Assignments

September 13, 2018

1 Assignments

Prj01 (L01)

A twin prime is a prime number that is either 2 less or 2 more than another prime number, see for details here: https://en.wikipedia.org/wiki/Twin_prime

- How many twin primes are bigger than one thousand but smaller than one million?
- What is the biggest twin prime you could find?

Prj02. (L04) This is an application of BSM evaluation to Geometric asian option price
Geometric asian call option with maturity T and strike K has its pay off as

$$C(T) = (A(T) - K)^+,$$

where $A(T)$ is geometric average of the stock price at times $0 \le t_1 < t_2, \ldots, < t_n = T$, i.e.

$$A(T) = (S(t_1)S(t_2)\ldots S(t_n))^{1/n}.$$

The call price can be thus written by

$$C_0 = \mathbb{E}[e^{-rT}(A(T) - K)^+].$$

To do Use "BSM_option_valuation" module to find the BSM asian option value with the following parameters

```
In [1]: %reset -f
        import numpy as np
        #An example is given here
        S0 = 100.0
        K = 110.0
        r=0.0475
        sigma = 0.20
        T = 1.
        Otype='C'
        n = 5
        t = np.linspace(0., T, n)
```

Hint We set $t_0 = 0$. Under the above BS model, one can show that the distribution of $A(T)$ is again a lognormal in the form of

$$\ln \frac{A(T)}{A(0)} \sim \mathcal{N}(\frac{\mu}{n} \sum_{i=1}^{n} t_i,\ \frac{\sigma^2}{n^2} \sum_{j=0}^{n-1} (n-j)^2 (t_{j+1} - t_j)),$$

where $A(0) = S(0)$, $\mu = r - \frac{1}{2}\sigma^2$. Therefore, we can rewrite the price formula as, with $Z \sim \mathcal{N}(0,1)$

$$C_0 = \mathbb{E}[e^{-rT}(S(0)e^{(\hat{r}-\frac{1}{2}\hat{\sigma}^2)T+\hat{\sigma}\sqrt{T}Z} - K)^+]$$

or equivalently

$$C_0 = e^{-(r-\hat{r})T}\mathbb{E}[e^{-\hat{r}T}(S(0)e^{(\hat{r}-\frac{1}{2}\hat{\sigma}^2)T+\hat{\sigma}\sqrt{T}Z} - K)^+]$$

with $\hat{\sigma}$ and $\hat{r}$ is determined from

$$\hat{\sigma}^2 T = \frac{\sigma^2}{n^2} \sum_{j=0}^{n-1} (n-j)^2 (t_{j+1} - t_j))$$

and

$$(\hat{r} - \frac{1}{2}\hat{\sigma}^2)T = \frac{\mu}{n} \sum_{i=1}^{n} t_i.$$