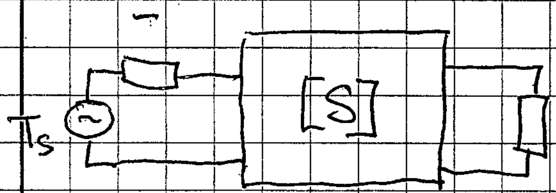
$$P_{av} = \frac{|V_S|^2}{8\,\mathrm{Re}(Z_S)} = \frac{|V_S|^2}{8Z_0}\,\frac{|1-\Gamma_S|^2}{1-|\Gamma_S|^2}$$




$$T_{sys} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_3}$$

Available gain

## Noise Parameters



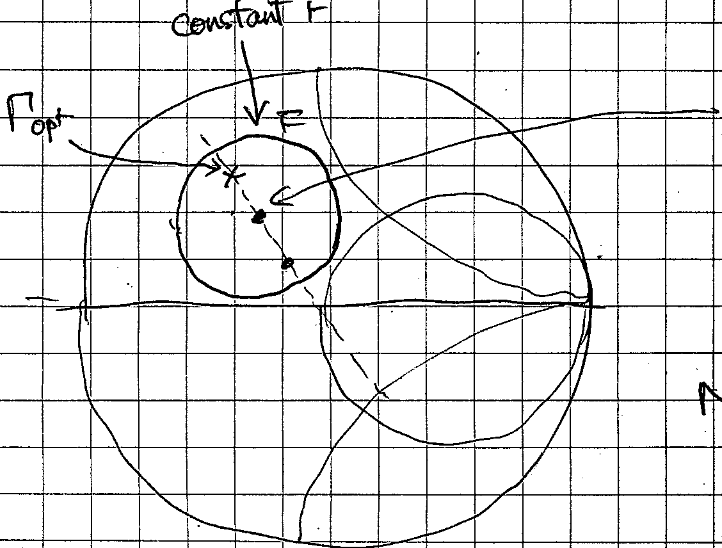

$$T = T_{min} + T_0 \frac{R_N}{G_s} |Y_s - Y_{opt}|^2 = T_{min} + T_0 \frac{r_n Z_0}{G_s} |Y_s - Y_{opt}|^2$$

$$= T_{min} + \frac{4R_N}{Z_0} \cdot \frac{T_0 \, |\Gamma_s - \Gamma_{opt}|^2}{(1-|\Gamma_s|^2)\,|1+\Gamma_{opt}|^2}$$

Noise figure $\quad F = \frac{T_n}{T_0} + 1$

Constant noise temp / figure appears as circles on Smith Chart

4 Noise parameters $T_{min}$, $\Gamma_{opt}$ ($Y_{opt}$) and $R_N$ ($r_n$)

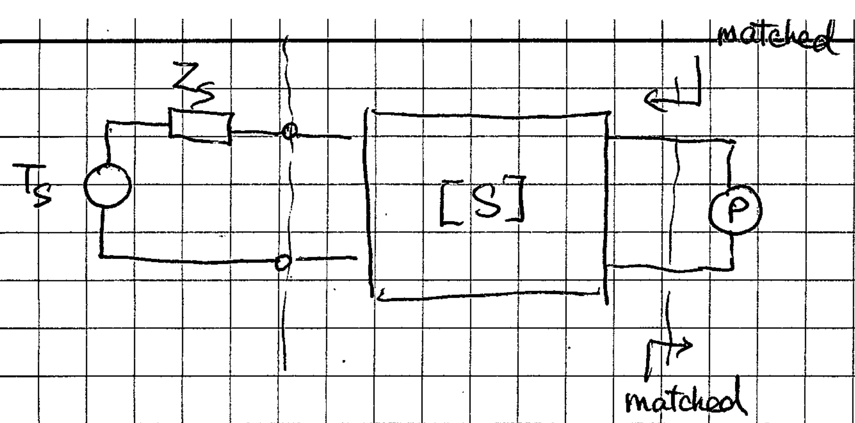



$$C_F = \frac{\Gamma_{opt}}{N+1}$$

$$R_F = \frac{\sqrt{N(N+1-|\Gamma_{opt}|^2)}}{N+1}$$

$$N = (F - F_{min}) \frac{|1+\Gamma_{opt}|^2}{4r_n}$$

Input Tuner used to derive these parameters




$$P_{out} = k G_{av} B (T_S + T_n) \qquad G_{av} = \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2} \cdot \left[1-|\Gamma_S|^2\right]$$

$$= kB \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2}\left[1-|\Gamma_S|^2\right]\left[T_S + T_{min} + T_0 \frac{R_N}{G_S}|Y_S - Y_{opt}|^2\right]$$

$$1-|\Gamma_S|^2 = \frac{4 G_S Y_0}{|Y_0+Y_S|^2} \qquad Y_0 = \frac{1}{Z_0} = \frac{1}{50\Omega}$$

$$P_{out} = kB\frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2}\left[\frac{4G_S Y_0}{|Y_0+Y_S|^2}(T_S+T_{min}) + \frac{4Y_0 R_N}{|Y_0+Y_S|^2} T_0 |Y_S - Y_{opt}|^2\right]$$

$$= \underbrace{kB\frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2} \cdot \frac{4Y_0}{|Y_0+Y_S|^2}}_{G_{net}}\left[G_S(T_S+T_{min}) + R_N T_0 |Y_S - Y_{opt}|^2\right]$$

$$P_{hot} - P_{cold} = \underbrace{G_{net}\, G_S}_{\hookrightarrow \text{ derived}}\,(T_{hot} - T_{cold}) \qquad G_S \approx \frac{1}{Z_0} \leftarrow 50\Omega$$

$$P_{out} = G_{net}\left[G_S(T_S+T_{min}) + R_N T_0\left((G_S-G_{opt})^2 + (B_S-B_{opt})^2\right)\right]$$

$$= G_{net}\left[G_S T_S + G_S(T_{min} - 2R_N T_0 G_{opt}) + R_N T_0 (G_{opt}^2 + B_{opt}^2) + R_N T_0 (G_S^2 + B_S^2) - 2R_N T_0 B_S B_{opt}\right]$$

Write $\quad Q_1 = \frac{T_{min}}{T_0} - 2R_N G_{opt} \qquad Q_2 = R_N$

$\qquad\qquad Q_3 = R_N(G_{opt}^2 + B_{opt}^2) \qquad Q_4 = -R_N B_{opt}$

$$P_{out} = G_{net} T_0 \left[G_S \frac{T_S}{T_0} + G_S Q_1 + Q_2 (G_S^2 + B_S^2) + Q_3 + 2B_S Q_4\right]$$

$$P_{out} = k G_{av} B \, (T_S + T_n)$$

$$= kB \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2}\left[1-|\Gamma_S|^2\right]\left[T_S + T_{min} + T_0 \frac{R_N}{G_S}|Y_S - Y_{opt}|^2\right]$$

$$= kB \frac{|S_{21}|^2}{|1-\Gamma_S S_{11}|^2} \cdot \frac{4G_S Y_0}{|Y_S+Y_0|^2}\left[T_S + T_{min} + T_0 \frac{R_N}{G_S}|Y_S - Y_{opt}|^2\right]$$

$$= kB|S_{21}|^2\left[\frac{4G_S Y_0}{|Y_S+Y_0|^2} \cdot \frac{1}{|1-\Gamma_S S_{11}|^2}(T_S + T_{min})\right.$$

$$\left. + \frac{1}{|1-\Gamma_S S_{11}|^2}\frac{4R_N Y_0}{|Y_S+Y_0|^2} T_0 |Y_S - Y_{opt}|^2\right]$$

For hot/cold measurement with constant source impedance

$$P_{hot} - P_{cold} = kB|S_{21}|^2 \cdot \frac{4G_S Y_0}{|Y_S+Y_0|^2} \cdot \frac{1}{|1-\Gamma_S S_{11}|^2}(T_{hot} - T_{cold})$$

Assume further that $Y_S = Y_0$ for this meas.

$$G_T \triangleq \frac{P_{hot} - P_{cold}}{T_{hot} - T_{cold}} = kB|S_{21}|^2 \cdot \cancel{\frac{1}{|1-S_{11}|^2}} \qquad (\Gamma_S = 0)$$

$$P_{out} = G_T\left[\frac{4G_S Y_0}{|Y_S+Y_0|^2} \cdot \frac{T_S + T_{min}}{|1-\Gamma_S S_{11}|^2} + \right.$$

$$\left. \frac{1}{|1-\Gamma_S S_{11}|^2} \cdot \frac{4R_N Y_0}{|Y_S+Y_0|^2} T_0 |Y_S - Y_{opt}|^2\right]$$

$$= \frac{G_T}{|1-\Gamma_S S_{11}|^2} \cdot 4Y_0\left[\frac{G_S(T_S + T_{min})}{|Y_S+Y_0|^2}\right.$$

$$1-|\Gamma_s|^2 = 1-\left|\frac{Y_o-Y_s}{Y_s+Y_o}\right|^2 = 1-\frac{|Y_o-Y_s|^2}{|Y_s+Y_o|^2}$$

$$= \frac{(Y_s+Y_o)(Y_s^*+Y_o) - (Y_s-Y_o)(Y_s^*-Y_o)}{|Y_s+Y_o|^2}$$

$$= \frac{(Y_sY_o+Y_s^*Y_o)+(Y_sY_o+Y_s^*Y_o)}{|Y_s+Y_o|^2}$$

$$= \frac{2Y_o(Y_s+Y_s^*)}{|Y_s+Y_o|^2} = \frac{4G_sY_o}{|Y_s+Y_o|^2}$$

$$|Y_s-Y_{opt}|^2 = (G_s-G_{opt})^2+(B_s-B_{opt})^2$$

$$= (G_s^2+B_s^2)+(G_{opt}^2+B_{opt}^2)-2G_sG_{opt}-2B_sB_{opt}$$

Assuming that $|\Gamma_S| \approx 0$ for hot-cold measurement

$$P_{hot} - P_{cold} = \frac{kB|S_{21}|^2}{|1-S_{11}|^2}(T_{hot} - T_{cold})$$

$$P_T = \frac{P_{hot} - P_{cold}}{T_{hot} - T_{cold}} = kB|S_{21}|^2$$

$$P_{out} = G_{av} kB (T_S + T_n)$$

$$= \frac{kB|S_{21}|^2}{|1-\Gamma_S S_{11}|^2}(1-|\Gamma_S|^2)\left[T_S + T_{min} + \frac{R_n}{G_S}|Y_S - Y_{opt}|^2 T_0\right]$$

$$T_{out} = \frac{P_{out}}{kB|S_{21}|^2}|1-\Gamma_S S_{11}|^2$$

$$1-|\Gamma_S|^2 = \frac{4G_S Y_0}{|Y_0+Y_S|^2}$$

$$T_{out} = (1-|\Gamma_S|^2)(T_S + T_{min}) + \frac{4R_n Y_0}{|Y_0+Y_S|^2}|Y_S - Y_{opt}|^2 T_0$$

$$|Y_S - Y_{opt}|^2 = (G_S^2 + B_S^2) + (G_{opt}^2 + B_{opt}^2) - 2G_S G_{opt} - 2B_S B_{opt}$$

$$\frac{T_{out} - (1-|\Gamma_S|^2)T_S}{T_0} = \frac{4G_S Y_0}{|Y_0+Y_S|^2}\frac{T_{min}}{T_0} + \frac{4R_n Y_0}{|Y_0+Y_S|^2}|Y_S - Y_{opt}|^2$$

$$Q = G_S\frac{T_{min}}{T_0} -$$

$$Q_1 = \frac{T_{min}}{T_0} - 2R_N G_{opt}$$

$$Q_2 = R_N \qquad\qquad Q_3 = R_N (G_{opt}^2 + B_{opt}^2)$$

$$Q_4 = -R_N B_{opt}$$

$$\frac{T_{out} - (1-|\Gamma_s|^2)T_s}{T_0} = \frac{4G_s Y_0}{|Y_0+Y_s|^2} Q_1 + \frac{4Y_0}{|Y_0+Y_s|^2}(G_s^2+B_s^2) Q_2$$

$$+ \frac{4Y_0}{|Y_0+Y_s|^2} Q_3 + \frac{4Y_0 B_s}{|Y_0+Y_s|^2} Q_4$$

$$\frac{T_{out} - (1-|\Gamma_s|^2)T_s}{4Y_0 T_0} = \frac{G_s}{|Y_0+Y_s|^2} Q_1 + \frac{G_s^2+B_s^2}{|Y_0+Y_s|^2} Q_2 + \frac{Q_3}{|Y_0+Y_s|^2} + \frac{B_s}{|Y_0+Y_s|^2} Q$$

$$R_N = Q_2 \qquad\qquad B_{opt} = -\frac{Q_4}{Q_2}$$

$$G_{opt} = \sqrt{\frac{Q_3}{Q_2} - \left(\frac{Q_4}{Q_2}\right)^2} = \frac{1}{Q_2}\sqrt{Q_2 Q_3 - Q_4^2}$$

$$T_{min} = T_0 \left[ Q_1 + 2Q_2 G_{opt} \right] = T_0 \left[ Q_1 + 2\sqrt{Q_2 Q_3 - Q_4^2} \right]$$

For short-circuit

$$B_s \to \infty \qquad \frac{1}{|Y_0+Y_s|^2} \to 0 \qquad \frac{B_s}{|Y_0+Y_s|^2} \to 0 \qquad \frac{G_s}{|Y_0+Y_s|^2} \to 0$$

$$\frac{G_s^2+B_s^2}{|Y_0+Y_s|^2} = \frac{|Y_s|^2}{|Y_0+Y_s|^2} \to 1$$

Short-circuit $\quad \dfrac{T_{out-sh}}{4Y_0 T_0} = Q_2 = R_N$

$$R_N = P_{out-sh} \frac{T_{hot} - T_{cold}}{P_{hot} - P_{cold}} |1-S_{11}|^2 \frac{1}{4Y_0 T_0} =$$

For open-circuit $G_s = 0 \quad B_s = 0$

$$\frac{T_{out\text{-}open}}{4Y_0T_0} = \frac{Q_3}{Y_0^2} \qquad Q_3 = \frac{T_{out\text{-}open}}{4T_0} Y_0$$

For matched load at $T_0$ (cold) $\quad G_s = Y_0 \quad B_s = 0$

$$\frac{T_{out} - T_{cold}}{4Y_0T_0} = \frac{Q_1}{4Y_0} + \frac{Q_2}{4} + \frac{Q_3}{4Y_0^2}$$

$$\frac{T_{out} - T_{cold}}{T_0} = Q_1 + Y_0 Q_2 + \frac{Q_3}{Y_0}$$

$$\frac{T_{out} - T_{cold}}{T_0} = Q_1 +$$

# Noise of passive circuit

$$[C] = kT_0\left[I - SS^*\right] \qquad \cancel{SS^* = [S_{11}S_{11}^*}$$

$$SS^* = \begin{bmatrix} S_{11}S_{11}^* + S_{12}S_{21}^* & S_{11}S_{12}^* + S_{12}S_{22}^* \\ S_{21}S_{11}^* + S_{22}S_{21}^* & S_{21}S_{12}^* + S_{22}S_{22}^* \end{bmatrix}$$

$$= \begin{bmatrix} |S_{11}|^2 + |S_{21}|^2 & S_{11}S_{21}^* + S_{21}S_{22}^* \\ S_{21}S_{11}^* + S_{22}S_{21}^* & |S_{21}|^2 + |S_{22}|^2 \end{bmatrix}$$

$$\langle c_1 c_1^* \rangle = kT_0\left(1 - |S_{11}|^2 - |S_{21}|^2\right)$$

$$\langle c_2 c_2^* \rangle = kT_0\left(1 - |S_{22}|^2 - |S_{21}|^2\right)$$

$$\langle c_1 c_2^* \rangle = -kT_0\left(S_{11}S_{21}^* + S_{21}S_{22}^*\right)$$

Noise Temp $\quad T_n = \dfrac{\beta C \beta^\dagger}{k} \qquad \beta = [$

## Fitting of over-constrained linear system

$y = a_1 X_1 + a_2 X_2 + a_3 X_3 + \dots$

$n = 1 \dots N$

$a_n$ — unknown coefficients

$X_n$ — ~~experimental parameters~~

$y_n$ — measurement data

If $M$ measurements are made $(M > N)$

$m = 1, \dots, M$

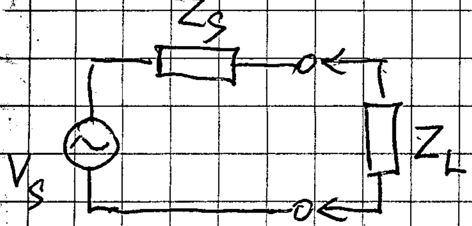
we have an over-constrained system
which is solved in the least square sense

$$\sigma^2 = \sum_m \left[ (a_1 (X_1)_m + a_2 (X_2)_m + a_3 (X_3)_m + \dots) - y_m \right]^2$$

$$\frac{\partial \sigma^2}{\partial a_1} = 2 \sum_m \left[ (a_1 (X_1)_m + a_2 (X_2)_m + a_3 (X_3)_m + \dots) - y_m \right] \cdot (X_1)_m = 0$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} \sum_m (X_1)_m^2 & \sum_m (X_1)_m (X_2)_m & \sum_m (X_1)_m (X_3)_m & \sim \\ \sum_m (X_1)_m (X_2)_m & \sum_m (X_2)_m^2 & \sum_m (X_2)_m (X_3)_m & \\ \sum_m (X_1)_m (X_3)_m & \sum_m (X_2)_m (X_3)_m & \sum_m (X_3)_m & \\ \vdots & \vdots & & \ddots \end{bmatrix}^{-1} \begin{bmatrix} \sum_m (X_1)_m y_m \\ \sum_m (X_2)_m y_m \\ \sum_m (X_3)_m y_m \\ \vdots \end{bmatrix}$$

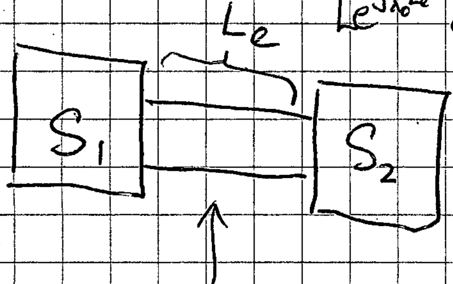
# Available Power




Max power $\quad Z_S = Z_L^*$

$$P_{av} = \frac{1}{8}\frac{|V_S|^2}{Re(Z_S)}$$

$$\begin{bmatrix} 0 & e^{-j\frac{2\pi}{\lambda_0}L_e} \\ e^{-j\frac{2\pi}{\lambda_0}L_e} & 0 \end{bmatrix}$$




$G_{av} =$

$$|S_{21}| = 1$$

$$|1 - S_{11}\Gamma_S| = 1 \qquad S_{11} = 0$$

$$\Gamma_S = S_1\big|_{out}$$

$$\Gamma_{out} = 0 + \frac{e^{-j\frac{4\pi}{\lambda_0}L_e} \cdot \Gamma_S}{\cancel{1}}$$

$$= \Gamma_S e^{-j\frac{4\pi}{\lambda_0}L_e}$$

$$G_{av} = \frac{1 - |\Gamma_{out_1}|^2}{|1 - |\Gamma_{out_1} e^{-j\frac{4\pi}{\lambda_0}L_e}|^2} = 1$$