Topic 1: Introduction, Vectors

May 3, 2020

Downloads

- All details for this course can be found on Moodle
- http://moodle.rhul.ac.uk/
- I will attempt to create a slidecast of each lecture that will be placed on moodle as well.

Who am I ?

- Hugh Shanahan (but you can call me Hugh).
- My office is in the Bedford Building.
- Here are my office hours - please come by the admin office and ask them to give me a call.
- Monday 15-16
- Tuesday 15-16
- If you want some guaranteed time with me to answer question etc., please email me to arrange an appointment.

Assignments

- Three quizzes - each worth 10% of the final grade.
- Each quiz will be 30 minutes in length so it can be done in an hour slot.
- A practice quiz will be set up this week.
- Deadlines are available on

Exam

- Answer all questions !

Labs

- Labs will run on a weekly basis.
- Thursday 10-12, Bedford 0-04 to 0-0.06, PC01.
- Class will be divided into two; each one taking one hour.
- No lab this week.
- We'll be using Python and NumPy for this course.
- More specifically we will be using Jupyter notebooks and Binder.

Motivation

- This course will focus on one, very regular class of data structures.
- Namely, vectors and matrices with almost invariably floating point entries.
- Linear Algebra is the branch of Mathematics to deal with this.
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Linear Algebra

- Linear Algebra is a vast branch of Mathematics with hugely powerful tools for us to use.
- Linear Algebra has a number of direct applications in Computer Science and every Computer Scientist should understand it. For example
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  - Computer Vision
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Mathematics

- This course is shamelessly Mathematical BUT
- I assume no prior knowledge of Linear Algebra.
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Definitions

Type	Symbol	Examples
Natural numbers	$\mathbb{N}$	0,1,2,...
Integers	$\mathbb{Z}$	$\ldots,-2,-1,0,1,2,\ldots$
Reals	$\mathbb{R}$	$-\pi$,0, e, 100.03
Complex numbers	$\mathbb{C}$	$3.0 + 4.95i$

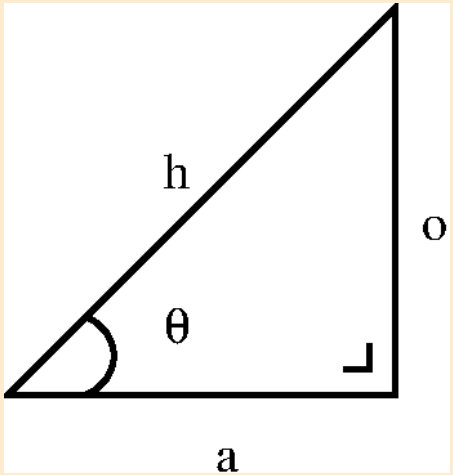
Table: Symbols used in this course.

Notation

Symbol	Meaning
$\equiv$	Defines
$\Longrightarrow$	This Implies
iff	If and Only if

Table: Notation used in this course.

Trigonometry



$$\cos\theta = \frac{a}{h} \quad , \quad \sin\theta = \frac{o}{h} \quad , \tag{1.1}$$

$$\cos(-\theta) = \cos\theta \quad , \quad \sin(-\theta) = -\sin\theta \tag{1.2}$$

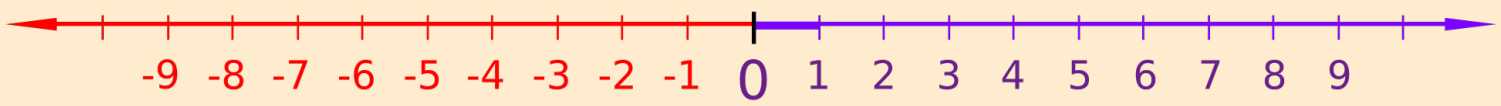
$$\cos^2\theta + \sin^2\theta = 1 \quad , \tag{1.3}$$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b \quad , \tag{1.4}$$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b \quad . \tag{1.5}$$

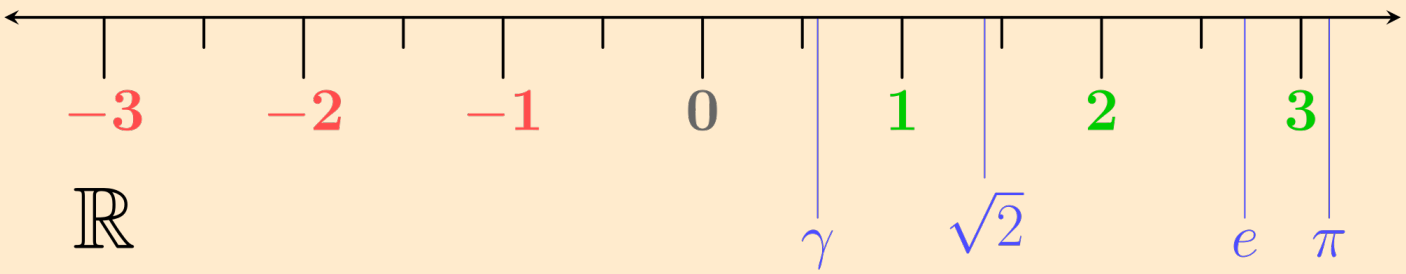
The number line

First steps in understanding numbers ($\mathbb{Z}$).



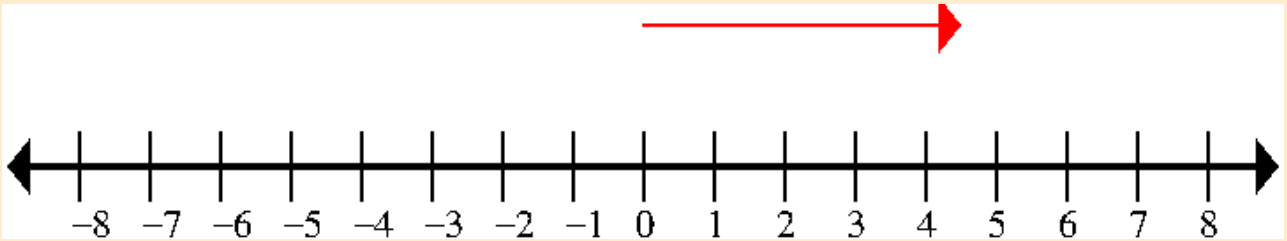
The reals

Can add in reals easily in this picture . . . ($\mathbb{R}$).



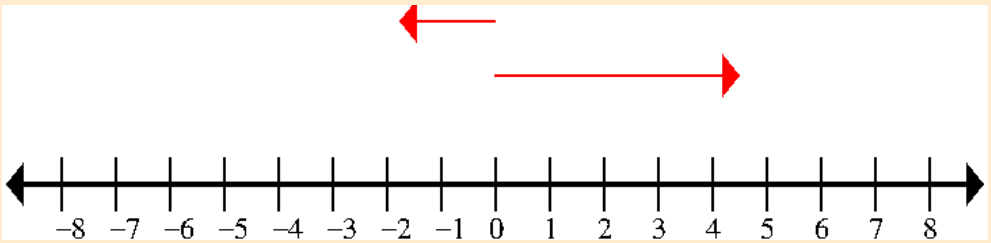
Direction

Can think of an arrow from the origin to each number on the number line.



Arrow is length 4.5 going in a positive direction.

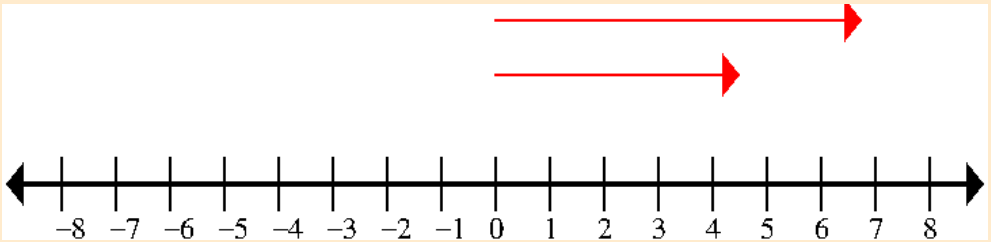
Direction



Other arrow is length 1.66 going in a negative direction.

Operations - scaling

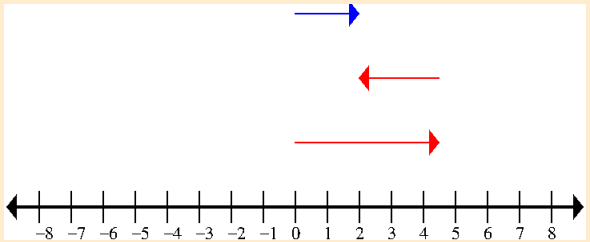
Can scale the length of an arrow.



Arrow of length 4.5 is scaled by a factor 1.5.

Operations - addition

Can add arrows - just put "tail to head".



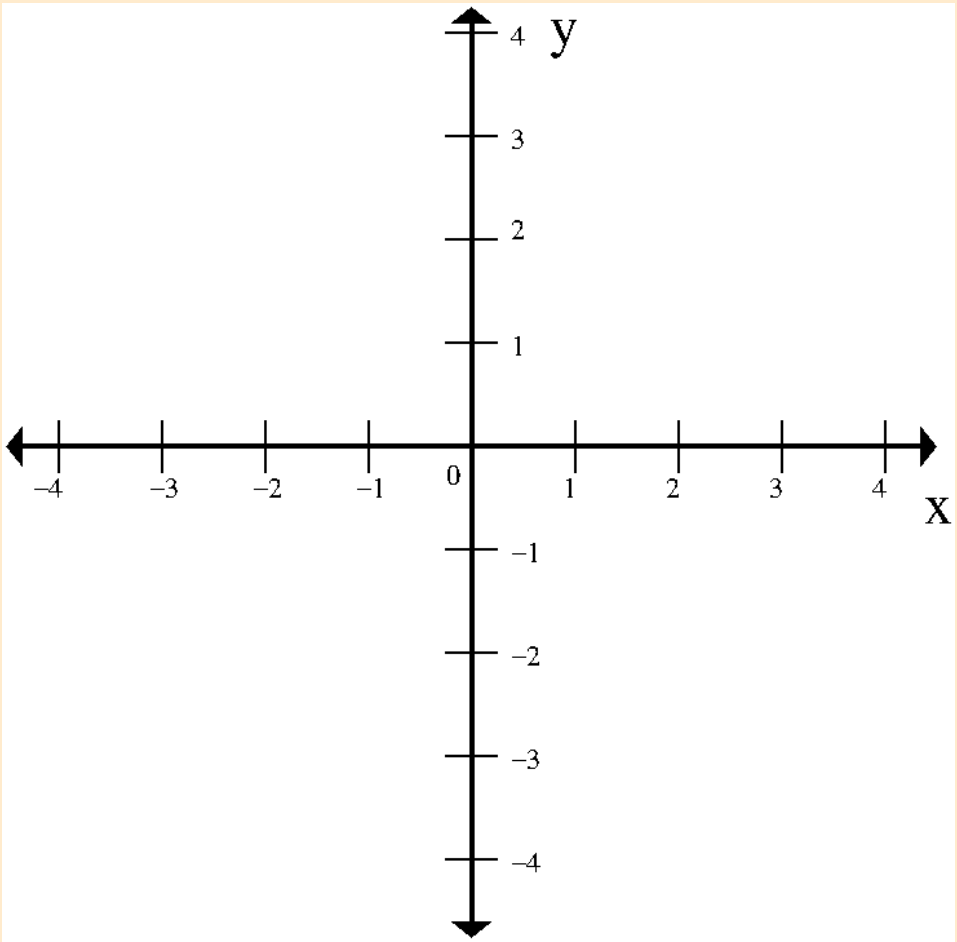
Arrow of length 4.5 going in positive direction plus arrow of length 2.5 going in a negative direction results in an arrow of length 2.0 going in a positive direction.

Operations - product

Multiplying the signs tells us if the arrows are going in the same or opposite direction.

Adding another number line

Move into 2 dimensions.



Moving up a dimension

- Instead of a position on the number line. You now have **coordinates**, represented by a pair of numbers (a, b).
- For the most part we will assume that these numbers are reals $a \in \mathbb{R}$, $b \in \mathbb{R}$.
- Formally, these are elements of the Cartesian product $\mathbb{R} \times \mathbb{R}$ (go and check your CS1860 notes).
- But we could have grids which are $\mathbb{N} \times \mathbb{N}$, $\mathbb{Z} \times \mathbb{Z}$, $\mathbb{C} \times \mathbb{C}$ and so on.

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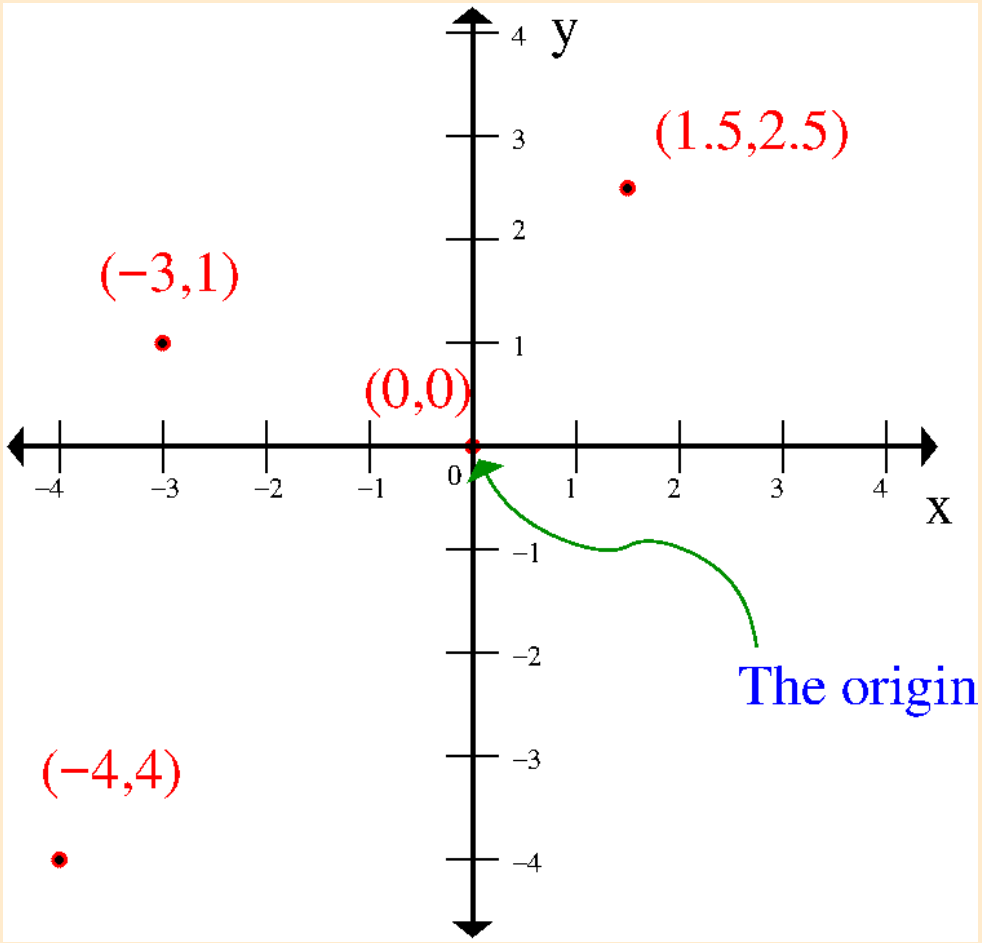
A coordinate



Vectors

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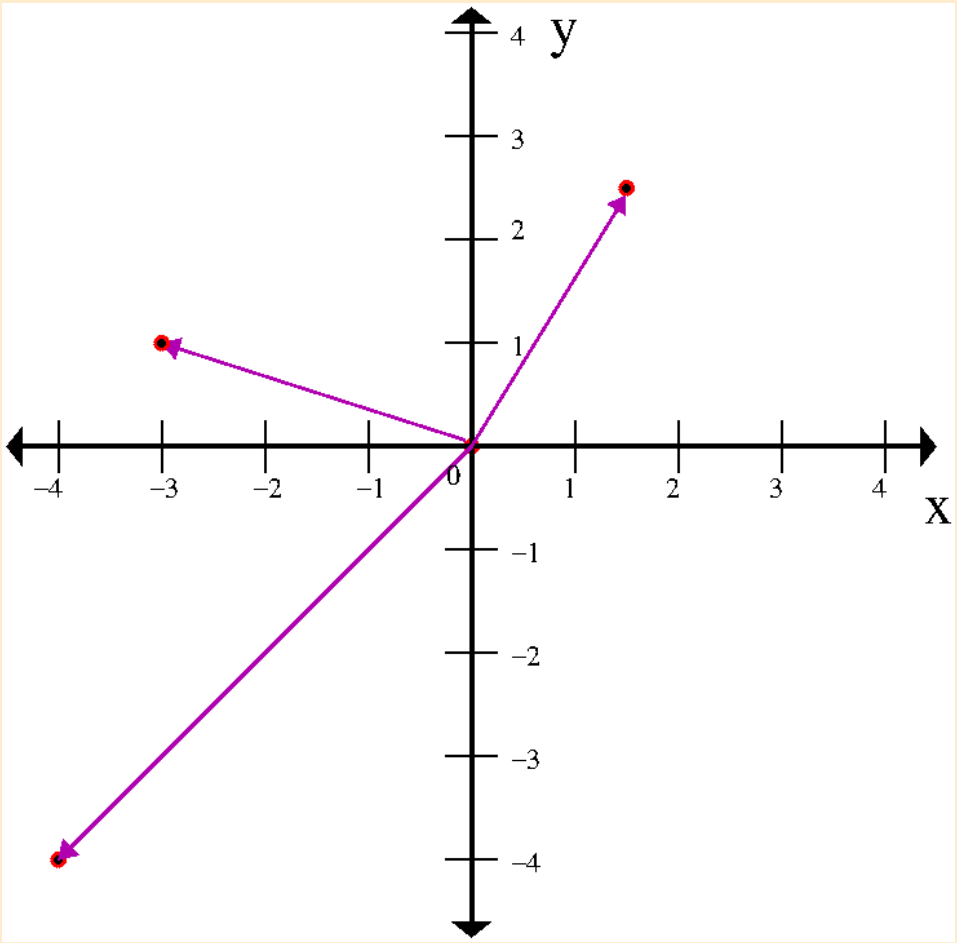

Notation

For reasons that will become clear we describe them as columns rather than rows.

$$\begin{pmatrix} 1.1 \\ 2.5 \end{pmatrix}, \begin{pmatrix} -3.7 \\ 0.34 \end{pmatrix} \tag{1.6}$$

As variables we use the following notation

$$\underline{v} = \begin{pmatrix} -4.67 \\ 0.76 \end{pmatrix} \tag{1.7}$$

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Notation contd.

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$$\mathbf{v} \quad , \quad \vec{v}$$

they all mean the same thing - a vector !

If we want a vector in row form then there is an explicit operation to get that.

$$\underline{v}^{\mathsf{T}} = \begin{pmatrix} -4.67 & 0.76 \end{pmatrix} \tag{1.8}$$

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Exercise

GO TO https://moodle.royalholloway.ac.uk/mod/quiz/view.php?id=462536 NOW