Question 4 Bayes theorem for rainy day

Bayes theorem tells us that

$$\text{posterior} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$\boxed{P(\text{rain} \mid \text{rain announced}) = \frac{P(\text{rain announced} \mid \text{rain})\,P(\text{rain})}{P(\text{rain announced})}}$$

where what we want is the posterior

$P(\text{rain} \mid \text{rain announced}) = ?$

$P(\text{rain})$ is given to us : $\frac{5 \text{ days}}{365 \text{ days}} \approx 0.0137$

$p(\text{rain announced} \mid \text{rain}) = 90\% = \frac{90}{100} = 0.9$

$$P(\text{rain announced}) = P(\text{rain announced} \mid \text{rain})\, p(\text{rain}) + \left(P(\text{rain announced} \mid \text{no rain}) \cdot P(\text{no rain})\right)$$

where $P(\text{no rain}) = 1 - P(\text{rain})$ and

$$P(\text{rain announced} \mid \text{no rain}) = 10\% = 0.1$$

so

$$P(\text{rain announced}) = (0.9 \cdot 0.0137) + ((1 - 0.0137) \cdot 0.1)$$

$$= 0.01233 + 0.09863$$

$$= 0.1106$$

so

$$\boxed{P(\text{rain} \mid \text{rain announced}) = \frac{0.0137 \cdot 0.9}{0.1106} = 0.\overline{11}}$$

This is 11% chance of rain (pretty low)