CAS MA575: Assignment #4

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Chapter 2.2

Statement (d) is true, and following are my reasons:

Through Figure 2.8, we can easily tell that $|y_i - \hat{y_i}|$ in model 1 is smaller compare to that in model 2. Since $RSS = \sum_{i=1}^{n}(y_i - \hat{y_i})^2$, RSS of model 1 should be less than RSS of model 2.

And because $RSS + SSreg = SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$is fixed, RSS of model 1 is less than RSS of model 2 means that SSreg of model 1 is greater than SSreg of model 2.

Chapter 2.3

Because y_i, y_j is independent when $i \neq j$, which means $cov(y_i, y_j) = 0$ when $i \neq j$.

$$\begin{aligned}
cov(y_{n+1}, y_{n+1} + n\hat{\mu_n}) &= cov(y_{n+1}, \sum_{i=1}^{n+1} y_i) \\
&= \sum_{i=1}^{n} cov(y_{n+1}, y_i) \\
&= cov(y_{n+1}, y_{n+1}) \\
&= \sigma^2
\end{aligned}$$

2.(b)

$$
\begin{aligned}
cov(\hat{\mu_n}, y_{n+1} + n\hat{\mu_n}) &= cov(\frac{1}{n}\sum_{i=1}^{n} y_i, \sum_{i=1}^{n+1} y_i) \\
&= \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n+1} cov(y_i, y_j) \\
&= \frac{1}{n}\sum_{i=1}^{n} cov(y_i, y_i) \\
&= \frac{1}{n} n\sigma^2 \\
&= \sigma^2
\end{aligned}
$$

2.(c)

$$
\begin{aligned}
cov(y_{n+1} - \hat{\mu_n}, y_{n+1} + n\hat{\mu_n}) &= cov(y_{n+1} - \frac{1}{n}\sum_{i=1}^{n} y_i, \sum_{i=1}^{n+1} y_i) \\
&= cov(y_{n+1}, \sum_{i=1}^{n+1} y_i) - cov(\frac{1}{n}\sum_{i=1}^{n} y_i, \sum_{i=1}^{n+1} y_i) \\
&= \sigma^2 - \sigma^2 \\
&= 0
\end{aligned}
$$

Since $y_{n+1} - \hat{\mu_n}$ and $y_{n+1} + n\hat{\mu_n}$ are normal distribution, $cov(y_{n+1} - \hat{\mu_n}, y_{n+1} + n\hat{\mu_n}) = 0$ means that they are independent.

Chapter 2.6

(a)

$$
\begin{aligned}
(y_i - \hat{y_i}) &= y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \\
&= y_i - (\bar{y} - \hat{\beta}_1\bar{x}) - \hat{\beta}_1 x_i \\
&= (y_i - \bar{y}) - \hat{\beta}_1(x_i - \bar{x})
\end{aligned}
$$

(b)

$$
\begin{aligned}
(\hat{y}_i - \bar{y}) &= \hat{\beta}_0 + \hat{\beta}_1 x_i - \bar{y} \\
&= \bar{y} - \hat{\beta}_1 \bar{x} + \hat{\beta}_1 x_i - \bar{y} \\
&= \hat{\beta}_1 (x_i - \bar{x})
\end{aligned}
$$

(c)

$$
\begin{aligned}
\sum_{i=1}^{n} (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) &= \sum_{i=1}^{n} \big[(y_i - \bar{y}) - \hat{\beta}_1 (x_i - \bar{x})\big] \hat{\beta}_1 (x_i - \bar{x}) \\
&= \hat{\beta}_1 \Big(\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x}) - \hat{\beta}_1 \sum_{i=1}^{n} (x_i - \bar{x})^2 \Big) \\
&= \hat{\beta}_1 \Big(SXY - \frac{SXY}{SXX} SXX \Big) \\
&= 0
\end{aligned}
$$