CAS MA575: Assignment #8

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4.2

$WRSS = \sum W_i(Y_i - bX_i)^2$

Let $\frac{\partial WRSS}{\partial b} = \sum W_i 2(Y_i - bX_i)(-X_i) = 0$

$\Rightarrow \hat{\beta} = \frac{\sum W_i X_i Y_i}{\sum W_i X_i^2}$ and $\frac{\sigma^2}{W_i} = X_i^2\sigma^2$ where $W_i = \frac{1}{X_i^2}$

5.3

(a)

Since the p-value of interaction term is $0.012 < 0.05$, the interaction term is statistically significant, and from the analysis of variance, we can deduce that the full model is better.

In other words, whether there has been any unwanted rain at vintage does affect the quality rating.

(b)

We can derive a regression model from the full model when there is no unwanted rain at vintage: $-1 = -0.03145 * X \Rightarrow X \approx 32$, so it takes about 32 days to decrease the quality rating by 1 point

Similarily, if there has been some unwanted rain at harvest, $-1 = -0.03145 * X - 0.08314 * X \Rightarrow X \approx 9$, so it takes about 9 days to decrease the quality rating by 1 point when there has been some unwanted rain at harvest.

2.

$$Left = \frac{Y^TY - (Y - X\hat{\beta})^T(Y - X\hat{\beta})}{p\hat{\sigma}^2}$$

$$Right = \frac{\hat{\beta}^T(X^TX)\hat{\beta}/p}{\hat{\sigma}^2} = \frac{((X^TX)^{-1}X^TY)^T(X^TX)(X^TX)^{-1}X^TY}{p\hat{\sigma}^2} = \frac{Y^TX(X^TX)^{-1}X^TY}{p\hat{\sigma}^2} = \frac{Y^TX\hat{\beta}}{p\hat{\sigma}^2}$$

we want to prove: $Y^TY - (Y - X\hat{\beta})^T(Y - X\hat{\beta}) = Y^TX\hat{\beta}$
$\Leftrightarrow Y^TX\hat{\beta} + (X\hat{\beta})^TY - (X\hat{\beta})^T(X\hat{\beta}) = Y^TX\hat{\beta} \Leftrightarrow (X\hat{\beta})^TY = (X\hat{\beta})^T(X\hat{\beta})$
$\Leftrightarrow \hat{\beta}^TX^TY = \hat{\beta}^TX^TX\hat{\beta} \Leftrightarrow \hat{\beta}^TX^TY = \hat{\beta}^TX^TX(X^TX)^{-1}X^TY = \hat{\beta}^TX^TY$
Prove done.