博士論文

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Superposition states in quantum optics: teleportation experiments, modeling theory, tomography algorithms

(非古典状態の量子テレポーテーション実験の研究—
条件付き操作、理論モデル、トモグラフィアルゴリズム)

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Being a foreigner in Japan certainly has its challenges. Overall, moving from France to Japan I feel I had the chance to receive a very warm and hearty welcome and could integrate myself smoothly and find my place in this wonderful country, thanks to the help from the people of my laboratory, the great support from the staff of the University of Tokyo, and the international student community here. Especially, I would like to thank all my friends at the University of Tokyo, those who have already graduated and those who will still be studying after me, for interesting discussions about life in Japan, for cheering me up, for listening to my incessant rambling, and overall many good moments and memories. I am also very grateful to Japan and the Ministry of Education for giving me the financial support and opportunity to study three years at the University of Tokyo and obtain the doctoral degree in such favorable conditions.

I cannot close this section without looking back on the long road which brought me here as I think that getting the Ph.D title eventually plays out in more than three years. First and foremost, it is my family I want to thank, to whom I owe my education and personal values, my curiosity and taste for understanding my surroundings. I also thank my home country which gave me the rare opportunity to study at such wonderful schools and learn from so many knowledgeable and wise teachers and professors who gave me the willingness to succeed and pursue in scientific studies as far as I did until today. In that regard, my greatest thanks go especially to the professoral staff of the Physics department at Ecole Polytechnique and Ecole Normale Supérieure.

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List of Symbols

Symbol	Description
Variables and parameters	
x,q	position variable
p	momentum variable
$\alpha = (x+ip)/\sqrt{2}$	complex amplitude
t	time
ω, Ω	angular frequency and sideband angular frequency
s	squeezing power
r	EPR correlation power
η	loss coefficient
Operators	
$\hat{x}$, $\hat{p}$	position and momentum operators
$\hat{a}$, $\hat{a}^\dagger$	annihilation and creation operators
$\hat{\rho}$	density matrix
$\hat{D}(\alpha)$, $\hat{D}_\alpha$	displacement operator
$\hat{S}(s)$, $\hat{S}_s$	squeezing operator
$\hat{U}(\theta)$, $\hat{U}_\theta$	phase-shifter operator
$\hat{B}(\theta)$, $\hat{B}_\theta$	beam-splitter operation
Phase-space quasi-distributions	
$W(q,p)$, $W_{q,p}$, $W(\alpha)$	Wigner function
$Q(q,p)$, $Q_{q,p}$, $Q(\alpha)$	Hushimi function
$P(q,p)$, $P_{q,p}$, $P(\alpha)$	Glauber-P function
$\chi(u,v)$, $\chi_{u,v}$, $\chi(\xi)$	Characteristic function
$\mathcal{R}$, $\mathcal{R}^{-1}$	Radon and inverse Radon transforms
Polynomial series and special functions	
$H_n(x)$	Hermite polynomials
$T_n(x)$, $U_n(x)$	Chebysheff's polynomials, first and second kind
$Z^n_s(r,\phi) = R^n_s(r)e^{in\phi}$	complex Zernike polynomials
$J_n(r)$	Bessel's functions of the first kind
erf(x)	error function
Mathematical operations	
$\circ$, $\star$	1-dimensional convolution
$\circ\circ$, $\star\star$	2-dimensional convolution
tr()	trace operation
$\bigotimes$	tensor product

Table 1: Recurrent mathematical symbols and notations.

Introduction

The field of Quantum Optics

Quantum optics could be defined as the study of the quantum properties of propagating waves of the electrical field. In other words, quantum optics is the description of light waves at the quantum level using quantum mechanics. Historically, the modern theory of quantum optics has been developed by Roy J. Glauber[24] in the sixties with his work on quantum coherence initiated to explain the Hanbury Brown-Twiss interferometer experiment and the photon bunching phenomena [5]. It was already known before that time that a quantum description of the electrical field was necessary to grasp the understanding of certain phenomena involving interaction between light and matter. Although the idea of the existence of particles of light appeared several times in the historical development of physics, the precise idea of quanta of light dates back to the beginning of the nineteenth century and to Plank[1] and Einstein's work[2]. As a matter of fact, the word "photon" itself was first proposed by Einstein to put a name on the concept of quanta of light. These ideas were later experimentally verified by Arthur Compton[3]. Despite these early development which also turned out to be the root of quantum mechanics itself and one of its first experimental hints, the modern theory of quantum optics was rather late to emerge. Before Glauber simplified the complex formalism of quantum field theory into a simpler theoretical framework for quantum optics, quantitative studies were only possible using the complete theory of quantum electrodynamics and radiation. Nowadays, we have gained a very good understanding of the quantum description of light, a question which for the most part is solved as far as fundamental sciences go.

Nevertheless quantum optics is nowadays a very active field of research. As it turns out, the electrical field is the most simple physical system for fundamental tests of quantum physics. The superposition principle, quantum entanglement, Bose-Einstein statistics, interferences; all essential properties of quantum mechanics can be observed and studied in quantum optics easily all the better thanks to the absence of direct photon-photon interactions. Experimental quantum optics was essentially started with the access to coherent light sources and Theodore H. Maiman's first ruby laser [6]. From the first pioneering laser systems of the sixties to current state-of-the-art experiments, xperimental quantum optics has advanced steadily and passed several important milestones. In the sixties, seventies and eighties, the development of more powerful laser systems at new wavelengths allowed for the study of non-linear optics[7]. In the eighties, progress progress in the study and understanding of parametric-down conversion and four wave mixing phenomena led to the observationand demonstration of squeezed light and "two-photons coherent states", as squeezing was called at that time[11, 12, 13]. The availability of squeezed quantum states eventually paved the road to many quantum optics experiments, starting with the first Gaussian information processing experiments[20] and qubit experiments[16] of the late nineties. In the last ten years experimental quantum optics has been the testing ground of further experiments and fundamental tests of quantum mechanics, focusing on the generation, measurement, characterisation and manipulation of exotic or entangled quantum states of light. Inspired by the "gedenken experiment" of the founding fathers of quantum mechanics, these exotic quantum states of light such as the elusive Schroedinger's cat state are now routinely produced in laboratories[23, 22, 25].

The rapid development of quantum optics was possible thanks to the rather unlikely combination of four ingredients: 1) lasers which produce very pure coherent light of all frequencies are now a mainstream technology, easy to produce, deploy and operate. 2) in the particular optical region of the electromagnetic spectrum, background light is virtually void of any black body radiations, so that a clean and quiet environment down to the level of the quantum vacuum can be easily obtained in the laboratory. 3) with electronics noise levels far below the optical shotnoise level, high energy conversion efficiency and linearity, silicon PIN photodiodes allow for easy measurements of the quantum state of light down to the quantum vacuum level. 4) easy passive manipulation of light through the index of refraction gives us a complete toolbox of so-called linear optical components which forms the backbone of most experimental setups: optical lenses, dielectric and Bragg mirrors, amplitude modulators, birefringent materials, waveguides.

Quantum Information Processing

Quantum mechanics calls for a different treatment of information processing than what Claude E. Shannon achieved with his classical theory of information[4]. With the swift development of computer technologies following the end of World War II and the invention of the transistor, it quickly appeared to physicists that quantum mechanics could certainly have a special status in information processing[9, 10]. Although the question is still open in computational complexity theory and although the exact relation between complexity classes associated with quantum computers like BQP (bounded error quantum polynomial time) and QMA (quantum analog of the deterministic complexity class NP or the probabilistic complexity class MA) and classical complexity classes is still unknown, it is firmly believed that quantum computers can genuinely solve a number of computational problems faster than classical computers . Some famous quantum algorithms like Shor's factoring algorithm[19] or Grover's search algorithm[18] are hints of this conjecture and of the advantage of quantum computers.

From the original idea of a quantum analogical computer by Feynman[9] and the experimental proof of violation of the Bell's inequalities[8] and therefore the genuine existence of quantum entanglement, the field of quantum information processing is now a major sub-field of quantum mechanics. Quantum information processing has now also become the major and most active research topic in the field of quantum optics. Of course, quantum information processing can be studied with several different physical systems, but light provides an especially adequate medium for the implementation of quantum communication and information algorithms, thanks to the absence of direct photon-photon interactions and the tendency of photons to stay undisturbed. In that respect, light waves have some genuine advantages for the practical applications and realization of quantum entanglement, quantum measurement and complex quantum information phenomena.

As a striking example of quantum communication protocols, teleportation was discovered early on in the development of the field of quantum information processing. With either qubit [14] and continuous variable flavors [15], experiments were soon to follow [16, 20]. Until now continuous variable teleportation has only been performed with the class of so-called Gaussian states [20, 26, 28, 29]. However, this alone is not sufficient for universal quantum computation where non-Gaussianity of some kind has been shown to be necessary: at least third order nonlinear operations are necessary for building a universal quantum computer[21], something Gaussian operations and Gaussian states alone cannot achieve. For instance photon-subtraction techniques which are based on discrete variable and qubit technology, can provide these useful non-linearities and are used to generate Schroedinger's cat states and other optical non-Gaussian states[17]. Schroedinger's cat states are of particular interest as they have been shown to be a useful resource for fault-tolerant QIP[27]. Although such non-Gaussian non-classical states of light that would allow for such universal operations have been available experimentally for some time in the continuous variable regime [23, 22, 25], the major challenge of actually manipulating these states in some Gaussian protocol context beyond simple generation has remained mostly unaddressed. Therefore, it appears crucial to extend the well understood Gaussian continuous variable technology and linear optics with non-Gaussian states and non-Gaussian operations.

Subject of the thesis

The main subject of this thesis is the experimental study of the quantum teleportation process with as input states non-Gaussian quantum states of light approximating small amplitude Schroedinger's cat states. More precisely, we are interested in the experimental demonstration of the quantum teleportation of non-classical states of light up to the point where the output quantum state are still without ambiguity non-classical quantum states. This experiment is ambitious for it lies on top of two different frontiers of experimental quantum optics. On the one side we inherit from the long lineage of continuous variable Gaussian experiments and have built a quantum teleportation apparatus which transcends the sideband regime. On the other side we have used the photon subtraction protocol to generate in the laboratory these highly non-classical non-Gaussian states necessary for universal quantum information processing. In this thesis manuscript we report on how we have combined these two sets of technologies and how we could demonstrate successful Gaussian manipulation of non-classical non-Gaussian states by achieving experimental quantum teleportation of Schroedinger's cat states of light. Using the photon-subtraction protocol we generate quantum states approximating closely Schroedinger's cat states in a manner similar to [23, 22, 25]. To accommodate with the required time-resolving photon detection techniques and handle the wave-packet nature of these optical Schroedinger's cat states, we have developed a hybrid teleporter built with continuous-wave light yet able to directly operate in the time domain. For this purpose we have constructed a time-gated source of EPR correlations as well as a zero phase-dispersion classical channel. We were able to bring all the experimental parameters up to the quantum regime and performed successful quantum teleportation in the sense that both our input and output states are strongly non-classical quantum states of light. As a prototype of these new techniques in a fully quantum regime, this experiment constitutes a first step towards more advanced QIP protocols and future non-classical state manipulation experiments in quantum optics.

This thesis manuscript explains in details and reports on the results of the experiment discribed above, but also includes several related theoretical studies and analysis linked to this experimental work. Overall, these theoretical studies can be divided in three groups. First, there is the question of understanding precisely what we observe in the teleportation experiment. This requires a complete phase space model of input state, teleportation process, and output teleported state, taking into account in addition the relevant experimental parameters. The second point, which is to some extent related to the first, is to correctly understand the multimode properties of the input photon subtracted state and of the broadband teleporter apparatus to model the transient and time-domain nature of this experiment. The third point is about studies on the process of quantum tomography, which is a necessity for current state-of-the-art experiments in continuous variable quantum optics for characterization of unknown quantum states.

Content of the manuscript

The main content of this thesis manuscript is divided into four chapters. Additional material mostly related to experimental details can be found in the appendix sections.

- The first chapter is a summary and succinct presentation of the theory of quantum optics. It mostly focuses on the physics of a single quantum harmonic oscillator, which is reviewed in the Heisenberg picture, Schroedinger picture, and phase space formalism. It also introduces models for the main experimental measurement techniques available in quantum optics used in this thesis.

- The second chapter focuses exclusively on the single photon subtraction protocol, the tool of choice nowadays to generate small amplitude Schroedinger's cat states of light and other low photon number non-Gaussian states. It presents our experimental setup and results as well as a detailed analysis and quantum theoretical model for this family of photon subtracted states so important for the field of quantum information processing.

- The third chapter deals with the problem of quantum tomography and its practical numerical implementation. The core of the chapter is a demonstration and further study of one of the main results of this thesis, an algorithm for the reconstruction of the Wigner function using polynomial series decomposition and stable linear inversion. It is followed by a detailed analysis of the statistical error in tomography reconstruction algorithms, illustrated with Monte-Carlo simulations.

- The fourth and last chapter presents the main experimental results of this thesis: the experimental deterministic and conditional teleportation of Schroedinger's cat states of light. It includes a simple presentation of the elementary theory of teleportation, and an extended model of wave-packet teleportation. In addition, it gives a description of the experimental setup, presents experimental results and their analysis using the model mentioned above.

The reading order proposed by this manuscript chapters arrangement does not have to be strictly observed, as chapters **??**, **??** and **??** are mostly independent, to the exception of experimental results of Chaps.**??** and **??**. Chapter **??** is included in this manuscript mostly as a necessary reference for the good understanding of equations and mathematical derivations and the rigorous definition of operators and mathematical objects appearing in the following chapters. However, some paragraphs in chapter **??**, most notably in Secs.**??** and **??** go beyond simple textbook material and are referenced at several latter points.

Academic Publications and Conferences

The results of this thesis have been published in several scientific journal publications. In chronological order, they are:

- N. Lee, H. Benichi, Y. Takeno, S. Takeda, J. Webbs, E. Hungtington, and A. Furusawa, "Quantum teleportation of nonclassical wave packets: An effective multimode theory", *Science* **332**, 330 (2011).

- H. Benichi, S. Takeda, N. Lee, and A. Furusawa, "Teleportation of Non-Classical Wave-Packets of light", *Phys. Rev. A* **84**, 012308 (2011).

- H. Benichi, and A. Furusawa, "Optical homodyne tomography with polynomial series expansion", *Phys. Rev. A* **84**, 032104 (2011).

The first of this article presents the main experimental results on Schroedinger's cat state teleportation, which is the subject of Chap.**??** for teleportation, and Chap.**??** for Schroedinger's cat state generation. The second article presents a detailed theoretical model of the experiment, with careful considerations of its broadband characteristics. This model can be found in Chap.**??**, over Secs.**??** and **??**. The last article presents a related work on quantum tomography and demonstrate a new reconstruction algorithm of the Wigner function, which is included in Chap.**??**. At the time of writing, another article reporting on the results of Gaussian conditional teleportation is in preparation. The work of this thesis has also been presented through posters and presentations at the following international and domestic conferences:

- ButsuriGakkai64, 64th fall meeting of the Japanese Society of Physics, Kumamoto University, Japan, Sep 2009 (presentation, 25pZF-2).
- QIT21, the 21st Quantum Information Technology Symposium, Tokyo, Japan, Nov 2009 (presentation).
- SPQT 2010, Quantum Measurement and Control workshop, Sydney, Australia, Feb 2010 (poster).
- ISPQT 2010, International symposium on Physics of Quantum Technology, Tokyo, Japan, Apr 2010 (poster, 8TH-01).
- QCMC 2010, Tenth International Conference on Quantum Communication, Measurement and Computation, Brisbane, Australia, Jul 2010 (poster, P3-10).
- FIRST 2010, Annual meeting of the FIRST project on quantum information processing, Atami, Japan, Dec 2010 (poster).
- CLEO 2011, Conference on Laser and Electro-Optics, Baltimore USA, May 2011 (presentation, JTuF4).

Finally, during the CLEO US 2011 conference, the work of this thesis has been awarded by the Optical Society of America (OSA) with the Theodore Maiman Outstanding Student Prize, first prize, including a $3000 cash prize.