

# Tutorial # 1.

## Machine , Simulations

### Exercise 1. Warming up

Q1. Write the 0 letters of the word on a 2<sup>nd</sup> tape.  
 Write the 1 letters of the word on a 3<sup>rd</sup> tape.  
 Compare the length of the words on the 2<sup>nd</sup> & 3<sup>rd</sup> tapes

Q2. Replace the # by a 1, remove the last 1.

Given a string  $w$  on the tape, write the last part on a 2<sup>nd</sup> tape.  
 Then compute addition and compare the results.

Q3. Write  $a$  and  $b$  on the 2<sup>nd</sup> & 3<sup>rd</sup> tape.

Until the start of  $a$  is reached, step to the left (2<sup>nd</sup> tape)  
 Write  $\langle 1^{\text{st}} \text{ tape} \rangle \# \langle 3^{\text{rd}} \text{ tape} \rangle$  on the 1<sup>st</sup> tape  
 and compute addition.

### Exercise 2. Universal Turing Machines

Q1.

tape #1	$a_1^1$	$a_2^1$	$a_3^1$	...
tape #2	$a_1^2$	$a_2^2$	$a_3^2$	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
tape #k	$a_1^k$	$a_2^k$	$a_3^k$	...

where we encode each letter in binary



and we have a tape whose size is at most

$$O(\log |\Gamma|) \times 2k \times s(|x|).$$

$$O(s(|x|)).$$

Q2. Use a tape for  $\langle \alpha, x \rangle$   
 and a tape for the state of  $M_\alpha$  }  $C_\alpha$   
 and 2 tapes for the tapes of  $M_\alpha$ .

$$\# \text{ used cells} \leq C_\alpha \times s(|x|)$$

### Exercise 3. Stimulating Simulation.

Q1.  $\dots | a_{-2} | a_{-1} | a_0 | a_1 | a_2 | \dots \rightsquigarrow \overline{a_0 | a_1 | a_2 | a_3 | a_4 | \dots}$

Q2. Write each letter of  $\Gamma$  in binary, the cost is  $\lceil \log_2 |\Gamma| \rceil$ .

Q3. Store the two tapes on an alphabet  $\Gamma^2 \times \{B, D\}$ .

We don't talk about the heads of both tapes.

Reduce the size of the alphabet w/ Q2.

Ans.