**Q1. Lotka-Volterra Equations**

(exercise statement translation made by deepL)

The Lotka-Volterra equations are an estimation model for the evolution of predator-prey populations. In other words, a way of predicting how the number of, for example, Iberian lynx (predator) and wild rabbits (prey) varies over time in a given nature reserve. These are two coupled 1st order differential equations, with the form:

$$dx/dt = x(a - by)$$

$$dy/dt = -y(c - dx)$$

where, in the case of lynxes and rabbits:

- $x$ : number of rabbits
- $y$ : number of lynxes
- $t$ : time. Note that $x$ and $y$ vary in time: $x(t)$, $y(t)$
- $a$ : birth rate of rabbits
- $c$ : lynx mortality rate
- $b$ : effect of predator-prey interaction on rabbit mortality
- $d$ : effect of predator-prey interaction on lynx birth rate

Except in rare cases, a system of coupled differential equations generally has no analytical solution, and the Lotka-Volterra equations are no exception. However, these systems can be solved numerically by applying a method similar to the one we used to solve 2nd order ordinary differential equations.

To numerically solve a system of coupled differential equations of the type:

$$dx/dt = f(t, x, y)$$

$$dy/dt = g(t, x, y)$$

where $f$ and $g$ are generic functions of the indicated variables, the following Heun iterations are applied:

$x_{i+1} = x_i + 1/2\,(k_{1x} + k_{2x})\,h$

$k_{1x} = f(t_i, x_i, y_i)$

$k_{2x} = f(t_i + h, x_i + k_{1x} h, y_i + k_{1y} h)$

$y_{i+1} = y_i + 1/2\,(k_{1y} + k_{2y})\,h$

$k_{1y} = g(t_i, x_i, y_i)$

$k_{2y} = g(t_i + h, x_i + k_{1x} h, y_i + k_{1y} h)$

As with a 2nd order differential equation, to start the iterations we need two initial values. In the case of the Lotka-Volterra equations, these are simply the initial values of the lynx and rabbit populations.

**Questions:**

(a) [3.5 points] Estimate the evolution of lynx and rabbit populations in the Guadiana Valley Natural Park (69,700 hectares [ha]) over 100 years, implementing the above iterations for the Lotka-Volterra equations and taking into account the following set of parameters:

- Initial rabbit population ($x_0$): 2 rabbits/ha (use thousands of rabbits as unit)
- Initial lynx population ($y_0$): 209 (exact value in 2021)
- a : 0.20 (20% increase in rabbits per year)
- c : 0.08 (8% decrease in lynx per year)
- b : 0.001 (each existing lynx increases mortality per thousand rabbits by 0.001; i.e., each lynx preys on one more rabbit in the time period h)
- d : 0.001 (each thousand rabbits increases the lynx birth rate by 0.001 cubs per existing lynx; i.e., each million rabbits generates one more lynx cub for each lynx existing in period h)
- h : 1 year

(b) [0.5 points] Graphically represent the results obtained and comment on them.