Empirical Bayes estimation of set-piece conversion rates in football

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January 2, 2025

1 Introduction

In football, set-piece scenarios such as penalties, corners, and free kicks play a decisive role in determining a team's success. Coaches commonly use metrics like conversion rates (successful attempts divided by total attempts) to assess player performance in these critical moments. However, conversion rates can be misleading. For instance, two players might share the same conversion rate but differ significantly in the number of attempts, leading to varying levels of confidence in their performance. Addressing this limitation requires incorporating prior information to better account for uncertainty.

The Empirical Bayes method offers a robust solution by combining individual player data with broader league-wide patterns. This approach stabilizes estimates, particularly for players with limited trials, by borrowing strength from the data . As a result, it provides a more nuanced understanding of conversion rates that accounts for both observed performance and the uncertainty of predictions, and facilitates interpretation and modeling in this scenario [1].

This study explores the application of Empirical Bayes techniques to model conversion rates in football's set-piece contexts, such as penalties and free kicks. Inspired by McKinley (2021) on Bayesian evaluation of penalty takers [2], the analysis builds upon real-world data. By showing how uncertainty affects decisions, this research helps highlight ways coaching teams can choose players more effectively for set-piece situations.

2 Data

2.1 Data Source

As in [2], the data for this project was gathered from the FBref online platform, which provides extensive statistical and historical data on football players and teams [2]. The data collected come from the Big Five leagues, which consist of the Premier League (England), La Liga (Spain), Serie A (Italy), Bundesliga (Germany) and Ligue 1 (France) [3]. Data were collected for the seasons from the 2016-2017 season up until the 2022-2023 season. Manual web-scrapping was therefore achieved to gather the data for this project.

2.2 Data Description

The data gathered contains player shooting statistics from 826 players and was filtered to only keep player names, the number of attempted penalty kicks, and the number of successful penalty kicks. Corners and direct free kicks were not used because these statistics were not consistently reported in the leagues and seasons of interest. Of those 826 players, only 537 had attempted one or more penalty kicks and 184 had attempted five or more.

3 Methodology

3.1 Empirical Bayes prior hyper-parameter estimation

To estimate the prior parameters α_0 and β_0 of the Beta distribution using Empirical Bayes, we utilize the observed data on penalty attempts and successes from the gathered data. Let x_i denote the number of successes and n_i denote the number of attempts for the i-th player. Assuming each player's success rate is drawn from a Beta Distribution $Beta(\alpha_0, \beta_0)$, the observed data for each player follows a Binomial likelihood, $x_i \sim Binomial(n_i, p_i)$, where p_i is the player's true success probability. Using the relationship between the mean and the variance of the Beta distribution, the prior mean $\mu = \frac{\alpha}{\alpha+\beta}$ and variance $\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ [4].

We estimate μ and σ^2 based on the observed data. The sample mean $\hat{\mu} = \frac{\sum x_i}{\sum n_i}$ provides an estimate for μ and the sample variance $\hat{\sigma^2} = \frac{\sum (x_i - \hat{\mu})^2}{\sum n_i - 1}$ provides an estimate for σ^2.

Given $\hat{\mu}$ and $\hat{\sigma}^2$, the parameters α_0 and β_0 are estimated using the method of moment [4]:

$$\alpha_0 = \hat{\mu}(\frac{\hat{\mu}(1-\hat{\mu})}{\hat{\sigma}^2} - 1), \beta_0 = (1-\hat{\mu})(\frac{\hat{\mu}(1-\hat{\mu})}{\hat{\sigma}^2} - 1). \tag{1}$$

These estimates maximize the likelihood of the observed data under the Empirical Bayes framework and provide a prior distribution that reflects the overall leagues-wide penalty success rate.

3.2 Posterior density calculation

Let θ represent the probability of a successful penalty kick by a player. θ is a random variable following a Beta distribution prior to observing data. Observed data X follows a Binomial distribution conditional on θ.

For the prior distribution, θ follows a Beta distribution $Beta(\theta|\alpha_0, \beta_0)$:

$$p(\theta) = \frac{\theta^{\alpha_0 - 1}(1-\theta)^{\beta_0 - 1}}{B(\alpha_0, \beta_0)} \tag{2}$$

where α is the number of successes, β is the number of failures, $B(\alpha_0, \beta_)) = \frac{\Gamma(\alpha_0)\Gamma(\beta_0)}{\Gamma(\alpha_0+\beta_0)}$ is the Beta function and Γ is the Gamma function.

For the likelihood function, we use a Binomial distribution $Bin(s|n, \theta)$ to model s the successful penalty kicks in n attempts:

$$\begin{aligned} p(s|\theta) &= \binom{n}{s}\theta^s(1-\theta)^{n-s} \\ &= \binom{s+f}{s}\theta^s(1-\theta)^f \end{aligned} \tag{3}$$

Following the method used in [2] and [5], we calculate the posterior success rate r using Bayes' theorem:

$$\begin{aligned} p(\theta|s) &= \frac{p(\theta) * p(s|\theta)}{p(s)} \\ &= \frac{p(\theta) * p(s|\theta)}{\int p(\theta) * p(s|\theta) d\theta} \end{aligned} \tag{4}$$

By plugging in the aforementioned equations for $p(\theta)$ and $p(s|\theta)$ we obtain:

$$
\begin{aligned}
p(\theta|s) &= \frac{\frac{\theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}}{B(\alpha_0,\beta_0)}\binom{s+f}{s}\theta^s(1-\theta)^f}{\int_0^1 \frac{\theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}}{B(\alpha_0,\beta_0)}\binom{s+f}{s}\theta^s(1-\theta)^f d\theta} \\
&= \frac{\theta^{s+\alpha_0-1}(1-\theta)^{f+\beta_0-1}}{B(s+\alpha_0, f+\beta_0)}
\end{aligned} \tag{5}
$$

which is a Beta distribution $B(\alpha_0 + s, \beta_0 + f)$ with mean

$$
\begin{aligned}
\mathbb{E}[\theta|s] &= \frac{\alpha_0 + s}{\alpha_0 + \beta_0 + s + f} \\
&= \frac{\alpha_0 + s}{\alpha_0 + \beta_0 + n}
\end{aligned} \tag{6}
$$

and variance

$$
\begin{aligned}
Var(\theta|s) &= \frac{(\alpha_0 + s)(\beta_0 + f)}{(\alpha_0 + s + \beta_0 + f)^2(\alpha_0 + s + \beta_0 + f + 1)} \\
&= \frac{(\alpha_0 + s)(\beta_0 + f)}{(\alpha_0 + \beta_0 + n)^2(\alpha_0 + \beta_0 + n + 1)}
\end{aligned} \tag{7}
$$

3.3 Uncertainty Estimation

To calculate the uncertainty in the posterior distribution, we calculate the 95% credible interval and the 95% highest posterior density interval (HPDI) for the posterior distribution of the penalty taker's conversion rate. The 95% credible interval is computed by finding the 2.5th and 97.5th percentiles of the posterior distribution. This interval contains 95% of the probability mass and is symmetric around the mean. In contrast, the 95% HPDI identifies the narrowest interval that captures 95% of the probability mass. Unlike the credible interval, the HPDI is not necessarily symmetric, making it particularly useful when the posterior distribution is skewed, such as in a Beta distribution.

4 Results

4.1 Prior Hyperparameter estimation

The obtained prior hyperparameters are $\alpha_0 = 4.97$ and $\beta_0 = 1.14$. The average conversion rate in the collected data is $\bar{\theta} = 0.813$.

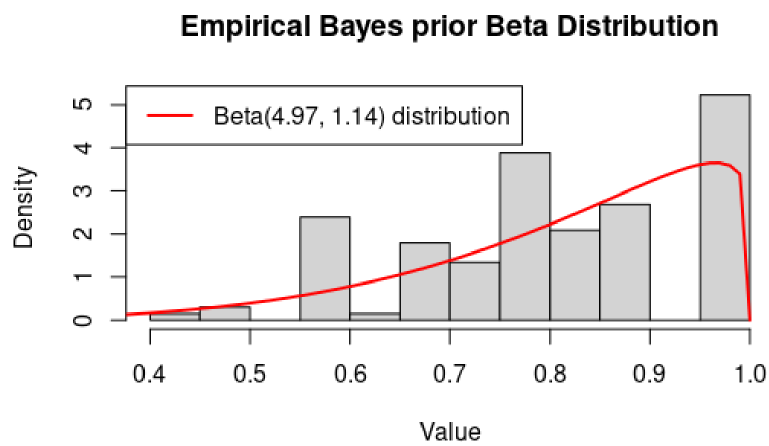

Figure 1: Estimated Empirical Bayes Beta Prior on top of observed data histogram

Player	Post. Mean	Post. Var.	95% Cred. Interval	95% HPDI
A	0.681	0.0127	[0.443, 0.877]	[0.450, 0.881]
B	0.623	0.0041	[0.493, 0.744]	[0.495, 0.750]

Table 1: Results: Example 1

4.2 Example 1: "worst than average" players.

In this situation, Player A had 6 successful penalty kicks in 10 attempts, while Player B had 30 successes in 50 attempts. While both player have a success rate (SR) of 0.6, Player B has a number of trials five times greater. Using the Empirical Bayes prior hyper-parameters estimated from the observed data, we can calculate their respective posterior densities. The results are shown in Table 1 and Figure 2.

4.3 Example 2: "Perfect beginner" vs. "Above average experienced".

In this situation, Player A has a "perfect" success rate of 2 successes in 2 attempts, while Player B has an "above average" success rate of 0.85 in 100 attempted penalty kicks. Using the Empirical Bayes prior hyper-parameters

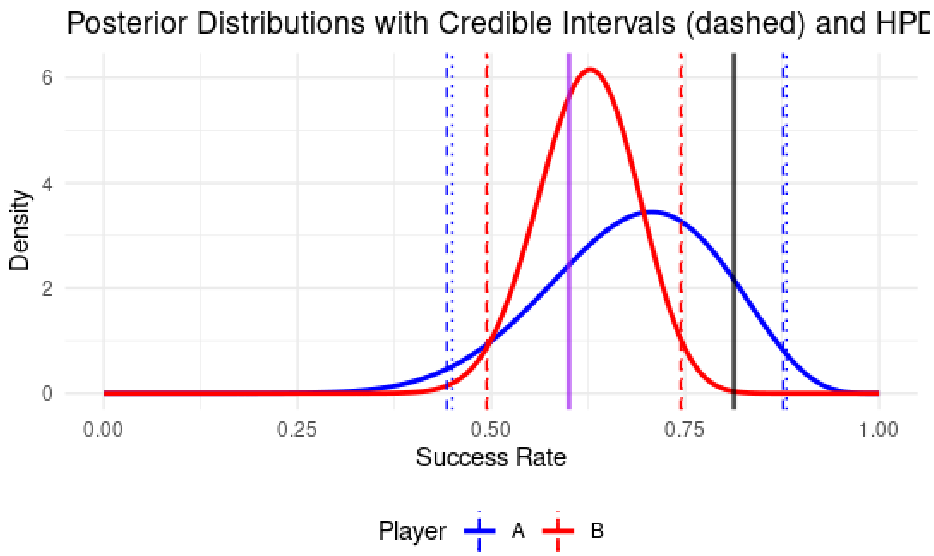

Figure 2: Example 1 posterior distributions. The purple vertical line represents the 0.6 SR while the black vertical line represents the league average of 0.83

Player	Post. Mean	Post. Var.	95% Cred. Interval	95% HPDI
A	0.859	0.0133	[0.568, 0.994]	[0.626, 0.999]
B	0.848	0.0012	[0.774, 0.909]	[0.779, 0.913]

Table 2: Results: Example 2

estimated from the observed data, we can calculate their respective posterior densities. The results are shown in Table 2 and Figure 3.

5 Discussion

5.1 Fitness of prior on observed data

The fit of the estimated beta distribution is not great, because the underlying data is not very smooth. The spikes in the histogram can be explained by a bias in the data towards specific success rate values, such as when the number of trials is small (ex: $3/5 = 0.6$, $4/5 = 0.8$, $5/5, = 1$, no in between values). Here the choice of 5 attempts minimum was made follow the choices made in

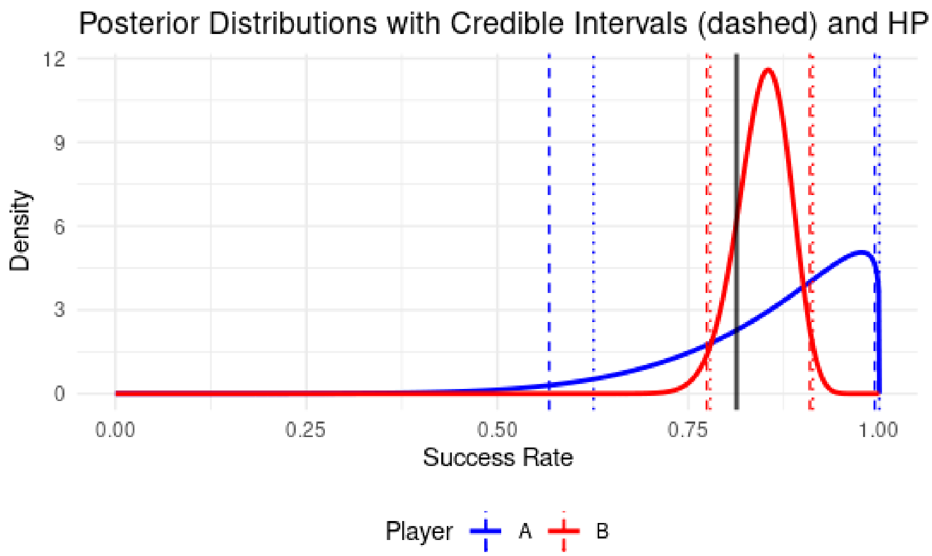

Figure 3: Example 2 posterior distributions

[2], but note that the number of data points decreases logarithmically with the minimum number of attempts, which yields the observed artifacts in the histogram.

5.2 Posterior observations

In both examples, it can be seen that the number of trials significantly impacts the variance of the posterior distribution. More trials reduce the variance, which also leads to narrower credible intervals. With weakly informative priors, the posterior is primarily shaped by the data. However, with fewer trials (Player A), the prior has a relatively larger influence. This effect can be observed in Example 1 where the blue curve is more shifted towards the observed data mean success rate.

5.3 Possible improvements

One way to improve this research would be to collect more data to estimate the prior hyperparameters. To obtain a data distribution that resembles more a Beta distribution, there is a need for a lot of players having attempted a lot of penalty kicks, which is something that can only be obtain by adding more leagues and more seasons to the data. The analysis carried in this research could also be extended to other metrics, such as free kicks, corner kicks, or goal saves; but these metrics are less consistently reported in the database consulted therefore the same challenge of gathering enough data will be encountered. Finally, this research only showed hypothetical examples to better illustrate the effects on the posterior distribution, but one could easily replace the hypothetical player data with their desired player data to make more informed decisions in set-piece situations.

6 Conclusion

This project demonstrates the value of Bayesian methods in improving decision-making for set-piece situations in football by addressing the limitations of simple conversion rates. While conversion rates provide an intuitive measure of performance, they can be misleading when comparing players with different number of trials. Using Empirical Bayes methods allowed to borrow prior information from the data and thus derive posterior distributions that account for both performance and uncertainty. The analysis showed that players with identical conversion rates can have very different levels of uncertainty, as seen in their posterior distributions and confidence intervals. Therefore, the Empirical Bayes frameworks allows coaching teams to make more informed decision under uncertainty.

Source Code

All the source code used for this project can be found on the following GitHub repository: `https://github.com/hugo-morvan/EmpBayes`

References

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