

Exercise 1. Lumberjack.

Q1.
$$\begin{cases} w + p \leq 100 \\ 10w + 50p \leq 4000 \\ w, p \geq 0 \end{cases}$$

maximize $50w + 120p =: \gamma$



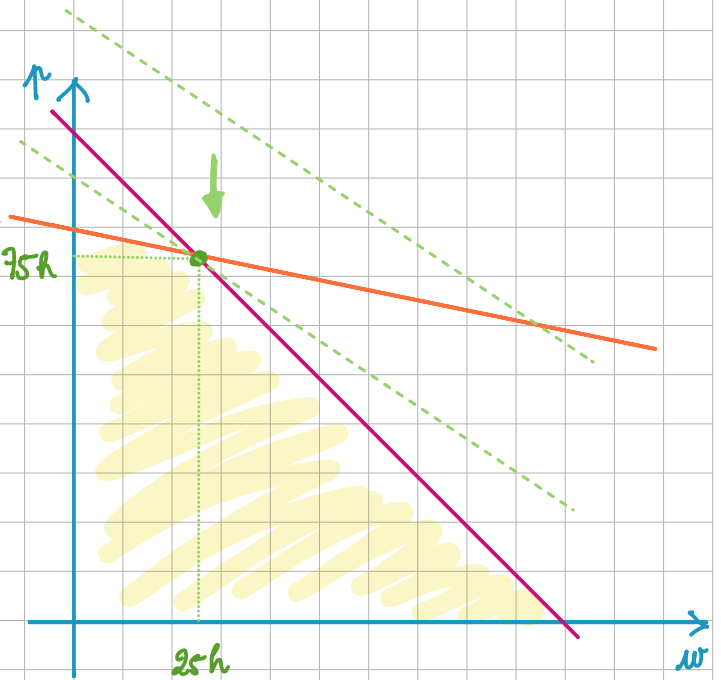
find a line

$$p = \frac{\gamma}{120} - \frac{50}{120} w$$

with the highest γ -intercept.

Q2.

75h



Q3. The best strategy is 25h with the wood cut and 75h re-seeded, for a profit of 10 250 k\$.

Exercise 2. Student diet problem

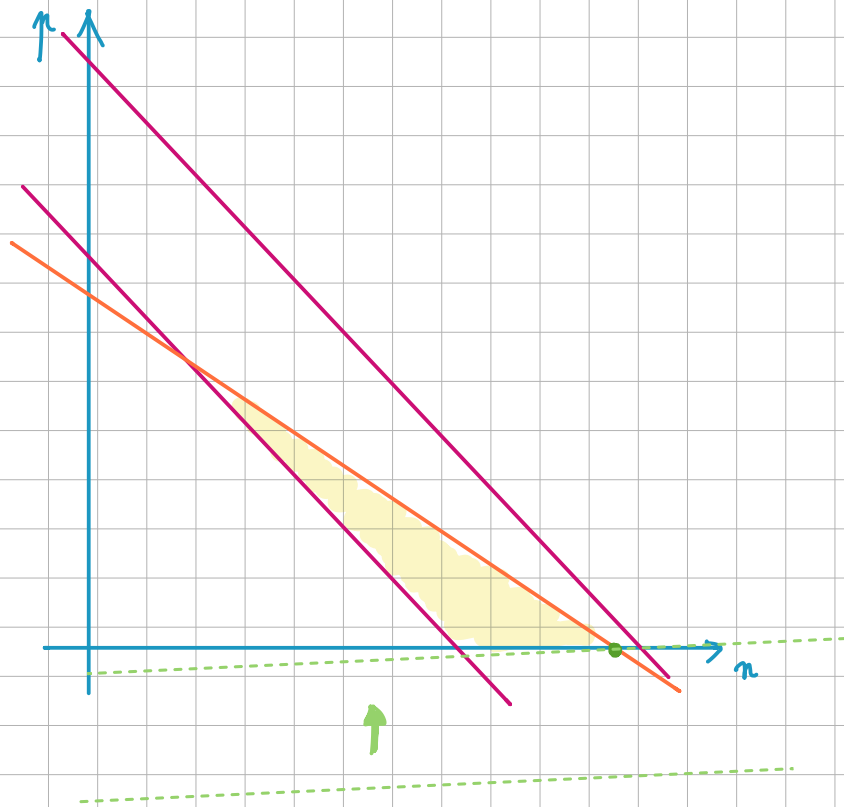
Q1.
$$\begin{cases} 160n + 700p + 400r \leq 3500 \\ 160n + 700p + 400r \geq 2500 \quad (1) \\ 21n + 20p \leq 120 \\ 21n + 20p \geq 80 \quad (2) \\ n, p, r \geq 0 \end{cases}$$

minimize $0,5n + 3,5p + 0,5r$

Q2. Instead of drawing in 3D, as we are minimizing a strictly increasing function, we consider two cases: if (1) is an equality or if (2) is an equality.

Case 1: $160n + 700p + 400r = 2500$

$$\Leftrightarrow r = 6,25 - 1,75p - 1,15n$$



Dans ce cas, l'optimal est $n = 125/23$, $p = 0$ et $x = 0$ (*)
avec un prix de 2,92 € environ.

Dans l'autre cas, on trouve $n = \frac{80}{21}$, $p = 0$ et $x = 25/4$, avec un prix
de 5,03 € environ. (**).

On conclut que (*) est l'optimal.

Exercice 3. Bank allocation.

c : crédit client, x : crédit voiture p : prêt.

$$\begin{aligned} x + p &\geq 60\% \times (x + p + c) &\Rightarrow \\ p &\geq 40\% (x + p + c) &\Rightarrow \\ 6\% c + 4\% x + 2\% p &\leq (x + p + c) 3,2\% &\Rightarrow \end{aligned} \quad \left\{ \begin{array}{l} x + c + p \leq 10^6 \\ 0,3x + 0,3p - 0,6c \geq 0 \\ 0,6p - 0,4x - 0,4c \geq 0 \\ 0,028c + 0,008x - 0,0012p \leq 0 \\ x, p, c \geq 0 \end{array} \right.$$

maximize $0,06c + 0,04x + 0,02p$

We can compute the solution by case by case analysis.

Exercise 6. Independent Set Problem

Variables: x_v for every vertex $v \in V$

Constraints: for any edge $uv \in E$, $x_u + x_v \leq 1$

for any vertex $v \in V$, $1 \geq x_v \geq 0$

Maximize $\sum_{v \in V} x_v =$ size of the independent set

Exercise 7. Dominating Set Problem

Variables x_u $u \in V$

Constraints $\forall u \in V$, $0 \leq x_u \leq 1$

$\forall u \in V$, $\sum_{v \in N[u]} x_v \geq 1$

\hookrightarrow closed neighborhood

Minimize $\sum_{u \in V} x_u$

Exercise 8. N-queens problem

Variables: $x_{i,j}$, $i, j \in [1, N]$

Constraints: $\forall i$, $\sum x_{i,j} \leq 1$

$\sum x_{j,i} \leq 1$

$\sum x_{i,i-j} \leq 1$

$\sum x_{i,i+j} \leq 1$

$\forall i, j$ $0 \leq x_{i,j} \leq 1$

Maximize $\sum_{i,j} x_{i,j}$

