

Assignment #2.

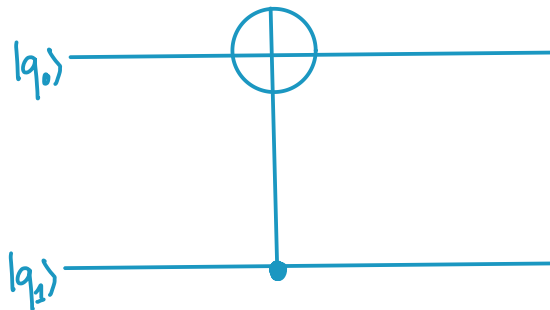
Hugo SALOU

Q1. We can compute explicitly the unitary matrix of the quantum circuit:

$$\frac{1}{4} \left(\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \left(\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right)$$

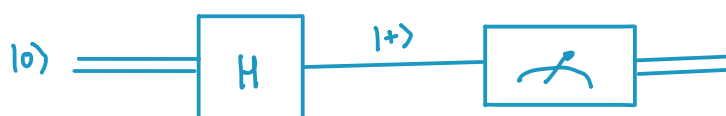
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

We can recognize the unitary matrix of the following quantum circuit:



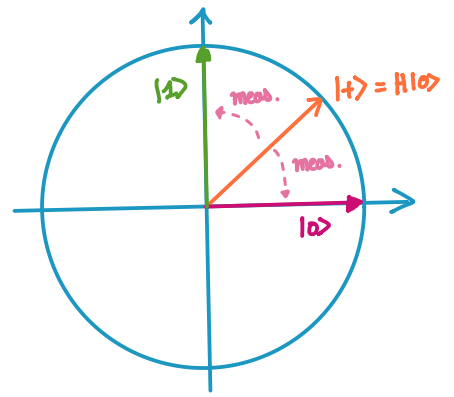
which is a lot simpler.

Q2. We get 1 qubit set at $|0\rangle$ from the machine. Then, we apply a Hadamard gate and measure in the $\{|0\rangle, |1\rangle\}$ basis.



The result of this procedure will be :

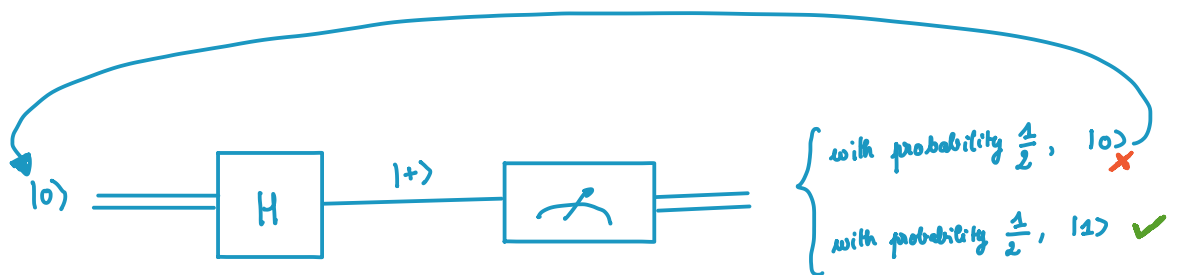
- $|1\rangle$ with probability $1/2$;
- $|0\rangle$ with probability $1/2$.



If we have a $|1\rangle$, we are done, but if we have a $|0\rangle$, we simply repeat the procedure again and again.

The number of iterations follows a geometric law $\mathcal{G}(1/2)$, thus we can "expect" to have to do $\mathbb{E}[X] = 2$ iterations to obtain a $|1\rangle$.

Repeating 7 times the procedure will give us a $|1\rangle$ with more than 99% probability.



A more complex procedure could give us better odds.

END of ASSIGNMENT #2