Mine crafting: Report

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1 Introduction

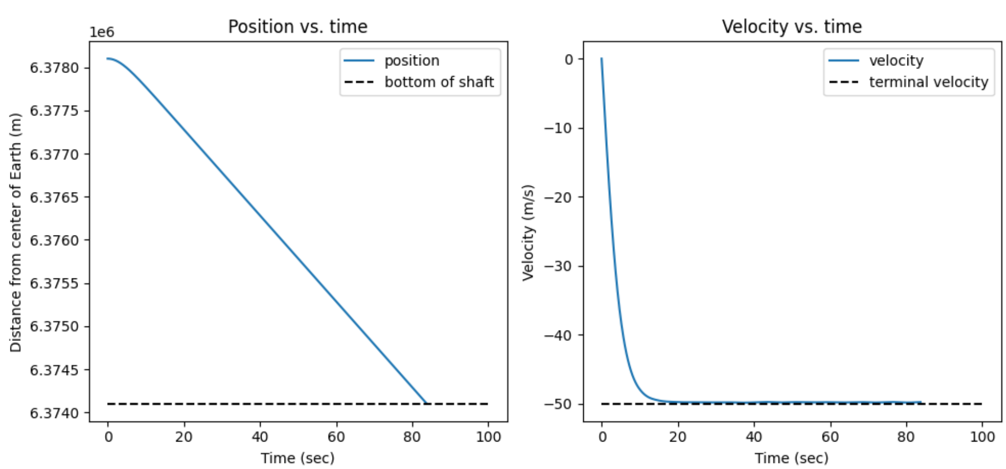
Our mining company operates one of the deepest mines on Earth. We know that it is roughly 4 kilometers deep, but we would like to obtain a more exact measurement. This paper explores the practicality of measuring the depth of the mine by dropping a test mass into the mine and measuring the time it takes for it to hit the bottom. It explores the tendencies of the test mass upon being dropped, under an increasingly complex series of assumptions.

2 Fall Time Calculations

Firstly, we attempted to obtain a rough estimate of the expected fall time of the mass. Firstly, a quick calculation assuming a constant gravitational force can be made by taking one of the basic kinematic equations and solving for time; the resulting estimate is 28.6 seconds. However, in reality, the gravitational force from the Earth on an object depends on the total mass of the Earth which lies at a radius less than that of the falling object. Thus, assuming a constant mass density, the force of gravity on an object is linearly proportional to its distance from the center of the Earth. However, the fall time, taking into account the linear relationship between gravity and the distance of the mass from the center of the Earth, is still 28.6 seconds. This is because the radius of the Earth is roughly $6.4 \cdot 10^3$ kilometers, which is more than a thousand times the depth of our mine. Therefore, the reduction in gravity as the mass falls down the mine is too small to be significant.

The biggest contributing factor toward lengthening the fall time is not the linear relationship between gravity and radius, but rather drag from air resistance. As the drag force is linearly proportional to the velocity, the mass will accelerate until the drag force equals the gravitational force, then cease to accelerate any further. This maximum velocity it reaches is called its terminal velocity, and assuming it to be around 50 meters per second, our fall time suddenly shoots up to almost three times longer, at 83.8 seconds.

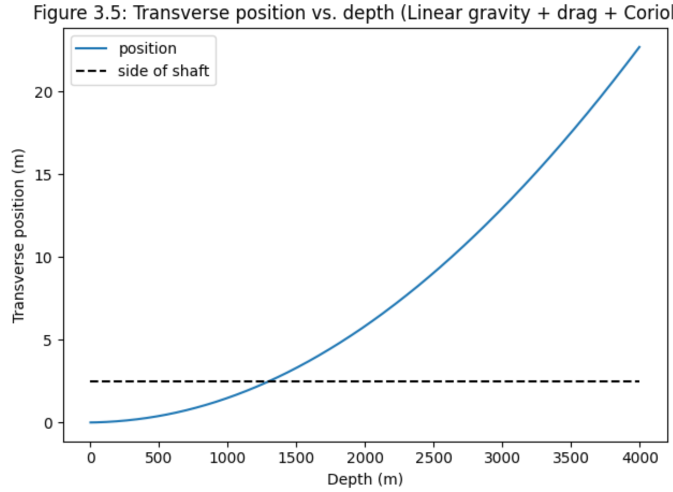
Fig 2.5: Position and velocity vs. time (linear gravity + drag)



3 Feasibility of Depth-measurement Approach

Despite the above discussion, the key reason it is impractical to measure the depth of our mine by measuring the fall time of a test mass lies in the Coriolis force, a fictitious force that appears in accelerating reference frames, such as the rotating surface of the Earth. Specifically, the Coriolis forces produced by the rotation of the Earth would cause our test mass, even if dropped with no sideways velocity, to accelerate toward the side of the mine. Taking into account linear gravity, air drag, and Coriolis forces, if we were to drop a test mass into a mine five meters wide at the equator, the mass would hit the wall at around 29.7 seconds, at a depth of only 1302.2 meters. Clearly, the Coriolis force from Earth's rotation would single-handedly prevent us from using this approach to measure the depth of our mine, unless our mine were within a few thousand kilometers of the North Pole. We would not recommend proceeding with this depth-measurement technique.

Figure 3.5: Transverse position vs. depth (Linear gravity + drag + Coriolis)

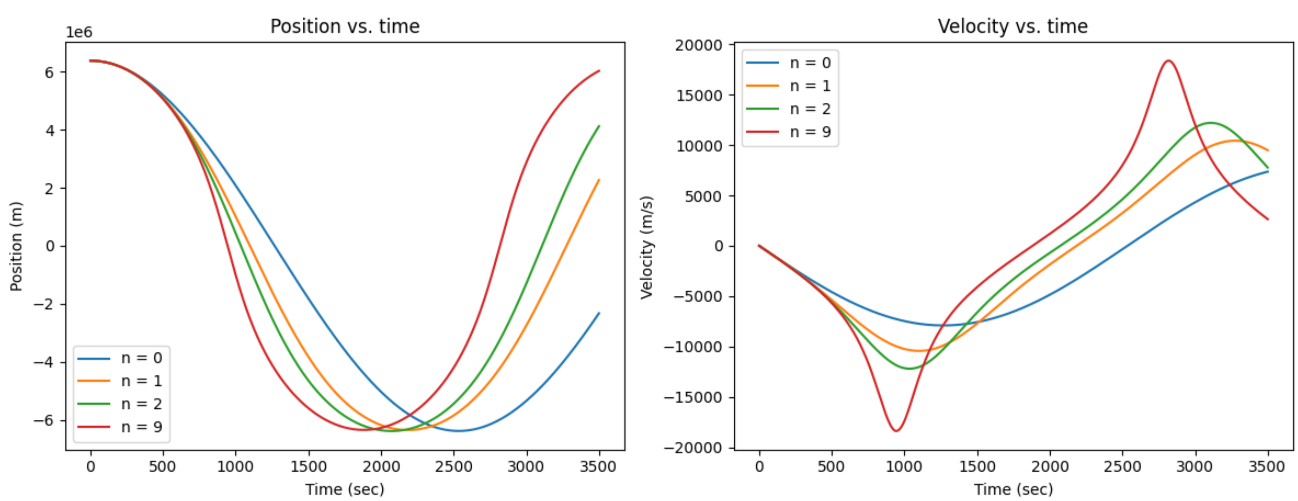


4 Crossing Times for Homogeneous and Nonhomogeneous Mass Density

Up until now, we have made the assumption in our calculations that the mass density of the Earth is constant. However, in reality, Earth's mass is more than three times more concentrated within a few thousand kilometers of its center than at the surface. The higher density in the center means that a higher total mass remains at a radius less than or equal to that of the falling mass for a longer time, maintaining a higher gravitational force and therefore a higher acceleration.

Let us imagine we hypothetically dropped a test mass into an infinitely deep mine, ignoring drag and Coriolis forces. Assuming a constant mass density, the mass would cross the center of the Earth after around 1267.2 seconds. However, assuming a more realistic mass density, where the density is more than three times larger than the density at the surface, the mass crosses the center after only 943.9 seconds.

Figure 7: Position and velocity vs. time (non-uniform density)



The effects of density on fall time can further be seen by comparing the fall time through a constant-density Earth to the fall time through a constant-density Moon. The density of Earth is about $5494.9 \mathrm{kg/m^3}$, which is almost twice the density of the Moon, at $3341.8 \mathrm{kg/m^3}$. It can be shown that the fall time through a constant-density planetary body is proportional to one over the square root of its mass density. As such, it makes sense that the time for a mass to cross the center of a constant-density Earth is only 1267.2 seconds, whereas it would take a mass 1624.9 seconds to cross the center of the Moon.

5 Discussion and Future Work

In making our calculations, we tried to take into account as many factors as possible. However, we at most took into account linear density, air drag, and the Coriolis force at the same time. Future work would do well to perform calculations for our variable-density Earth, alongside the air drag and Coriolis force factors.

Additionally, our calculations involving the Coriolis force were performed under the assumption that the mine were on the Equator. This assumption was made in order to observe the maximum impact that the Coriolis force could have on the fall time and trajectory of the mass. Further work would do well to calculate the trajectory of the mass at multiple latitudes, ultimately making a plot of transverse displacement as a function of latitude. This would allow us to know how far North (or South) our mine would have to be in order for the Coriolis force to cease to be a problem for this depth-measuring technique.

Finally, throughout all our experiments, we assumed a perfectly spherical Earth. However, this is sadly also untrue. Earth's features range from Mount Everest to the Mariana Trench, and all these natural features impact the actual gravitational force felt by a falling object (which would also change based on the location of the mine). As such, future research should take explore the maximum possible impacts such natural features could have on the trajectory of the falling mass, such as if the mine were on top of Mount Everest or if the mine were dug into the Mariana Trench.

In summary, this paper has shown that the Coriolis force makes it impractical to measure the depth of a mine by measuring the fall time of a dropped test mass. It has also shown the effects of a variable-density Earth on the fall time of an object. However, it did so through various simplifying assumptions. It is left for future work to take into account more factors, and to ultimately arrive at even more accurate estimates.