# Project 3 : Markov switching models

Statistics for Smart Data

*Mattia Guerri - Hugo Brehier*

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# Contents

# Introduction

In this project, we study Markov Switching Models. First, a simulation study is done. Next, we define the model and the estimation algorithm. Finally, we apply it to real financial data.

## Simulated Data

Install the depmixS4 and quantmod libraries

```
library('depmixS4')
library('quantmod')
library(ggplot2)
library(scales)
set.seed(1)
```

Create the parameters for the bull and bear market returns distributions

```
Nklower <- 50
Nkupper <- 150
bullmean <- 0.1
bullvar <- 0.1
bearmean <- -0.05
bearvar <- 0.2
```

Create the list of durations (in days) for each regime

```
days = replicate(5, sample(Nklower :Nkupper, 1))
```

Create the various bull and bear markets returns

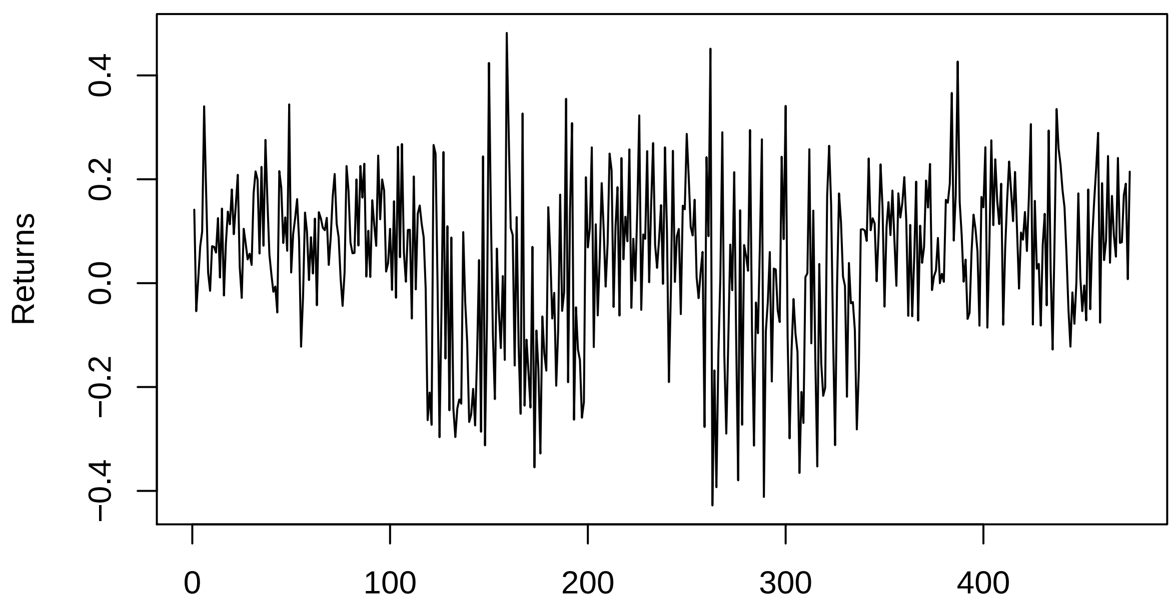
```
marketbull1 <- rnorm( days[1], bullmean, bullvar )
marketbear2 <- rnorm( days[2], bearmean, bearvar )
marketbull3 <- rnorm( days[3], bullmean, bullvar )
marketbear4 <- rnorm( days[4], bearmean, bearvar )
marketbull5 <- rnorm( days[5], bullmean, bullvar )
```

Create the list of true regime states and full returns list

```
trueregimes <-c( rep(2,days[1]), rep(1,days[2]), rep(2,days[3]), rep(1,days[4]), rep(2,days[5]))
returns <-c( marketbull1, marketbear2, marketbull3, marketbear4, marketbull5)
```

Plotting the returns shows the clear changes in mean and variance between the regime switches.

```
plot(returns, type="l", xlab='', ylab="Returns")
```



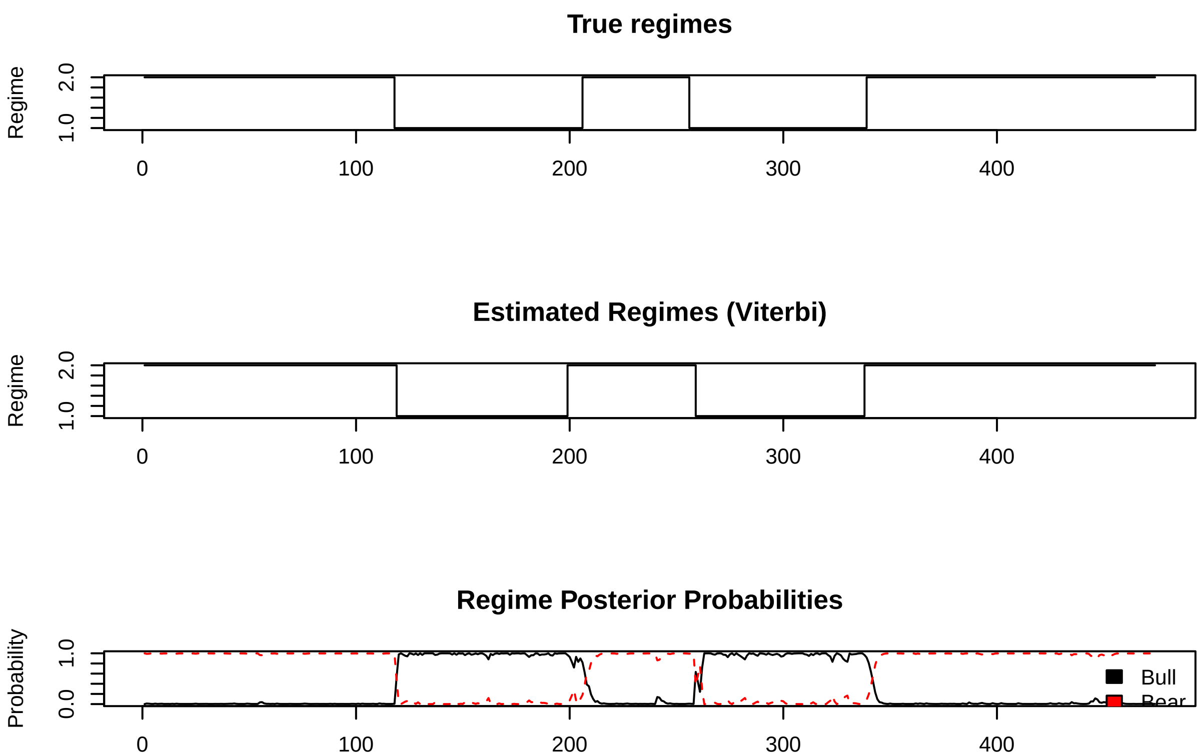
Create and fit the Hidden Markov Model

```r
hmm = depmix(returns ~ 1, family = gaussian(), nstates = 2, data=data.frame(returns))
hmmfit = fit(hmm, verbose = FALSE)
```

```
## converged at iteration 24 with logLik: 289.6389
```

Output both the true regimes and the posterior probabilities of the regimes

```r
postprobs <- posterior(hmmfit)
layout(1 :3)
plot(trueregimes,type = 's',main = 'True regimes',xlab='', ylab='Regime')
plot(postprobs$state, type='s', main='Estimated Regimes (Viterbi)', xlab='', ylab='Regime')
matplot(postprobs[, -1], type='l', main='Regime Posterior Probabilities',ylab='Probability')
legend(x='topright', c('Bull','Bear'), fill=1:2, bty='n')
```



# Model behind the code

## Markov Switching Models

Write the MS model. Documented from [3] and [2] and [1] .

## EM algorithm

EM Estimation method, maximum likelihood. Also Backward-Forward ?

## Application to real financial data

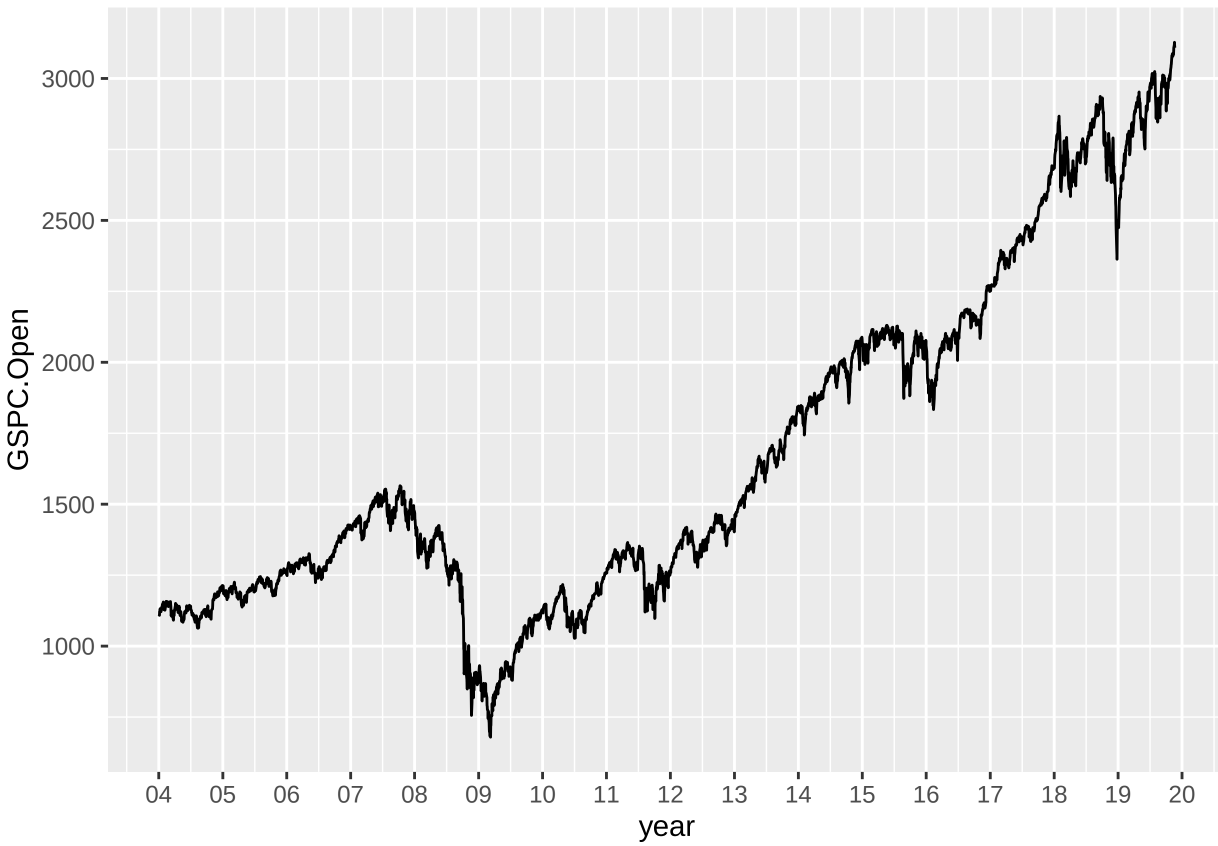
### S&P Index

Obtain S&P500 data from 2004 onwards

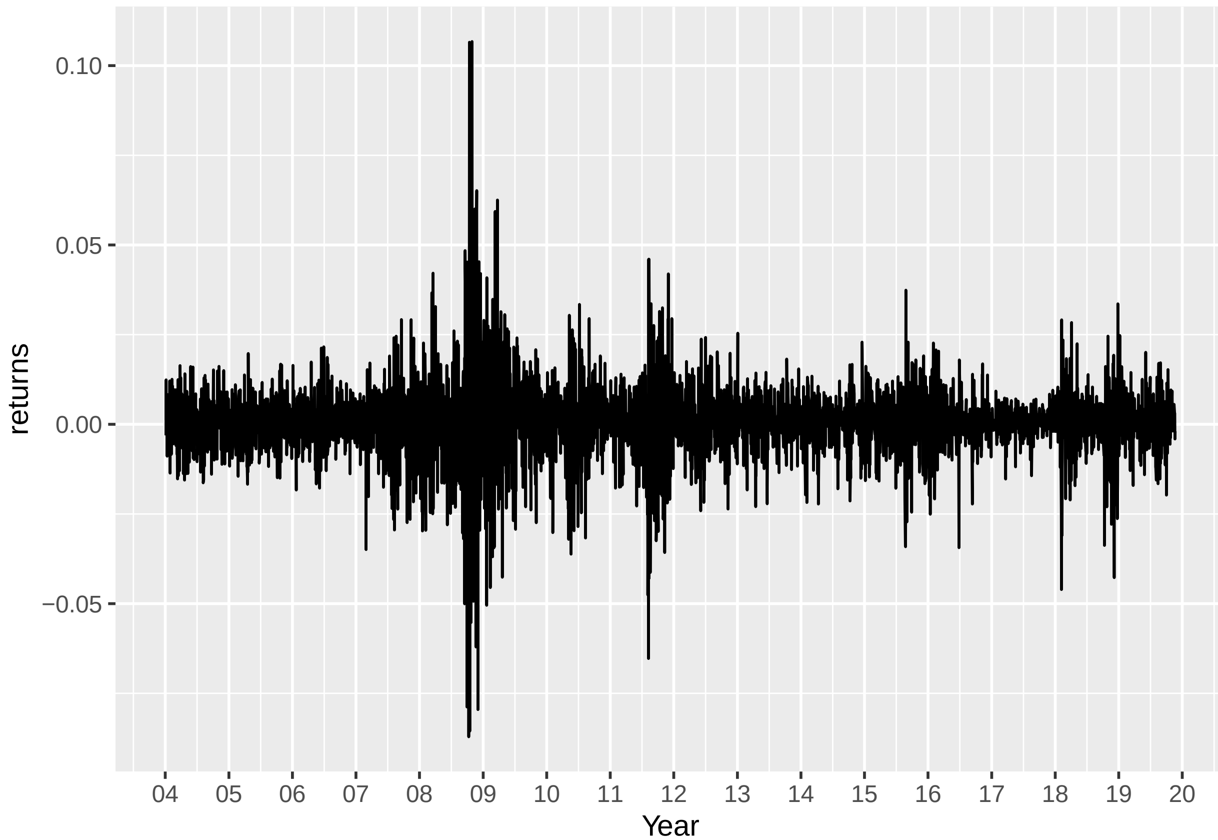
```
getSymbols( '^GSPC',src="yahoo", from="2004-01-01")
```

```
## [1] "^GSPC"
```

```
values = as.data.frame(GSPC)
n= nrow(values)
values$date = as.Date(rownames(values))

ggplot(values, aes(x=date, y=GSPC.Open,group = 1)) + geom_line() + scale_x_date(breaks = date_breaks("y
  labels = date_format('%y')) + xlab('year')
```




```
gspc.returns = as.data.frame((values[2:n, 1] - values[1:(n-1), 1]) /  values[1:(n-1), 1])
gspc.returns$date = as.Date(rownames(values)[2:n])

colnames(gspc.returns) = c('returns','date')

ggplot(gspc.returns, aes(x=date, y=returns,group = 1)) + geom_line() + scale_x_date(breaks = date_breaks
  labels = date_format('%y')) + xlab('Year')
```




```r
k=10
AICs = numeric(length(2:k))

for (i in 2:k) {
  hmm <- depmix(returns ~ 1, family = gaussian(), nstates = i, data=data.frame(gspc.returns))
  hmmfit <- fit(hmm, verbose = FALSE)
  AICs[i-1] = AIC(hmmfit)
}
```

```
## converged at iteration 54 with logLik: 13371.82
## converged at iteration 90 with logLik: 13555.77
## converged at iteration 453 with logLik: 13589.83
## converged at iteration 341 with logLik: 13609.15
```

```r
bestk = which.min(AICs) + 1
bestk
```

```
## [1] 9
```

```r
hmm <- depmix(returns ~ 1, family = gaussian(), nstates = bestk, data=data.frame(gspc.returns))
hmmfit <- fit(hmm, verbose = FALSE)
```

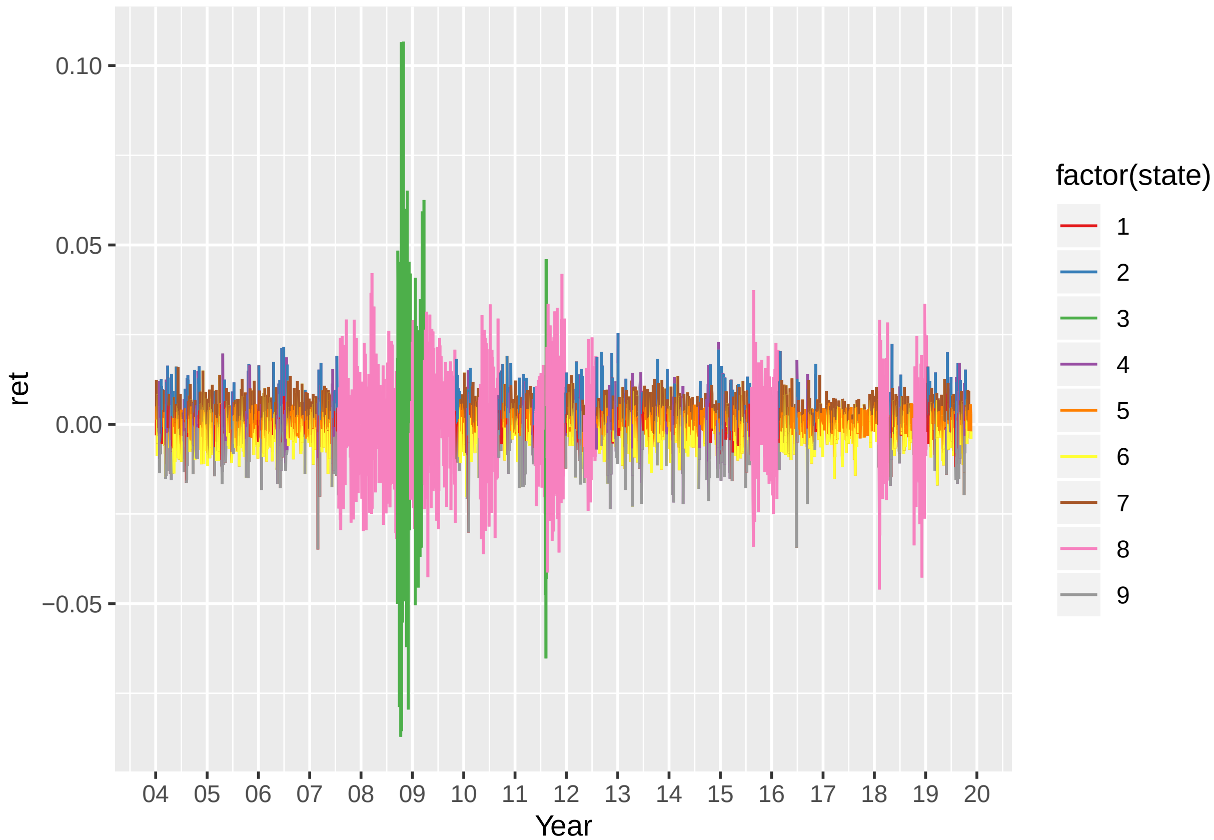
```r
summary(hmmfit)
```

```
## Initial state probabilties model
## pr1 pr2 pr3 pr4 pr5 pr6 pr7 pr8 pr9
##   1   0   0   0   0   0   0   0   0
##
## Transition matrix
##         toS1  toS2  toS3  toS4  toS5  toS6  toS7  toS8  toS9
```

```
## fromS1 0.442 0.130 0.000 0.000 0.000 0.000 0.214 0.000 0.214
## fromS2 0.760 0.029 0.000 0.000 0.114 0.000 0.096 0.000 0.001
## fromS3 0.000 0.000 0.969 0.000 0.000 0.000 0.000 0.031 0.000
## fromS4 0.000 0.271 0.000 0.386 0.000 0.000 0.000 0.000 0.343
## fromS5 0.000 0.003 0.000 0.000 0.571 0.313 0.113 0.000 0.000
## fromS6 0.099 0.000 0.000 0.000 0.165 0.242 0.419 0.000 0.075
## fromS7 0.000 0.000 0.000 0.000 0.582 0.218 0.183 0.000 0.016
## fromS8 0.000 0.000 0.004 0.023 0.000 0.000 0.000 0.972 0.000
## fromS9 0.000 0.000 0.004 0.734 0.000 0.000 0.000 0.038 0.224
##
## Response parameters
## Resp 1 : gaussian
##     Re1.(Intercept) Re1.sd
## St1           0.000  0.004
## St2           0.012  0.005
## St3          -0.003  0.036
## St4           0.003  0.007
## St5           0.001  0.002
## St6          -0.004  0.004
## St7           0.006  0.003
## St8           0.000  0.015
## St9          -0.010  0.007
```

```r
dat = posterior(hmmfit)
dat$date = gspc.returns$date
dat$ret = gspc.returns$returns
```

```r
ggplot(dat, aes(x=date, y=ret,color=factor(state),group = 1)) + geom_line() + scale_x_date(breaks = dat
  labels = date_format('%y')) + xlab('Year') + scale_color_brewer(palette="Set1")
```




The most volatil bear state appears almost only during the subprime crisis. It shows the market was in an exceptional state.

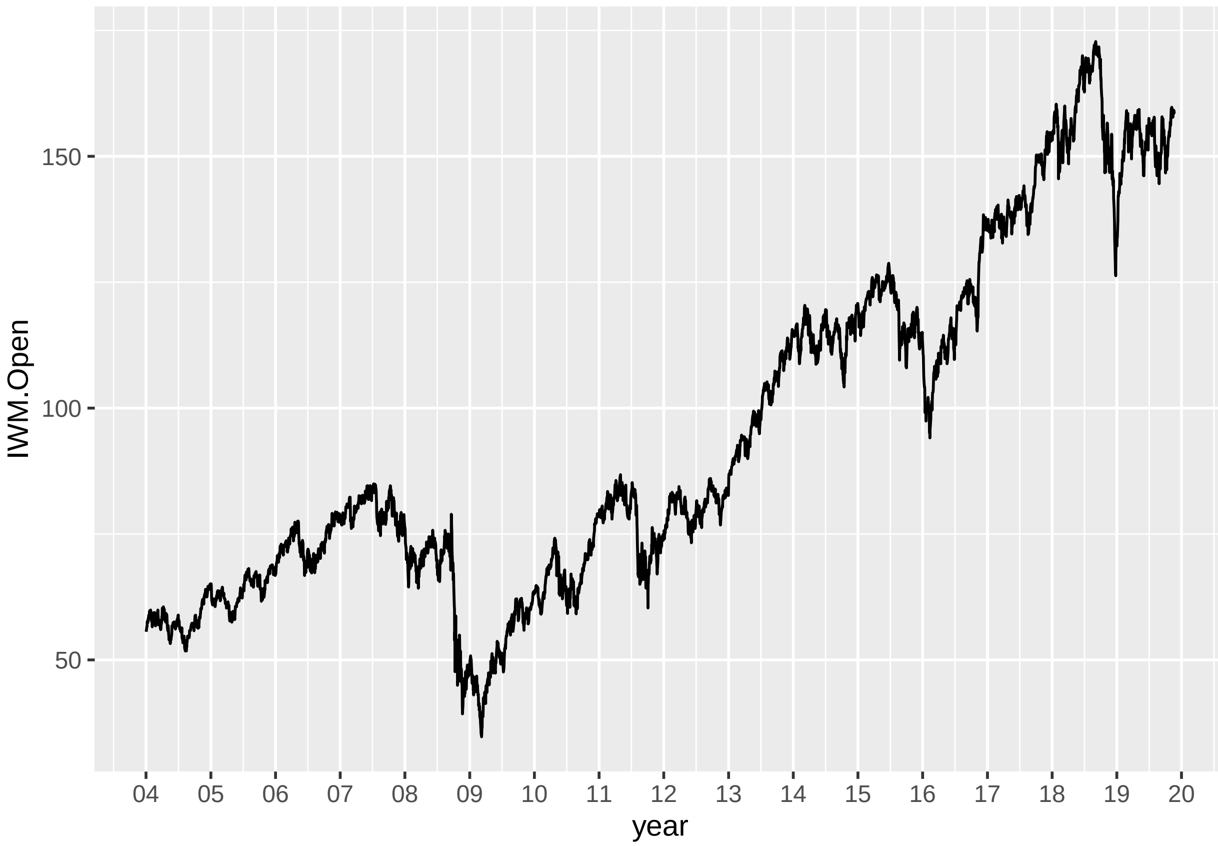
## IWM

From etf.com: IWM tracks a market-cap-weighted index of US small-cap stocks. The index selects stocks ranked 1,001-3,000 by market cap.

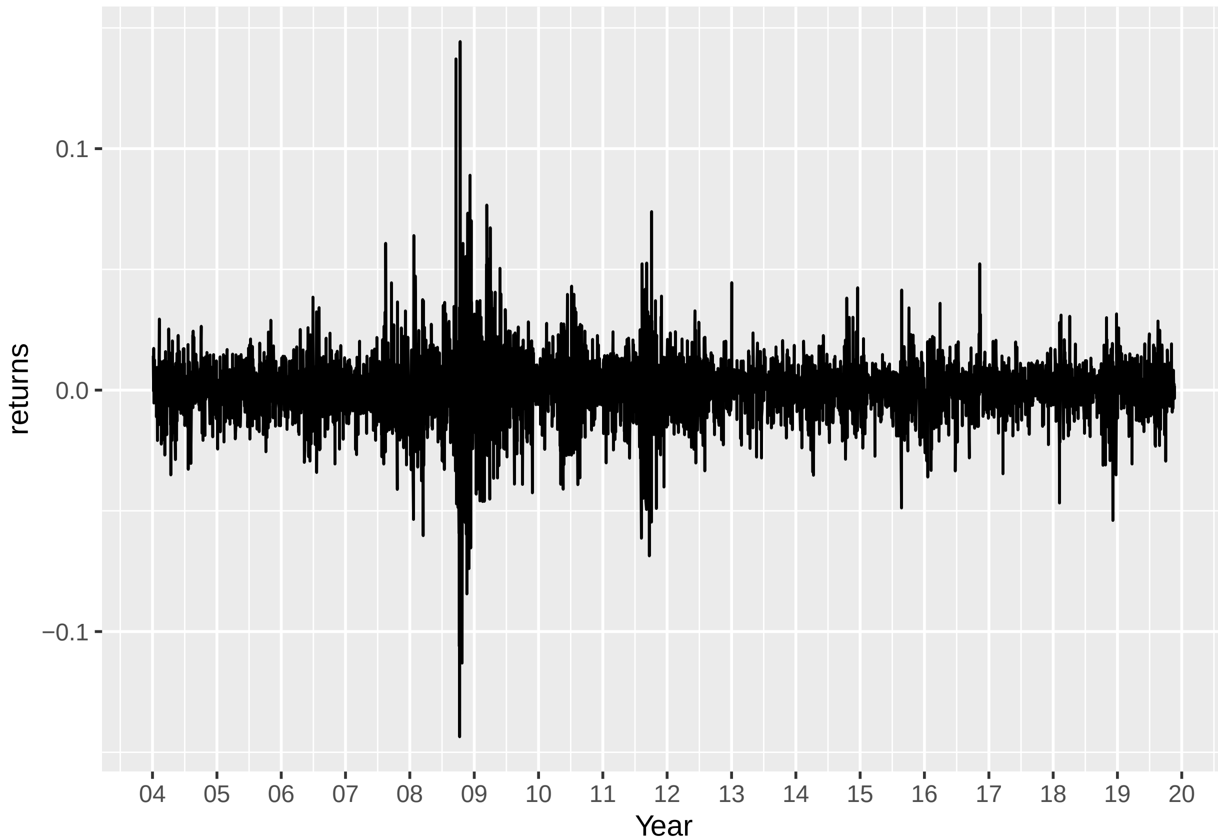
```
getSymbols( 'IWM',src="yahoo", from="2004-01-01" )
```

```
## [1] "IWM"
```

```
values = as.data.frame(IWM)
n= nrow(values)
values$date = as.Date(rownames(values))

ggplot(values, aes(x=date, y=IWM.Open,group = 1)) + geom_line() + scale_x_date(breaks = date_breaks("yea
  labels = date_format('%y')) + xlab('year')
```




```r
iwm.returns = as.data.frame((values[2:n, 1] - values[1:(n-1), 1]) /  values[1:(n-1), 1])
iwm.returns$date = as.Date(rownames(values)[2:n])

colnames(iwm.returns) = c('returns','date')

ggplot(iwm.returns, aes(x=date, y=returns,group = 1)) + geom_line() + scale_x_date(breaks = date_breaks
  labels = date_format('%y')) + xlab('Year')
```




```
k=10
AICs = numeric(length(2:k))

for (i in 2:k) {
  hmm <- depmix(returns ~ 1, family = gaussian(), nstates = i, data=data.frame(iwm.returns))
  hmmfit <- fit(hmm, verbose = FALSE)
  AICs[i-1] = AIC(hmmfit)
}
```

```
## converged at iteration 42 with logLik: 11952.08
## converged at iteration 144 with logLik: 12075.47
## converged at iteration 452 with logLik: 12104.5
```

```
bestk = which.min(AICs) +1
bestk
```

```
## [1] 7
```

```
hmm <- depmix(returns ~ 1, family = gaussian(), nstates = bestk, data=data.frame(iwm.returns))
hmmfit <- fit(hmm, verbose = FALSE)
```

```
summary(hmmfit)
```

```
## Initial state probabilties model
## pr1 pr2 pr3 pr4 pr5 pr6 pr7
##   0   0   0   1   0   0   0
##
## Transition matrix
##         toS1  toS2  toS3  toS4  toS5  toS6  toS7
## fromS1 0.538 0.000 0.020 0.000 0.000 0.443 0.000
```

```
## fromS2 0.014 0.316 0.000 0.645 0.000 0.000 0.024
## fromS3 0.000 0.000 0.964 0.000 0.000 0.036 0.000
## fromS4 0.000 0.266 0.000 0.516 0.219 0.000 0.000
## fromS5 0.000 0.144 0.000 0.000 0.856 0.000 0.000
## fromS6 0.544 0.000 0.000 0.000 0.034 0.312 0.110
## fromS7 0.014 0.000 0.000 0.013 0.000 0.000 0.973
##
## Response parameters
## Resp 1 : gaussian
##     Re1.(Intercept) Re1.sd
## St1          -0.014  0.016
## St2          -0.009  0.008
## St3          -0.001  0.045
## St4           0.006  0.007
## St5           0.002  0.006
## St6           0.019  0.014
## St7           0.000  0.013
```

```r
dat = posterior(hmmfit)
dat$date = iwm.returns$date
dat$ret = iwm.returns$returns
```

```r
ggplot(dat, aes(x=date, y=ret,color=factor(state),group = 1)) + geom_line() + scale_x_date(breaks = dat
  labels = date_format('%y')) + xlab('Year') + scale_color_brewer(palette="Set1")
```


Like with the S&P index, the most volatil bear state appears during the crisis.

# References


[1] F. Arnaud. Markov switching models, 2012.

[2] G. de Truchis and E. Dumitrescu. Économétrie non-linéaire, 2016.

[3] J. D. Hamilton. Regime-switching models. 2005.