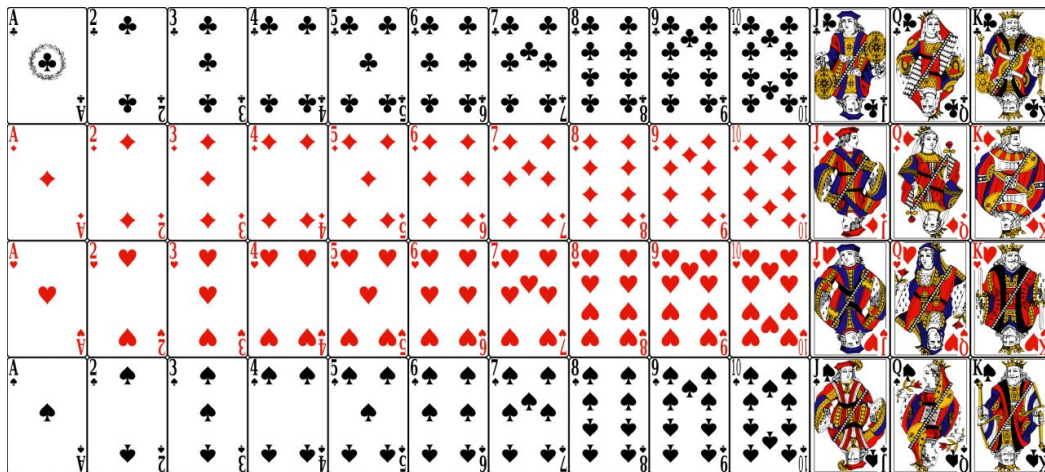


# Permutations and Weak order

OPAHC : Partial Order and Hopf Algebras in Combinatorics

# Permutations: an easy way to understand

Shuffling a deck of cards



# Permutations

Let's take less cards: 8



# Permutations

Let's take less cards: 8



12345678

15738642

18472635

# Permutations

Definition: A **permutation** of size  $n$  is a word on the alphabet  $[n] := \{1, \dots, n\}$  where each letter appears exactly once.

Example, all 6 permutations of size 3 :

123, 132, 213, 231, 312, 321



# Bubble sort

15738642



# Bubble sort

15738642 -> 15**37**8642



# Bubble sort

15738642 -> 15**37**8642 -> 1**35**78642





# Bubble sort

15738642 -> 15**37**8642 -> 1**35**78642 -> 1357**68**42



# Bubble sort

15738642 -> 15**37**8642 -> 1**35**78642 -> 1357**68**42 -> ..... -> 12345678



# Bubble sort

15738642 -> 15**37**8642 -> 1**35**78642 -> 1357**68**42 -> ..... -> 12345678

15738642 -> 157386**24** -> 15738**26**4 -> 157382**46** -> ..... -> 12345678



# Bubble sort

15738642 -> 15**37**8642 -> 1**35**78642 -> 1357**68**42 -> ..... -> 12345678

15738642 -> 157386**24** -> 15738**26**4 -> 157382**46** -> ..... -> 12345678

Less letters: 3

321 -> **23**1 -> 2**13** -> **123**

321 -> 3**12** -> **132** -> **123**



# Bubble sort

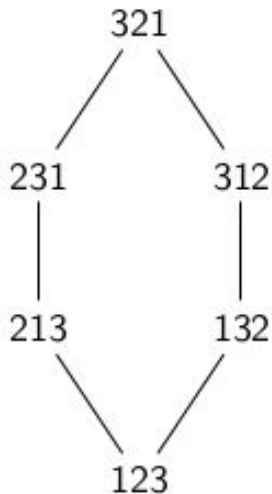
15738642 -> 15**37**8642 -> 1**35**78642 -> 1357**68**42 -> ..... -> 12345678

15738642 -> 157386**24** -> 15738**264** -> 157382**46** -> ..... -> 12345678

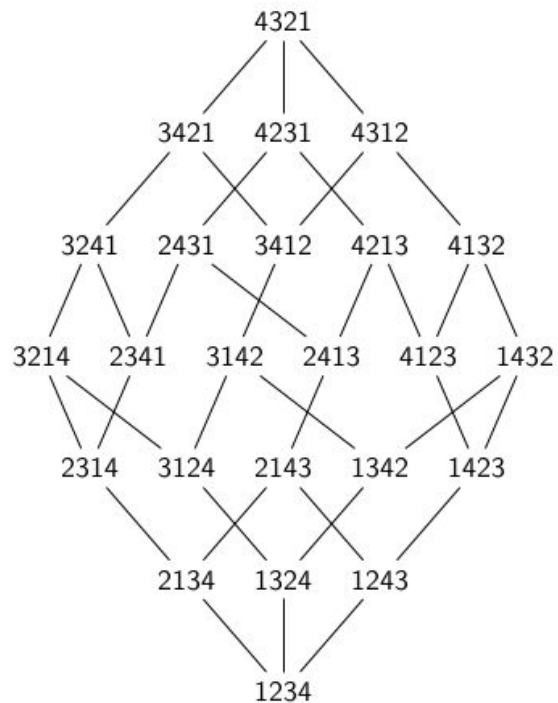
Less letters: 3

321 -> **231** -> **213** -> **123**

321 -> **312** -> **132** -> **123**



# Weak order size 4



# Weak order size 4 (permutohedron)

