UTRECHT UNIVERSITY

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Theoretical Physics master thesis

D-brane gauge theories with spontaneous supersymmetry braking through freely acting orbifolds

First examiner:
Stefan Vandoren

Second examiner:
Thomas Grimm

Candidate:
Hugo Calvo Castro

In cooperation with:
George Gkountoumis

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Abstract

In the context of String Theory, freely acting orbifolds have proven to be an effective method of spontaneously breaking supersymmetry (cite). The effects on the spectrum of the closed string in type IIB String Theory have been studied in detail in (cite), and this thesis aims to explore the effects of the SUSY breaking in the open string spectrum. Here we first show how the open string spectrum is affected in general by the orbifold action, and we calculate the full orbifold projection on a specific example of D1/D5 brane system. This system is closely linked to black hole solutions of the low energy supergravity, and in the last section we give predictions as to how the orbifold projection acts on the low energy worldvolume CFT and thus the black hole theormodynamics in the system with broken supersymmetry.

Contents

1. Introduction

Physics aims to describe the dynamics between all the fundamental constutuents of nature. In the one hand, we can use Quantum Field Theory to describe Particle Physics, and in the other we can use General Relativity to describe astronomical interactions. These two theories are fundamentally different in the sense that the first is a quantum theory, while the second one is not, and the most naive attempts to convert it to quantum language fail fundamentally.

String theory is a paradigm change to the way Particle Physics is built, in the sense that now the fundamental objects are no longer point-like, but extended one dimensional *strings*. Among an impressive list of results that were derived not long after String Theory was invented, the most notable one might be that this fully quantum theory is a theory of gravity. Thus, being a promising candidate for a unifying theory of physics.

One of the issues of String Theory is that in order to have a consistent theory (no tachyons) we need to add supersymmetry in the sense of fermionic excitations of the string. It turns out that the low energy effective theory of this system is Super Gravity (SUGRA), which is the supersymmetric extension of Einstein's General Relativity. But, as we all know, supersymmetry is not actually a low energy symmetry of nature in our universe.

At this point, we can look for ways of breaking the supersymmetry of String Theory. The one considered for this thesis is compactification by freely acting orbifolds. In essence, there is a discrete symmetry in the compact dimensions that gets quotiented away, projecting a part of the spectrum that has a nontrivial group charge.

String Theories in orbifolded backgrounds have been studied extensively (cite), with a focus on the closed string spectrum and the low energy SUGRA (cite). It was discovered that the orbifold projection can be regarded as a

Higgs-like mechanism for the charged fields.

In this thesis we will focus on the open string spectrum, which has not yet beef fully understood in orbifold backgrounds. A full description of the spectrum will be given in a general orbifold for some specific examples, namely the D1/D5 system.

The main goal is to calculate the central charge of the effective CFT in the infrared (IR) of this D1/D5 system, that is dual to a certain black hole solution of the corresponding SUGRA. This process is still not well understood but a prediction can be made based on the projections of the spectrum.

1.1 Outline

This thesis will be organized as follows. In Chapter 2 we will briefly describe general aspects of String Theory relevant for delevoping the later calculations. In Chapter 3 we will describe the massless spectrum of type IIB String Theory from a group theoretical point of view. In Chapter 4 we will use the group theoretical description to understand how the orbifold modifies the spectrum and thus breaks supersymmetry.

Lastly, in Chapters 5 and **??** we will give a dynamical description to the spectrum found in previous chapters, with the goal of calculating thermodynamic quantities of the black hole that describes the D1/D5 system in the orbifold background.

1.2 Conventions

2. Preliminaries

In this chapter we will present some basic concepts to give a foundation to this thesis. Only the necessary steps will be presented in order to give context for future chapters. For a more extensive review on the topic of Superstring Theory, the reader can refer to [1], [2].

2.1 Type IIB string theory

String Theory is, in its most simple realization, one of the most straight forward generalizations of the quantum theory behind Particle Physics. Instead of the fundamental object being a particle, which classically traces a 1D world-line when it propagates through space-time, a string describes a 2D world-sheet. This world-sheet is parametrized by coordinates $\sigma^\alpha = (\sigma, \tau)$, through the embeddings $X^\mu(\sigma, \tau)$.

Superstring theory, as the name implies, also has fermionic degrees of freedom, that give rise to world-sheet SUSY ψ^μ and $\tilde{\psi}^\mu$. This world-sheet SUSY, as it turns out, gives rise to space-time SUSY when the String Theory is treated with carefully. This procedure is known as the GSO projection, and will be an integral part in future chapters.

Consider the following action,

$$S = \frac{1}{4\pi} \int_{\mathcal{M}} d^2 z \left\{ \frac{1}{\alpha'} \partial X^\mu \bar{\partial} X_\mu + \psi^\mu \bar{\partial} \psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu \right\}$$

with $\mathcal{M}$ being a complex cylinder, $z \in \mathbb{C}$, $z + 2\pi \sim z$.

We can start by solving the classical equations of motion, which can be read as $\partial\bar{\partial} X^\mu = \bar{\partial}\psi^\mu = \partial\tilde{\psi}^\mu = 0$. These mean the fields can be written in terms of holomorphic and anti-holomorphic functions as follows,

$$
\begin{aligned}
X^\mu &= X^\mu_L(z) + X^\mu_R(\bar{z}) \\
\psi^\mu &= \psi^\mu(z) \\
\tilde{\psi}^\mu &= \tilde{\psi}^\mu(\bar{z})
\end{aligned}
$$

Now, to find a mode expansion we have to impose boundary conditions for these fields. In order to find the closed string spectrum, we impose the following periodicity conditions. For the bosonic field these are $X^\mu(z + 2\pi) = X^\mu(z)$, while the fermions can close up to a $\pm$ sign. This allows for two sectors in the spectrum, called Rammond (R) and Naveu-Schwarz (NS), given by the periodicity conditions,

$$
\begin{aligned}
\mathrm{R} : \psi^\mu(z + 2\pi) &= +\psi^\mu(z) \\
\mathrm{NS} : \psi^\mu(z + 2\pi) &= -\psi^\mu(z)
\end{aligned}
$$

and the same for $\tilde{\psi}^\mu$. Al fields can then be expressed in terms of Fourier modes. The bosonic modes will be called a^μ_n, $\tilde{a}^\mu_n$, $n \in \mathbb{Z}$, while the fermionic modes are b^μ_r, $\tilde{b}^\mu_r$, with r being in $\mathbb{Z}$ in the NS sector, or in $\mathbb{Z} + 1/2$ in the R sector.

Focusing on one half of the spectrum, say the left moving spectrum, we identify the NS vacuum to be a space-time scalar $|0\rangle_R$, while the R vacuum is degenerate under the action of b^μ_0. The zero-modes of the R sector generate the $D = 10$ Clifford algebra, so the R vacuum is characterized by a spinor, $|a\rangle_R$. This spinor can be characterized by $SO(2)$ eigenvalues $|s_0, s_1, s_2, s_3, s_4\rangle$, $s_i = \pm 1/2$, forming a 32 dimensional complex space. This is a Dirac spinor

32 of $SO(1,9)$.

The closed string spectrum will consist of both the right and left moving spectrum along with the level matching condition. Focusing again on the right part of the spectrum, we can find the following mass formula,

$$M^2 = \frac{1}{\alpha'}\left(\sum_{n,r} a{-}n \cdot a_n + r b_{-r} \cdot b_r - a\right) \tag{2.1}$$

whith $a = -1/2$ in the NS sector and $a = 0$ in the R sector. We can extract useful information from this. First of all, there is a tachyon $|0\rangle_R$ with mass $M^2 = -1/2\alpha'$. We will assign fermion number $(-1)^F = -1$ to this state. The only massless states are then $|a\rangle_R$ and $b^\mu_{-1/2}|0\rangle_R$. The vector $b^\mu_{-1/2}|0\rangle_R$ has $(-1)^F = 1$, while for the fermion $\mathbf{32} = \mathbf{16}_s \oplus \mathbf{16}_c$[1] we have a choice. We can choose the value of $(-1)^F$ for one chirality, and the other will automatically adopt the oposite one.

All this discussion of fermion numbers is related to the GSO projection. In order to make the theory well defined, we require the spectrum to only contain states that have an even fermion number. Thus, the tachyon is projected, and at the massless level, only one chirality of the fermion remains. The same reasoning follows for the left moving spectrum, leading to the two different type II Superstring Theories. Type IIA has both chiralities in the spectrum, while IIB has only one (conventionally the $\mathbf{16}_s$). To write out the spectrum, it is common to go to lightcone gauge, in which the massless excitations will form representations of the little group of $SO(1,9)$, namely, $SO(8)$. The massless spectrum of type IIB string theory can then be written as,

$$(\mathbf{8}_v, \mathbf{8}_v) \oplus (\mathbf{8}_v, \mathbf{8}_s) \oplus (\mathbf{8}_s, \mathbf{8}_v) \oplus (\mathbf{8}_s, \mathbf{8}_s)$$

[1] This is the typical decomposition into Weyl spinors, for details refer to Appendix A

The takeaway from this short discussion should be the halving of the spectrum via GSO projection, and the description of the spectrum of type IIB. In future chapters, GSO projection will be used to calculate the spectrum of the D1/D5 brane system.

2.2 Orbifolds

Intuitively, orbifolds can be understood as a generalization or manifolds where we allow for the existence of singular points. Mathematically, they are defined by a differentiable manifold $\mathcal{M}$, and a symmetry group G inside $\mathcal{M}$. Then the quotient $\mathcal{M}/G$ is an orbifold.

As an example, we can consider $\mathbb{R}^2$, whith reflections $(x, y) \to (-x, -y)$ identified. This is a realization of the orbifold $\mathbb{R}^2/\mathbb{Z}_2$. This particular orbifold has a conical singularity at the fixed point at the origin, so the orbifold is actually not a manifold.

In what will follow we will consider freely acting orbifolds, which are orbifolds by construction as a quotient between a manifold and a symmetry group, but don't have any fixed points. The resulting structure then turns out to be a manifold again. In relation to string theory, freely acting orbifolds are interesting because

2.3 D-branes

When we introduced String Theory, we assumed periodic boundary conditions to represent a closed string propagating through spacetime. But this choice could be extended to non-periodic boundary conditions. Strings can indeed end on hypersurfaces that are called D-branes.

Consider the bosonic part coming from the world-sheet action given by equation 2.1. Instead of imposing periodic boundary conditions, we can instead consider the string endings fixed on a surface of dimeension p, so that $\partial_\sigma X^a(\sigma, \tau)|_{\sigma=0,\pi} = 0$, $\partial_\tau X^i(\sigma, \tau)|_{\sigma=0,\pi} = 0$, with $a = 0, ...p$, $i = p + 1, ..., D$.

If we express the coordinates in terms of the modes a_n^μ, the Neumann and Dirichlet conditions imply a relation between left and right moving modes as $a_n^a = \overline{a}_n^a$ and $a_n^i = -\overline{a}_n^i$. The spectrum then, can be constructed from the set of, for instance, left moving oscillators, as the right moving degrees of freedom are related by the boundary conditions.

The same procedure can be performed to the fermionic oscillations, and we find the same conclusion. The take away from the splitting into Neumann+Dirichlet boundary conditions, is that the full Lorentz group is now not a symmetry of the spectrum, because rotations between a and i indices will mix different boundary conditions. The splitting can be schematically represented as $SO(1,9) \to SO(1,p) \times SO(9-p)$. In terms of the representation theory of the spectrum, in the closed string we found that the stated arranged themselves into representations of the Lorentz group $SO(1,9)$, but now in turn, they will be arranged into representations of the subgroups mentioned.

To work a quick example, we can think of a D9 brane covering all directions of the background space. In this case, the open string spectrum is just the left moving part of what we presented as the closed string spectrum. The massless spectrum is then given by the states $b^\mu_{-1/2}|0\rangle$ and the Rammond vacuum $|a\rangle$, which in lightcone gauge they fall into the $\mathbf{8}_v$ and $\mathbf{8}_s$ of $SO(1,9)$.

All the rest of the single Dp-brane spectrums can be computed from the D9 spectrum by dimensional reduction. The procedure is best understood from the vector. The index $\mu = 0, ..., 9$ splits into two indices $a = 0, ..., p$ and $i = p+1, ..., 10$, so that a vector $b^\mu_{-1/2}|0\rangle \to b^a_{-1/2} \otimes b^i_{-1/2}|0\rangle$. The procedure for spinor representations is outlined in detail in Appendix A. In summary, if we wanted to calculate the D5 spectrum with the data from the D9 spectrum, the dimensional reduction would go as follows,

$$SO(1,9) \to SO(1,5) \times SO(4)$$
$$\mathbf{10} \to (\mathbf{6},\mathbf{1}) \oplus (\mathbf{1},\mathbf{4})$$
$$\mathbf{16}_s \to (\mathbf{4}_s,\mathbf{2}_s) \oplus (\mathbf{4}_c,\mathbf{2}_c)$$

2.4 Supersymmetry

Supersymmetry is the name given to systems that exhibit a transformation between bosons and fermions that leave the theory invariant. As a global symmetry, the existance of a conserved charge, called supercharge, follows from Noether's theorem. The most basic example of supersymmetry can be found in a 2D theory with a massless MW spinor and a scalar,

$$S = \int d^2z \left(\partial\phi\bar{\partial}\phi + \psi\partial\psi\right). \tag{2.2}$$

This theory is invariant (on-shell) under the spinor valued transformation,

$$\begin{aligned} \delta_\epsilon\phi &= \epsilon\psi \\ \delta_\epsilon\psi &= \epsilon\bar{\partial}\phi \end{aligned} \tag{2.3}$$

In this case, there will be only one supercharge Q in the same representation as the spinor field ψ following the algebra $\{Q,Q\} = 2P$, where P is the generator of translations. This illustrative example can be generalized to the case of D dimensions and $\mathcal{N}$ supercharges. These supercharges will follow the algebra,

$$\{Q_i, \overline{Q}_j\} = 2\delta_{ij}\Gamma^{\mu}P_{\mu}, \tag{2.4}$$

where $\overline{Q}_j = Q_j^{\dagger}\Gamma^0$, and Γ^{μ} are the gamma matrices suitable to the representation of Q_i.

The number of degrees of freedom carried by Q_i represents the amount of conserved supercharges of the theory. Assuming these supercharges are Weyl spinors, and $D = 2\nu$, each Q_i would have 2^{ν} real coponents, for a total of $2^{\nu}\mathcal{N}$ supercharges.

There is an extra internal symmetry between the supercharges called R-symmetry. In general it is given by the indices of the supercharges. In our case, we will make a slight abuse and say that the R-symmetry is $SO(n)$, while in reality it is its double cover $Spin(n)$, and the indices of the extended supercharges will be spinor indices of the spin group.

Now imagine that we want to write a theory containing a massless vector boson and a massless fermion. Massless vectors in $D = 2\nu$ dimensions have $D - 2$ degrees of freedom, while Weyl fermions have $2^{\nu-1}$. In $D = 10$ it turns out that fermions can be Majorana an Weyl at the same time, so the number of components gets further reduced to $2^{\nu-2}$. Equating these quantities, we find that $\mathcal{N} = 1$ supersymmetry without extra boson fields can only be realized in $D = 10$, and for $D > 10$, we would need spin 2 fields to be able to match degrees of freedom. This is what we will call maximal supersymmetry, the case where only one supercharge pairs a fermion to a boson of a particular spin value supersymmetrically.

Maximal supersymmetric gauge theories will then have 16 supercharges and in this thesis will all come from dimensional reduction of $D = 10$ $\mathcal{N} = 1$ SYM. Maximal supergravity has 32 supercharges and come naturally from $D = 11$ $\mathcal{N} = 1$ supergravity.

For example, superstring theory in a Minkowski background has 2 superharges Q_+, Q_-, which are MW spinors in $D = 10$ of oposite chiralities.

Each of them has 16 real components, for a total of 32 supercharges.

3. Open string spectrum

In this chapter we will describe the spectrum of D-brane systems in the context of type IIB String Theory. We start by discussing single brane spectrums, and then move on to general brane configurations, to conclude with the main example of this thesis, the D1/D5 brane system.

3.1 Dp-Dp spectrum

Consider a single D9-brane. This equates to considering Neumann boundary conditions in all directions of a string. The NS vacuum is a scalar $|0\rangle$, while the R sector is a spinor $|a\rangle$, given by $SO(2)$ eigenvalues s_0, s_1, s_2, s_3, s_4. The massless spectrum can be found from the mass formula $\alpha' M^2 = N + 1/2$. This leads to the modes $b^{\mu}_{-1/2}|0\rangle$ and $|a\rangle$.

The representation theory of the massless spectrum turns out to be straight forward. There is a vector and a fermion in D = 10, composed by $b^{\mu}_{-1/2}|0\rangle$ and $|a\rangle$ respectively.

To obtain the spectrum of an arbitrary Dp-brane we can perform dimensional reduction over the D9 spectrum we already constructed. By dimensional reduction we mean splitting the D9 symmetry group $SO(1,9)$ into the transverse and world-volume symmetry groups of the lower dimensional Dp-brane, namely, $SO(1,p) \times SO(9-p)$.

Starting with a vector in the $\mathbf{10}$ of $SO(1,9)$, it decomposes into a $(\mathbf{p},\mathbf{1}) \oplus (\mathbf{1},\mathbf{9}-\mathbf{p})$ of $SO(1,p) \times SO(9-p)$. The original R vacua was a Majorana-Weyl spinor of $SO(1,9)$, and depending on p it will decompose into the corresponding irreducible spinors of $SO(1,p) \times SO(9-p)$. The dimensional reduction of spinors is discussed in detail in A.

This is already the full spectrum of a single Dp-brane. Extending it to a stack of Q_p Dp-branes adds a Chan-Paton factor to both string ends, which labels the adjoint representation of $U(Q_p)$.

We can count on-shell degrees of freedom to make sure the spectrum is supersymmetric. For a explicit example, we can consider a single D5-brane. By on-shell we refer to adopting light-cone gauge, meaning that $SO(1,5) \to SO(4)$, effectively identifying 2 degrees of freedom for the vector, and specifying a s_0 eigenvalue for the spinor.

$$
\begin{aligned}
(\mathbf{6},\mathbf{1}) &\to (\mathbf{4},\mathbf{1} \\
(\mathbf{1},\mathbf{4}) &\to (\mathbf{1},\mathbf{4}) \\
(\mathbf{4}_s,\mathbf{2}_s) &\to (\mathbf{2}_s,\mathbf{2}_s) \\
(\mathbf{4}_c,\mathbf{2}_c) &\to (\mathbf{2}_c,\mathbf{2}_c)
\end{aligned}
$$

Now if we count degrees of freedom in the right side, we find that there are 8 matching bosonic and fermionic degrees of freedom, meaning that there can be 8 on-shell supercharges in the theory. A general discussion can be made for any p to find the same 8 possible on-shell supercharges.

3.2 Dp-Dp' spectrum

Now consider two D branes of arbitrary dimensions p, p'. In general, some of the directions will have the same boundary conditions, i.e. DD or NN, while some will have mixed boundary conditions, i.e. DN or ND. This is illustrated in Figure 3.1.

As we did for the single D brane, the boundary conditions will lead to a relation between left and right moving modes.

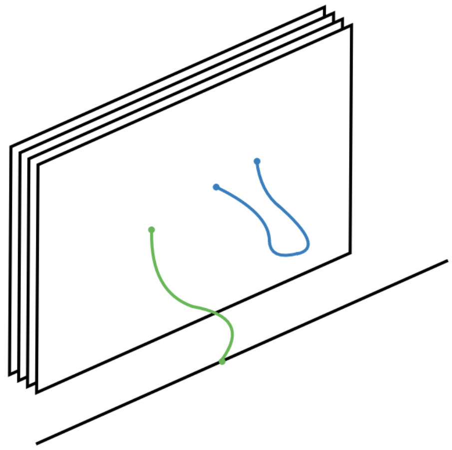

Figure 3.1: Schematic representation of a stack of Dp branes and a single Dp′ brane with strings attached. The blue string will have the same boundary conditions in all directions while the green string will have mixed conditions in some. Figure taken from [3].

$$\text{NN: } X^\mu(z,\bar{z}) = x^\mu - i\alpha' p^\mu \ln(z\bar{z}) + i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \frac{a_m^\mu}{m}\left(z^{-m} + \bar{z}^{-m}\right) \tag{3.1}$$

$$\text{DN, ND: } X^\mu(z,\bar{z}) = i\sqrt{\frac{\alpha'}{2}} \sum_{r\in\mathbb{Z}+1/2} \frac{a_r^\mu}{r}\left(z^{-r} \pm \bar{z}^{-r}\right) \tag{3.2}$$

$$\text{DD: } \quad X^\mu(z,\bar{z}) = -i\frac{\delta X^\mu}{2\pi} \ln(z/\bar{z}) + i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \frac{a_m^\mu}{m}\left(z^{-m} - \bar{z}^{-m}\right) \tag{3.3}$$

Notice that the moding for the ND/DN directions gets shifted by 1/2 to accomodate the mixed boundary conditions. This property is also apparent on the fermions

Once again, we aim to calculate the massless spectrum. Thus, we want to find the zero point energy in both the R and the NS sector. As per usual, the R vacuum is massless, while the zero point energy for the NS sector is,

$$-\frac{1}{2} + \frac{\nu}{8} \tag{3.4}$$

3.3 D1/D5 spectrum

The D1/D5 brane system will be the main example treated in this thesis. It deserves a propper introduction, so first of all let's define it in detail. Consider a stack of Q_5 D5 branes covering the 5, 6, 7, 8, 9 directions. Next, consider a stack of Q_1 D1 branes laying inside the D5 stack in the 5 direction. Table 3.1 gives a summary of what was described.

	0	1	2	3	4	5	6	7	8	9
D1	-	.	.	.	.	-	.	.	.	.
D5	-	.	.	.	.	-	-	-	-	-

Table 3.1: Schematic representation of the D1/D5 system.

As we can see, the number of mixed directions is $\nu = 4$, so from equation 3.4, the zero point energy of the NS sector of 1-5 and 5-1 string vanishes. Then, the massless spectrum of excitations of these strings will be both the R vacuum and the NS vacuum. Take for instance a 1-5 string, meaning that the endpoint at $\sigma = 0$ lays in the D1 brane, while the one at $\sigma = \pi$ lays in the D5 brane. The bassless modes will be generated by,

$$\text{NS: } b_0^i, \quad i = 6,7,8,9 \tag{3.5}$$

$$\text{R: } b_0^M, \quad M = 0,1,2,3,4,5 \tag{3.6}$$

Both of these sectors generate Clifford algebras separately of the respective subgroups of $SO(1,9)$. At first glance, we could think the important subgroup is $SO(1,5) \times SO(4) \subset SO(1,9)$, but as it was mentionied previously, we want to focus on the theory on the intersection world-volume, so we want to look at the subgroup $SO(1,1) \times SO(4)_I \times SO(4)_E \subset SO(1,9)$, where the subindices indicate the directions 1, 2, 3, 4 for I and 6, 7, 8, 9 for E.

According to this splitting, the Clifford algebras of Equations 3.5 generate spinors of $SO(1,5)$ and $SO(4)_I$ respectively. The $SO(1,5)$ spinor then splits into various $SO(1,1) \times SO(4)_E$ spinors.

Let's consider the R vacuum given by the $SO(1,5)$ spinor $|s_0, s_1, s_2\rangle$. The GSO projection picks one $SO(1,5)$ chirality, and in this case we decide it picks the $\mathbf{4}_s$.

We pick a basis where the first eigenvalue s_0 is associated to the 0, 5 directions, while the other are associated to the 1, 2, 3, 4 directions. This is convenient because the spinor splits as $|s_0\rangle \otimes |s_1, s_2\rangle$ under $SO(1,9) \to SO(1,1) \times SO(4)_E$. Now, counting chiralities in $SO(1,1)$ and $SO(4)_E$ we can see that this spinor fills the representations $(\mathbf{1}_s, \mathbf{2}_s) \oplus (\mathbf{1}_c, \mathbf{2}_c)$.

A similar, albeit easier calculation, shows that the NS vacuum is characterized by a $SO(4)_I$ spinor in the $\mathbf{2}_s$.

At last, the spectrum of the 1-5 string can be obtained by noticing that in the NS sector all states are $SO(4)_I$ singlets, while in the R sector they are $SO(1,1) \times SO(4)_E$ singlets, i.e,

$$\text{NS: } (\mathbf{1}_s, \mathbf{2}_s, \mathbf{1}) \oplus (\mathbf{1}_c, \mathbf{2}_c, \mathbf{1}) \tag{3.7}$$

$$\text{R: } (\mathbf{1}, \mathbf{1}, \mathbf{2}_s) \tag{3.8}$$

Note that the 5-1 strings will produce the same zero modes, so that the vacuum will be identical.

The only missing piece at this point it the 1-1 and 5-5 massless spectrums. We already know that these come from the dimensional reduction of the $\mathbf{10}$ and the $\mathbf{16}_s$ of $SO(1,9)$. It is straight forward to see that the vector decomposes as $(\mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{4}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{4})$. The procedure to decompose the spinor is very similar to what we did with the $\mathbf{4}_s$ of $SO(1,5)$ previously. Now we need to split the $\mathbf{16}_s$ via the eigenvalues s_0 in $SO(1,1)$, s_1, s_2 in $SO(4)_E$ and s_3, s_4 in $SO(4)_I$. Again, counting chiralities we end up with,

$$\mathbf{16}_s \to (\mathbf{1}_s, \mathbf{2}_s, \mathbf{2}_s) \oplus (\mathbf{1}_s, \mathbf{2}_c, \mathbf{2}_c) \oplus (\mathbf{1}_c, \mathbf{2}_c, \mathbf{2}_s) \oplus (\mathbf{1}_c, \mathbf{2}_s, \mathbf{2}_c)$$

4. Orbifolds

String theory can be defined on a vast number of different target spaces. String Theory on torus target spaces is an exactly solvable theory [4], but it does not produce phenomenologically plausible results for several reasons, including that tori preserve all supercharges of the noncompact background [5].

Using orbifolds as target spaces, we fix this last issue because they can break supersymmetry [6]. The process can be throught of as gauging away global symmetries on the worldsheet in a way that the orbifolded spectrum is left with less supercharges than the original spectrum. This idea was first presented in [7] in the special case of a toroidal orbifold. This specific type of orbifold will be the focus of this chapter.

In general, this argument makes orbifolds a good candidate for study as target spaces, and toroidal orbifolds are the first deviations we can make from plain torus compactification, making them the easiest examples to work explicitly.

4.1 Orbifold compactification

We are interested in compactifications of the type,

$$\mathbb{R}^{1,4} \times S^1 \times T^4. \tag{4.1}$$

Consider the worldsheet scalars $X^M = (X^\mu, Z, Y^m)$ split according to the decomposition of the target space above $M = 0, \ldots, 9$, $\mu = 0, 1, 2, 3, 4$, $m = 1, 2, 3, 4$. We can arrange the torus scalars as,

$$W^i = \frac{1}{\sqrt{2}}(Y^{2i-1} + iY^{2i}), \quad i = 1, 2. \tag{4.2}$$

The torus structure is contained in each complex torus coordinate as an identification of the type $W^i \sim W^i + 1 \sim W^i + \tau^i$ for some $\tau \in \mathbb{C}$. The equations of motion for these scalars allow us to split them into left and right movers as,

$$W^i(\tau, \sigma) = W^i_L(\tau + \sigma) + W^i_R(\tau - \sigma). \tag{4.3}$$

It is now apparent that there can be 4 independent $\mathbb{Z}_p$ rotations acting on each of the torus coordinates that we have split into right and left movers.

$$\begin{aligned} W^1_L &\to e^{2\pi i u_1/p} W^1_L \\ W^1_R &\to e^{2\pi i u_2/p} W^1_R \\ W^2_L &\to e^{2\pi i u_3/p} W^2_L \\ W^2_R &\to e^{2\pi i u_4/p} W^2_R, \end{aligned} \tag{4.4}$$

The S^1 coordinate will be identified with a shift, $Z \sim Z + 2\pi r/p$, making the orbifold freely acting.

This story is not complete, as we should have introduced the discrete symmetry as a subgroup of the T-duality of the 4-torus $SO(4, 4\mathbb{Z})$ and chosen the integers u_i accordingly to suit the rotational subgroups of the T-duality group. The details are contained in [3], [8], and the integers are characterized by mass parameter m_i,

$$\frac{2\pi u_1}{p} = m_1 + m_3, \quad \frac{2\pi u_2}{p} = m_2 + m_4 \tag{4.5}$$
$$\frac{2\pi u_3}{p} = m_1 - m_3, \quad \frac{2\pi u_4}{p} = m_2 - m_4. \tag{4.6}$$

Notice that the mass parameters may not be equal in certain cases. When the right movers and left movers are rotated in unequivalent ways, we refer to the space as an asymmetric orbifold. When they rotate in the same way though, we refer to those as symmetric orifolds.

Only some values of p are actually permited for 4-tori. In our case, we want to restrict to orbifolds that can preserve some supercharges, so we restrict to $p = 2, 3, 4, 6$.

In order to fully characterize the action of the orbifold on the spectrum of the superstring, we still need to know the charges of the fermionic modes and of the R vacuum (the NS vacuum is a scalar so it is uncharged by definition). The first is straight forward, because they tansform the same way as the scalar modes by supersymmetry. The latter requires a bit of extra work.

Take a basis element $|s_0, s_1, s_2, s_3, s_4\rangle_{L/R}$ of the R vacuum. The eigenvalues s_i, $i = 1, 2, 3, 4$, by construction, are eigenvalues of the $SO(2)$ rotations of the plane $(2i + 1, 2i + 2)$. Since the orbifold acts precisely as separate discrete subgroups of these $SO(2)$ rotations for both left and right movers, we can conclude that,

$$|s_0, s_1, s_2, s_3, s_4\rangle_L \to e^{2\pi i(\tilde{u}_1 s_3 + \tilde{u}_2 s_4)} |s_0, s_1, s_2, s_3, s_4\rangle_L \tag{4.7}$$
$$|s_0, s_1, s_2, s_3, s_4\rangle_R \to e^{2\pi i(\tilde{u}_3 s_3 + \tilde{u}_4 s_4)} |s_0, s_1, s_2, s_3, s_4\rangle_R \tag{4.8}$$

Looking at the charges of the scalars, we can read that,

$$\tilde{u}_3 = \frac{m_1 + m_3}{2\pi}, u_3 = \frac{m_2 + m_4}{2\pi}, \tag{4.9}$$

$$\tilde{u}_4 = \frac{m_1 - m_3}{2\pi}, u_4 = \frac{m_2 - m_4}{2\pi}. \tag{4.10}$$

The classification is complete if we specify values for (s_3, s_4), for a left moving vacuum the charges are,

$$(+1/2, +1/2) : m_1, \tag{4.11}$$

$$(+1/2, -1/2) : -m_1, \tag{4.12}$$

$$(-1/2, +1/2) : m_3, \tag{4.13}$$

$$(-1/2, -1/2) : -m_3, \tag{4.14}$$

while for a right moving vacuum they are,

$$(+1/2, +1/2) : m_2 \tag{4.15}$$

$$(+1/2, -1/2) : -m_2 \tag{4.16}$$

$$(-1/2, +1/2) : m_4 \tag{4.17}$$

$$(-1/2, -1/2) : -m_4 \tag{4.18}$$

We can notice that only 2 aspects mattered to find out how all these objects were charged by the orbifold action. Firstly, we need to know if it is left or right moving. Secondly, since it acts as rotations in the $SO(4)$ subgroup related to the torus directions of the Lorentz group $SO(1, 9)$, we only need to know in which representation of the Lorentz group an object sits, and we will automatically know how it is charged by the orbifold action.

Sector	State	L charge	R charge
NS	$b^1_{-1/2}\|0\rangle$	$m_1 + m_3$	$m_2 + m_4$
	$\bar{b}^1_{-1/2}\|0\rangle$	$-(m_1 + m_3)$	$-(m_2 + m_4)$
	$b^2_{-1/2}\|0\rangle$	$m_1 - m_3$	$m_2 - m_4$
	$\bar{b}^2_{-1/2}\|0\rangle$	$-(m_1 - m_3)$	$-(m_2 - m_4)$
R	$\|s_0, s_1, s_2, +1/2, +1/2\rangle$	m_1	m_2
	$\|s_0, s_1, s_2, -1/2, -1/2\rangle$	$-m_1$	$-m_2$
	$\|s_0, s_1, s_2, -1/2, +1/2\rangle$	m_3	m_4
	$\|s_0, s_1, s_2, -1/2, -1/2\rangle$	$-m_3$	$-m_4$

It is more useful for this thesis to organize all these objects as representations of $SO(2) \times SO(2) \subset SO(4)$. The bosonic part comes from a $\mathbf{4}$ of $SO(4)$, and it splits into $(\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2})$ of $SO(2) \times SO(2)$. Just to make it explicit, take 4.2, we have,

$$Y^1_L = \frac{1}{\sqrt{2}}(W^1_L + \overline{W^1}_L) \tag{4.19}$$

$$Y^2_L = \frac{1}{\sqrt{2}i}(W^1_L - \overline{W^1}_L) \tag{4.20}$$

The charges of the complex torus coordinates lead us to just a usual $SO(2)$ rotation,

$$\begin{bmatrix} Y^1_L \\ Y^2_L \end{bmatrix} \rightarrow \begin{bmatrix} \cos(m_1 + m_3) & -\sin(m_1 + m_3) \\ \sin(m_1 + m_3) & \cos(m_1 + m_3) \end{bmatrix} \begin{bmatrix} Y^1_L \\ Y^2_L \end{bmatrix} \tag{4.21}$$

An equivalent calculation with W^2 leads to the same rotation but with the respective charges for the real coordinates (Y^3, Y^4). We could read this from the table from the fact that the charges present themselves in the same linear combination for each $SU(2)$ vector. For the spinors, a similar phenomenom happens.

We now can arrange the spinors into chiral representations of $SO(4)$. Take for instance the components with charge m_1, they can be arranged into the $\mathbf{2}_s$ of $SO(4)$. The other half of the spinor, with charge m_3 forms a $\mathbf{2}_c$ of $SO(4)$.

SO(4) rep	L charge	R charge
$(\mathbf{2},\mathbf{1})$	m_1+m_3	m_2+m_4
$(\mathbf{1},\mathbf{2})$	m_1-m_3	m_2-m_4
$\mathbf{2}_s$	m_1	m_2
$\mathbf{2}_c$	m_3	m_4

Table 4.1: Orbifold charges for relevant representations of the subgroup of the Lorentz group corresponding to the torus $SO(4) \subset SO(1,9)$.

Usualy, vectors of $SO(4)$ will not be split into their $SO(2) \times SO(2)$ components unless $m_1 = m_2 = m_3 = m_4 \neq 0$.

4.2 Branes in orbifold backgrounds

Building D-branes on orbifold backgrounds is not as easy as one could have expected. The orbifold action may break the boundary conditions of the open strings attached to the D-brane, forbidding it from existing in the theory. As we will se in the following, both the D1 and D5 branes we are interested in can survive if we restrict our study to symmetric orbifolds.

First of all, consider the general scenario of a single D brane as discussed in 2.3. We introduced the conditions on the open string modes that enable us to define D-branes,

$$a^a = \tilde{a}^a, \tag{4.22}$$

if the boundary conditions were Neumann, or

$$a^i = -\tilde{a}^i, \tag{4.23}$$

if they were Dirichlet. The a and i indices are in the vector representation of the corresponding subgroups of $SO(1,9)$. The D-brane corresponding to these boundary conditions will be able to exist in the orbifold background if said conditions are respected by the orbifold group action.

As the orbifold group only charges the T^4 directions, we can restrict the study the boundary conditions on these directions. First of all, taking from the complex coordinate definition 4.2, we define the modes,

$$w^i = \frac{1}{\sqrt{2}}(a^{2i-1} + ia^{2i}) \tag{4.24}$$

$$\tilde{w}^1 = \frac{1}{\sqrt{2}}(\tilde{a}^{2i-1} + i\tilde{a}^{2i}) \tag{4.25}$$

The orbifold action on this complex modes is as in 4.4,

$$\begin{aligned} w^i &\to e^{i(m_1 \pm m_3)} w^1 \\ \tilde{w}^1 &\to e^{i(m_2 \pm m_4)} \tilde{w}^1 \end{aligned} \tag{4.26}$$

where the $i = 1$ takes the $+$, and $i = 2$, the $-$ signs.

Now, take for instance the first complex coordinate w^1 and $\tilde{w}^1$. If the brane we wanted to construct had only N or D conditions on this T^2, say $a^1 = \pm\tilde{a}^1$ and $a^2 = \pm\tilde{a}^2$, this translates as $w^1 = \pm\tilde{w}^1$. In the other hand, if we had mixed conditions on these directions, $a^1 = \pm\tilde{a}^1$ and $a^2 = \mp\tilde{a}^2$, they would imply $\overline{w}^1 = \pm\tilde{w}^1$.[1]

Since the orbifold action is a phase on the complex modes, the only difference between different brane constructions will be if they imply complex conjugation in the boundary conditions. This leads to essentially 4 different cases that we can treat without loss of generality.

[1]The overline represents complex conjugation.

Case 1: $w^1 = \tilde{w}^1$ and $w^2 = \tilde{w}^2$. The group action on these conditions is,

$$\begin{cases} e^{i(m_1+m_3)}w^1 = e^{i(m_2+m_4)}\tilde{w}^1 \\ e^{i(m_1-m_3)}w^2 = e^{i(m_2-m_4)}\tilde{w}^2 \end{cases} \tag{4.27}$$

From which we extract the condition on the mass parameters,

$$\begin{cases} m_1 = m_2 \\ m_3 = m_4 \end{cases} \tag{4.28}$$

This is what is known as a symmetric orbifold, one in which left and right movers are rotated in the same manner. The D1 and D5 branes defined in **??** are examples that follow this case.

Case 2: $\overline{w}^1 = \tilde{w}^1$ and $\overline{w}^2 = \tilde{w}^2$. The group action on these conditions is,

$$\begin{cases} e^{-i(m_1+m_3)}\overline{w}^1 = e^{i(m_2+m_4)}\tilde{w}^1 \\ e^{-i(m_1-m_3)}\overline{w}^2 = e^{i(m_2-m_4)}\tilde{w}^2 \end{cases} \tag{4.29}$$

From which we extract the condition on the mass parameters,

$$\begin{cases} m_1 = -m_2 \\ m_3 = -m_4 \end{cases} \tag{4.30}$$

This kind of orbifold is usually called anti-symmetric, in the sense that the right movers rotate in the oposite direction of the left movers.

The last two cases are not particularly useful for this thesis but are left here for the sake of completeness. There is, as far as I know, no special name for the orbifolds that allow these kinds of branes.

Case 3: $w^1 = \tilde{w}^1$ and $\overline{w}^2 = \tilde{w}^2$. The group action on these conditions is,

$$\begin{cases} e^{i(m_1+m_3)}w^1 = e^{i(m_2+m_4)}\tilde{w}^1 \\ e^{-i(m_1-m_3)}\overline{w}^2 = e^{i(m_2-m_4)}\tilde{w}^2 \end{cases} \tag{4.31}$$

From which we extract the condition on the mass parameters,

$$\begin{cases} m_1 = m_4 \\ m_2 = m_3 \end{cases} \tag{4.32}$$

Case 4: $\overline{w}^1 = \tilde{w}^1$ and $w^2 = \tilde{w}^2$. The group action on these conditions is,

$$\begin{cases} e^{-i(m_1+m_3)}\overline{w}^1 = e^{i(m_2+m_4)}\tilde{w}^1 \\ e^{i(m_1-m_3)}w^2 = e^{i(m_2-m_4)}\tilde{w}^2 \end{cases} \tag{4.33}$$

From which we extract the condition on the mass parameters,

$$\begin{cases} m_1 = -m_4 \\ m_2 = -m_3 \end{cases} \tag{4.34}$$

In the following section we will study in more detail the D1/D5 system defined in **??**, and as we have seen in **Case 1**, only symmetric orbifolds allow the existence of the system. We will restrict to $m_1 = m_2$ and $m_3 = m_4$ for the rest of the thesis.

4.3 Orbifold group action on the spectrum of the D1/D5 system

The effect of orbifolding on the spectrum of a theory projects out states that are not invariant under the orbifold action, gauging away the global symmetry that defined the orbifold in the first place [6]. It was discovered that for freely acting orbifolds, this process happens through a Higgs-like mechanism that gives mass to states charged under the obrifold action [8] (thus the name *mass parameters*).

In this section we will study the resulting spectrum of the D1/D5 system defined in **??** after orbifolding the target space. The remaining field content will lead us to discover the amount of supercharges in the orbifolded theory, from which we will read and classify the supersymmetry breaking for different orbifolds.

Firstly, let us summarize the results of section 3.3. Note that all representations labeled as $(\cdot,\cdot,\cdot)$ are representing the $SO(1,1) \times SO(4)_E \times SO(4)_I$ representations in that order.

	1-1 strings	5-5 strings	1-5 strings
Bosonic	$(2,1,1)+(1,4,1)+(1,1,4)$	$(2,1,1)+(1,4,1)+(1,1,4)$	$2(1,1,2_s)$
Fermionic	$(1_s,2_s,2_s)+(1_s,2_c,2_c)+$ $(1_c,2_s,2_c)+(1_c,2_c,2_s)$	$(1_s,2_s,2_s)+(1_s,2_c,2_c)+$ $(1_c,2_s,2_c)+(1_c,2_c,2_s)$	$2(1_s,2_s,1)+2(1_c,2_c,1$

Table 4.2: Massless spectrum of the D1/D5 system before projecting charged states.

Case 1: $m_i = 0$, corresponds to the spectrum in **??**, so the spectrum has $\mathcal{N} = (4,4)$ supercharges.

Case 2: $m_1 = m_2 = 0$ and $m_3 = m_4 \neq 0$. In this case, all representations containing the $\mathbf{2}_c$ or $\mathbf{4}$ of $SO(4)_I$ are projected out. This projection leads to the following spectrum,

Counting states in each sector of the spectrum we see that supersymme-

	1-1 strings	5-5 strings	1-5 strings
Bosonic	$(2,1,1)+(1,4,1)$	$(2,1,1)+(1,4,1)$	$2(1,1,2_s)$
Fermionic	$(1_s,2_s,2_s)+(1_c,2_c,2_s)$	$(1_s,2_s,2_s)+(1_c,2_c,2_s)$	$2(1_s,2_s,1)+2(1_c,2_c,1)$

try can still be present and indeed the field content is compatible with $(4,4)$ supersymmetry, implying that even if some fields were projected away, all supercharges can remain in the theory.

Case 3: $m_1 = m_2 \neq 0$ and $m_3 = m_4 = 0$. This case is similar to the previous but now the $\mathbf{2}_c$ survives, and the $\mathbf{2}_s$ is the one projeted out.

	1-1 strings	5-5 strings	1-5 strings
Bosonic	$(2,1,1)+(1,4,1)$	$(2,1,1)+(1,4,1)$	-
Fermionic	$(1_s,2_c,2_c)+(1_c,2_c,2_c)$	$(1_s,2_c,2_c)+(1_c,2_c,2_c)$	$2(1_s,2_s,1)+2(1_c,2_c,1)$

In this case, just looking at the 1-5 sector we notice that only fermions survive. This is a clear indication that all supercharges were broken, as the superpartners of the 1-5 fermions are not in the spectrum anymore. The remaining supercharges are $\mathcal{N}=(0,0)$.

Case 3.1: $m_1 = m_2 \neq 0$ and $m_3 = m_4 \neq 0$, with $m_1 \neq m_3$. In this case, every object that is not a singlet under $SO(4)_I$ is charged under the orbifold action.

	1-1 strings	5-5 strings	1-5 strings
Bosonic	$(2,1,1)+(1,4,1)$	$(2,1,1)+(1,4,1)$	-
Fermionic	-	-	$2(1_s,2_s,1)+2(1_c,2_c,1)$

It is clear again that no supercharges remain in the theory. It was always expected that turning all the mass parameters would break all supersymmetry as it was already known for the closed string.

Case 3.2: $m_1 = m_2 = m_3 = m_4 \neq 0$. In thi special case when all parameters are aqual, we see that we should split the representations according to $SO(2)\times SO(2) \subset SO(4)_I$. In this special case, half of the $\mathbf{4}$ of $SO(4)_I$ is uncharged.

Although the field content is enlarged in this case compared to the previous, again all supersymmetry is broken to $\mathcal{N}=(0,0)$.

	1-1 strings	5-5 strings	1-5 strings
Bosonic	$(2,1,1)+(1,4,1)+(1,1,(1,2))$	$(2,1,1)+(1,4,1)$	-
Fermionic	-	-	$2(1_s,2_s,1)+2(1_c,2_c,1)$

5. D-brane gauge theories

In this chapter we will discuss the actions that correspond to the spectrums discussed in Chapter 3. In the context of D-branes, string endpoints can move through the worldvolume of the brane. The massless excitations describe a SYM type of action for single stacks of branes. Later on, we will study the worldvolume theory of the D1/D5 system, this will be a gauge theory on the intersection of the two branes, leading to a 2D action. Lastly, we will discuss two different limits of the action that describe a bound system of branes an a decaying one.

5.1 Gauge theory on a single brane and dimensional reduction

There is a straight forward way of justifying a SYM action for the world volume theory of the D-brane fields. We can start by introducing the DBI action for a single brane in a bosonic theory [9],

$$S = -T_p \int d^{p+1}\xi \, e^{-\phi} \sqrt{-\det(G_{\alpha\beta} + B_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})}. \tag{5.1}$$

where the G, B and ϕ are the pullbacks of the 10D metric, B field and dilaton to the D-brane world volume. Meanwhile, F is the field strength tensor associated to the gauge field A_α. Expanding to first order,

$$S = -\tau_p V_p - \frac{1}{4g_{\mathrm{YM}}^2} \int d^{p+1}\xi \left(F_{\alpha\beta}F^{\alpha\beta} + \frac{2}{(2\pi\alpha')^2}\partial_\alpha X^a \partial^\alpha X^a \right) + \mathcal{O}(F^4), \tag{5.2}$$

from which we read a $U(1)$ gauge theory in $p+1$ dimensions with $9-p$ scalars, with a YM coupling given by,

$$g_{\mathrm{YM}}^2 = \frac{1}{4\pi^2\alpha'^2\tau_p} = \frac{g}{\sqrt{\alpha'}}(2\pi\sqrt{\alpha'})^{p-2}. \tag{5.3}$$

The supersymmetric extension of the DBI action is possible to write but unnecessary, because to first order it is effectively the supersymmetryc completion of the previous action. The extension to multiple coincident Dp-branes is also straightforward as we just need to consider $U(N)$ gauge fields instead of $U(1)$. Consider the case $p=9$, this is a D9-brane covering all the target space. To first order, the worldvolume theory is $D=10$, $\mathcal{N}=1$, $U(N)$ SYM,

$$S = \int d^{10}\xi \left(-\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\mathrm{Tr}\, \bar{\psi}\Gamma^{\mu}D_{\mu}\psi \right) \tag{5.4}$$

Here we see that the field content is a **10** vector and a $\mathbf{16}_s$ MW spinor of $SO(1,9)$, in agreeance with the spectrum found on Chapter 3. Note that the gauge field A^{μ} and the spinor ψ are both in the adjoint representation of $U(N)$, so the field strength will be,

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig_{YM}[A_{\mu}, A_{\nu}], \tag{5.5}$$

and the covariant derivative will be,

$$D_{\mu}\psi = \partial_{\mu}\psi - ig_{\mathrm{YM}}[A_{\mu}, \psi]. \tag{5.6}$$

Dimensional reduction of the action 5.4 can be performed by assuming that all coordinate dependence of the fields is on ξ^α with $\alpha = 0, \ldots, p$. In this case the index μ is split into (α, a) with $a = p+1, \ldots, 9$, effectively splitting the Lorentz symmetry $SO(1,9) \to SO(1,p) \times SO(9-p)$. All derivatives ∂_a drop out of the action, and we are left with,

$$S = \frac{1}{4g_{\text{YM}}^2} \int d^{p+1}\xi \; \text{Tr} \left(-F_{\alpha\beta}F^{\alpha\beta} - 2(D_\alpha X^a)^2 + [X^a, X^b]^2 + \text{fermions} \right). \quad (5.7)$$

The fermion portion of the action deserves a careful treatment in order to justify it splits as discussed in Chapter 3. In order to do this, it is convenient to write the Gamma matrices in the chiral basis [10] and Wick rotate the time component to be in Euclidean signature,

$$\begin{aligned}
\Gamma_0 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \\
\Gamma_2 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \\
\Gamma_3 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \otimes \mathbb{1} \\
\Gamma_4 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \otimes \mathbb{1} \\
\Gamma_5 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_6 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_7 &= \sigma_1 \otimes \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_8 &= \sigma_1 \otimes \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_9 &= \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}
\end{aligned} \quad (5.8)$$

Now, for a specific example, let us consider a free 10D Dirac spinor following the action,

$$S = \int d^{10}\xi \overline{\lambda} \partial_{\hat{\mu}} \Gamma^{\hat{\mu}} \lambda, \tag{5.9}$$

with $\hat{\mu} = 0, ..., 9$. Now dimensionally reduce by effectively dropping the ξ^8 and ξ^{10} dependenc of the spinor. We are left with,

$$S = \int d^8\xi \overline{\lambda} \partial_\mu \Gamma^\mu \lambda \tag{5.10}$$

with $\mu = 0, ..., 8$. Now, if we take a close look at the first 8 Gamma matrices in the basis written in B.4, we can see that they contain the full 8D Clifford algebra as a subalgebra in the form,

$$\Gamma_\mu = \sigma_1 \otimes \Gamma^8_\mu = \begin{pmatrix} 0 & \Gamma^8_\mu \\ \Gamma^8_\mu & 0 \end{pmatrix}, \tag{5.11}$$

where Γ^8_μ is the 8D Clifford algebra in Euclidean signature. Now, the 10D chirality matrix is defined as $\Gamma^{10}_c = (-i)\Gamma_0 \ldots \Gamma_9$ and it is explicitly,

$$\Gamma^{10}_c = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \tag{5.12}$$

Thus, it is compatible with 5.11, in the sense that the 10D Dirac spinor splits into its $\mathbf{16}_s$ and $\mathbf{16}_c$ Weyl components, and each follow the 8D free Dirac equation. We can name the Weyl components λ_+ and λ_-, from which we read,

$$S = \int d^8\xi \left(\overline{\lambda}_- \partial_\mu \Gamma^8_\mu \lambda_- + \overline{\lambda}_+ \partial \Gamma^8_\mu \lambda_+ \right). \quad (5.13)$$

These 8D Dirac spinors can once again be split into their Weyl components using $\Gamma^8_c = \Gamma^8_0 \dots \Gamma^8_8$. The result is analogous to the previous,

$$\lambda_- = \begin{pmatrix} \phi_- \\ \phi_+ \end{pmatrix}, \quad \lambda_+ = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}. \quad (5.14)$$

What is left is to check their 2D chiralities coming from the 2D subalgebra,

$$\begin{aligned} \Gamma^2_c = -i\Gamma_8\Gamma_9 = \sigma_3 \otimes \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} = \\ \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \end{aligned} \quad (5.15)$$

From this we read that the 2D chiralities are $+, -, -, +$ respectively for $\phi_-, \phi_+, \psi_-, \psi_+$, confirming that their $SO(1,7) \times SO(2)$ representations are indeed $(\mathbf{8}_s, \mathbf{1}_c) + (\mathbf{8}_c, \mathbf{1}_s) + (\mathbf{8}_c, \mathbf{1}_c) + (\mathbf{8}_s, \mathbf{1}_s)$ in full agreance with Chapter 3.

This procedure that was explicitly discussed for the case $SO(1,9) \rightarrow SO(1,7) \times SO(2)$ can be generalized for dimensional reductions of all kinds in a similar maner[1]. Notice how in this case, the 10D Dirac spinor split into 2 8D Dirac spinors. If we reduced instead to 6D we would have 4

[1]For an in depth derivation of the dimensional reduction $SO(1,9) \rightarrow SO(1,7) \times SO(4)$, refer to Appendix B. All non-abelian terms are treated carefully and the example is less trivial, with more structure in the R-symmetry group.

Dirac spinors and they would carry an $SO(4)$ R-symmetry and the procedure would follow although being a bit more complicated from the need of explicitly writing the internal indices.

The agreeance with the bosonic spectrum is easy to check, as the vector A^α is on the $(\mathbf{p}, \mathbf{1})$ and the scalars form the $(\mathbf{1}, \mathbf{9}-\mathbf{p})$ just by looking at the indices α and a. The fermions are not as easy to check from the action, but they also follow the decomposition described in 3. Since we started with 16 supercharges, 16 supercharges remain irregardless of the vaulue of p. For example, in the case of a stack of D4-branes, the worldvolume theory would be maximal $\mathcal{N}=4$ SYM in $D=4$. If we were to study a stack of D1-branes, the action would be that of $\mathcal{N}=(8,8)$ SYM in $D=2$. The fact is that since we started with maximally supersymmetric SYM, we always land in a theory with maximal supersymmetry.

5.2 Gauge theory of the D1/D5 system

Now that we know that single brane stacks are described by SYM theories with maximal supersymmetry, we want to discuss the case of the worldvolume theory in the intersection of the D1 and D5 branes. As hinted by the spectrum 3, we will have 3 sectors,

1-1 strings: As discussed in the previous section, the theory of this sector will be the dimensional reduction of $\mathcal{N}=1$, $U(Q_1)$ SYM in $D=10$ to 1+1 dimensions. In our case the theory will be defined in the (t, x^5) directions. The bosonic content of this sector is,

$$\begin{aligned}&\text{Vector multiplet: } A_0^{(1)}, A_5^{(1)}, Y_m^{(1)}, m=1,2,3,4\\&\text{Hypermultiplet: } Y_i^{(1)}, i=6,7,8,9\end{aligned} \tag{5.16}$$

5-5 strings: The field content of this sector is essentially the same of the 1-1 string when we dimensionally reduce to the 1+1 theory, but instead having $U(Q_5)$ gauge group.

$$\begin{aligned}&\text{Vector multiplet: } A_0^{(5)}, A_5^{(5)}, Y_m^{(1)}, m = 1,2,3,4\\&\text{Hypermultiplet: } Y_i^{(5)}, i = 6,7,8,9\end{aligned} \tag{5.17}$$

Up untili now we have essentially two copies of $\mathcal{N} = (8,8)$ SYM in 1+1 dimensions, with R-symmetry group $SO(4)_I$. The next ingredient in the theory will break supersymmetry by half to The $SU(2)_R$ of $SU(4) = SU(2)_R \times SU(2)_L$.

1-5 and 5-1 strings: From Chapter 3 we argued that the bosonic spectrum of 1-5 and 5-1 strings should follow $(1,1,\mathbf{2}_s)$, this is, a MW spinor of $SO(4)_I$ that is a singlet under $SO(1,1) \times SO(4)_E$. They can be described by χ^1 and χ^2 and can be joined as $\chi = 1/\sqrt{2}(\chi^1 + i\chi^2)$ to form a Weyl spinor of $SO(4)$ with + chirality. It will prove usefull to split the spinor into its components,

$$\chi = \begin{pmatrix} A \\ B^{\dagger} \end{pmatrix} \tag{5.18}$$

It is only left to define the gauge indices. Since both the 1-5 and 5-1 string have one end in the D1 and other in the D5 branes, the Chan-Paton indices will be in the fundamental of $U(Q_1)$ and of $U(Q_5)$. This is commonly called the bifundamental representation in the sense that the object has 2 indices, one in each of the fundamentals. The important difference is that now when this scalars get coupled to the gauge fields $A^{(1)}$ or $A^{(5)}$ they will do it as fundamental matter instead of adjoint matter.

We are now ready to write the action, we will go term by term,

$$S_{1-1} = k_{11} \int d^2\xi \ \mathrm{Tr}\left(-F^{(1)}_{\alpha\beta} F^{(1)\alpha\beta} - 2(D_\alpha Y^{(1)a})^2 + [Y^{(1)a}, Y^{(1)b}]^2\right) \tag{5.19}$$

$$S_{5-5} = k_{55} \int d^2\xi \; \mathrm{Tr} \; (-F^{(5)}_{\alpha\beta} F^{(5)\alpha\beta} - 2(D_\alpha Y^{(5)a})^2 + [Y^{(5)a}, Y^{(5)b}]^2) \qquad (5.20)$$

$$S_{1-5} = \int d^2\xi \; \mathrm{Tr} \; \left(|D_\alpha \chi|^2 + \frac{\chi^\dagger \chi}{2\pi\alpha'} (Y^{(1)}_m - Y^{(5)}_m)^2 + g \sum_{I=1}^{3} (\chi^\dagger \tau^I \chi)^2 \right) \qquad (5.21)$$

In this last action, the first term is just the kinetic term with covariant derivative $D_\alpha \chi = (\partial_\alpha + i A^{(1)}_\alpha - i A^{(5)}_\alpha)\chi$. The last 2 terms come from supersymmetry.

6. Orbifold gauge theory via Sherk-Schwarz reduction

Up until now, we discussed the open string spectrum in Chapter 3, where we found how the orbifold acts on the different representations of the internal symmetry $SO(4)$. After successfully finding the massless spectrum in orbifold backgrounds, we presented the worldvolume effective actions of D-brane stacks and a very special brane system in Chapter 5, with the caveat that all those theories lived in a flat background.

The goal of this chapter is to merge those two ideas and arrive at an effective theory of brane worldvolumes on orbifold backgrounds. We will se that we can impose periodicity conditions on the S^1 coordinate with a duality twist for the charged fields, that will give them masses according to the open string spectrum.

6.1 Orbifold gauge theory of the D9 brane stack

The starting point for this section is going to be the D9 effective action in a flat background,

$$S = \int d^{10}\xi \mathrm{Tr}\left(-\frac{1}{4}F^2 + \frac{i}{2}\overline{\lambda}D_{\hat{\mu}}\lambda\right). \tag{6.1}$$

Next, we are going to compactify on a T^4. If we assume the volume of the torus is small $V_4 < 1$, then we can ignore all the KK momentum modes and pick only the zero mode, leading to the same theory as the one of a D6 described in Appendix B. For the sake of applying the knowledge developed in Chapter 3, we also want to have the T^4 components of the vector in the $(2,1)+(1,2)$, so we define the complex fields $N_1 = 1/\sqrt{2}\,(A_6 + iA_7)$ and $N_2 = 1/\sqrt{2}\,(A_8 + iA_9)$. In terms of these fields the worldvolume action is,

$$S = S_{kin} + S_{pot} \tag{6.2}$$

where

$$\begin{aligned} S_{kin} &= \int d^6\xi \mathrm{Tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 2D_\mu N_i D^\mu \overline{N}^i + \phi_+^{\dagger\alpha} D_\mu \gamma_\mu \phi_+^\alpha + \phi_-^{\dagger\dot{\alpha}} D_\mu \overline{\gamma}_\mu \phi_-^{\dot{\alpha}}\right) \\ S_{pot} &= i\int d^6\xi \left([N_i, N_j][\overline{N}^i, \overline{N}^j] + [N_i, \overline{N}_j][\overline{N}^i, N^j] + \phi_+^{\dagger\alpha}\sigma^i_{\alpha\beta}\gamma_5[A_i, \phi_-^\beta] + \phi_-^{\dagger\alpha}\overline{\sigma}^i_{\alpha\beta}[A_i, \phi_+^\beta]\right) \end{aligned} \tag{6.3}$$

Now, in order to give masses to these fields, we resort to the Scherk-Schwarz reduction [11]. Depending on the $SO(4)$ representation the fields belong to, different monodromies (in this case just phases) will be acceptable. The idea is that when compactifying by a S^1, fields can be expanded in Fourier modes as,

$$f(x,z) = e^{iMz}\sum f_n(x)e^{2\pi inz/R}, \tag{6.4}$$

which is just a generalized Fourier expansion. Now, we can see that the field is not periodic, $f(x, z+2\pi) = e^{iM}f(x,z)$, so an expansion of this type is only admisible if the transformation $f \to e^{iM}f$ is a global symmetry.

In our case, it's clear that after T^4 compatifications, some discrete subgrups of $SO(4)$ are actually global symmetries of our system, and the possible charges associated to different representations of this group have already been classified. Thus, we can use the SS reduction to spontaneously break supersymmetry in the gauge theory describing the branes. What results is the effective worldvolume theory when the string theory is defined in an orbifolded background.

Now, in order to retrieve the spectrum desired, we need to consider the limit where $R \to 0$, effectively selecting the zeroth KK mode of the SS expansion[1]. Besides that, remember that the action descends from a theory with an $SO(1,9)$ global symmetry, thus $SO(4) \subset SO(1,9)$ will also be a global symmetry, thus any potential term will remain unchanged.

The proposed expansions for the charged fields are,

$$\begin{aligned} N_1(x,z) &= e^{i(m_1+m_3)z} N_1(x) \\ N_2(x,z) &= e^{i(m_1-m_3)z} N_2(x) \\ \phi_+^\alpha(x,z) &= e^{im_1 z}\phi_+^\alpha(x) \\ \phi_-^\alpha(x,z) &= e^{im_2 z}\phi_-^\alpha(x) \end{aligned} \tag{6.5}$$

Now, the only change happens in the action, where the derivatives ∂_z now turn into masses,

$$\begin{aligned} D_z N_1 D^z \overline{N}_1 &= (m_1+m_3)^2 |N_1|^2 + V_1 \\ D_z N_2 D^z N_2 &= (m_1-m_3)^2 |N_2|^2 + V_2 \\ \phi_+^{\dagger\alpha} D_z \gamma_z \phi_+^\alpha &= \phi_+^{\dagger\alpha} i m_1 \gamma_z \phi_+^\alpha \\ \phi_-^{\dagger\alpha} D_z \gamma_z \phi_-^\alpha &= \phi_-^{\dagger\alpha} i m_3 \gamma_z \phi_-^\alpha \end{aligned} \tag{6.6}$$

as we can see, the fields acquire a mass corresponding to their charge under the orbifold action accordin to their $SO(4)$ represntation. Now, depending on the orbifold, the massless fields will correspond to those men-

[1]There is an interesting story here about higher momentum modes. It turns out that in certain orbifolds, for specific values of the S^1 radius R, some momentum modes remain massless since the orbifold charge and the KK momentum cancel out. Only in those specific cases, supersymmetry is restored.

tiones in (tables of chap3), and supersymmetry will be broken (partially or completely).

7. Conclusions

This thesis explored the extension of the well understood orbifold supersymmetry breaking of the closed string spectrum to the open string spectrum. Firstly, we studied the spectrum of the theory, and matched the orbifold charges of the different representations of $SO(4)$ to those already known to the closed string. Secondly, in an attempt to reproduce the spontaneous symmetry breaking known to the closed string effective supergravity through a SS reduction, we proposed an analogour procedure in the effective worldvolume theory.

As it turned out, this process seems well behaved, and it can be tuned to reproduce the spectrum found in the high energy string theory.

In single brane stacks, the number of supercharges can be broken to 16, 8 or 0. While for the D1/D5 brane systen, it can only be totally broken from 8 to 0.

Some questions still linger. At the hearth of this thesis, we wanted to calculate the infrared limit of the D1/D5 orbifold worldvolume theory, but due to a lack of tools to describe that IR limit, it was not possible to perform it. Besides, there is a story about theories with an S^1 radius $R > 1$ that can restore the supersymmetry normaly broken by the orbifold, which could be worthwile to study.

Appendices

A. Spinors in various dimensions

In this appendix we will juntify some claims about spinors in general even dimensions that are used throughout the thesis.

It is well known that the Dirac representation is not irreducible in even dimensions, in which a chirality projection exists into the two different Weyl basis. There is also always a Majorana condition that can induces a real structure in the spinor spaces of all dimensions, but is only truncates the degrees of freedom of Weyl spinors in some dimensions. All these topics will be covered in the following sections in detail with the main objective of describing irreducible spinors in all even dimensions $D \leq 10$.

A.1 Weyl Spinors in $D = 2, 4, 6, 8, 10$

We start with the Clifford algebra,

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}, \tag{A.1}$$

and let the metric be $\eta^{\mu\nu} = \text{diag}\,(-1, 1, \ldots, 1)$ the $D = 2k + 2$ dimensional Minkowski metric.

We first make a change of basis for the algebra elements so that the fundamental representation can be directly extracted from the algebra. Take the following linear combinations,

$$\Gamma^{0\pm} = \frac{1}{2}\left(\pm\Gamma^0 + \Gamma^1\right) \tag{A.2}$$

$$\Gamma^{a\pm} = \frac{1}{2}\left(\Gamma^{2a} \pm i\Gamma^{2a+1}\right), \quad a = 1, \ldots, k \tag{A.3}$$

Now the Clifford algebra A.1 can be stated in terms of the new operators as,

$$\begin{aligned}\left\{\Gamma^{a+}, \Gamma^{b-}\right\} &= \delta^{ab} \\ \left\{\Gamma^{a+}, \Gamma^{b+}\right\} &= \left\{\Gamma^{a-}, \Gamma^{b-}\right\} = 0.\end{aligned} \tag{A.4}$$

We can see that these operators are raising and lowering operators for $k+1$ different eigenvalues. We can write an arbitrary basis element of this representation as $|s_0, \ldots, s_k\rangle$, with $s_i = \pm 1$. An arbitrary spinor in this representation is then in the complex span of this basis, which has 2^{k+1} complex components. This is what is called a Dirac spinor, or $\mathbf{2^{k+1}}_{\text{Dirac}}$.

These eigenvalues s_i are actually eigenvalues of rotations in planes given by the grouping of A.2. To see this explicitly, we need to recover the Lorentz algebra from the Clifford algebra. Define,

$$\Sigma^{\mu v} = -\frac{i}{4}\left[\Gamma^\mu, \Gamma^v\right] \tag{A.5}$$

The elements $\Sigma^{\mu v}$ define the Lorentz algebra os $SO(1, 2k+1)$. The generators $\Sigma^{2a,2a+1}$ commute and have eigenvalues proportional to s_a when acting on the basis element $|s_0, \ldots, s_k\rangle$. To be precise, the operator,

$$S_a \equiv i^{\delta_{a,0}} \Sigma^{2a,2a+1} = \Gamma^{a+}\Gamma^{a-} - \frac{1}{2} \tag{A.6}$$

has eigenvalue s_a.

Next, we can define the chirality operator,

$$\Gamma = i^{-k}\Gamma^0\Gamma^1 \dots \Gamma^{d-1} \tag{A.7}$$

which has the properties,

$$(\Gamma)^2 = 1, \quad \{\Gamma, \Gamma^\mu\} = 0, \quad [\Gamma, \Sigma^{\mu v}] = 0 \tag{A.8}$$

From the first property, we see that Γ has eigenvalues ± 1. From the rest we see that we can split the basis $|s_0, \dots, s_k\rangle$ into two subspaces according to the eigenvalues of Γ. We can rewrite the chirality operator in terms of the rotation generators S_a as follows,

$$\Gamma = 2^{k+1} S_0 S_1 \dots S_k \tag{A.9}$$

which allows us to identify the two chiralities as $+1$ when the product of the s_a is positive and -1 when it is negative. The two subspaces defined by the chirality operator are called Weyl representations and are labeled as $\mathbf{2^k}_s$ and $\mathbf{2^k}_c$ for the $+1$ and -1 subspaces respectively. So we can finally state that the dirac representation splits into Weyl representations as,

$$\mathbf{2^{k+1}}_{\text{Dirac}} = \mathbf{2^{k}}_{s} \oplus \mathbf{2^{k}}_{c} \tag{A.10}$$

The staple example in string theory is $D = 10$, where we have the decomposition,

$$\mathbf{32}_{\text{Dirac}} = \mathbf{16}_{s} \oplus \mathbf{16}_{c} \tag{A.11}$$

A.2 Majorana condition

Up until now, we have been able to define Weyl spinors, that have a well defined chirality and have half the degrees of freedom of Dirac spinors. It will be shown in this section that a real structure can be imposed for some values of D, effectively halving the degrees of freedom of a Dirac spinor.

From the definition A.2, and the action on the basis elements $|s_0, \ldots, s_k\rangle$, we see that as a matrix, $\Gamma^{a\pm}$ are real. For the original gamma matrices defined in A.1, we see that the matrices Γ^{2a} are real, while the matrices Γ^{2a+1} are purely imaginary.

Now, take all of the purely imaginary matrices, and define the operators,

$$B_1 = \Gamma^3\Gamma^5 \ldots \Gamma^{d-1}, \quad B_2 = \Gamma B_1 \tag{A.12}$$

From the commutation relations of the Gamma matrices we have that,

$$B_1 \Gamma^\mu B_1^{-1} = (-1)^k \Gamma^{\mu *}, \quad B_2 \Gamma^\mu B_2^{-1} = (-1)^{k+1} \Gamma^{\mu *}, \tag{A.13}$$

and for the Lorentz generators,

$$B\Sigma^{\mu\nu}B^{-1} = -\Sigma^{\mu\nu *}, \tag{A.14}$$

for either B_1 or B_2.

Now, take a Dirac spinor ξ, and make a change of basis given by $\xi \to B\xi$. Then, by the trnasformation rules of ξ and the relation A.14, we see that $B\xi$ trnasforms as a conjugate spinor ξ^*.

The fact that we are able to linearly transform into conjugate spinors means that we may be able to relate the real and imaginary components of a spinor in a way consistent with Lorentz transformations. Concretely, we propose the Majorana condition,

$$\xi^* = B\xi. \tag{A.15}$$

Now, taking the conjugates $(B\xi)^* = \xi = B^*B\xi$, so $B^*B = 1$. Now, we can calculate explicitly using the definitions for B_1 and B_2 that,

$$B_1^*B_1 = (-1)^{k(k+1)/2}, \quad B_2^*B_2 = (-1)^{k(k-1)/2}. \tag{A.16}$$

So the condition $B^*B = 1$ can only be satisfied with $k = 0, 1, 3$ (mod 4). Which means that Majorana spinors can exist in $D = 2, 4, 8$ (mod 8) but not in $D = 6$ (mod 8).

We are ultimately interested in Majorana-Wey spinors, so we need to discuss whether a Majorana condition can be applied to a Weyl spinor. Take the chirality matrix Γ. We need to check if a Majorana change of basis can keep the chirality consistant. Otherwise, there would be mixing between right and left moving spinors, making them unconsistant with Lorentz trnasofrmations.

From the properties of the chirality matrix A.8, we calculate,

$$B_1 \Gamma B_1^{-1} = B_2 \Gamma B_2^{-1} = (-1)^k \Gamma^*, \tag{A.17}$$

so when k is even, each Weyl representation transforms as its own conjugate, while for k odd, the transformation rules get exchanged.

This automatically forbids Majorana-Weyl spinors from existing in $D = 4, 6 \pmod 8$, so that both conditions are only compatible in $D = 2 \pmod 8$. This is a very interesting result, because superstring theory is formulated from MW spinors in the worldsheet, this is $D = 2$, and produces MW spinors in spacetime, which is necessarily $D = 10$, it is quite a beautiful coincidence.

A.3 Table of irreducible spinors in even dimensions

To wrap up this brief lesson on spinors, we can list all of the minimal degrees of freedom of a spinor in any (even) dimension mod 8. For the sake of completeness I will also list the minimal representations in odd dimensions without any derivation (see [1] for details),

d	Majorana	Weyl	Majorana-Weyl	minimal dof
2	yes	self	yes	1
3	yes	-	-	2
4	yes	complex	-	4
5	-		-	8
6	-	self	-	8
7	-	-	-	16
8	yes	complex	-	16
9	yes	-	-	16
10	yes	self	yes	16

B. Dimensional reduction of SYM

Consider the SYM action in $D = 10$ with gauge group $U(N)$, with Euclidean signature (Wick rotation will yield results for other signature),

$$S = \int d^{10}\xi \mathrm{Tr} \left(-\frac{1}{4} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} + i\overline{\lambda} D_{\hat{\mu}} \Gamma^{\hat{\mu}} \lambda \right), \tag{B.1}$$

where all field content is adjoint, $\hat{\mu} = 0, \ldots, 9$,

$$\begin{aligned} F_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A_{\hat{\nu}} - \partial_{\hat{\nu}} A_{\hat{\mu}} + i[A_{\hat{\mu}}, A_{\hat{\nu}}], \\ D_{\hat{\mu}} = \partial_{\hat{\mu}} + i[A_{\hat{\mu}}, \cdot] \end{aligned} \tag{B.2}$$

and the fermions are MW in the sense,

$$\begin{aligned} \Gamma_c^{10} \lambda = \lambda, \\ C\lambda = \overline{\lambda}^T, \end{aligned} \tag{B.3}$$

so that they have 16 real components.

We choose the chiral basis of the gamma matrices in order to be able to split the spinor via chirality as it was stated in Appendix A,

$$\begin{aligned}
\Gamma_0 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \\
\Gamma_2 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \\
\Gamma_3 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \otimes \mathbb{1} \\
\Gamma_4 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \otimes \mathbb{1} \\
\Gamma_5 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_6 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_7 &= \sigma_1 \otimes \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_8 &= \sigma_1 \otimes \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_9 &= \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}
\end{aligned} \tag{B.4}$$

And the chirality and charge conjugation matrices are defined as,

$$\begin{aligned}
\Gamma_c^{10} &= i \prod_{\hat{\mu}} \Gamma_{\hat{\mu}} = \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \\
C &= \prod_{i=1}^{5} \Gamma_{2i-1} = -\sigma_2 \otimes \sigma_3 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_2
\end{aligned} \tag{B.5}$$

Now consider that we dimensionally reduce from $D = 10$ to $D' = 6$ as it is the easiest non-trivial exaple. The indices split into $\hat{\mu} = (\mu, i)$, $\mu = 0, \ldots, 5$, $i = 6, \ldots, 9$. Let's discuss the two parts of the action independently. First the vector goes as follows,

$$\begin{aligned}
S_{\text{boson}} &= -\frac{1}{4} \int d^6\xi \mathrm{Tr} \left(F_{\mu\nu} F^{\mu\nu} + 2 F_{\mu i} F^{\mu i} + F_{ij} F^{ij} \right) = \\
&= -\frac{1}{4} \int d^6\xi \left(F_{\mu\nu} F^{\mu\nu} + 2 D_\mu A_i D^\mu A^i + i[A_i, A_j][A^i, A^j] \right),
\end{aligned} \tag{B.6}$$

where we can see that A_μ is in the $(\mathbf{6}, \mathbf{1})$, and A_i in the $(\mathbf{1}, \mathbf{4})$.

The fermion part after dimensional reduction is,

$$S_{\text{fermion}} = i \int d^6 \text{Tr} \left(\xi \lambda^\dagger D^\mu \Gamma_0 \Gamma_\mu \lambda + \lambda^\dagger \Gamma_0 \Gamma_i [A^i, \lambda] \right) \tag{B.7}$$

Now, let us define the following lower dimensional gamma matrices,

$$\begin{aligned} \Gamma_0^6 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \\ \Gamma_1^6 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \\ \Gamma_2^6 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_3 \\ \Gamma_3^6 &= \sigma_1 \otimes \sigma_2 \otimes \mathbb{1} \\ \Gamma_4^6 &= \sigma_1 \otimes \sigma_3 \otimes \mathbb{1} \\ \Gamma_5^6 &= \sigma_2 \otimes \mathbb{1} \otimes \mathbb{1} \end{aligned} \tag{B.8}$$

then we can write,

$$\Gamma_0 \Gamma_\mu = \mathbb{1} \otimes \mathbb{1} \otimes \Gamma_0^6 \Gamma_\mu^6 \tag{B.9}$$

The spinors λ can be split according to their $SO(1,5)$ and $SO(4)$ chirality, as eigenspinors of,

$$\begin{aligned} \Gamma_c^6 &= \mathbb{1} \otimes \mathbb{1} \otimes \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \\ \Gamma_c^4 &= \sigma_3 \otimes \mathbb{1} \otimes \sigma_3 \otimes \mathbb{1} \otimes \mathbb{1} \end{aligned} \tag{B.10}$$

the projection split the spinor as,

$$\lambda = \begin{pmatrix} \lambda_+ \\ 0 \end{pmatrix}, \quad \lambda_+ = \begin{pmatrix} \phi_+^{\alpha} \\ \phi_-^{\dot{\alpha}} \end{pmatrix} \tag{B.11}$$

the spinors ϕ_+^{α} and $\phi_-^{\dot{\alpha}}$ belong to the $(\mathbf{4}_s, \mathbf{2}_s)$ and $(\mathbf{4}_c, \mathbf{2}_c)$. With this construction in mind, we can calculate the following matrices,

$$\begin{aligned}
\Gamma_0^6 \Gamma_0^6 &= \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \\
\Gamma_0^6 \Gamma_1^6 &= \mathbb{1} \otimes \mathbb{1} \otimes \sigma_3 \\
\Gamma_0^6 \Gamma_2^6 &= \mathbb{1} \otimes \mathbb{1} \otimes \sigma_2 \\
\Gamma_0^6 \Gamma_3^6 &= \mathbb{1} \otimes \sigma_3 \otimes \sigma_1 \\
\Gamma_0^6 \Gamma_4^6 &= \mathbb{1} \otimes \sigma_2 \otimes \sigma_1 \\
\Gamma_0^6 \Gamma_5^6 &= \sigma_3 \otimes \sigma_1 \otimes \sigma_1
\end{aligned} \tag{B.12}$$

So we see that depending on the 6D chirality of the spinor, they will couple through different vectors, akin to a 6D generalization of the Pauli matrices. Let us define γ_μ and $\overline{\gamma}_\mu$, the 4x4 matrices resulting from,

$$\Gamma_0^6 \Gamma_\mu^6 = \begin{pmatrix} \gamma_\mu & 0 \\ 0 & \overline{\gamma}_\mu \end{pmatrix} \tag{B.13}$$

We are finally ready to write the kinetic term of the fermion action, which results in,

$$S_{\text{kinetic}} = i \int d^6\xi \operatorname{Tr} \left(\phi_+^{\dagger\alpha} D_\mu \gamma_\mu \phi_+^{\alpha} + \phi_-^{\dagger\dot{\alpha}} D_\mu \overline{\gamma}_\mu \phi_-^{\dot{\alpha}} \right). \tag{B.14}$$

The potential term involves the matrices,

$$
\begin{aligned}
\Gamma_0\Gamma_6 &= \mathbb{1} \otimes \mathbb{1} \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_0\Gamma_7 &= \mathbb{1} \otimes \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_0\Gamma_8 &= \mathbb{1} \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_0\Gamma_9 &= \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1
\end{aligned}
\tag{B.15}
$$

Taking into account that for the first index, every spinor has eigenvalue +1, we can reduce the problem to,

$$
\begin{aligned}
\Gamma_0\Gamma_6 &= 1 \otimes \mathbb{1} \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_0\Gamma_7 &= 1 \otimes \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_0\Gamma_8 &= 1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \\
\Gamma_0\Gamma_9 &= 1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1
\end{aligned}
\tag{B.16}
$$

From the discussion of the kinetic term we defined $\gamma_5 = \sigma_1 \otimes \sigma_1$, which we can now use to find,

$$
S_{\text{potential}} = i \int d^6\xi \left(\phi_+^{\dagger\alpha} \sigma^i_{\alpha\beta} \gamma_5 [A_i, \phi_-^\beta] + \phi_-^{\dagger\alpha} \overline{\sigma}^i_{\alpha\beta} [A_i, \phi_+^\beta] \right) \tag{B.17}
$$

The matrices $\sigma^i_{\alpha\beta}$ and $\overline{\sigma}^i_{\alpha\beta}$ are the corresponding intertwiners between the different representation of $SO(4)$ that exist in the action.

Further dimensional reductions follow the same procedure described in this appendix. Note that we chose to show the details for $D = 10 \to 6$ because this is the smallest dimension in which the R-symmetry group has indices with more than one component, so intertwiners appear in a nontrivial manner.

B.1 Supercharges under dimensional reduction

The action resulting from the dimensional reduction described in Appendix B does not break any supersymmetry (this is apparent since all fields remain massless), so the total number of supercharges is always 16. Before dimensional reduction, these could be written as,

$$\begin{aligned} \delta_\epsilon A_{\hat{\mu}} &= \bar{\epsilon}\Gamma_{\hat{\mu}}\lambda \\ \delta_\epsilon \lambda &= -\frac{1}{2}F_{\hat{\mu}\hat{\nu}}\Gamma^{\hat{\mu}\hat{\nu}}\epsilon \end{aligned} \tag{B.18}$$

where ϵ is A MW spinor in the same representation as λ, the $\mathbf{16}_s$. Since the expression for the fermions get rather cumbersome, we will only derive in detail the dimensional reduction of the vector transformations. The only matrix calculations we need to check for the following are the commutators $\Gamma^{\mu\nu}$.

$$\Gamma_{\mu\nu} = \Gamma_{[\mu}\Gamma_{\nu]} = \Gamma_{[\mu}(\Gamma_0)^2\Gamma_{\nu]} = \mathbb{1}\otimes\mathbb{1}\otimes\Gamma^6_{[\mu}\Gamma^6_{\nu]} \tag{B.19}$$

Then, the transformation rules are simply,

$$\begin{aligned} \delta_\epsilon A_\mu &= \epsilon_+^{\dagger\alpha}\gamma_\mu\phi_+^\alpha + \epsilon_-^{\dagger\alpha}\overline{\gamma}_\mu\phi_-^\alpha \\ \delta_\epsilon A^i &= \epsilon_+^{\dagger\alpha}\sigma^i_{\alpha\beta}\gamma_5\phi_+^\beta + \epsilon_-^{\dagger\alpha}\overline{\sigma}^i_{\alpha\beta}\gamma_5\phi_+^\beta \end{aligned} \tag{B.20}$$

where we have split ϵ into ϵ_+^α and ϵ_-^α in the same manner we split λ. As we can see, the transformation $\delta_\epsilon \to \delta_{\epsilon_+} + \delta_{\epsilon_-}$. The 16 spercharges remain, but they are split as $\mathcal{N} = (2,2)$ in the sense that they are carried by 2 $\mathbf{4}_s$ and

2 $\mathbf{4}_c$, with the appropriate R-symmetry group.

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