Write

$$\bar{p} = \frac{1}{G}\frac{\sum_{l=kG}^{G} l\phi(l,G,p)}{\sum_{l=kG}^{G}\phi(l,G,p)} = \frac{1}{G}\frac{\sum_{l=kG}^{G} l\phi(l,G,p)}{P}$$

where

$$\phi(l,G,p) = \binom{G}{l} p^{l}(1-p)^{G-l}$$

is the binomial distribution of the number of group reciprocators in a group, and P is the probability that $p_g > k$.

Let's write the binomial as $\phi(l)$ for short. Differentiating it with respect to p reveals

$$\phi'(l) = \frac{l-Gp}{p(1-p)}\phi(l).$$

Writing $\sum\phi$ as short for $\sum_{l=kG}^{G}\phi(l)$, et cetera, we differentiate $\bar{p}$:

$$\begin{aligned}
\bar{p}' &= \frac{1}{G}\frac{1}{P^2}\left(\left[\sum\phi\right]\left[\frac{1}{p(1-p)}\sum l(l-Gp)\phi\right] - \left[\sum l\phi\right]\frac{1}{p(1-p)}\left[\sum(l-Gp)\phi\right]\right)\\
&= \frac{1}{G}\frac{1}{P^2}\frac{1}{p(1-p)}\left(\left[\sum\phi\right]\left[\sum l(l-Gp)\phi\right] - \left[\sum l\phi\right]\left[\sum(l-Gp)\phi\right]\right)\\
&= \frac{1}{G}\frac{1}{P^2}\frac{1}{p(1-p)}\left(\left[\sum\phi\right]\sum l^2\phi - \left[\sum\phi\right]Gp\sum l\phi - \left[\sum l\phi\right]^2 + \left[\sum l\phi\right]\left[\sum\phi\right]Gp\right)\\
&= \frac{1}{G}\frac{1}{P^2}\frac{1}{p(1-p)}\left(\left[\sum\phi\right]\left[\sum l^2\phi\right] - \left[\sum l\phi\right]^2\right)
\end{aligned}$$

This is signed by the final term. Multiplying out the sums gives

$$\begin{aligned}
&\left[\sum\phi(l)\right]\left[\sum l^2\phi(l)\right] - \left[\sum l\phi(l)\right]^2\\
&= \left[\sum l^2\phi(l)\right]\left[\sum\phi(l)\right] - \left[\sum l\phi(l)\right]\left[\sum l\phi(l)\right]\\
&= \left[R^2\phi(R) + (R+1)^2\phi(R+1) + \ldots + G^2\phi(G)\right]\left[\phi(R) + \ldots + \phi(G)\right] -\\
&\quad [R\phi(R) + (R+1)\phi(R+1) + \ldots + G\phi(G)]\,[R\phi(R) + (R+1)\phi(R+1) + \ldots + G\phi(G)]\\
&\quad \text{where I wrote } R = kG;\\
&= \sum_{l=R}^{G}\sum_{m=R}^{G} l^2\phi(l)\phi(m) - \sum_{l=R}^{G}\sum_{m=R}^{G} lm\phi(l)\phi(m)\\
&= \sum_{l=R}^{G}\sum_{m=R}^{G} l(l-m)\phi(l)\phi(m).
\end{aligned}$$

Now observe that the terms in this sum equal zero whenever $l = m$. When $l \neq m$, we can put the terms in pairs:

$$\begin{aligned}
&l(l-m)\phi(l)\phi(m) + m(m-l)\phi(l)\phi(m)\\
&= [l^2 + m^2 - 2ml]\phi(l)\phi(m)\\
&= [l-m]^2\phi(l)\phi(m)
\end{aligned}$$

Thus we can rewrite the double sum as

$$\sum_{l=R}^{G} \sum_{m=l+1}^{G} [l-m]^2 \phi(l)\phi(m)$$

which is positive.