Recall prop.

"prop1" If $MSE(\hat{\alpha}_n) \to 0$, then $\{\hat{\alpha}_n\}$ is consistent.

prop1 is the consequence of the following fact.

prop2 $L_2\text{-}\lim_{n\to\infty} \alpha_n = \alpha$ implies $p\text{-}\lim_{n} \alpha_n = \alpha$.

pf $\forall \varepsilon > 0$

$$\mathbb{P}(|\alpha_n - \alpha| > \varepsilon) = \mathbb{P}\left(\frac{|\alpha_n - \alpha|}{\varepsilon} > 1\right)$$

$$\leq \mathbb{E}\,\frac{|\alpha_n - \alpha|^2}{\varepsilon^2} \quad \text{by Chebyshev inequality,}$$

$$\longrightarrow 0 \quad \text{b/c } L_2\text{-}\lim_{n\to\infty} \alpha_n = \alpha.$$

$\boxed{S}$

Rk Chebyshev inequality is used above, i.e.

$$\mathbb{P}(|X| > 1) \leq \underbrace{\mathbb{E}[|X|]}$$

pf RHS $= \int_{\mathbb{R}} |x|\, p(x)\, dx$, when $p \sim X$

$$\overset{?}{\geq} \int_{-\infty}^{-1} p(x)dx + \int_{1}^{\infty} p(x)\, dx = \text{LHS}$$

prop

If F is a strictly increasing CDF,

$U \sim U(0,1)$, uniform r.v. on $[0,1]$,

then $X = F^{-1}(U)$ has its CDF F.

pf $\mathbb{P}(X < a) \overset{?}{=} F(a)$

$= \mathbb{P}(F^{-1}(U) < a)$

$= \mathbb{P}(U < F(a))$

$= F(a)$

$\square$

$U \sim U(0,1)$

$\mathbb{P}(U < \frac{1}{2}) = \frac{1}{2}$

$\mathbb{P}(U < \tilde{a}) = \tilde{a}$ if $\tilde{a} \in (0,1)$

A company sells option

If ICC crashes within 10 Yrs, Buyer will obtain \$100 otherwise ~~Nothing~~ Zero \$.

Suppose $\mathbb{P}(\text{ICC crashes within 10 Yr}) = 1\%$.

then what is fair price of the option?