Lab 1 Topic 1 Block 2 Machine Learning

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State of contribution

Assignment1: Niklas Larsson

Assignment2: Hugo Knape

Assignment3: Zahra Jalilpour

Assignment 1: Ensembled methods

Task A

Generate test data:

```
library(randomForest)

## randomForest 4.6-14

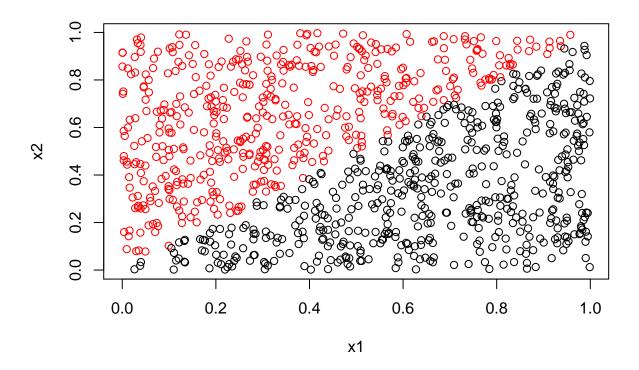
## Type rfNews() to see new features/changes/bug fixes.

##
## Attaching package: 'randomForest'

## The following object is masked from 'package:ggplot2':
##
## margin

set.seed(1234)

x1<-runif(1000)
x2<-runif(1000)
tedataA<-cbind(x1,x2)
y<-as.numeric(x1<x2)
telabelsA<-as.factor(y)
plot(x1,x2,col=(y+1))</pre>
```

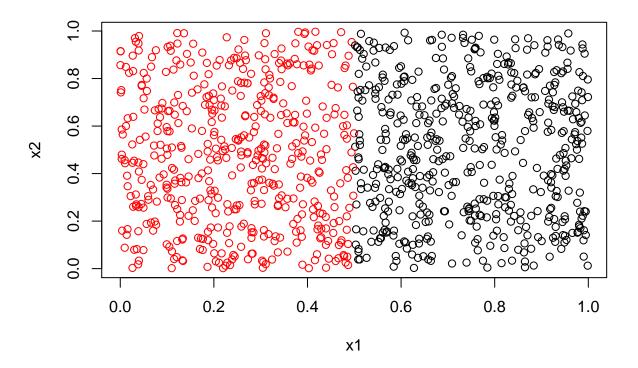


```
forestsize = c(1,10,100)
results = matrix(0,100,3)
for (i in 1:3){
  ntree = forestsize[i]
  for(j in 1:100){
    x1<-runif(100)
    x2<-runif(100)
    trdata<-cbind(x1,x2)</pre>
    y < -as.numeric(x1 < x2)
    trlabels<-as.factor(y)</pre>
    fitA = randomForest(trdata, trlabels, ntree=ntree, nodesize = 25, keep.forest = TRUE)
    predA = predict(fitA, tedataA)
    results[j,i] = sum(as.numeric(predA != telabelsA))/length(telabelsA)
  }
}
resdataA = matrix(0,3,2)
dimnames(resdataA) = list(c("[1]","[10]","[100]"),c("Mean","Variance"))
for(i in 1:3){
  resdataA[i,1] = mean(results[,i])
  resdataA[i,2] = var(results[,i])
```

$\label{eq:angle_bound} \textbf{Task B}$ New data generation

```
set.seed(1234)

x1<-runif(1000)
x2<-runif(1000)
tedataB<-cbind(x1,x2)
y<-as.numeric(x1<0.5)
telabelsB<-as.factor(y)
plot(x1,x2,col=(y+1))</pre>
```



```
forestsize = c(1,10,100)
results = matrix(0,100,3)

for (i in 1:3){
   ntree = forestsize[i]

   for(j in 1:100){
      x1<-runif(100)
      x2<-runif(100)
      trdata<-cbind(x1,x2)
      y<-as.numeric(x1<0.5)</pre>
```

```
trlabels<-as.factor(y)

fitB = randomForest(trdata, trlabels, ntree=ntree, nodesize = 25, keep.forest = TRUE)
  predB = predict(fitB, tedataB)
    results[j,i] = sum(as.numeric(predB != telabelsB))/length(telabelsB)
}

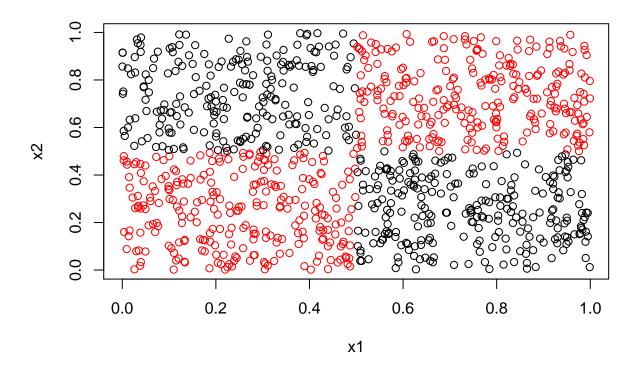
resdataB = matrix(0,3,2)
dimnames(resdataB) = list(c("[1]","[10]","[100]"),c("Mean","Variance"))
for(i in 1:3){
  resdataB[i,1] = mean(results[,i])
  resdataB[i,2] = var(results[,i])
}</pre>
```

Task C

New data generation

```
set.seed(1234)

x1<-runif(1000)
x2<-runif(1000)
tedataC<-cbind(x1,x2)
y<-as.numeric((x1<0.5 & x2<0.5) | (x1>0.5 & x2>0.5))
telabelsC<-as.factor(y)
plot(x1,x2,col=(y+1))</pre>
```



```
forestsize = c(1,10,100)
results = matrix(0,100,3)
for (i in 1:3){
  ntree = forestsize[i]
  for(j in 1:100){
    x1<-runif(100)
    x2<-runif(100)
    trdata<-cbind(x1,x2)</pre>
    y<-as.numeric( (x1<0.5 \& x2<0.5) | (x1>0.5 \& x2>0.5) )
    trlabels<-as.factor(y)</pre>
    fitC = randomForest(trdata, trlabels, ntree=ntree, nodesize = 12, keep.forest = TRUE)
    predC = predict(fitC, tedataC)
    results[j,i] = sum(as.numeric(predC != telabelsC))/length(telabelsC)
  }
}
resdataC = matrix(0,3,2)
dimnames(resdataC) = list(c("[1]","[10]","[100]"),c("Mean","Variance"))
for(i in 1:3){
  resdataC[i,1] = mean(results[,i])
  resdataC[i,2] = var(results[,i])
```

Task D

Question A

Question: What happens with the mean and variance of the error rate when the number of trees in the random forest grows?

As seen in all three cases the mean error rate decreases with an increasing number of trees in the forest. The same applies for the variance of the error except for case 1 where the variance increased just slightly between the 10- and 100-trees models.

```
print("Task A results:", quote = FALSE)
## [1] Task A results:
print(resdataA)
                     Variance
##
            Mean
         0.20471 0.0034907130
## [1]
## [10] 0.13291 0.0008548302
## [100] 0.11047 0.0009660900
print("Task B results:", quote = FALSE)
## [1] Task B results:
print(resdataB)
                     Variance
            Mean
         0.09324 0.0195394570
## [1]
## [10]
        0.01463 0.0003040334
## [100] 0.00692 0.0001089228
print("Task B results:", quote = FALSE)
## [1] Task B results:
print(resdataC)
##
            Mean
                    Variance
## [1]
         0.23886 0.011984303
## [10]
        0.11890 0.003328333
## [100] 0.07728 0.001224567
```

Question B

Due to the node size parameter. Using a smaller minimum node size allows the tree to grow more, in other words the model can divide the data into more specific sections where it can label data. Using smaller node size will need more computation power and could potentially overfit to the data while too large size does the opposite.

Question C

The variance is a measure which tells how much the resulting misclassification are deviating from the mean error. Having a lower variance gives a higher certainty regarding the mean error rate.

Assignment 2: Mixture models

```
EM_algo <- function(k_num){</pre>
set.seed(1234567890)
max_it <- 100 # max number of EM iterations</pre>
min_change <- 0.1 # min change in log likelihood between two consecutive EM iterations
N=1000 # number of training points
D=10 # number of dimensions
x <- matrix(nrow=N, ncol=D) # training data
true_pi <- vector(length = 3) # true mixing coefficients</pre>
true_mu <- matrix(nrow=3, ncol=D) # true conditional distributions</pre>
true_pi=c(1/3, 1/3, 1/3)
true_mu[1,]=c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true mu[2,]=c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
true_mu[3,]=c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
# Producing the training data
for(n in 1:N) {
 k <- sample(1:3,1,prob=true_pi)</pre>
 for(d in 1:D) {
    x[n,d] \leftarrow rbinom(1,1,true_mu[k,d])
}
K <- k num# number of quessed components
z <- matrix(nrow=N, ncol=K) # fractional component assignments
pi <- vector(length = K) # mixing coefficients</pre>
mu <- matrix(nrow=K, ncol=D) # conditional distributions</pre>
llik <- vector(length = max_it) # log likelihood of the EM iterations</pre>
# Random initialization of the paramters
pi \leftarrow runif(K, 0.49, 0.51)
pi <- pi / sum(pi)
for(k in 1:K) {
  mu[k,] \leftarrow runif(D,0.49,0.51)
}
#pi
#mu
for(it in 1:max_it) {
  #plot(mu[1,], type="o", col="blue", ylim=c(0,1))
  #points(mu[2,], type="o", col="red")
  #points(mu[3,], type="o", col="green")
  #points(mu[4,], type="o", col="yellow")
```

```
Sys.sleep(0.5)
  # E-step: Computation of the fractional component assignments
  for (j in 1:k) {
    for (i in 1:n) {
      z[i,j] \leftarrow pi[j]*prod((mu[j,]^x[i,])*((1-mu[j,])^(1-x[i,])))
  for(j in 1:nrow(z)) {
    z[j,] \leftarrow z[j,]/sum(z[j,])
  #Log likelihood computation.
  for(i in 1:n ){
    for(j in 1:k){
      llik[it] \leftarrow llik[it] + z[i,j] * (log(pi[j]) + sum(x[i,]) * log(mu[j,]) + (1- x[i,]) * log(1- mu[j,])
    }
  }
  cat("iteration: ", it, "log likelihood: ", llik[it], "\n")
  flush.console()
  # Stop if the lok likelihood has not changed significantly
  if(it > 1){
    if(( abs(llik[it] - llik[it-1]) < min_change)){</pre>
      break
    }
  }
  #M-step: ML parameter estimation from the data and fractional component assignments
  row_sum_z <- c(rep(NA, ncol(z)))</pre>
  for (i in 1:ncol(z)) {
    row_sum_z[i] \leftarrow sum(z[,i])
  pi <- row_sum_z/N
  mu <- t(z) %*% x /row_sum_z</pre>
return(list(pi = pi))
```

For the E-step for mixtures of multivariate Bernoulli distributions we compute:

$$p\left(z_{nk} \mid \boldsymbol{x}_{n}, \boldsymbol{\mu}, \boldsymbol{\pi}\right) = \frac{p\left(\boldsymbol{z}_{nk}, \boldsymbol{x}_{n} \mid \boldsymbol{\mu}, \boldsymbol{\pi}\right)}{\sum_{k} p\left(z_{nk}, \boldsymbol{x}_{n} \mid \boldsymbol{\mu}, \boldsymbol{\pi}\right)} = \frac{\pi_{k} p\left(\boldsymbol{x}_{n} \mid \boldsymbol{\mu}_{k}\right)}{\sum_{k} \pi_{k} p\left(\boldsymbol{x}_{n} \mid \boldsymbol{\mu}_{k}\right)}$$

for all n and k

where

$$p(\boldsymbol{x}_n \mid \boldsymbol{\mu}_k) = \prod_i \mu_{ki}^{x_i} (1 - \mu_{ki})^{(1-x_i)}$$

The code is:

```
# E-step: Computation of the fractional component assignments
for (j in 1:k) {
    for (i in 1:n) {
        z[i,j] <- pi[j]*prod((mu[j,]^x[i,])*((1-mu[j,])^(1-x[i,])))
    }
}
for(j in 1:nrow(z)) {
    z[j,] <- z[j,]/sum(z[j,])
}</pre>
```

For computing the Log likelihood we use:

$$\sum_{n} \sum_{k} p\left(z_{nk} \mid \boldsymbol{x}_{n}, \boldsymbol{\mu}, \boldsymbol{\pi}\right) \left[\log \pi_{k} + \sum_{i} \left[x_{ni} \log \mu_{ki} + (1 - x_{ni}) \log \left(1 - \mu_{ki}\right)\right] \right]$$

The code is:

```
#Log likelihood computation.
for(i in 1:n ){
  for(j in 1:k){
    llik[it] <- llik[it] + z[i,j] * (log(pi[j]) + sum(x[i,] * log(mu[j,]) + (1- x[i,])*log(1- mu[j,])
  }
}</pre>
```

ML parameter estimation from the data and fractional component assignments we use:

$$\pi_k^{ML} = \frac{\sum_n p\left(z_{nk} \mid \boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi}\right)}{N}$$
$$\mu_{ki}^{ML} = \frac{\sum_n x_{ni} p\left(z_{nk} \mid \boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi}\right)}{\sum_n p\left(z_{nk} \mid \boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi}\right)}$$

The code is:

```
#M-step: ML parameter estimation from the data and fractional component assignments
row_sum_z <- c(rep(NA, ncol(z)))
for (i in 1:ncol(z)) {
   row_sum_z[i] <- sum(z[,i])
}</pre>
```

EM_algo(2)

```
-7623.873
## iteration: 1 log likelihood:
## iteration:
              2 log likelihood:
                                 -7621.944
                                 -7620.533
## iteration: 3 log likelihood:
## iteration: 4 log likelihood:
                                 -7609.638
## iteration: 5 log likelihood:
                                 -7532.2
## iteration: 6 log likelihood:
                                 -7173.56
## iteration: 7 log likelihood:
                                 -6661.821
## iteration: 8 log likelihood:
                                 -6520.028
## iteration: 9 log likelihood:
                                 -6503.563
## iteration: 10 log likelihood:
                                 -6499.807
## iteration: 11 log likelihood:
                                  -6498.296
```

```
12 log likelihood:
                                    -6497.535
## iteration:
## iteration:
               13 log likelihood:
                                    -6497.12
## iteration:
               14 log likelihood:
                                    -6496.883
               15 log likelihood:
## iteration:
                                    -6496.745
## iteration:
               16 log likelihood:
                                    -6496.662
## $pi
## [1] 0.4981919 0.5018081
```

When K is equal to 2 the EM-algorithm stops after 16 iterations and the pi values are very close to each other. In this case we miss one true pi, which maybe can be seen as under fitting.

EM_algo(3)

```
1 log likelihood:
                                   -8029.723
## iteration:
               2 log likelihood:
## iteration:
                                   -8027.183
## iteration:
               3 log likelihood:
                                   -8024.696
               4 log likelihood:
## iteration:
                                   -8005.631
## iteration:
               5 log likelihood:
                                   -7877.606
## iteration:
               6 log likelihood:
                                   -7403.513
               7 log likelihood:
## iteration:
                                   -6936.919
                                   -6818.582
## iteration:
               8 log likelihood:
## iteration:
               9 log likelihood:
                                    -6791.377
## iteration:
               10 log likelihood:
                                    -6780.713
## iteration:
               11 log likelihood:
                                    -6774.958
## iteration:
               12 log likelihood:
                                    -6771.261
## iteration:
               13 log likelihood:
                                    -6768.606
## iteration:
               14 log likelihood:
                                    -6766.535
## iteration:
               15 log likelihood:
                                    -6764.815
               16 log likelihood:
## iteration:
                                    -6763.316
               17 log likelihood:
## iteration:
                                    -6761.967
## iteration:
               18 log likelihood:
                                    -6760.727
               19 log likelihood:
## iteration:
                                    -6759.572
               20 log likelihood:
## iteration:
                                    -6758.491
               21 log likelihood:
## iteration:
                                    -6757.475
## iteration:
               22 log likelihood:
                                    -6756.521
## iteration:
               23 log likelihood:
                                    -6755.625
               24 log likelihood:
## iteration:
                                    -6754.784
## iteration:
               25 log likelihood:
                                    -6753.996
## iteration:
               26 log likelihood:
                                    -6753.26
## iteration:
               27 log likelihood:
                                    -6752.571
## iteration:
               28 log likelihood:
                                    -6751.928
               29 log likelihood:
## iteration:
                                    -6751.328
## iteration:
               30 log likelihood:
                                    -6750.768
## iteration:
               31 log likelihood:
                                    -6750.246
## iteration:
               32 log likelihood:
                                    -6749.758
               33 log likelihood:
## iteration:
                                    -6749.304
               34 log likelihood:
                                    -6748.88
## iteration:
               35 log likelihood:
                                    -6748.484
## iteration:
## iteration:
               36 log likelihood:
                                    -6748.114
## iteration:
               37 log likelihood:
                                    -6747.767
               38 log likelihood:
## iteration:
                                    -6747.444
               39 log likelihood:
## iteration:
                                    -6747.14
```

```
## iteration:
               40 log likelihood:
                                    -6746.856
## iteration:
               41 log likelihood:
                                    -6746.589
               42 log likelihood:
## iteration:
                                    -6746.338
               43 log likelihood:
## iteration:
                                    -6746.102
## iteration:
               44 log likelihood:
                                    -6745.88
## iteration:
               45 log likelihood:
                                    -6745.67
## iteration:
               46 log likelihood:
                                    -6745.472
## iteration:
               47 log likelihood:
                                    -6745.285
## iteration:
               48 log likelihood:
                                    -6745.108
## iteration:
               49 log likelihood:
                                    -6744.939
## iteration:
               50 log likelihood:
                                    -6744.78
## iteration:
               51 log likelihood:
                                    -6744.627
## iteration:
               52 log likelihood:
                                    -6744.483
## iteration:
               53 log likelihood:
                                    -6744.344
               54 log likelihood:
## iteration:
                                    -6744.212
## iteration:
               55 log likelihood:
                                    -6744.086
               56 log likelihood:
                                    -6743.964
## iteration:
               57 log likelihood:
                                    -6743.848
## iteration:
## iteration:
               58 log likelihood:
                                    -6743.736
## iteration:
               59 log likelihood:
                                    -6743.628
## iteration:
               60 log likelihood:
                                    -6743.524
               61 log likelihood:
## iteration:
                                    -6743.423
               62 log likelihood:
## iteration:
                                    -6743.326
## $pi
## [1] 0.3259592 0.3044579 0.3695828
```

When K is equal to 3 the EM-algorithm stops after 62 iterations and the pi values are pretty close to each other.

EM_algo(4)

```
## iteration:
               1 log likelihood:
                                   -8317.187
## iteration:
               2 log likelihood:
                                   -8314.81
## iteration:
               3 log likelihood:
                                   -8312.256
## iteration:
               4 log likelihood:
                                   -8292.606
## iteration:
               5 log likelihood:
                                   -8159.059
               6 log likelihood:
## iteration:
                                   -7666.637
## iteration:
               7 log likelihood:
                                   -7196.701
## iteration:
               8 log likelihood:
                                   -7061.15
## iteration:
               9 log likelihood:
                                   -7018.948
## iteration:
               10 log likelihood:
                                    -6999.971
               11 log likelihood:
## iteration:
                                    -6989.735
## iteration:
               12 log likelihood:
                                    -6983.5
## iteration:
               13 log likelihood:
                                    -6979.315
## iteration:
               14 log likelihood:
                                    -6976.279
               15 log likelihood:
## iteration:
                                    -6973.932
               16 log likelihood:
                                    -6972.026
## iteration:
## iteration:
               17 log likelihood:
                                    -6970.415
               18 log likelihood:
                                    -6969.009
## iteration:
## iteration:
               19 log likelihood:
                                    -6967.751
               20 log likelihood:
## iteration:
                                    -6966.598
               21 log likelihood:
## iteration:
                                    -6965.517
```

```
22 log likelihood:
                                     -6964.48
## iteration:
## iteration:
               23 log likelihood:
                                     -6963.457
## iteration:
               24 log likelihood:
                                     -6962.415
               25 log likelihood:
## iteration:
                                     -6961.313
## iteration:
               26 log likelihood:
                                     -6960.098
## iteration:
               27 log likelihood:
                                     -6958.703
               28 log likelihood:
## iteration:
                                     -6957.042
## iteration:
               29 log likelihood:
                                     -6955.01
## iteration:
               30 log likelihood:
                                     -6952.485
## iteration:
               31 log likelihood:
                                     -6949.342
## iteration:
               32 log likelihood:
                                     -6945.475
## iteration:
               33 log likelihood:
                                     -6940.834
               34 log likelihood:
                                     -6935.458
## iteration:
## iteration:
               35 log likelihood:
                                     -6929.501
               36 log likelihood:
## iteration:
                                     -6923.217
               37 log likelihood:
                                     -6916.917
## iteration:
               38 log likelihood:
                                     -6910.896
## iteration:
               39 log likelihood:
                                     -6905.381
## iteration:
## iteration:
               40 log likelihood:
                                     -6900.502
## iteration:
               41 log likelihood:
                                     -6896.299
## iteration:
               42 log likelihood:
                                     -6892.745
               43 log likelihood:
## iteration:
                                     -6889.776
               44 log likelihood:
## iteration:
                                     -6887.313
               45 log likelihood:
## iteration:
                                     -6885.273
## iteration:
               46 log likelihood:
                                     -6883.583
## iteration:
               47 log likelihood:
                                     -6882.178
               48 log likelihood:
## iteration:
                                     -6881.007
## iteration:
               49 log likelihood:
                                     -6880.024
## iteration:
               50 log likelihood:
                                     -6879.196
               51 log likelihood:
                                     -6878.494
## iteration:
## iteration:
               52 log likelihood:
                                     -6877.895
               53 log likelihood:
                                     -6877.383
## iteration:
## iteration:
               54 log likelihood:
                                     -6876.941
               55 log likelihood:
                                     -6876.56
## iteration:
               56 log likelihood:
## iteration:
                                     -6876.228
## iteration:
               57 log likelihood:
                                     -6875.939
## iteration:
               58 log likelihood:
                                     -6875.687
               59 log likelihood:
## iteration:
                                     -6875.465
               60 log likelihood:
## iteration:
                                     -6875.27
               61 log likelihood:
## iteration:
                                     -6875.098
               62 log likelihood:
## iteration:
                                     -6874.947
               63 log likelihood:
## iteration:
                                     -6874.813
## iteration:
               64 log likelihood:
                                     -6874.694
               65 log likelihood:
  iteration:
                                     -6874.59
## iteration:
               66 log likelihood:
                                     -6874.497
## $pi
## [1] 0.1614155 0.1383613 0.3609912 0.3392319
```

When K is equal to 4 the EM-algorithm stops after 16 iterations and the pi values are pretty far from each other. In this task we have one extra pi, which maybe can be seen as over fitting.

Assignment3. High Dimentional Methods

1 Diviving data to train and test(70/30) and performing NSC classification.

Then run to train a nearest shrunken centroid classifier.(NSC) For 30 different values of the threshold the number of nonzero genes and the number of misclassifications on the training set will be listed

model

```
## Call:
## pamr.train(data = mydata)
##
      threshold nonzero errors
## 1
       0.000
                 2058
                         18
## 2
       0.801
                 1040
                         21
## 3
       1.603
                  374
                         23
## 4
       2.404
                  226
                         25
       3.206
## 5
                  147
                         21
## 6
       4.007
                  102
                         20
## 7
       4.809
                   78
                         19
## 8
       5.610
                   54
                         20
## 9
       6.412
                   35
                         20
## 10
      7.213
                   25
                         19
## 11
       8.014
                   19
                         18
       8.816
## 12
                         19
                   11
## 13
       9.617
                   10
                         27
## 14 10.419
                    8
                         40
## 15 11.220
                    7
                         73
## 16 12.022
                    4
                         72
## 17 12.823
                    4
                         72
## 18 13.624
                    3
                         72
## 19 14.426
                    2
                         73
## 20 15.227
                    1
                         73
## 21 16.029
                    1
                         73
## 22 16.830
                         72
                    1
## 23 17.632
                    1
                         72
## 24 18.433
                         72
## 25 19.235
                    1
                         72
## 26 20.036
                         70
## 27 20.837
                         70
                    1
## 28 21.639
                    1
                         70
## 29 22.440
                    1
                         69
## 30 23.242
                         137
```

cvmodel

```
## Call:
## pamr.cv(fit = model, data = mydata)
##
      threshold nonzero errors
## 1
       0.000
                 2058
                         24
       0.801
## 2
                 1040
                         22
## 3
       1.603
                 374
                         25
## 4
       2.404
                 226
                         25
## 5
       3.206
                  147
                         25
```

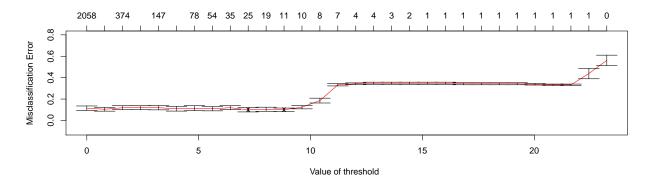
```
## 6
       4.007
                   102
                           23
## 7
       4.809
                    78
                           24
## 8
       5.610
                    54
                           23
                    35
## 9
       6.412
                           25
## 10
       7.213
                    25
                           21
## 11
       8.014
                    19
                           22
## 12
       8.816
                    11
                           21
                    10
## 13
       9.617
                           26
## 14 10.419
                     8
                           39
                     7
                           70
## 15 11.220
## 16 12.022
                     4
                           72
                     4
## 17 12.823
                           73
                     3
                           73
## 18 13.624
                     2
## 19 14.426
                           73
## 20 15.227
                     1
                           73
## 21 16.029
                     1
                           73
## 22 16.830
                     1
                           72
## 23 17.632
                     1
                           72
## 24 18.433
                     1
                           72
## 25 19.235
                     1
                           72
## 26 20.036
                     1
                           71
## 27 20.837
                     1
                           70
## 28 21.639
                     1
                           70
## 29 22.440
                     1
                           92
## 30 23.242
                     0
                           118
```

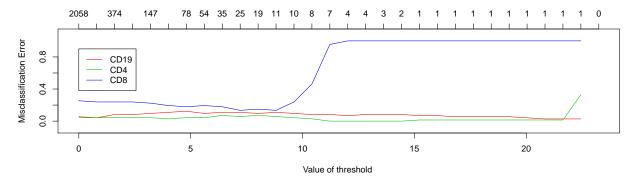
The output of this function looks very similar to the model. The numbers in the errors column are now the summed errors of all 10 cross validation steps. The CV error usually is bigger than the training error.

Missclassification Error Plot. In both figures, the x-axis represents different values of threshold (corresponding to different numbers of nonzero genes as shown on top of each figure and the y-axis shows the number of misclassifications. The upper figure describes the whole dataset, the lower one describes each class individually

```
pamr.plotcv(cvmodel)
```







Minimum threshold:

[1] "Best threshold equals: 7.2129439923563"

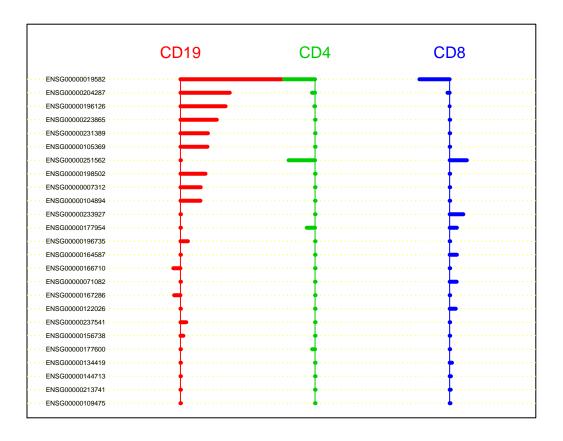
[1] "Number of features: 25"

The value of 7.2 of the threshold has the lowest error of cv.model.

Centroid plot(threshold = 7.2):

The function pamr.plotcen() plots the shrunken class centroids for each class.NSC classification shrink each of the class centroid toward the overall centroid for all classes by threshold. This shrinkage consists of moving the centroid toward zero by threshold, setting it equal to zero if it hits zero. Here the threshold is 7.2, hence the class centroid would be shrink be minus by the threshold. It is the reason that we see positive and negative values in the centroid plot.

pamr.plotcen(model, mydata, threshold = best_Th)



Number of genes selected by this model:

```
id CD19-score CD4-score CD8-score
   [1,] 2
             1.5153
                        -0.4617
                                  -0.4489
   [2,] 15 0.7328
                        -0.0448
                                  -0.032
                        -0.0105
## [3,] 31 0.6715
                                  -0.001
## [4,] 32 0.5403
## [5,] 37 0.4089
                                  0
                        0
## [6,] 138 0.4019
                        0
                                  0
## [7,] 1
                        -0.3933
            0
                                  0.2553
## [8,] 90 0.3727
                                  0
                        0
## [9,] 172 0.3016
                        0
                                  0
## [10,] 126 0.2974
                        0
## [11,] 79 0
                                  0.2028
## [12,] 11 0
                        -0.1239
                                  0.1074
## [13,] 110 0.1109
                        0
                                  0
## [14,] 21 0
                                  0.1078
                        0
## [15,] 3
            -0.1057
## [16,] 50 0
                        0
                                  0.1051
## [17,] 207 -0.0933
                        0
## [18,] 29 0
                        0
                                  0.0914
## [19,] 192 0.0862
## [20,] 309 0.0438
                                  0
## [21,] 28 0
                        -0.0426
## [22,] 38 0
                        0
                                  0.0346
## [23,] 18 0
                        0
                                  0.0189
## [24,] 127 0
                                  0.0156
```

```
## [25,] 36 0
                                   0.0119
## [1] 25
paste("Number of genes selected by this model : ", nrow(a))
## [1] "Number of genes selected by this model :
task_2 names of the 2 most contributing genes
            CD19-score CD4-score CD8-score
    [1,] 2
             1.5153
                         -0.4617
                                   -0.4489
##
    [2,] 15
             0.7328
                         -0.0448
                                   -0.032
##
                         -0.0105
    [3,] 31
                                   -0.001
##
             0.6715
##
    [4,] 32
             0.5403
                         0
                                   0
##
    [5,] 37
             0.4089
                         0
                                   0
##
    [6,] 138 0.4019
                         0
                                   0
   [7,] 1
                         -0.3933
##
                                   0.2553
   [8,] 90 0.3727
                         0
                                   0
##
    [9,] 172 0.3016
##
                         0
                                   0
## [10,] 126 0.2974
                         0
                                   0
## [11,] 79
             0
                         0
                                   0.2028
                         -0.1239
                                   0.1074
## [12,] 11
             0
## [13,] 110 0.1109
                         0
                                   0.1078
## [14,] 21 0
                         0
## [15,] 3
             -0.1057
                         0
## [16,] 50 0
                         0
                                   0.1051
## [17,] 207 -0.0933
                         0
                                   0
                                   0.0914
## [18,] 29 0
                         0
## [19,] 192 0.0862
                         0
                                   0
## [20,] 309 0.0438
                         0
                                   0
## [21,] 28
                         -0.0426
                                   0
             0
## [22,] 38
                         0
                                   0.0346
## [23,] 18
                         0
                                   0.0189
             0
## [24,] 127 0
                         0
                                   0.0156
## [25,] 36 0
                                   0.0119
## [1] "ENSG00000019582" "ENSG00000204287"
paste("Names of the two most contributing genes: ",mydata$genenames[k] )
```

[1] "Names of the two most contributing genes: ENSG00000019582" "Names of the two most contributing

The name of two most contributing genes are, " CD74" and "HLA-DRA". And both of them are considered as marker genes.

Confusion Matrix. Here we see 73 samples belong to class CD19, 65 sample is classified correctly and 8 are missclassified as CD4. 70 samples belong to CD4. 65 sample is classified correctly and 5 are missclassified as CD8 67 samples belong to CD8, 59 is classified correctly and 8 is missclassified as CD4. This makes an overall error rate 0.1%.

```
pamr.confusion(cvmodel, threshold = best_Th)
        CD19 CD4 CD8 Class Error rate
##
                            0.10958904
## CD19
          65
               8
                    0
## CD4
                             0.05714286
           0 66
                    4
## CD8
           0
               9 58
                             0.13432836
## Overall error rate= 0.1
reporting test error.
## [1] "Confusion Matrix for test data :"
##
         pam.diagnosis
## y_test CD19 CD4 CD8
##
     CD19
            27
                  0
                      0
##
     CD4
             0
                26
                     4
##
     CD8
             0
                  6 27
## ME rate for test data :
                               0.1111111111111111
It seems that 27 samples are classified correctly to CD19, 30 samples belong to CD4, in which 4 samples are
miss classified. 33 samples belong to CD8,6 samples from them are miss qualified .
task_3
library(glmnet)
Elastic net with multinomial response:
## Loading required package: Matrix
## Loaded glmnet 4.0-2
set.seed(12345)
x_e <- as.matrix(train[, -ncol(train)])</pre>
y_e <- as.matrix(train[[ncol(train)]])</pre>
# fit the elastic net
elastic <- cv.glmnet(x_e, y_e, alpha = 0.5, family = "multinomial")</pre>
testx=as.matrix((test[ , -ncol(test)]))
predenet=predict(elastic, newx = testx, s = elastic$lambda.min, type = "class")
cm_elastic_net <- table(y_test, predenet)</pre>
cm_elastic_net
##
         predenet
## y_test CD19 CD4 CD8
            27
                 0
##
     CD19
                      0
     CD4
             0
                29
##
                     1
```

3 29

1

##

CD8

```
test_error_elasic_net <- 1-sum(diag(cm_elastic_net))/sum(cm_elastic_net)</pre>
coef1 <-coef(elastic, s="lambda.min")</pre>
num_feature<-length(coef1$CD8@x)+length(coef1$CD19@x)+length(coef1$CD4@x)
cat(paste("ME rate for test data : ", test_error_elasic_net))
## ME rate for test data :
                             0.05555555555556
paste("Number of feature :",num_feature )
## [1] "Number of feature : 163"
library("kernlab")
SVM:
##
## Attaching package: 'kernlab'
## The following object is masked from 'package:ggplot2':
##
##
       alpha
set.seed(12345)
train.x <-as.matrix(train[, -ncol(train)])</pre>
train.y <- as.matrix(train[[ncol(train)]])</pre>
svm.trained <- ksvm(train.x, train.y , type = "C-svc", kernel='vanilladot')</pre>
## Setting default kernel parameters
## Warning in .local(x, ...): Variable(s) '' constant. Cannot scale data.
svm_pred <- predict(svm.trained, testx)</pre>
cm_svm <- table(y_test, svm_pred)</pre>
cm_svm
##
         svm_pred
## y_test CD19 CD4 CD8
    CD19 27
##
               0 0
##
     CD4
            0 30 0
     CD8
             0 2 31
##
test_error_svm = 1-sum(diag(cm_svm))/sum(cm_svm)
paste("number of feature:", (ncol(df)-1))
## [1] "number of feature: 2085"
```

```
cat(paste("ME rate for test data : ", test_error_svm))
```

ME rate for test data : 0.02222222222223

Table 1: Comparsion of Three models

	Nearest Shrunken Centroid Model	Elastic_Net Model	SVM Model
Number of Features	25.0000000	163.0000000	2.08500e+03
Error rate	0.1111111	0.0555556	2.2222e-02

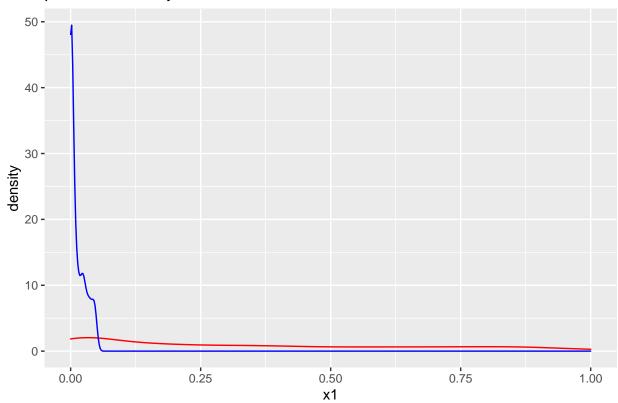
Analysis: By comparison three different models, we can see the SVM has the lowest error rate compare two other models, but the this model select 2085 features for classifying three different classes. The error rate in Elastic net model is lower than NSC.but number of features in three classes are more than NSC. As the difference between error rate in NSC and Elastic net could be ignored, by considering number of features, NSC could be a good model for classification. #### task4: Benjamin Hochberg

[1] "number of features in CD4: 513"

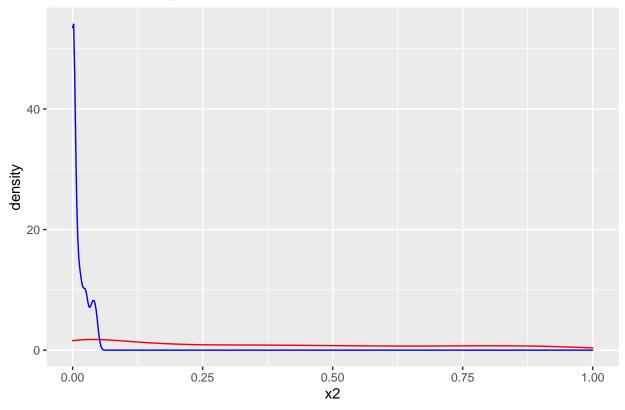
[1] "number of features in CD8: 430"

[1] "number of features in CD19: 685"

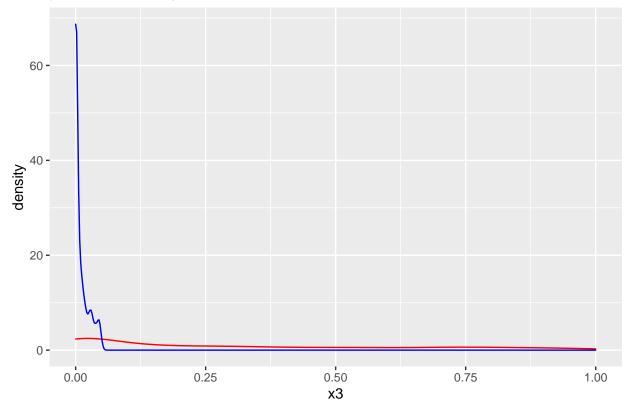
p_values and rejected area for CD4



p_values and rejected area for CD8



p_values and rejected area for CD19



The Benjamin_Hochberg method is implemented. By using t.test, varoius p_value are calculated for three different classes. The number of features that are not related to three classes are more than three other models.

Appendix:

```
knitr::opts_chunk$set(echo = TRUE)
library(kknn)
library(ggplot2)

library(randomForest)
set.seed(1234)

x1<-runif(1000)
x2<-runif(1000)
tedataA<-cbind(x1,x2)
y<-as.numeric(x1<x2)
telabelsA<-as.factor(y)
plot(x1,x2,col=(y+1))

forestsize = c(1,10,100)
results = matrix(0,100,3)</pre>
```

```
for (i in 1:3){
  ntree = forestsize[i]
  for(j in 1:100){
    x1<-runif(100)
    x2<-runif(100)
    trdata<-cbind(x1,x2)
    y<-as.numeric(x1<x2)
    trlabels<-as.factor(y)</pre>
    fitA = randomForest(trdata, trlabels, ntree=ntree, nodesize = 25, keep.forest = TRUE)
    predA = predict(fitA, tedataA)
    results[j,i] = sum(as.numeric(predA != telabelsA))/length(telabelsA)
}
resdataA = matrix(0,3,2)
dimnames(resdataA) = list(c("[1]","[10]","[100]"),c("Mean","Variance"))
for(i in 1:3){
  resdataA[i,1] = mean(results[,i])
  resdataA[i,2] = var(results[,i])
set.seed(1234)
x1<-runif(1000)
x2<-runif(1000)
tedataB<-cbind(x1,x2)
y < -as.numeric(x1 < 0.5)
telabelsB<-as.factor(y)</pre>
plot(x1,x2,col=(y+1))
forestsize = c(1,10,100)
results = matrix(0,100,3)
for (i in 1:3){
  ntree = forestsize[i]
  for(j in 1:100){
    x1<-runif(100)
    x2<-runif(100)
    trdata<-cbind(x1,x2)
    y < -as.numeric(x1 < 0.5)
    trlabels<-as.factor(y)</pre>
    fitB = randomForest(trdata, trlabels, ntree=ntree, nodesize = 25, keep.forest = TRUE)
    predB = predict(fitB, tedataB)
    results[j,i] = sum(as.numeric(predB != telabelsB))/length(telabelsB)
  }
}
resdataB = matrix(0,3,2)
dimnames(resdataB) = list(c("[1]","[10]","[100]"),c("Mean","Variance"))
```

```
for(i in 1:3){
  resdataB[i,1] = mean(results[,i])
  resdataB[i,2] = var(results[,i])
set.seed(1234)
x1<-runif(1000)
x2<-runif(1000)
tedataC<-cbind(x1,x2)
y < -as.numeric((x1 < 0.5 & x2 < 0.5) | (x1 > 0.5 & x2 > 0.5))
telabelsC<-as.factor(y)</pre>
plot(x1,x2,col=(y+1))
forestsize = c(1,10,100)
results = matrix(0,100,3)
for (i in 1:3){
  ntree = forestsize[i]
 for(j in 1:100){
    x1<-runif(100)
    x2<-runif(100)
    trdata<-cbind(x1,x2)</pre>
    y<-as.numeric((x1<0.5 & x2<0.5) | (x1>0.5 & x2>0.5))
    trlabels<-as.factor(y)</pre>
    fitC = randomForest(trdata, trlabels, ntree=ntree, nodesize = 12, keep.forest = TRUE)
    predC = predict(fitC, tedataC)
    results[j,i] = sum(as.numeric(predC != telabelsC))/length(telabelsC)
  }
}
resdataC = matrix(0,3,2)
dimnames(resdataC) = list(c("[1]","[10]","[100]"),c("Mean","Variance"))
for(i in 1:3){
  resdataC[i,1] = mean(results[,i])
 resdataC[i,2] = var(results[,i])
}
print("Task A results:", quote = FALSE)
print(resdataA)
print("Task B results:", quote = FALSE)
print(resdataB)
print("Task B results:", quote = FALSE)
print(resdataC)
EM_algo <- function(k_num){</pre>
set.seed(1234567890)
max_it <- 100 # max number of EM iterations</pre>
min_change <- 0.1 # min change in log likelihood between two consecutive EM iterations
N=1000 # number of training points
```

```
D=10 # number of dimensions
x <- matrix(nrow=N, ncol=D) # training data
true_pi <- vector(length = 3) # true mixing coefficients</pre>
true_mu <- matrix(nrow=3, ncol=D) # true conditional distributions</pre>
true_pi=c(1/3, 1/3, 1/3)
true_mu[1,]=c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,]=c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
true_mu[3,]=c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
# Producing the training data
for(n in 1:N) {
  k <- sample(1:3,1,prob=true_pi)</pre>
  for(d in 1:D) {
    x[n,d] <- rbinom(1,1,true_mu[k,d])
}
K <- k_num# number of guessed components
z <- matrix(nrow=N, ncol=K) # fractional component assignments
pi <- vector(length = K) # mixing coefficients</pre>
mu <- matrix(nrow=K, ncol=D) # conditional distributions</pre>
llik <- vector(length = max_it) # log likelihood of the EM iterations</pre>
# Random initialization of the paramters
pi \leftarrow runif(K, 0.49, 0.51)
pi <- pi / sum(pi)
for(k in 1:K) {
  mu[k,] \leftarrow runif(D,0.49,0.51)
#pi
#mu
for(it in 1:max it) {
  #plot(mu[1,], type="o", col="blue", ylim=c(0,1))
  #points(mu[2,], type="o", col="red")
  #points(mu[3,], type="o", col="green")
  #points(mu[4,], type="o", col="yellow")
  Sys.sleep(0.5)
  # E-step: Computation of the fractional component assignments
  for (j in 1:k) {
    for (i in 1:n) {
      z[i,j] <- pi[j]*prod((mu[j,]^x[i,])*((1-mu[j,])^(1-x[i,])))
  }
  for(j in 1:nrow(z)) {
    z[j,] \leftarrow z[j,]/sum(z[j,])
  #Log likelihood computation.
  for(i in 1:n ){
```

```
for(j in 1:k){
      llik[it] \leftarrow llik[it] + z[i,j] * (log(pi[j]) + sum(x[i,]) * log(mu[j,]) + (1- x[i,]) * log(1- mu[j,])
    }
  }
  cat("iteration: ", it, "log likelihood: ", llik[it], "\n")
  flush.console()
  # Stop if the lok likelihood has not changed significantly
  if(it > 1){
    if(( abs(llik[it]- llik[it-1]) < min_change)){</pre>
      break
    }
  }
  #M-step: ML parameter estimation from the data and fractional component assignments
  row_sum_z <- c(rep(NA, ncol(z)))</pre>
  for (i in 1:ncol(z)) {
    row_sum_z[i] \leftarrow sum(z[,i])
  pi <- row_sum_z/N
  mu <- t(z) %*% x /row_sum_z</pre>
return(list(pi = pi))
  # E-step: Computation of the fractional component assignments
  for (j in 1:k) {
    for (i in 1:n) {
      z[i,j] <- pi[j]*prod((mu[j,]^x[i,])*((1-mu[j,])^(1-x[i,])))
  }
  for(j in 1:nrow(z)) {
   z[j,] \leftarrow z[j,]/sum(z[j,])
  #Log likelihood computation.
  for(i in 1:n ){
    for(j in 1:k){
      llik[it] \leftarrow llik[it] + z[i,j] * (log(pi[j]) + sum(x[i,]) * log(mu[j,]) + (1- x[i,]) * log(1- mu[j,])
    }
  }
  #M-step: ML parameter estimation from the data and fractional component assignments
  row_sum_z <- c(rep(NA, ncol(z)))</pre>
  for (i in 1:ncol(z)) {
    row_sum_z[i] \leftarrow sum(z[,i])
  }
EM_algo(2)
EM_algo(3)
EM_algo(4)
setwd("D:/Linkoping university/first semester/Machine Learning/lab/block2")
new_df <- read.csv("geneexp.csv")</pre>
```

```
df = new_df[,-1] # the first and the last are factor
set.seed(12345)
m <- nrow(df)
# divide data to train and test(70:30)
id <- sample(1:m, floor(m*0.7))</pre>
train <- df[id,]</pre>
test <- df[-id,]</pre>
### train ###
rownames(train) = 1:nrow(train)
x = t(train[, -ncol(train)])
y = train[[ncol(train)]]
mydata <- list(x=x, y=as.factor(y),geneid=as.character(1:nrow(x)),genenames=rownames(x))</pre>
### test ###
rownames(test) = 1:nrow(test)
x_test = t(test[, -ncol(test)])
y_test = test[[ncol(test)]]
test_mydata <- list(x=x_test, y=as.factor(y_test),geneid=as.character(1:nrow(x_test)),genenames=rowname
library(pamr)
model = pamr.train(mydata)
# choice the threshold by cv
cvmodel = pamr.cv(model,mydata)
model
cvmodel
pamr.plotcv(cvmodel)
min_Error <- which.min(cvmodel$error)</pre>
best_Th <- cvmodel$threshold[min_Error]</pre>
paste("Best threshold equals: ", best_Th)
numberofgen <- model$nonzero[min_Error]</pre>
paste("Number of features: ", numberofgen)
pamr.plotcen(model, mydata, threshold = best_Th)
a<-pamr.listgenes(model, mydata, threshold = best_Th, genenames=FALSE)</pre>
nrow(a)
paste("Number of genes selected by this model : ", nrow(a))
a<-(pamr.listgenes(model, mydata, threshold = best_Th))</pre>
top 2 = a[1:2,1]
k <- as.numeric(top_2)</pre>
mydata$genenames[k]
paste("Names of the two most contributing genes: ",mydata$genenames[k] )
pamr.confusion(cvmodel, threshold = best_Th)
pam.diagnosis <- pamr.predict(model, x_test, threshold = best_Th)</pre>
new_matrix <- table(y_test, pam.diagnosis)</pre>
paste("Confusion Matrix for test data :")
new_matrix
test_ME <- 1-(sum(diag(new_matrix)))/sum(new_matrix)</pre>
```

```
cat(paste("ME rate for test data : ", test_ME))
library(glmnet)
set.seed(12345)
x_e <- as.matrix(train[, -ncol(train)])</pre>
y_e <- as.matrix(train[[ncol(train)]])</pre>
# fit the elastic net
elastic <- cv.glmnet(x_e, y_e, alpha = 0.5, family = "multinomial")</pre>
testx=as.matrix((test[ , -ncol(test)]))
predenet=predict(elastic, newx = testx, s = elastic$lambda.min, type = "class")
cm_elastic_net <- table(y_test, predenet)</pre>
cm elastic net
test_error_elasic_net <- 1-sum(diag(cm_elastic_net))/sum(cm_elastic_net)</pre>
coef1 <-coef(elastic, s="lambda.min")</pre>
num_feature<-length(coef1\$CD8\circ{0}{x})+length(coef1\$CD19\circ{0}{x})+length(coef1\$CD4\circ{0}{x})
cat(paste("ME rate for test data : ", test_error_elasic_net))
paste("Number of feature :",num_feature )
library("kernlab")
set.seed(12345)
train.x <-as.matrix(train[, -ncol(train)])</pre>
train.y <- as.matrix(train[[ncol(train)]])</pre>
svm.trained <- ksvm(train.x, train.y , type = "C-svc", kernel='vanilladot')</pre>
svm_pred <- predict(svm.trained, testx)</pre>
cm_svm <- table(y_test, svm_pred)</pre>
cm svm
test_error_svm = 1-sum(diag(cm_svm))/sum(cm_svm)
paste("number of feature:", (ncol(df)-1))
cat(paste("ME rate for test data : ", test_error_svm))
final_result <- cbind(numberofgen, num_feature, (ncol(df)-1))</pre>
feature_count <- cbind(test_ME, test_error_elasic_net,test_error_svm)</pre>
final_matrix <- rbind(final_result,feature_count)</pre>
colnames(final_matrix) <- c("Nearest Shrunken Centroid Model", "Elastic_Net Model","SVM Model")</pre>
rownames(final_matrix) <- c("Number of Features", "Error rate")</pre>
knitr::kable(final_matrix, caption = "Comparsion of Three models")
df = new_df[,-1]
y_1 <- ifelse(df$CellType=="CD4",1,0)</pre>
y_2 <- ifelse(df$CellType=="CD8",1,0)
y_3 <- ifelse(df$CellType=="CD19",1,0)
p1 \leftarrow sapply(1:(ncol(df)-1), function(x) t.test(df[,x] \sim y_1, data = df) p.val)
p2 \leftarrow sapply(1:(ncol(df)-1), function(x) t.test(df[,x] \sim y_2, data = df) p.val)
p3 \leftarrow sapply(1:(ncol(df)-1), function(x) t.test(df[,x] ~ y_3, data = df) p.val)
pvall 1 <- as.data.frame(p1)</pre>
pvall_2 <- as.data.frame(p2)</pre>
pvall_3 <- as.data.frame(p3)</pre>
no_CD4 <-length(pvall_1$p1[which(p1 < 0.05)])</pre>
paste("number of features in CD4: ", no_CD4)
no_CD8 <-length(pvall_2$p2[which(p2 < 0.05)])</pre>
paste("number of features in CD8: ", no_CD8)
no_CD19 \leftarrow length(pvall_3p3[which(p3 < 0.05)])
paste("number of features in CD19: ", no_CD19)
feature_CD4<-as.data.frame(pvall_1$p1[which(p1 < 0.05)])</pre>
names(feature_CD4)[1]<-"feature_sel"</pre>
```

```
feature_CD8<-as.data.frame(pvall_2$p2[which(p2 < 0.05)])</pre>
names(feature_CD8)[1]<-"feature_sel"</pre>
feature_CD19<-as.data.frame(pvall_3$p3[which(p3 < 0.05)])</pre>
names(feature_CD19)[1]<-"feature_sel"</pre>
x1 <- sort(as.numeric(p1))</pre>
x2 <- sort(as.numeric(p2))</pre>
x3 <- sort(as.numeric(p3))</pre>
y1<- sort(as.numeric(feature_CD4$feature_sel))</pre>
y2<- sort(as.numeric(feature_CD8$feature_sel))</pre>
y3 <- sort(as.numeric(feature_CD19$feature_sel))
library(ggplot2)
ggplot()+
  geom_density(aes(x1),col="red")+
  geom_density(aes(y1) , col="blue")+
  ggtitle("p_values and rejected area for CD4")
ggplot()+
  geom_density(aes(x2),col="red")+
  geom_density(aes(y2) , col="blue")+
  ggtitle("p_values and rejected area for CD8")
ggplot()+
  geom_density(aes(x3),col="red")+
  geom_density(aes(y3) , col="blue")+
  ggtitle("p_values and rejected area for CD19")
```