HUGH HAN
600.363 Introduction to Algorithms / 600.463 Algorithms I **Fall 2015**
Homework #1 **Due:** September 8, 2015, 1:30pm

Remember: you may work in groups of up to three people, but must write up your solution entirely on your own. Collaboration is limited to discussing the problems – you may not look at, compare, reuse, etc. any text from anyone else in the class. Please include your list of collaborators on the first page of your submission. You may use the internet to look up formulas, definitions, etc., but may not simply look up the answers online.

Please include proofs with all of your answers, unless stated otherwise.

1 Asymptotic Notation (32 points)

For each of the following statements explain if it true or false and prove your answer. The base of log is 2 unless otherwise specified, and ln is $\log_e$.

(a) $\log(n^{70}) = O(\log(n^{1/2}))$

True. **Proof:**

$$\begin{aligned} \log(n^{1/2}) &= \frac{1}{2}\log n \\ O(\log(n^{1/2})) &= O(\log n) \\ \log(n^{70}) &= 70\log n \end{aligned}$$

Let $g(n) = 70\log n$, $f(n) = \log n$.

$$\begin{aligned} g(n) &= 70\log n \\ g(n) &= c \cdot f(n) \end{aligned}$$

for $c = 70$, and all $n \geq n_0$, where $n_0 = 1$.

We see that:

$$70\log n = O(\log n).$$

Thus,

$$\log(n^{70}) = O(log(n^{1/2})).$$

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