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We consider a two-qubit error and a one qubit-error

$$E_0 := \left(Z \otimes \mathbb{1} \otimes \mathbb{1}\right) \otimes \left(\mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}\right) \otimes \left(Z \otimes \mathbb{1} \otimes \mathbb{1}\right),$$

$$E_1 := \left(\mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}\right) \otimes \left(Z \otimes \mathbb{1} \otimes \mathbb{1}\right) \otimes \left(\mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}\right).$$

Then,

$$E_0 |\bar{0}\rangle = \frac{1}{2\sqrt{2}} \left(|000\rangle - |111\rangle\right) \otimes \left(|000\rangle + |111\rangle\right) \otimes \left(|000\rangle - |111\rangle\right)$$

$$\shortparallel$$

$$E_1 |\bar{1}\rangle = \frac{1}{2\sqrt{2}} \left(|000\rangle - |111\rangle\right) \otimes \left(|000\rangle + |111\rangle\right) \otimes \left(|000\rangle - |111\rangle\right)$$

As Shor's code would be able to correct
the one-qubit error E_1 back into $|\bar{1}\rangle$, it would also
(falsely) "correct" the two-qubits error E_0 into $|\bar{1}\rangle$ since $E_0|\bar{0}\rangle = E_1|\bar{1}\rangle$.

We can conclude that Shor's code is not resistant
to two-qubits errors.

END of Assignment #7.