Real Analysis

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1 Lebesgue Spaces

or $L^p(X, \mathcal{S}, \mu)$

[1] Given a measure space $(X, \mathcal{S}, \mu)$, and $0 < p \leq \infty$, the Lebesgue Space $\boxed{L^p}$ is the set of $\mathcal{S}$-measurable functions $f : X \to \mathbb{F}$ such that $\boxed{\|f\|_p} < \infty$ is defined as follows:

p-norm of f

- If $0 < p < \infty$, $\|f\|_p = \left(\int |f|^p d\mu\right)^{\frac{1}{p}}$
- $\|f\|_\infty = \inf\{t > 0 : \mu(\{x \in X : |f(x)| > t\}) = 0\}$

With:

- The $\|f\|_p$ for $0 < p < \infty$ measures the "average" size of the function f, raising values to the power p, integrating, and taking the p-th root.
- The $\|f\|_\infty$ finds the *essential supremum* of f, which is the smallest value t such that the function is less than or equal to t almost everywhere (except on sets with measure zero).

References

[1] Tyler Perlman, The Development of the Hardy Inequality `https://www.sas.rochester.edu/mth/undergraduate/honorspaperspdfs/tylerperlman2021.pdf`