DBDM

Exercise 2.1.

(Natural) join $\bowtie$ is associative and comutative.

Exercise 2.2.

It's the cartesian product.

Exercise 2.3.

1) We suppose $\{a_1, \dots, a_n\} \cap \{b_1, \dots, b_n\} = \emptyset$ and $\forall i \neq j,\ \begin{matrix} a_i \neq a_j \\ b_i \neq b_j \end{matrix}$

$$R \underset{\substack{a_1 = b_1 \\ \vdots \\ a_n = b_n}}{\bowtie} S = \sigma_{a_1 = b_1}\left(\sigma_{a_2 = b_2}\left(\dots \sigma_{a_n = b_n}(R \times S) \dots\right)\right)$$

$$\pi_{(A_R \cup A_S) \setminus \{a_1, \dots, a_n\}} \quad R \underset{\substack{a_1 = b_1 \\ \vdots \\ a_n = b_n}}{\bowtie} S = \rho_f(R) \bowtie S$$

$$\begin{aligned} f : A_R &\longrightarrow A_R' \\ a_i &\longmapsto b_i \\ \{a_1, \dots, a_n\} \not\ni a &\longmapsto a \end{aligned}$$

2) Define $T = A_R \cap A_S$, and

$$\begin{aligned} f : A_S &\longrightarrow Att \\ T \not\ni a &\longmapsto a \\ T \ni a &\longmapsto a' \end{aligned}$$

fresh attribute

$$R \bowtie S = \pi_{A_R \cup A_S}\left(R \underset{a = a'}{\bowtie} S\right)$$

Exercise 2.4.

$$\sigma_{travel \neq travel'}\left(\pi_{travel, travel'}\left(usedFor \bowtie \rho_f(usedFor)\right)\right)$$

$$\begin{aligned} f : train &\longmapsto train' \\ date &\longmapsto date' \\ travel &\longmapsto travel' \end{aligned}$$

$$\sigma_{a \neq b}(q) = q \setminus \sigma_{a = b}(q)$$

Exercise 2.5.

$$\sigma_{j=a}(R) = R \bowtie S.$$
$$= \{ u \in D^{A_R} \mid \underbrace{u(j)=a}_{u \in S} \}.$$

Exercise 2.6.

1. $R_1, \ldots, R_m$ joins completely
   iff $\forall t \in R_i,\ \exists t' \in R_1 \bowtie \cdots \bowtie R_m,\ t'[A_{R_i}] = t$
   iff $\forall t \in R_i,\ t \in \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$
   iff* $R_i = \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$

$*: \Rightarrow \quad R_i \subseteq \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$ by assumption
$\quad R_i \supseteq \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$ by def.

2. We simply have to prove that $R_i = \pi_{S_i}(R_1 \bowtie \cdots \bowtie R_m)$.
   We have that $R_1 \bowtie \cdots \bowtie R_m = \pi_{\bigcup_i S_i}(R)$ and so
   $$\pi_{S_i}(R_1 \bowtie \cdots \bowtie R_m) = \pi_{S_i}(\pi_{\bigcup_i S_i}(R)) = \pi_{S_i}(R) = R_i.$$

Exercise 2.7.

We have to have $A \cap B$ as primary keys of R.
It is not sufficient.

Exercise 2.8.

$$(R \bowtie S) \div S = (R \times S) \div S = S$$

Exercise 2.9. ♥

1.