TD n°1 Modelization & linear Programming

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Exercise 1. Lumberjack.

Q1. $\begin{cases} w + p \leq 100 \\ 10w + 50p \leq 4000 \\ w, p \geq 0 \end{cases}$

maximize $50w + 120p =: z$

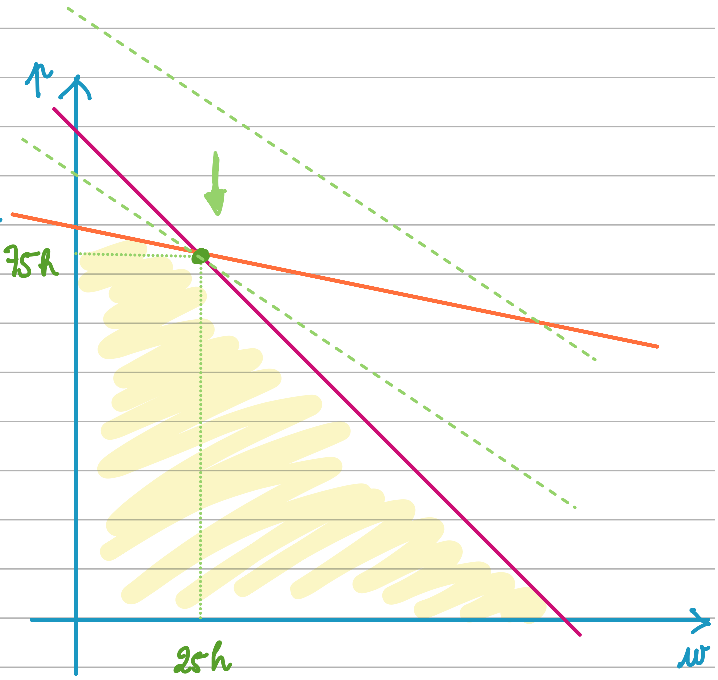
$\Updownarrow$

find a line

$$p = \frac{z}{120} - \frac{50}{120}a$$

with the highest y-intercept.

Q2.



Q3. The best strategy is 25h with the wood cut and 75h re-seeded, for a profit of 10 250 k\$.

Exercise 2. Student diet problem

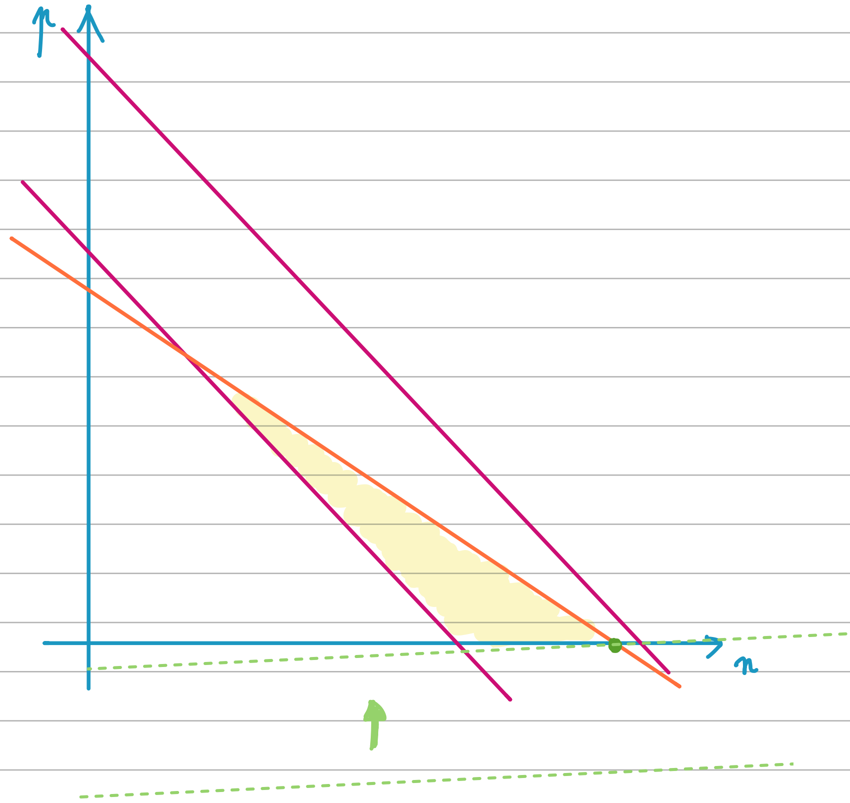
Q1. $\begin{cases} 460n + 700p + 400r \leq 3500 \\ 460n + 700p + 400r \geq 2500 \quad (1) \\ 21n + 20p \leq 120 \\ 21n + 20p \geq 80 \quad (2) \\ n, p, r \geq 0 \end{cases}$

minimize $0{,}5n + 3{,}5p + 0{,}5r$

Q2. Instead of drawing in 3D, as we are minimizing a strictly increasing function, we consider two cases: if (1) is an equality or if (2) is an equality.

Case 1: $460n + 700p + 400r = 2500$

$\iff r = 6{,}25 - 1{,}75p - 1{,}15n$



Dans ce cas, l'optimal est $n = 125/23$, $p = 0$ et $r = 0$ (*)

avec un prix de 2,92 € environ.

Dans l'autre cas, on trouve $n = \frac{80}{21}$, $p = 0$ et $r = 25/4$, avec un prix de 5,03€ environ. (**).

On conclut que (*) est l'optimal.

Exercice 3. Bank allocation.

c : crédit client, v : crédit voiture p : prêt.

$$v + p \geq 60\% \times (v + p + c) \Rightarrow$$
$$p \geq 40\% \ (v + p + c) \Rightarrow$$
$$6\% c + 4\% v + 2\% p \leq (v+p+c)\, 3{,}2\% \Rightarrow$$

$$\begin{cases} v + c + p \leq 10^6 \\ 0{,}3 v + 0{,}3 p - 0{,}6 c \geq 0 \\ 0{,}6 p - 0{,}4 v - 0{,}4 p \geq 0 \\ 0{,}028 c + 0{,}0008 v - 0{,}0012 p \leq 0 \\ v, p, c \leq 0 \end{cases}$$

Maximize $0{,}06 c + 0{,}04 v + 0{,}02 p$

We can compute the solution by case by case analysis.

Exercise 6. Independant Set Problem

Variables : x_v for every vertex $v \in V$

Constraints : for any edge $uv \in E$, $x_u + x_v \leq 1$

for any vertex $v \in V$, $1 \geq x_v \geq 0$

Maximize $\sum_{v \in V} x_v$ = size of the independant set

Exercise 7. Dominating Set Problem

Variables x_u $u \in V$

Constraints $\forall u \in V$, $0 \leq x_u \leq 1$

$\forall u \in V$, $\sum_{v \in N[u]} x_v \geq 1$

↳ closed neighborhood

Minimize $\sum_{u \in V} x_u$

Exercise 8. N-queens problem

Variables : $x_{i,j}$, $i,j \in [\![1, N]\!]$

Constraints : $\forall i$, $\sum x_{i,j} \leq 1$

$\sum x_{j,i} \leq 1$

$\sum x_{i,i-j} \leq 1$

$\sum x_{i,i+j} \leq 1$

$\forall i,j$ $0 \leq x_{ij} \leq 1$

Maximize $\sum_{ij} x_{ij}$

TD n° 2 Linear Programming and the simplex method.

Exercise 1. Computer production

Q1.

$$\begin{cases} n + d \leq 10\,000 & m \\ n + 2d \leq 15\,000 & m \\ 4n + 3d \leq 38\,000 & m \\ n, d \geq 0 \end{cases}$$

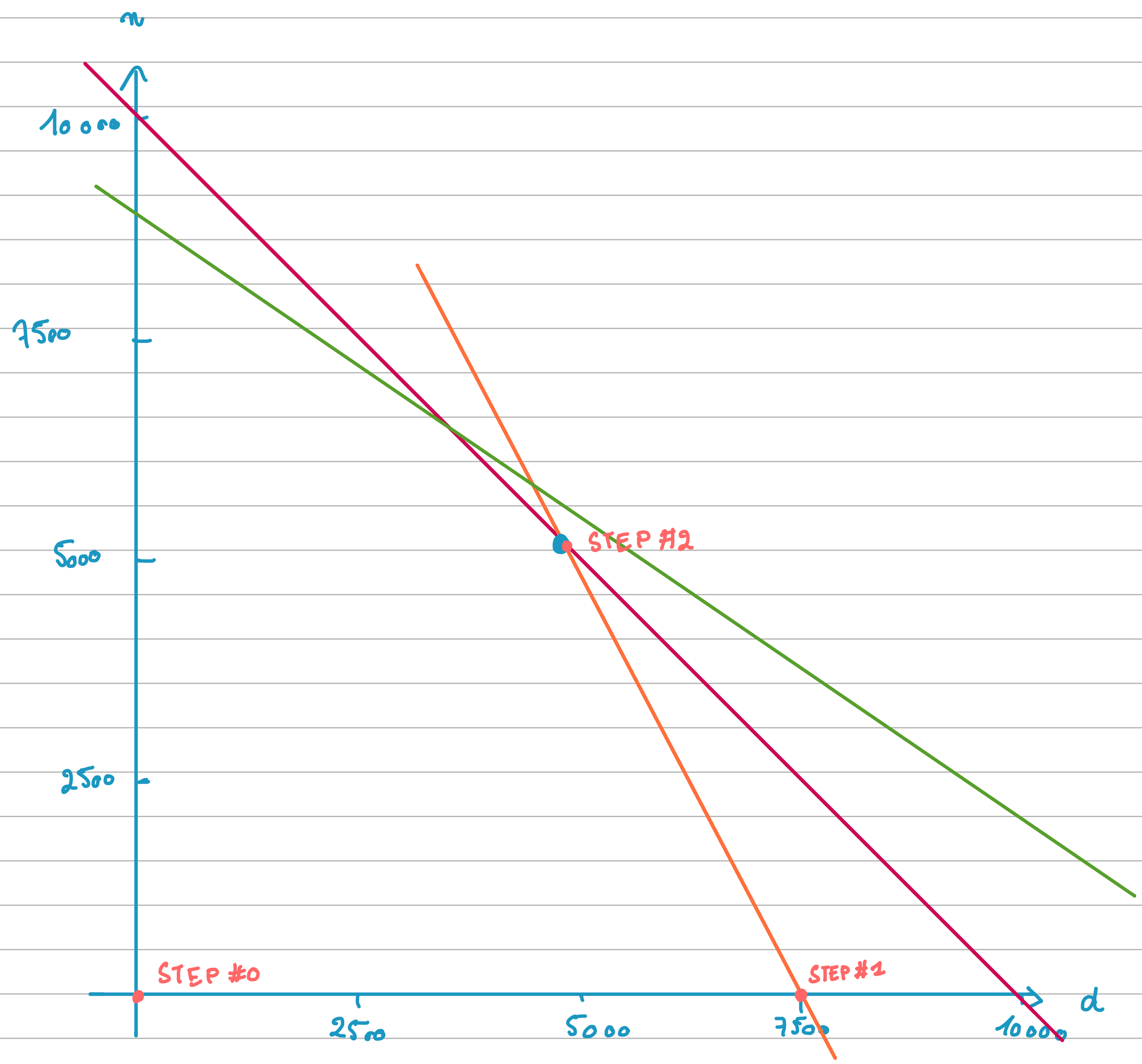
$$n = 9500 - \frac{3}{4} d$$

$$\frac{3}{4} d = 9500$$

max $750 n + 1000 d$

Q2.

L'optimal est $n = 5000$ et $d = 5000$.



$$\begin{cases} u = 10 - n - d \\ v = 15 - n \cdot 2d \\ w = 38 - 4n - 3d \\ z = 75n + 100d \end{cases}$$

$\boxed{\text{STEP \#1}}$ Choice for pivot : d enters and v leaves.

We have $d = 7.5 - \frac{n}{2} - \frac{v}{2}$ thus, we substitute

$$\begin{cases} u = 2{,}5 - \frac{n}{2} + \frac{v}{2} \\ d = 7{,}5 - \frac{n}{2} - \frac{v}{2} \\ w = 15{,}5 - 2{,}5n - 2{,}5v \\ z = 750 + 25n - 50v \end{cases}$$

$\boxed{\text{STEP \#2}}$ Choice for pivot : n enters and u leaves

Solution: $5000 = n = d$ for a reward of 87500

Exercise 2. Multiple optimal solutions