The Apollo Missions: Report

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1 Introduction

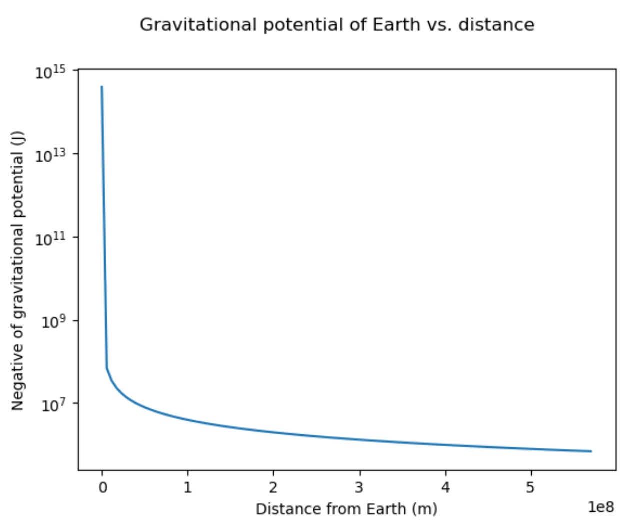
The following is a report detailing the findings of several calculations and computer simulations of the currently in-development Saturn V rocket, specifically its first stage.

2 Gravitational Potential of the Earth-Moon system

Gravitational potential represents the amount of energy it would take to completely overcome all gravitational effects, starting at a certain point. It is given by the expression:

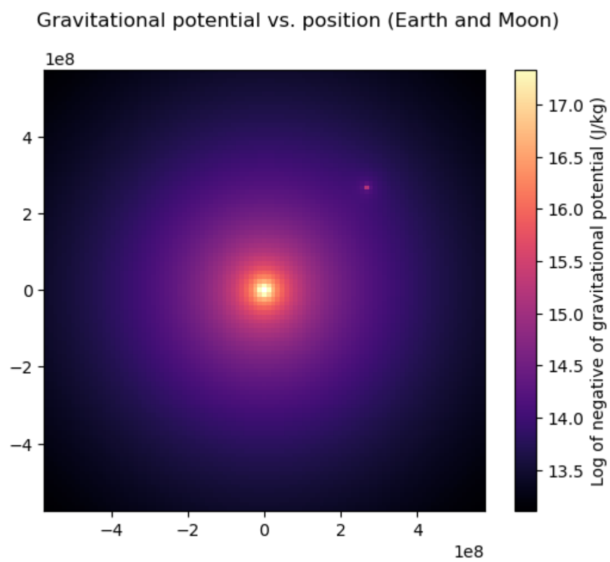
$$-\frac{GM}{r}$$

where G is the Gravitational Constant (a very small number), M is the mass of the object, and r is the distance from the point to the object's center. To illustrate the behavior of gravitational potential, here is a plot of the gravitational potential as a function of the distance of a point from Earth's center:



It is worth noting that gravitational potential increases much more rapidly close to the Earth than farther away from the Earth. It is important to note that in this chart, the Earth was approximated as a point particle; while this approximation is accurate at distances greater than the radius of the earth (6378km), it becomes inaccurate for distances less than that.

Below is a better representation of the gravitational potential in the space surrounding the Earth and Moon:



Note the small speck representing the gravitational potential contribution from the Moon. It is quite insignificant in comparison to that of the Earth.

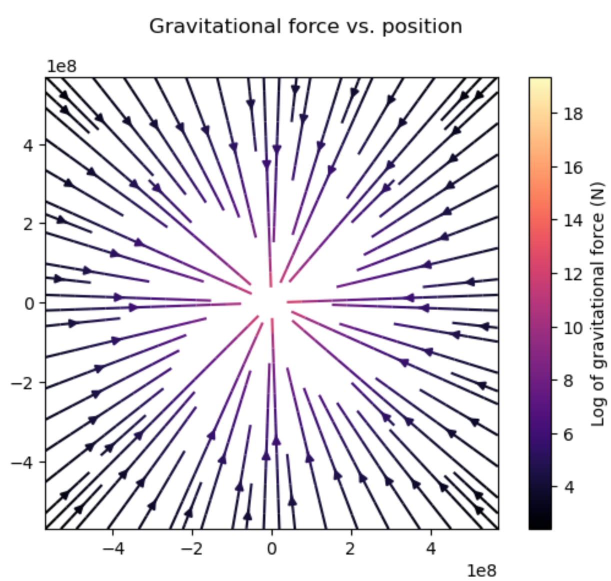
3 Gravitational Force of the Earth-Moon system

The gravitational force exerted by one mass on another is given by the following expression:

$$-G\frac{M_1 m_2}{r^2}\hat{r}_{12}$$

where G is the Gravitational Constant, M_1 is the mass of the attracting object, m_2 is the mass of the object being attracted, r is the distance between the two objects, and $\hat{r}_{12}$ is a unit vector representing direction. Essentially this expression says that the force points from the attracted object to the attracting object.

Below is a chart of the gravitational force in the space surrounding the Earth and Moon:



Again, the strength of the force scales rapidly upon closeness to the Earth, and the contribution from the Moon is negligible.

4 Projected Performance of the Saturn V Stage 1

The burn time, or the time it takes for the rocket to burn through all its fuel, can be calculated from the following expression:

$$\frac{m_0 - m_f}{\dot{m}}$$

where m_0 is the wet mass of the rocket (including all its fuel), m_f is the dry mass of the rocket (after it has expended all of its fuel), and $\dot{m}$ is the fuel burn rate (rate at which the rocket's fuel is expended, in units of kg/s). Plugging in the values we were given, we calculated a total burn time of around 158 seconds, which is very close to the findings from the recent test results (160 seconds). The slight underestimation is likely due to simple random error.

Rockets function by conservation of momentum: the momentum of the expelled fuel equals the momentum propelling the rocket. The formula for the velocity of a rocket as a function of time t given by:

$$v_e \ln\left(\frac{m_0}{m_0 - \dot{m}t}\right) - gt$$

where v_e is the fuel exhaust velocity, m_0 is the wet mass, $\dot{m}$ is the fuel burn rate, and g is the gravitational acceleration on Earth. Integrating the velocity over time (from 0 to the end of the burn time) gives the altitude the rocket is able to reach. Plugging in the values we were given, we calculated an altitude of about 74km, which is a little higher than the findings from the recent test results (70km). This is likely due the formula's failure to consider factors such as air drag.

5 Discussion and Future Work

The biggest area for future work is in calculating the launch altitude, as more accurate estimations require more complex calculations. In particular, the Tsiolkovsky equation, which we used, assumes gravity is uniform at all points in space in the launch, and also assumes it to be the only force acting on the rocket (aside from the rocket's own propulsion). Looking back at the charts of gravitational potential, it is clear that, if one starts close to the Earth (which in our case we do), the strength of Earth's gravity decreases quite rapidly as one moves away from the Earth. Additionally, at the speeds at which the rocket is moving, air drag is hardly ignorable. Future work should focus on creating a more accurate expression which takes into account more environmental factors and gives a more accurate estimate for the actual launch altitude of the rocket. Of course, empirical testing should continue being conducted simultaneously, as it is the best way to verify if our calculations are correct.