Real Analysis

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1 Lebesgue Spaces

or $L^p(X, \mathcal{S}, \mu)$

[1] Given a measure space $(X, \mathcal{S}, \mu)$, and $0 < p \leq \infty$, the Lebesgue Space L^p is the set of $\mathcal{S}$-measurable functions $f : X \to \mathbb{F}$ such that $\|f\|_p < \infty$ is defined as follows:

p-norm of f

measures the "average" size of the function by raising values to the power p, integrating, and taking the p-th root.

- If $0 < p < \infty$, $\|f\|_p = \left(\int |f|^p d\mu\right)^{\frac{1}{p}}$
- $\|f\|_\infty = \inf\{t > 0 : \mu(\{x \in X : |f(x)| > t\}) = 0\}$

supremum norm: maximum absolute value of the function f over its domain

greatest lower bound of a set: searching for the smallest that satisfies the condition

References

[1] Tyler Perlman, The Development of the Hardy Inequality `https://www.sas.rochester.edu/mth/undergraduate/honorspaperspdfs/tylerperlman2021.pdf`