# L04s02

September 20, 2018

## 1 Calibration of BSM Volatility

[Ref] Chap 8 of [Hil15]

In simple terms, problem of calibration is to find parameters for the proposed model such that observed market quotes of liquidly traded plain vanilla options are replicated as closely as possible. The general approach is that, one defines an error function that is to be minimized.

In this below, we will use - Sep-30-2014 Market quote on call options underlying Euro Stoxx 50 - BSM model - RMSE as an error function

to illustrate the calibration of volatility.

```
In [1]: import numpy as np
        import pandas as pd
        import scipy.optimize as sop
```

```
In [2]: # Market Data from www.eurexchange.com
        # as of 30. September 2014
        # European call & put option data (3 maturities)

        options = pd.read_csv('SX5E_Options_140930.csv')
        S0 = 3225.93  # EURO STOXX 50 level
        r = 0.0005  # ECB base rate
        options['Date'] = pd.to_datetime(options['Date'])
        options['Maturity'] = pd.to_datetime(options['Maturity'])

        options
```

```
Out[2]:          Date  Strike   Call   Maturity    Put
        0  2014-09-30  3175.0  126.8 2014-12-19   78.8
        1  2014-09-30  3200.0  110.9 2014-12-19   87.9
        2  2014-09-30  3225.0   96.1 2014-12-19   98.1
        3  2014-09-30  3250.0   82.3 2014-12-19  109.3
        4  2014-09-30  3275.0   69.6 2014-12-19  121.6
        5  2014-09-30  3175.0  171.0 2015-03-20  129.2
        6  2014-09-30  3200.0  156.1 2015-03-20  139.4
        7  2014-09-30  3225.0  142.0 2015-03-20  150.3
        8  2014-09-30  3250.0  128.5 2015-03-20  161.8
        9  2014-09-30  3275.0  115.8 2015-03-20  174.0
        10 2014-09-30  3175.0   82.3 2014-10-17   24.5
```

```
11 2014-09-30  3200.0   64.3 2014-10-17   31.5
12 2014-09-30  3225.0   48.3 2014-10-17   40.5
13 2014-09-30  3250.0   34.6 2014-10-17   51.8
14 2014-09-30  3275.0   23.5 2014-10-17   65.8
```

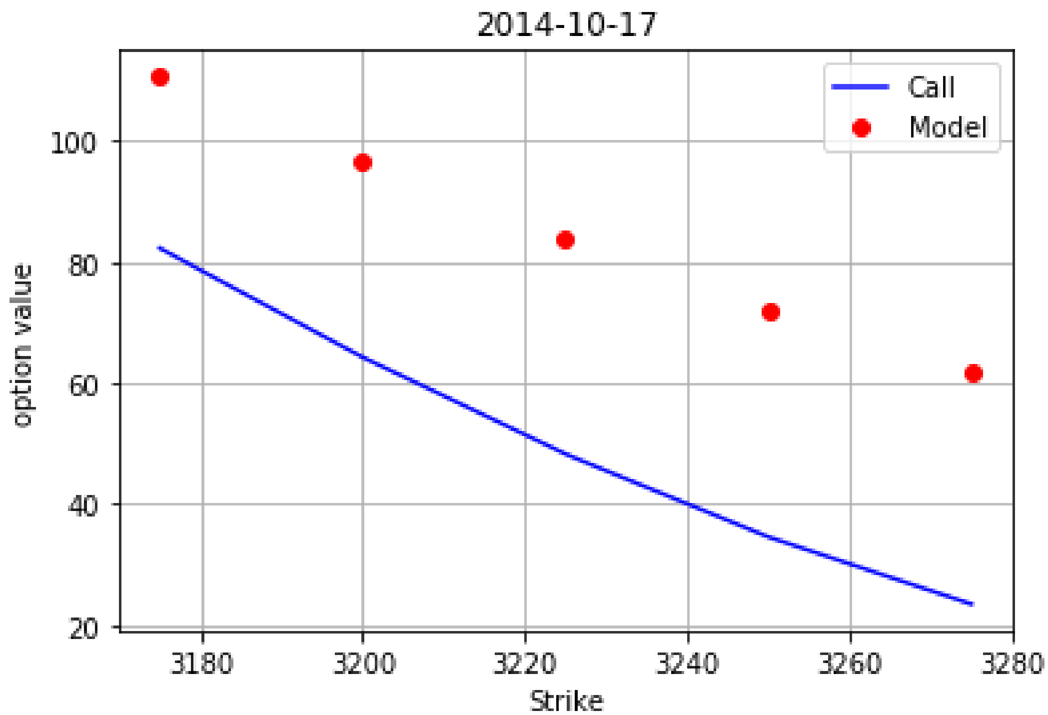
The above data is the market quote on Sep-30-2014 on call and put options underlying Euro Stoxx 50. The question of the calibration is, given that the market follows BSM, what is the volatility $\sigma$ fitting to the above market quote?

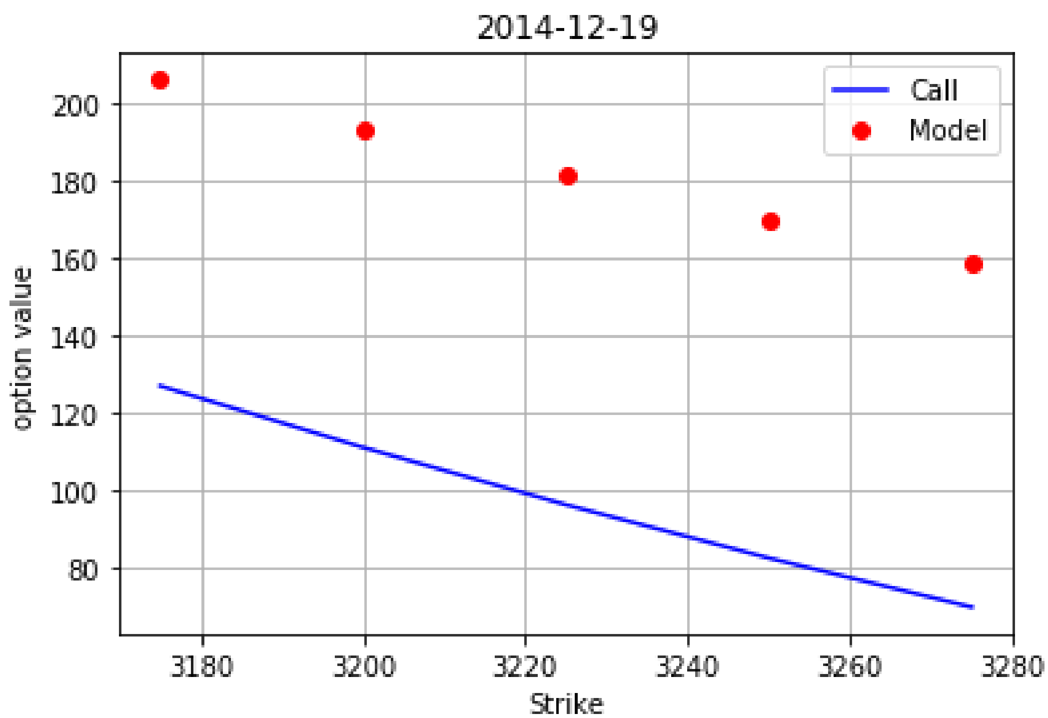
```
In [3]: from BSM_option_valuation import *
```

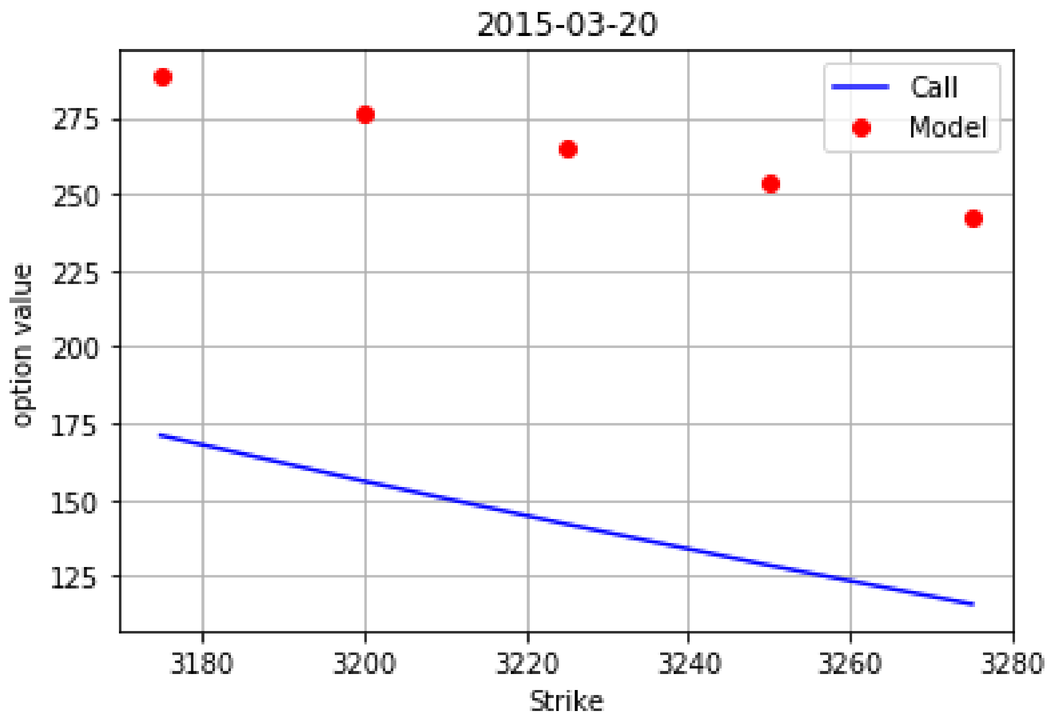
```
In [4]: def generate_plot(opt, options):
            #
            # Calculating Model Prices
            #
            sigma = opt
            options['Model'] = 0.0
            for row, option in options.iterrows():
                T = (option['Maturity'] - option['Date']).days / 365.
                options.loc[row, 'Model'] = BSM_call_value(S0, option['Strike'], 0, T, r, sigma)

            #
            # Plotting
            #
            mats = sorted(set(options['Maturity']))
            options = options.set_index('Strike')
            for i, mat in enumerate(mats):
                options[options['Maturity'] == mat][['Call', 'Model']].\
                    plot(style=['b-', 'ro'], title='%s' % str(mat)[:10],
                         grid=True)
                plt.ylabel('option value')
                plt.savefig('BSM_calibration_3_%s.pdf' % i)
```

The following graph shows that how the market data differs from the theoretical price by just making a guess of 30% volatility.

```
In [5]: generate_plot(.3, options); #Arbitrary input for the volatility
```


2014-10-17
Call
Model
option value
100
80
60
40
20
3180
3200
3220
3240
3260
3280
Strike


2014-12-19
Call
Model
option value
200
180
160
140
120
100
80
3180
3200
3220
3240
3260
3280
Strike




### 1.1 Calibration by minimizing RMSE

Suppose there are $N$ options available in the market quote with various strike and maturities. We denote the market quotes of $N$ options by

$$\{C_n^* : \quad n = 1, 2, \ldots, N.\}.$$

Correpondinly, there are $N$ BSM theoretical prices available as a function of the volatility. We denote it by

$$\{C_n(\sigma) : \quad n = 1, 2, \ldots, N.\}$$

The error function RMSE is defined by

$$RMSE(\sigma) = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\Big|C_n^* - C_n(\sigma)\Big|^2}.$$

Our goal is to find out the calibrated volatility $\hat{\sigma}$, which is given by

$$\hat{\sigma} = \arg\min_{\sigma} RMSE(\sigma).$$

```
In [6]: # Error Function
        #
        def BSM_error_function(p0):
            ''' Error Function for parameter calibration in BSM Model via
```

```
        Parameters
        ==========
        sigma: float
            volatility factor in diffusion term

        Returns
        =======
        RMSE: float
            root mean squared error
        '''
        global i, min_RMSE
        sigma = p0
        if sigma < 0.0:
            return 500.0
        se = []
        for row, option in options.iterrows():
            T = (option['Maturity'] - option['Date']).days / 365.
            model_value = BSM_call_value(S0, option['Strike'], 0, T, r, sigma)
            se.append((model_value - option['Call']) ** 2)
        RMSE = math.sqrt(sum(se) / len(se))
        min_RMSE = min(min_RMSE, RMSE)
        if i % 5 == 0:
            print('%4d |' % i, np.array(p0), '| %7.3f | %7.3f' % (RMSE, min_RMSE))
        i += 1
        return RMSE
```

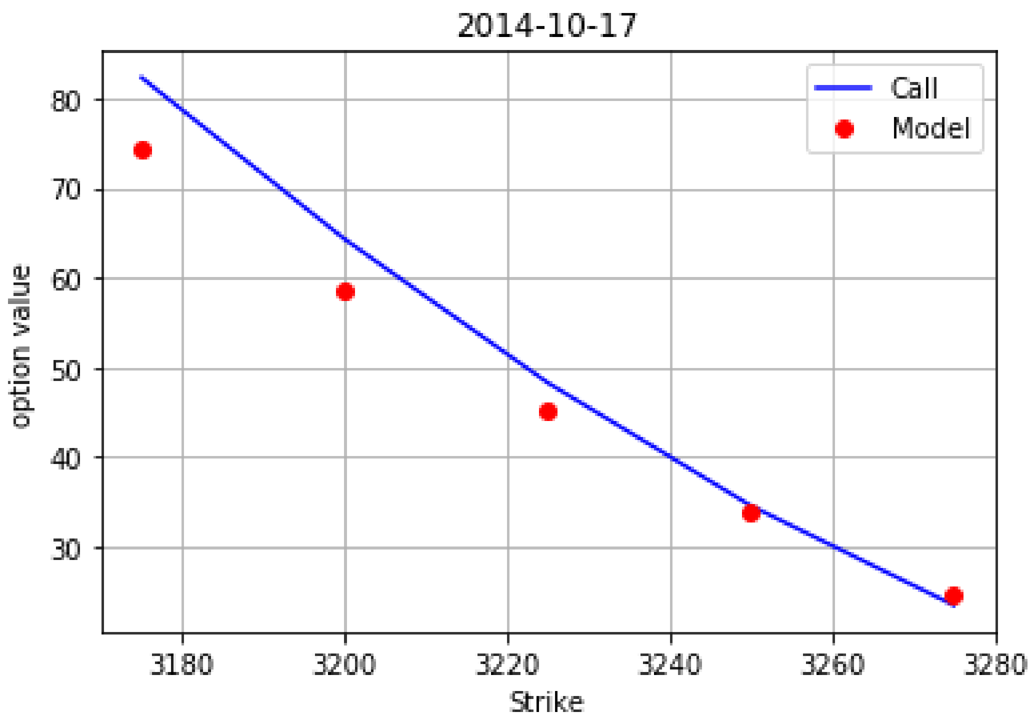
```
In [7]: # Calibration initialization
        #
        i = 0  # counter initialization
        min_RMSE = 200  # minimal RMSE initialization
        opt = sop.fmin(BSM_error_function, 0.5,
                       maxiter=500, maxfun=750,
                       xtol=0.000001, ftol=0.000001)
```

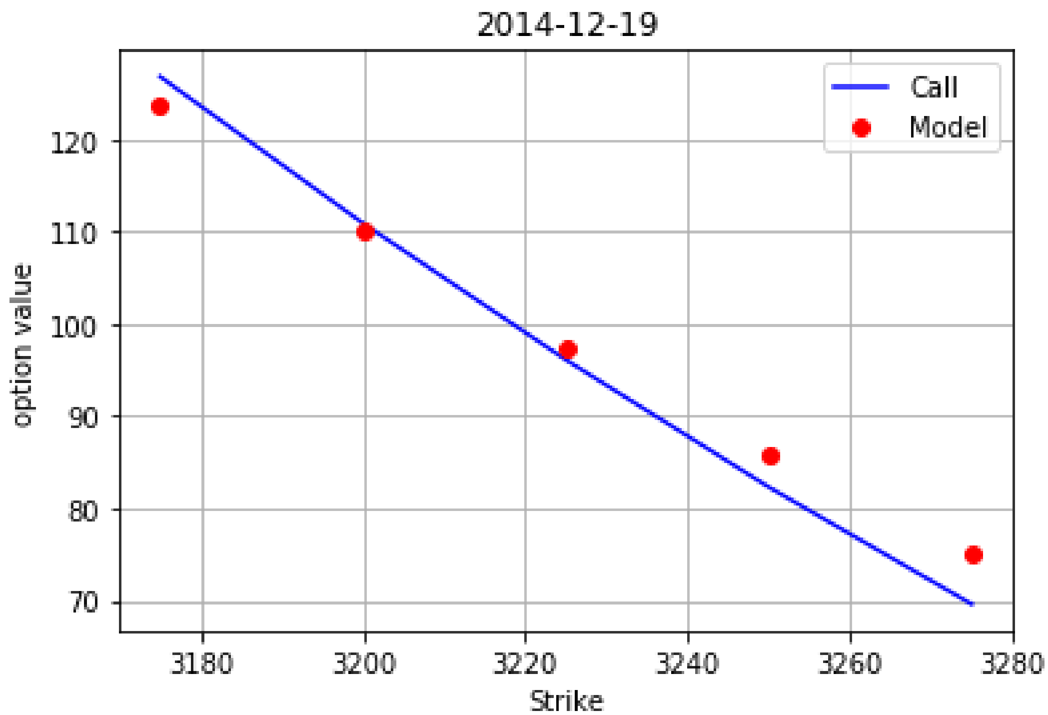
```
   0 | [0.5] | 214.284 | 200.000
   5 | [0.35] | 119.761 | 119.761
  10 | [0.2] |  25.162 |   7.525
  15 | [0.175] |   9.808 |   3.796
  20 | [0.1609375] |   3.594 |   3.594
  25 | [0.1609375] |   3.594 |   3.584
  30 | [0.16049805] |   3.584 |   3.584
  35 | [0.16049805] |   3.584 |   3.584
  40 | [0.16051178] |   3.584 |   3.584
Optimization terminated successfully.
         Current function value: 3.584264
         Iterations: 22
         Function evaluations: 44
```

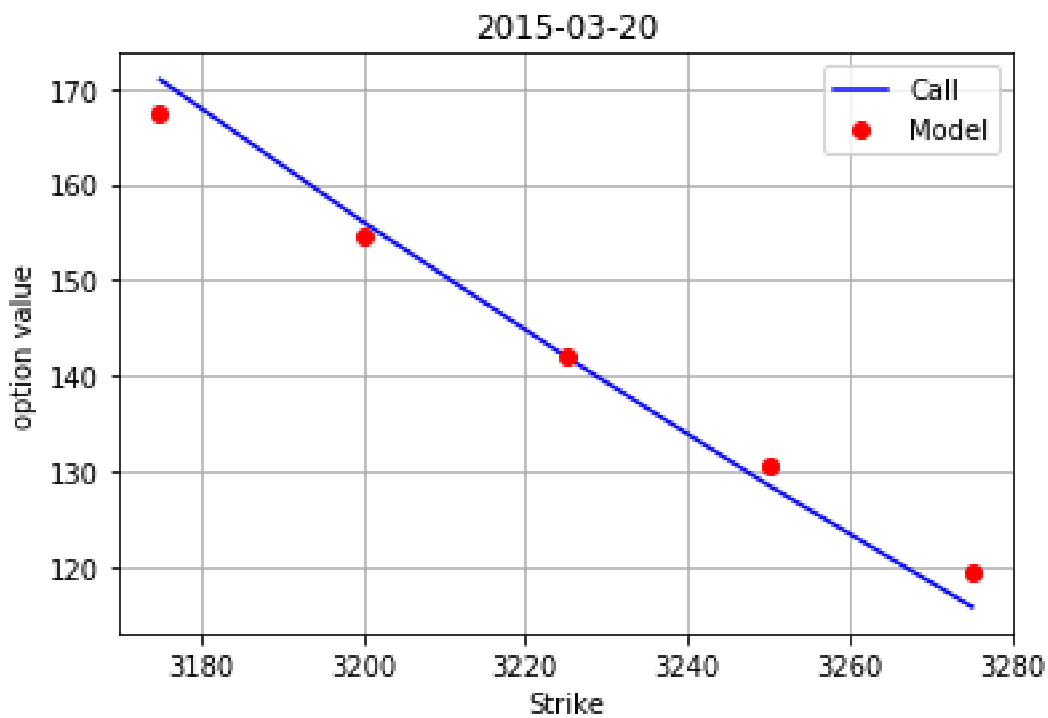
```
In [8]: print(opt)
```

```
[0.16051331]
```

```
In [9]: generate_plot(opt, options);
```




2014-12-19
Call
Model
option value
Strike


2015-03-20
Call
Model
option value
Strike