Application of linear programming.

Consider the two player game of Morra:

- ▷ Alice hides one or two coins;
- ▷ Bob hides one or two coins;
- ▷ Alice and Bob announce one or two coins.

Each player will have a pair $(i, j) \in [2] \times [2]$ where i is the hidden number of coins, and j is the guess. This is a zero-sum game:

...fill this ...

Alice wants a probability vector which maximizes the gain for every possible move of Bob. In our example, it means maximizing

$$\min(-2x_2 + 3x_3, 2x_1 + 3x_4, -3x_1 + 4x_4, 3x_2 - 4x_3)$$

under the constraints $x_1 + x_2 + x_3 + x_4 = 1$ and $x_1, x_2, x_3, x_4 \geq 0$. This is not exactly a linear program, but we can easily translate it into one:

$$\text{maximize } y \text{ such that } \begin{cases} -2x_2 + 3x_3 \geq y \\ 2x_1 + 3x_4 \geq y \\ -3x_1 + 4x_4 \geq y \\ 3x_2 - 4x_3 \geq y \\ x_1 + x_2 + x_3 + x_4 = 1 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}.$$

We can find an unconventional solution (alternatively, we can use the Simplex algorithm to get the solution). The game is symmetric thus $y = 0$. By multiplying the second by 3 and third inequality by 2, we get that $x_4 = 0$ and $x_1 = 0$. Finally, by $x_1 = 1 - x_3$, we can conclude that

$$x_3 \geq \frac{2}{5} = 0.4 \text{ and } x_1 \leq \frac{3}{7} = 0.\overline{428571}.$$

The optimal strategy for Alice is to chose $t \in [0.4, 0.\overline{428571}]$ and to play $(2, 1)$ with probability t and $(1, 2)$ with probability $1 - t$.

Let us go back to the simplex algorithm.