Assignment # 8 Hugo SALOU

As the code $\mathcal{C}$ can correct errors on any subsets of t or fewer qubits, so it can correct

$$\#\text{Correctable Errors} := \sum_{j=0}^{t} \binom{n}{j} 3^{j}.$$

size of the subset of errors

choices for j qubits

choices for j errors X, Y, Z

Also, $\dim(Q_i \mathcal{C}) = \dim(\mathcal{C}) = 2^k$ as each error Q_i is invertible. And, by non-degeneracy of $\mathcal{C}$, we have that $Q_i \mathcal{C} \perp Q_j \mathcal{C}$ whenever $i \neq j$, so

$$\sum_{i=1}^{\#\text{Correctable Errors}} \underbrace{\dim(Q_i \mathcal{C})}_{2^k} \leq \underbrace{\dim\left((\mathbb{C}^2)^{\otimes n}\right)}_{2^n}$$

and we finally have

$$\sum_{j=0}^{t} \binom{n}{j} 3^{j} \, 2^{k} \leq 2^{n}.$$

End of Assignment #8