

Assignment #8

Hugo SALOU

As the code \mathcal{C} can correct errors on any subsets of t or fewer qubits, so it can correct

$$\# \text{Correctable Errors} := \sum_{j=0}^t \binom{n}{j} 3^j.$$

\{ size of the subset of errors \{ choices for j qubits \{ choices for j errors X,Y,Z

Also, $\dim(Q_i \mathcal{C}) = \dim(\mathcal{C}) = 2^k$ as each error Q_i is invertible. And, by non-degeneracy of \mathcal{C} , we have that $Q_i \mathcal{C} \perp Q_j \mathcal{C}$ whenever $i \neq j$, so

$$\# \text{Correctable Errors} \leq \underbrace{\dim(Q_i \mathcal{C})}_{2^k} \leq \underbrace{\dim((\mathbb{C}^2)^{\otimes n})}_{2^n}$$

and we finally have

$$\sum_{j=0}^t \binom{n}{j} 3^j 2^k \leq 2^n.$$

End of Assignment #8