CAS MA575: Assignment #3

Hu Hengchang(U34497427)
hhc@bu.edu

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1.

Statement (d) is true, and following are my reasons:

Through Figure 2.8, we can easily tell that $|y_i - \hat{y_i}|$ in model 1 is smaller compare to that in model 2. Since $RSS = \sum_{i=1}^{n}(y_i - \hat{y_i})^2$, RSS of model 1 should be less than RSS of model 2.

And because $RSS + SSreg = SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$is fixed, RSS of model 1 is less than RSS of model 2 means that SSreg of model 1 is greater than SSreg of model 2.

2.(a)

Because y_i, y_j is independent when $i \neq j$, which means $cov(y_i, y_j) = 0$ when $i \neq j$.

$$
\begin{aligned}
cov(y_{n+1}, y_{n+1} + n\hat{\mu_n}) &= cov(y_{n+1}, \sum_{i=1}^{n+1} y_i) \\
&= \sum_{i=1}^{n} cov(y_{n+1}, y_i) \\
&= cov(y_{n+1}, y_{n+1}) \\
&= \sigma^2
\end{aligned}
$$

2.(b)

$$
\begin{aligned}
cov(\hat{\mu_n}, y_{n+1} + n\hat{\mu_n}) &= cov(\frac{1}{n}\sum_{i=1}^{n} y_i, \sum_{i=1}^{n+1} y_i) \\
&= \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n+1} cov(y_i, y_j) \\
&= \frac{1}{n}\sum_{i=1}^{n} cov(y_i, y_i) \\
&= \frac{1}{n} n\sigma^2 \\
&= \sigma^2
\end{aligned}
$$

2.(c)

$$
\begin{aligned}
cov(y_{n+1} - \hat{\mu_n}, y_{n+1} + n\hat{\mu_n}) &= cov(y_{n+1} - \frac{1}{n}\sum_{i=1}^{n} y_i, \sum_{i=1}^{n+1} y_i) \\
&= cov(y_{n+1}, \sum_{i=1}^{n+1} y_i) - cov(\frac{1}{n}\sum_{i=1}^{n} y_i, \sum_{i=1}^{n+1} y_i) \\
&= \sigma^2 - \sigma^2 \\
&= 0
\end{aligned}
$$

Since $y_{n+1} - \hat{\mu_n}$ and $y_{n+1} + n\hat{\mu_n}$ are normal distribution, $cov(y_{n+1} - \hat{\mu_n}, y_{n+1} + n\hat{\mu_n}) = 0$ means that they are independent.

3.(a)

Because $y_i, i = 1, ...n$ is $N(0, \sigma^2)$, $\frac{y_i}{\sigma}$ is $N(0, 1)$.Therefore $\sigma^{-2}\sum_{i=1}^{m} y_i^2 = \sum_{i=1}^{m}(\frac{y_i}{\sigma})^2$ is square sum of m independent $N(0, 1)$, which is χ_m^2, for any $1 \leq m \leq n$.

3.(b)

First of all, $\frac{y_{m+1}}{\sigma}$ is $N(0, 1)$, for $1 \leq m \leq n - 1$.

And $\sum_{i=1}^{m}(\frac{y_i}{\sigma})^2$ is $\chi_m^2 \rightarrow \frac{\frac{y_{m+1}}{\sigma}}{\sqrt{\frac{\sum_{i=1}^{m} y_i^2}{m\sigma^2}}}$ is t_m.

Which equals $\frac{y_{m+1}}{\sqrt{m^{-1}\sum_{i=1}^{m} y_i^2}}$ is t_m, for any $1 \leq m \leq n - 1$.

3.(c)

Because $\sum_{i=m+1}^{n}(\frac{y_i}{\sigma})^2$ is χ^2_{n-m}, $\sum_{i=1}^{m}(\frac{y_i}{\sigma})^2$ is χ^2_m.

And since $y_i, i = 1, ...n$ are independent, $\sum_{i=m+1}^{n}(\frac{y_i}{\sigma})^2$ and $\sum_{i=1}^{m}(\frac{y_i}{\sigma})^2$ are independent.

Therefore $\frac{\sum_{i=m+1}^{n}(\frac{y_i}{\sigma})^2/(n-m)}{\sum_{i=1}^{m}(\frac{y_i}{\sigma})^2/m} = \frac{(n-m)^{-1}\sum_{i=m+1}^{n} y_i^2}{m^{-1}\sum_{i=1}^{m} y_i^2}$ is $F_{n-m,m}$.