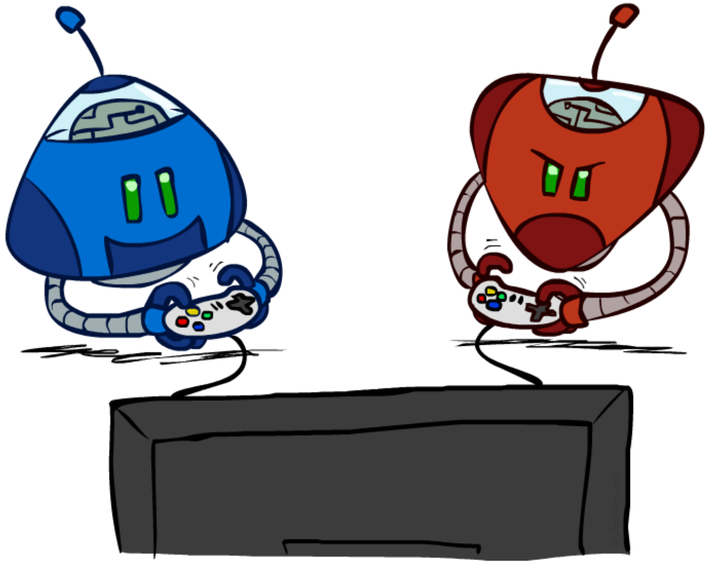
Artificial Intelligence - INFOF311

Games and adversarial search



Instructor : Tom Lenaerts

Acknowledgement

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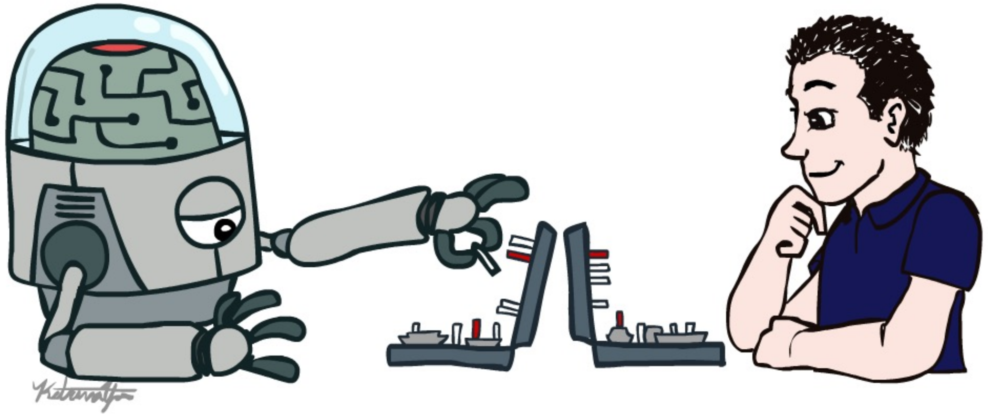
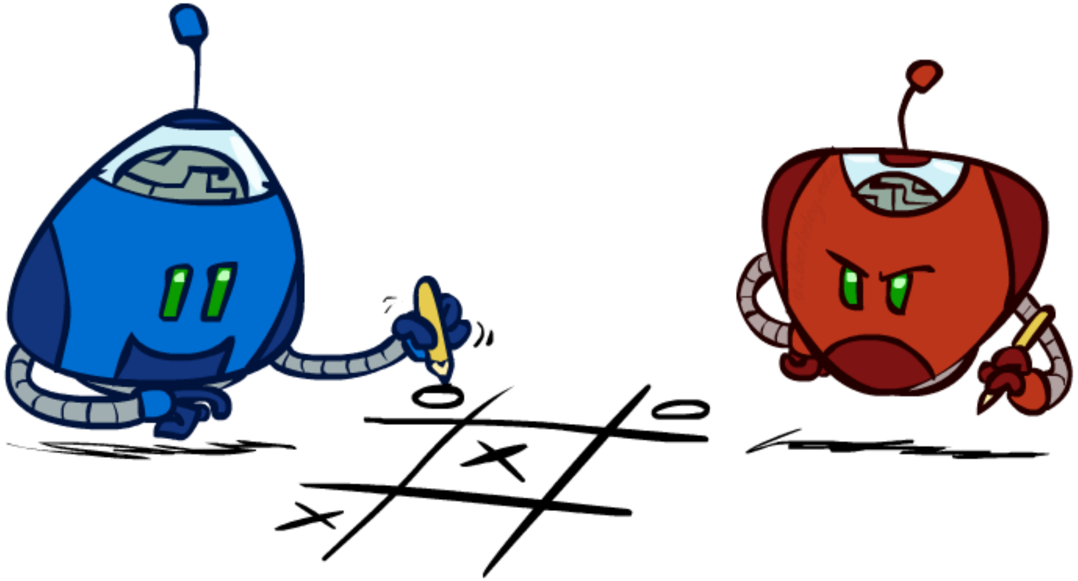


The slides for INFOF311 are slightly modified versions of the slides of the spring and summer CS188 sessions in 2021 and 2022

Outline

- History / Overview
- Minimax for Zero-Sum Games
- α-β Pruning
- Finite lookahead and evaluation



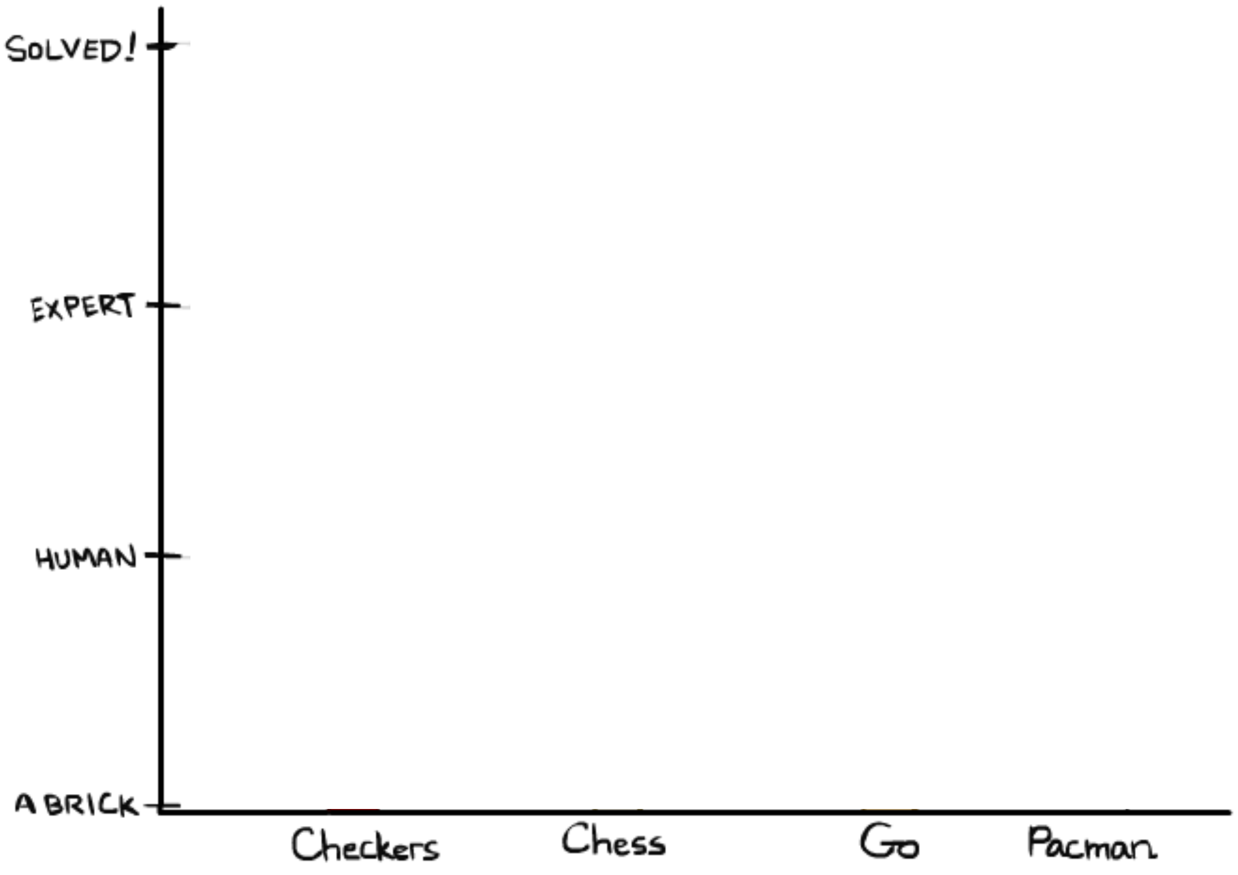
A brief history

- **Checkers:**
  - 1950: First computer player.
  - 1959: Samuel's self-taught program.
  - 1994: First computer world champion: Chinook defeats Tinsley
  - 2007: Checkers solved! Endgame database of 39 trillion states

- **Chess:**
  - 1945-1960: Zuse, Wiener, Shannon, Turing, Newell & Simon, McCarthy.
  - 1960s onward: gradual improvement under "standard model"
  - 1997: Deep Blue defeats human champion Garry Kasparov
  - 2022: Stockfish (15+) rating 3541 (vs 2882 for Magnus Carlsen 2015).

- **Go:**
  - 1968: Zobrist's program plays legal Go, barely (b>300!)
  - 1968-2005: various ad hoc approaches tried, novice level
  - 2005-2014: Monte Carlo tree search -> strong amateur
  - 2016-2017: AlphaGo defeats human world champions

- **Pacman**



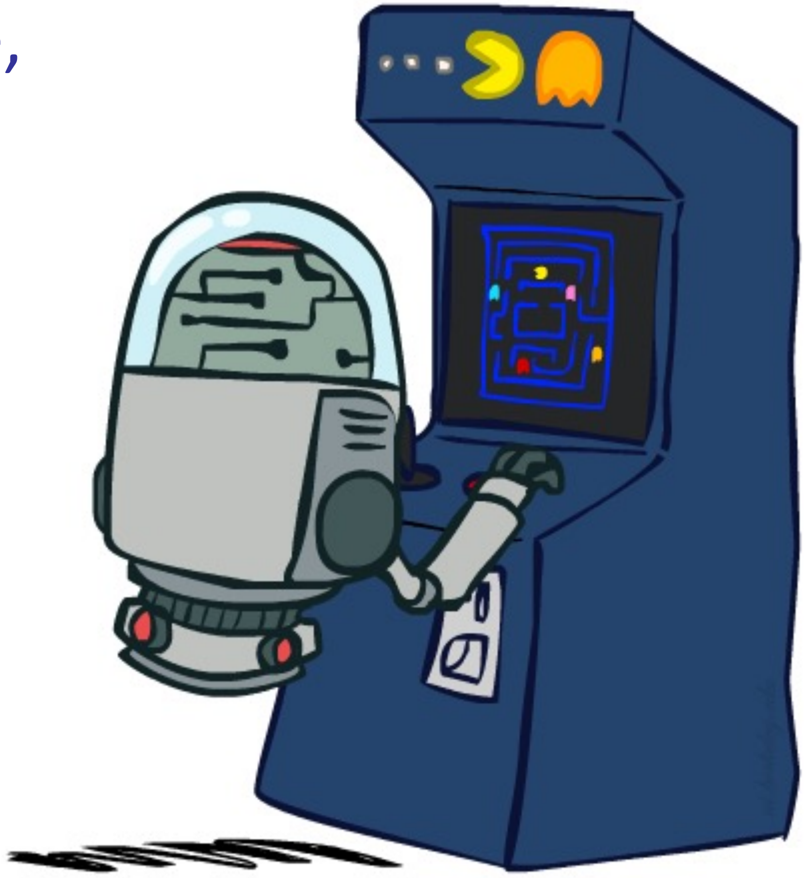
Types of Games

- Game = task environment with > 1 agent
- Axes:
  - Deterministic or stochastic?
  - Perfect information (fully observable)?
  - Two, three, or more players?
  - Teams or individuals?
  - Turn-taking or simultaneous?
  - Zero sum?

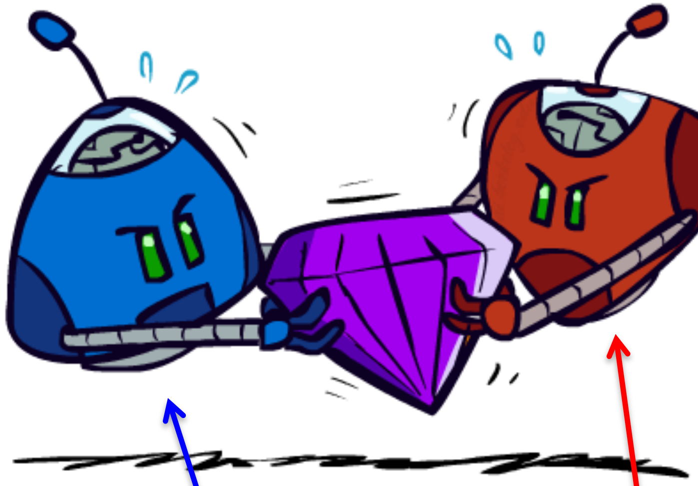
- Want algorithms for calculating a ***contingent plan*** (a.k.a. strategy or policy) which recommends a move for every possible eventuality

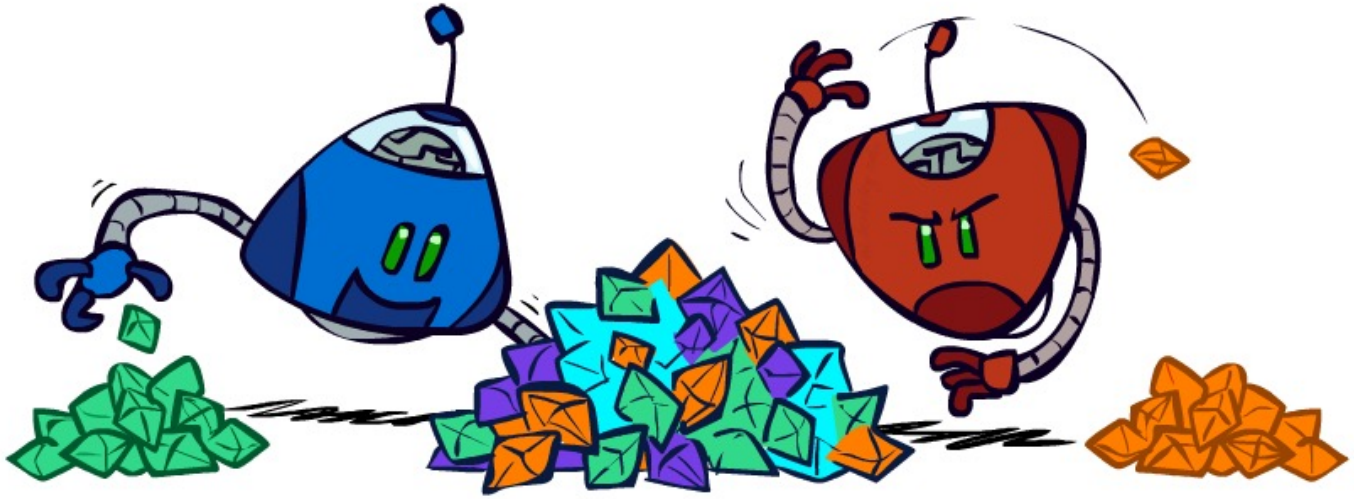
“Standard” Games

- Standard games are deterministic, observable, two-player, turn-taking, zero-sum
- Game formulation:
  - Initial state: s_0
  - Players: Player(s) indicates whose move it is
  - Actions: Actions(s) for player on move
  - Transition model: Result(s,a)
  - Terminal test: Terminal-Test(s)
  - Terminal values: Utility(s,p) for player p
    - Or just Utility(s) for player making the decision at root
- Solution for a player is a policy : S → A



Zero-Sum Games

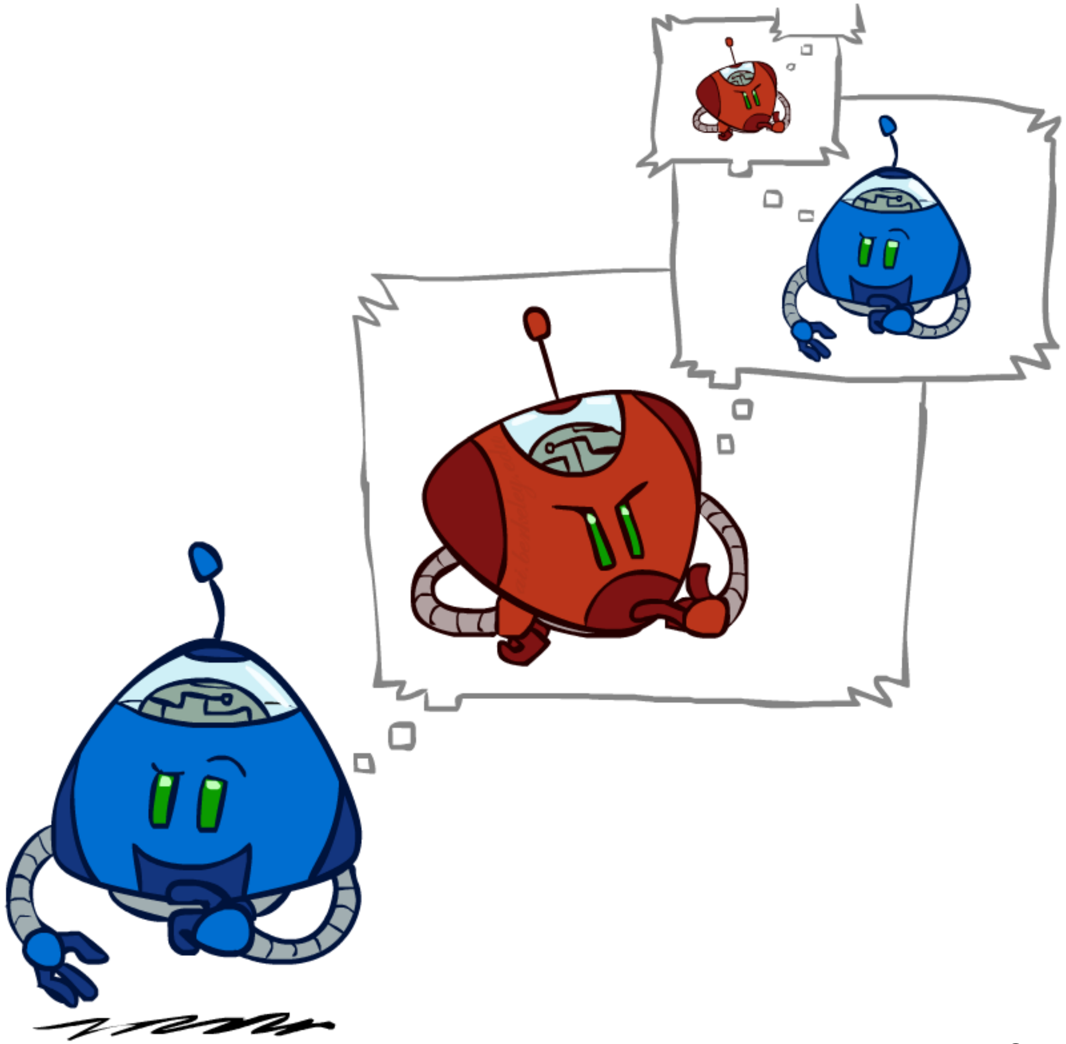




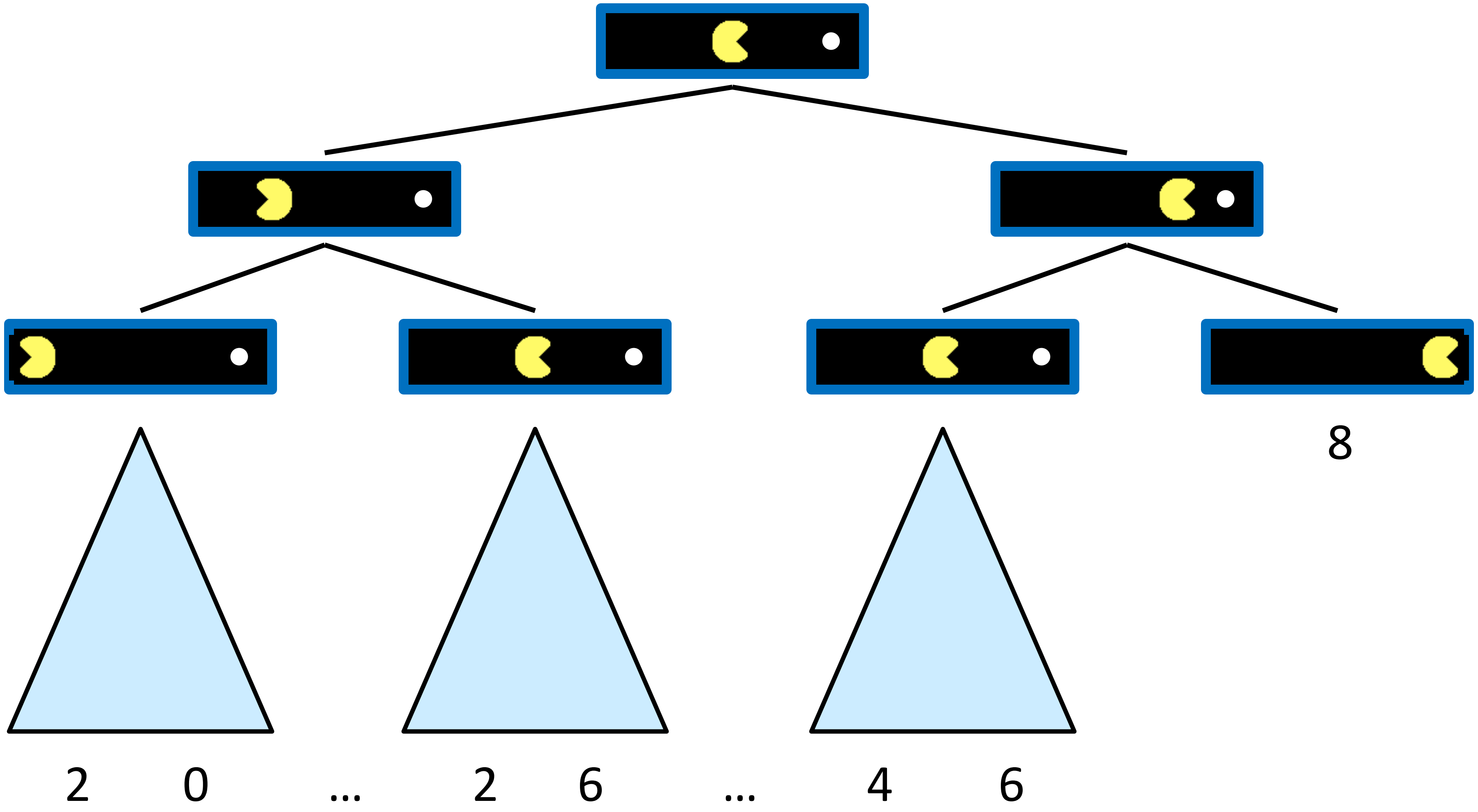
- Zero-Sum Games
  - Agents have ***opposite*** utilities
  - Pure competition:
    - One ***maximizes***, the other ***minimizes***

- General-Sum Games
  - Agents have ***independent*** utilities
  - Cooperation, indifference, competition, shifting alliances, and more are all possible
- Team Games
  - Common payoff for all team members

Adversarial Search



Single-Agent Trees

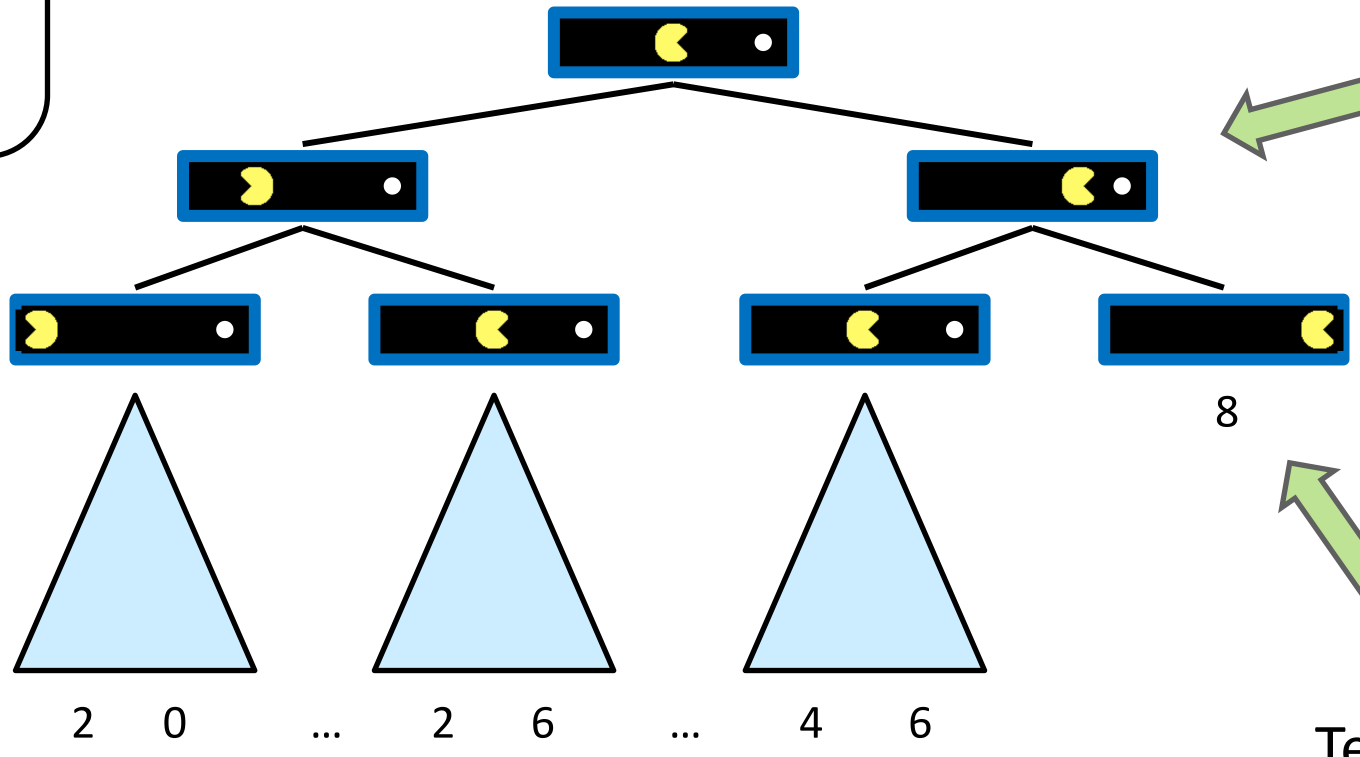


Value of a State

Value of a state:
The best achievable outcome (utility) from that state

Non-Terminal States:

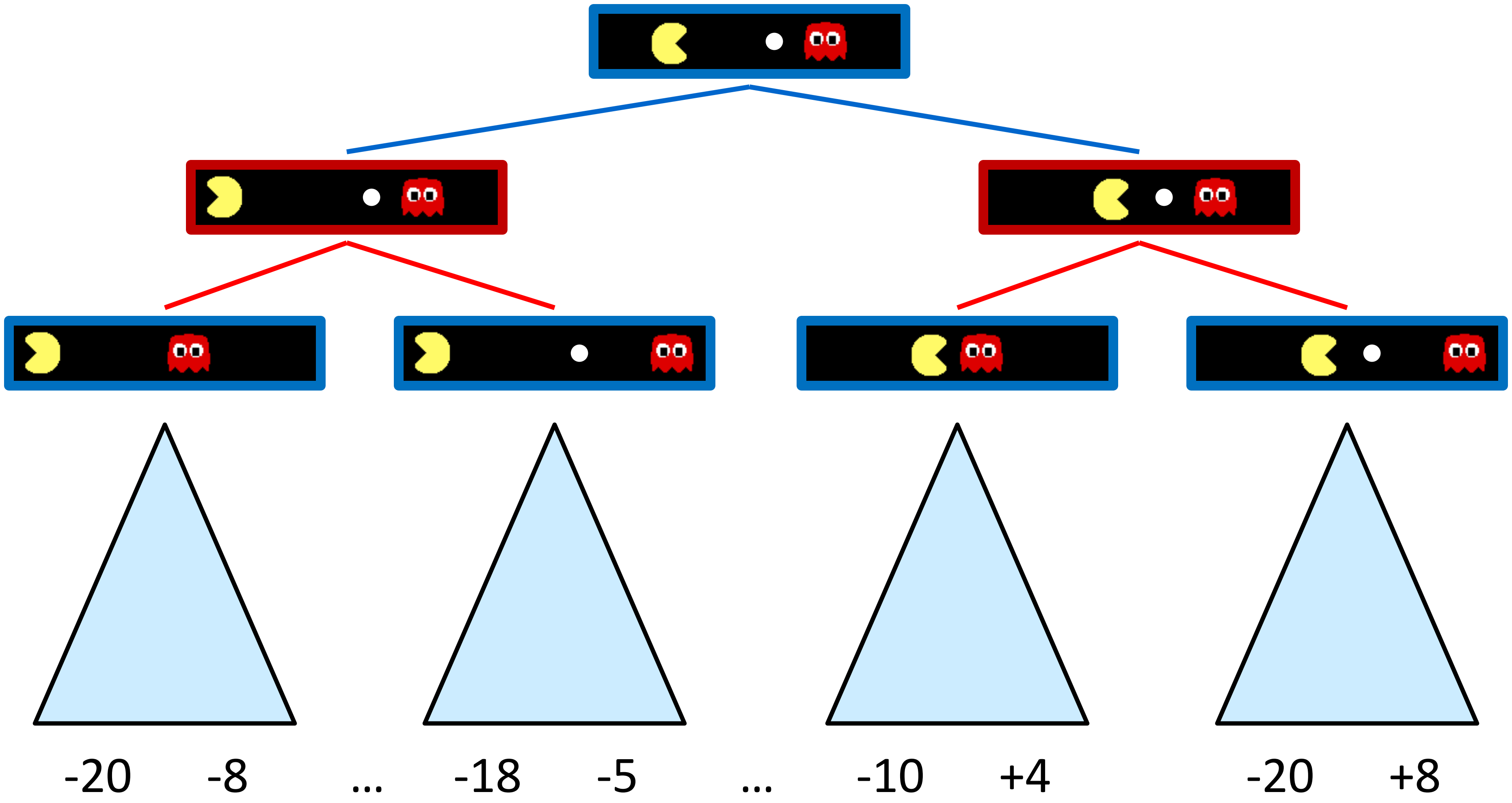
$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$



Terminal States:

$V(s)$ = known

Adversarial Game Trees



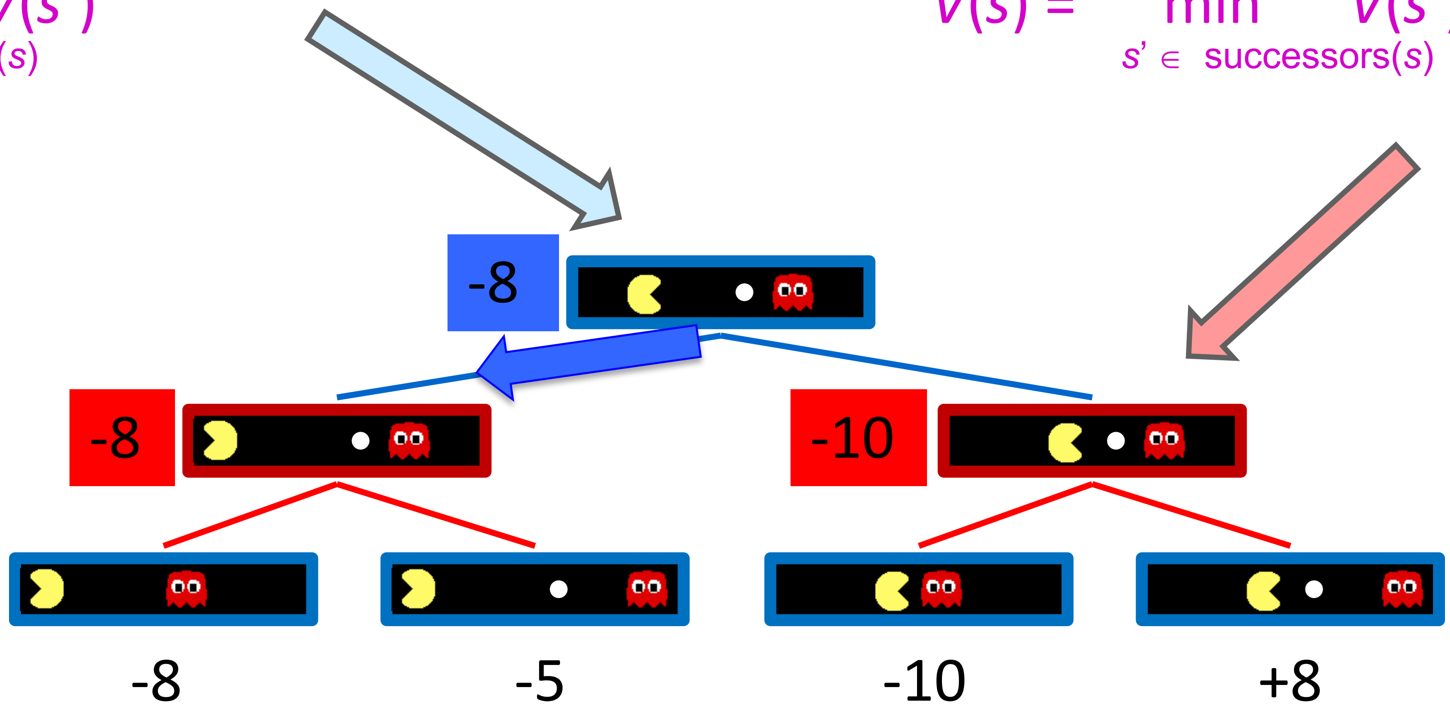
Minimax Values

MAX nodes: under Agent's control

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

MIN nodes: under Opponent's control

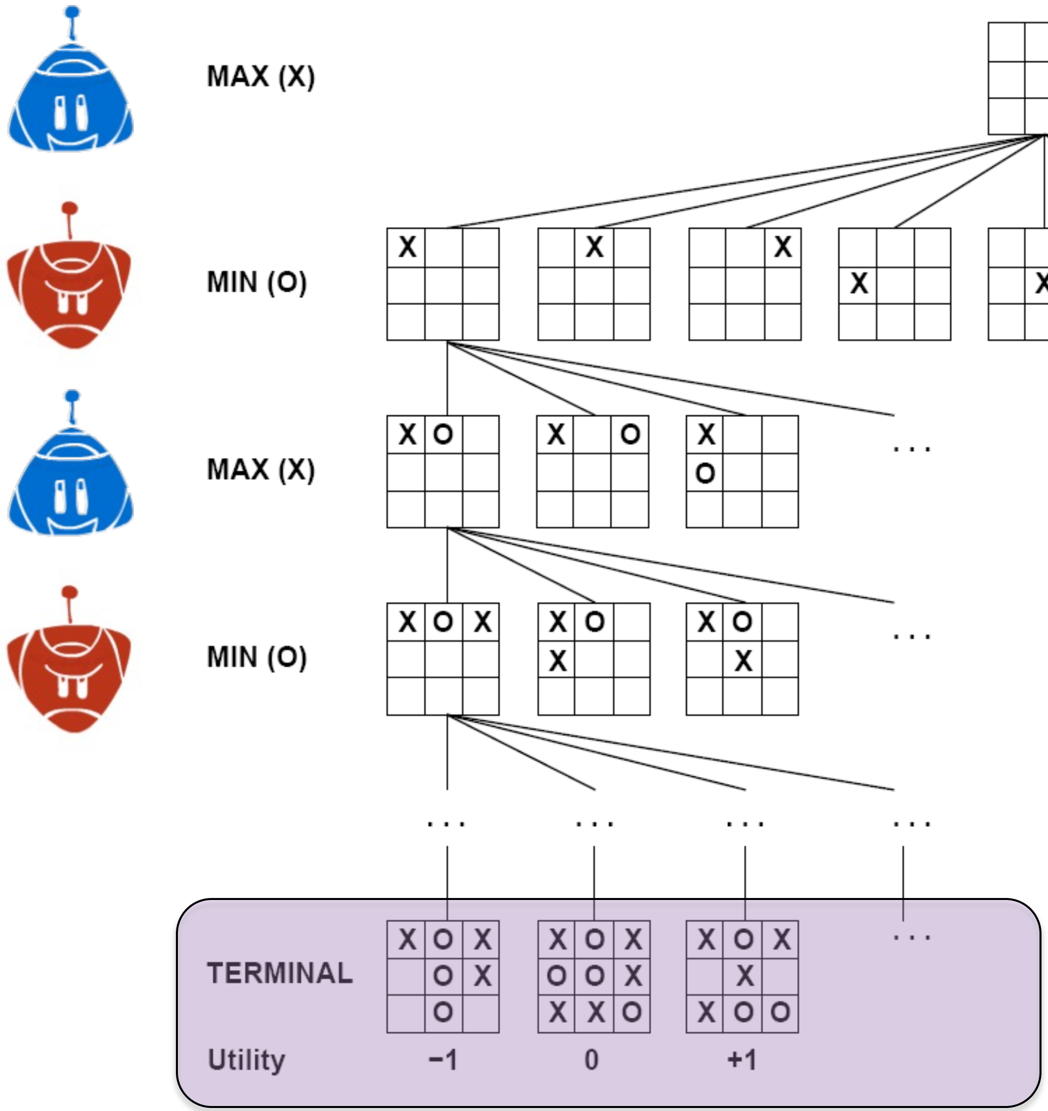
$$V(s) = \min_{s' \in \text{successors}(s)} V(s')$$

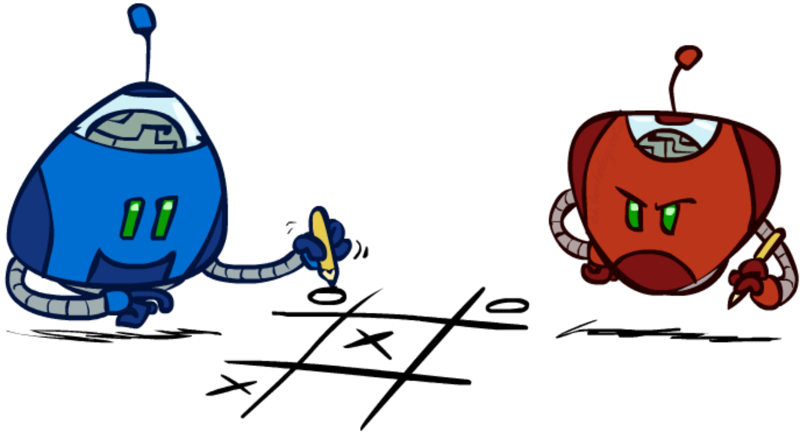


Terminal States:

$V(s)$ = known

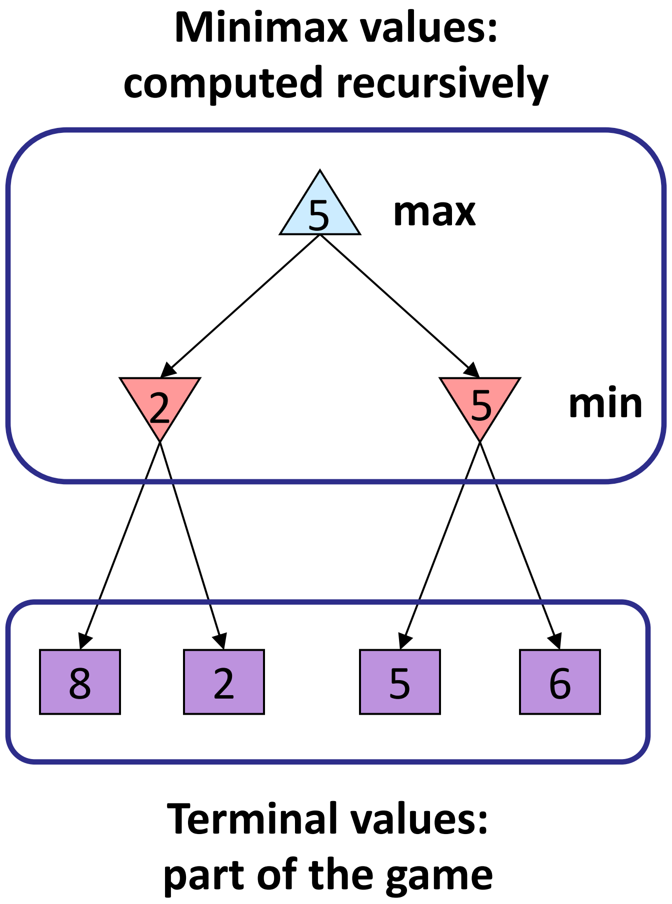
Tic-Tac-Toe Game Tree





Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

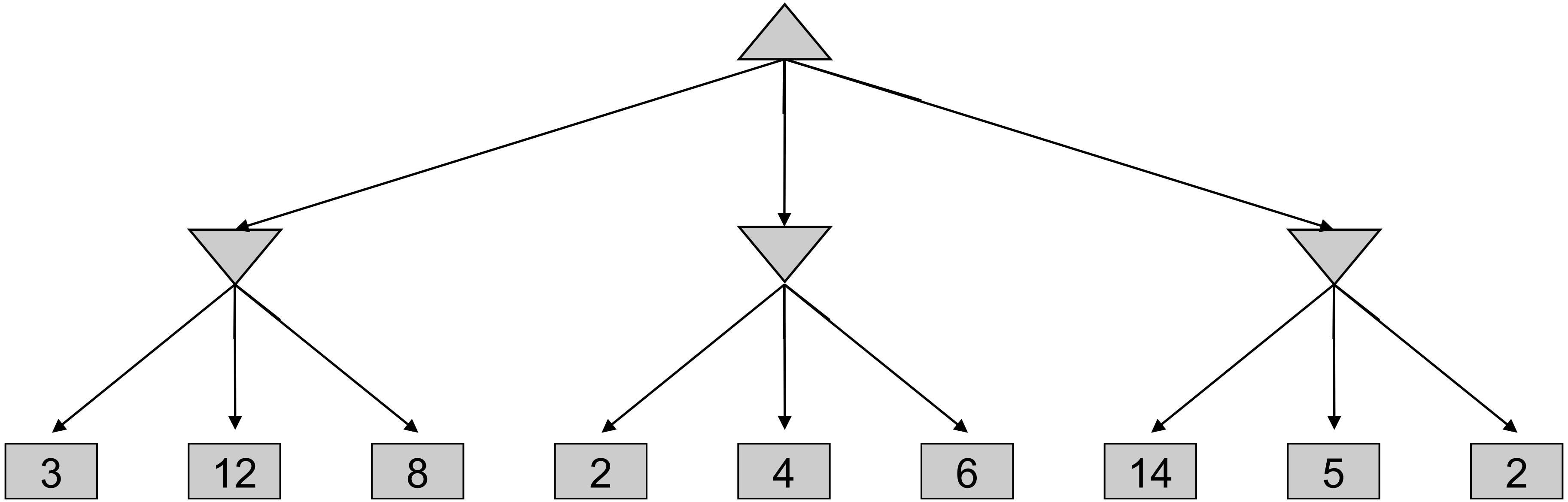
Minimax Implementation (Dispatch)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```

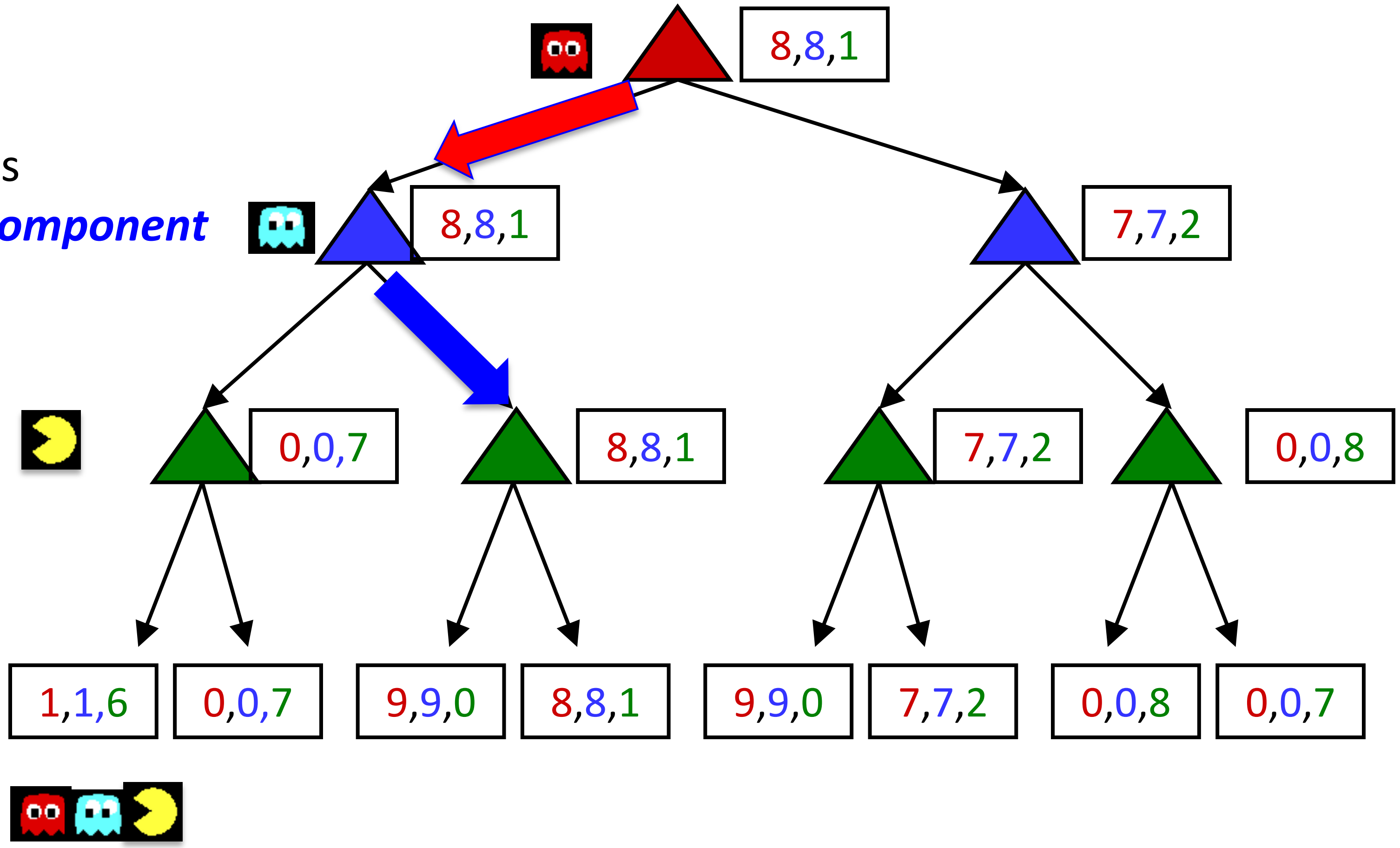
Minimax Example

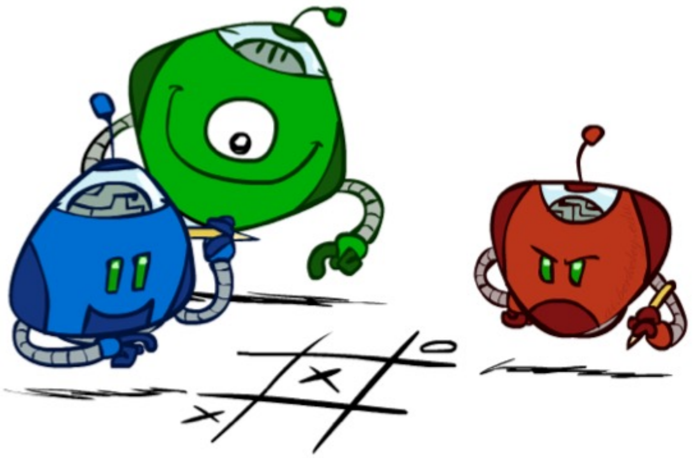


Generalized minimax

- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have ***utility tuples***
  - Node values are also utility tuples
  - ***Each player maximizes its own component***
  - Can give rise to cooperation and competition dynamically…



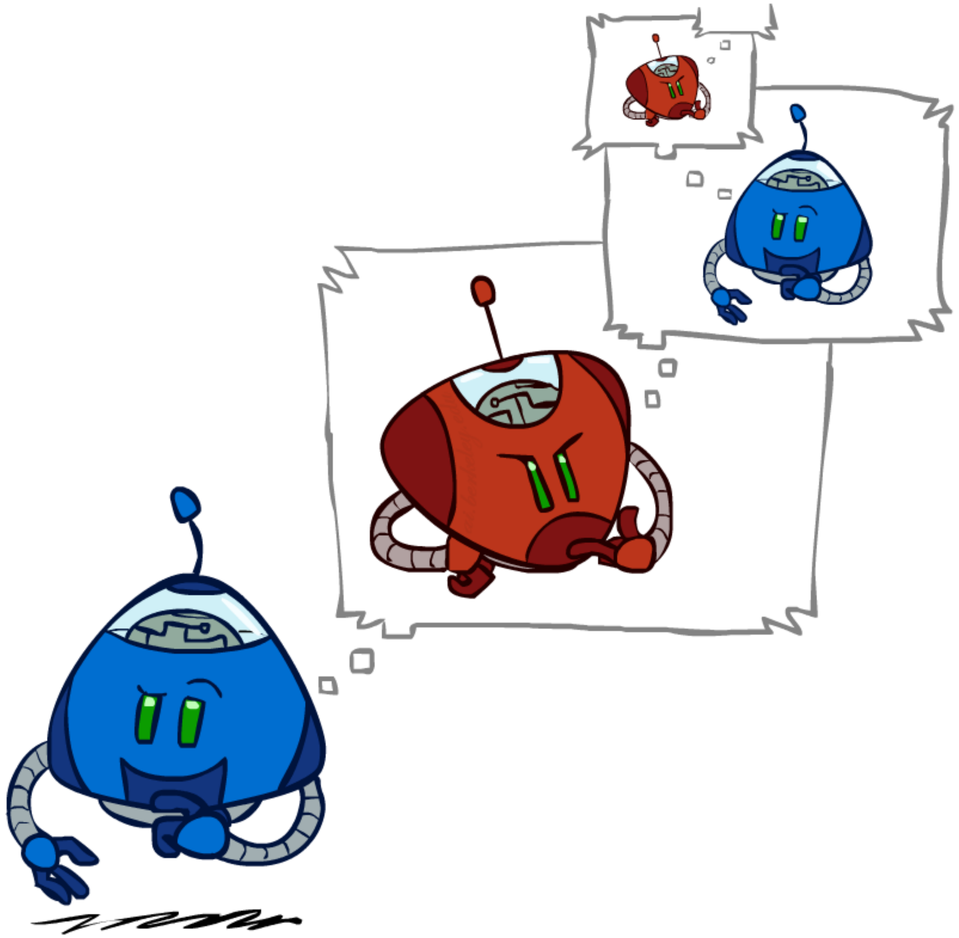


Emergent coordination in ghosts

Minimax Efficiency

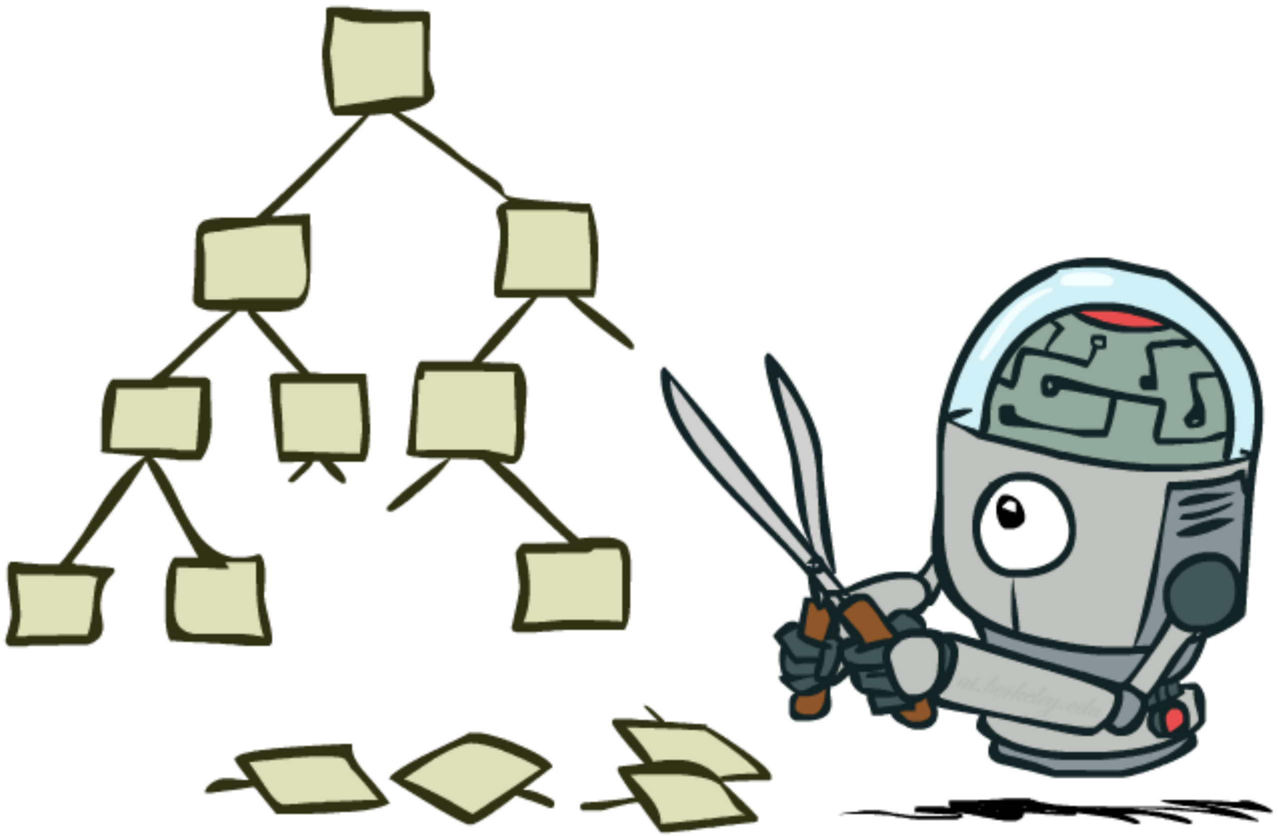
- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - Humans can't do this either, so how do we play chess?

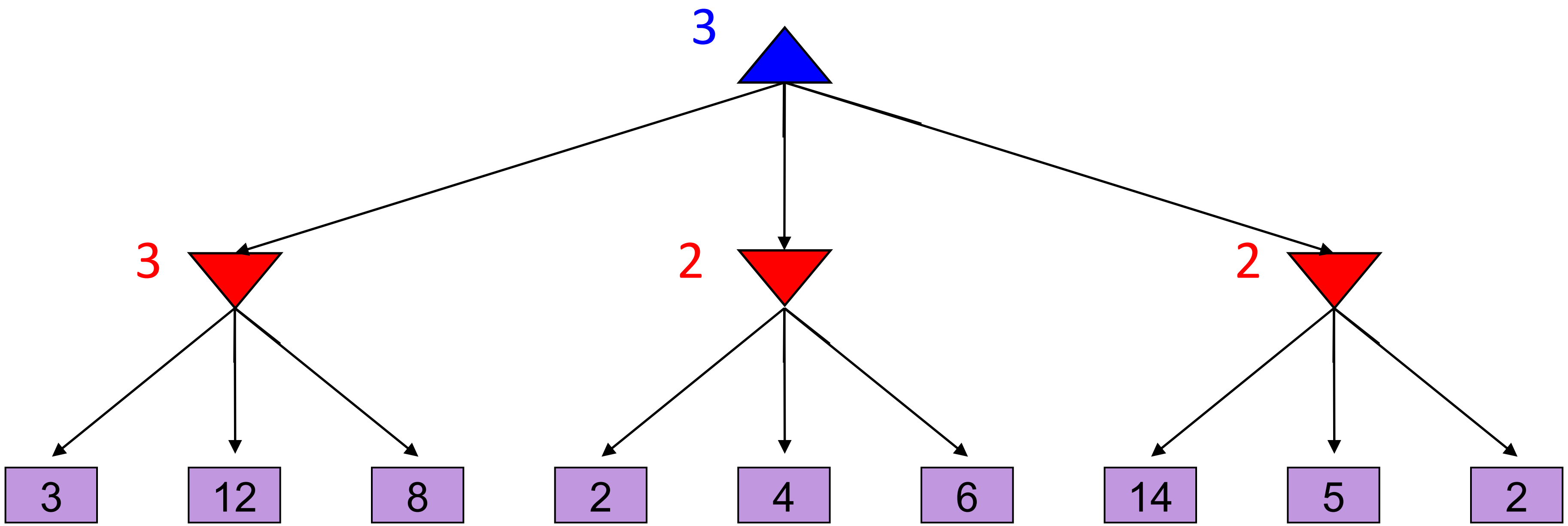


Resource Limits

Game Tree Pruning

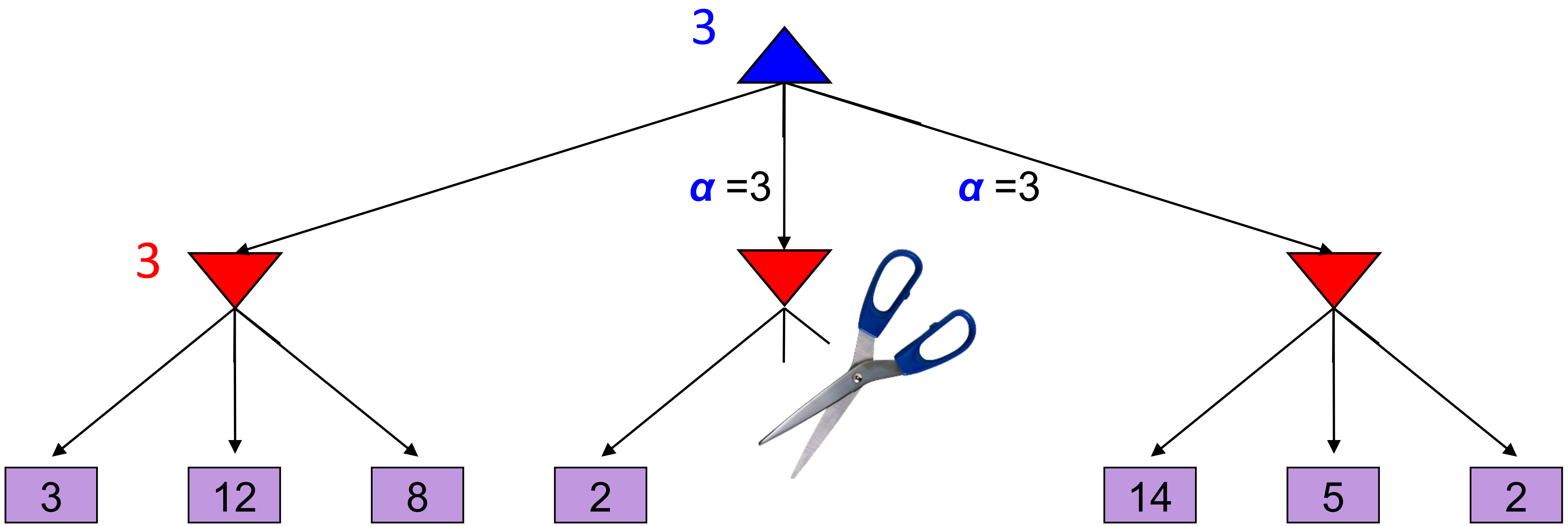


Minimax Example



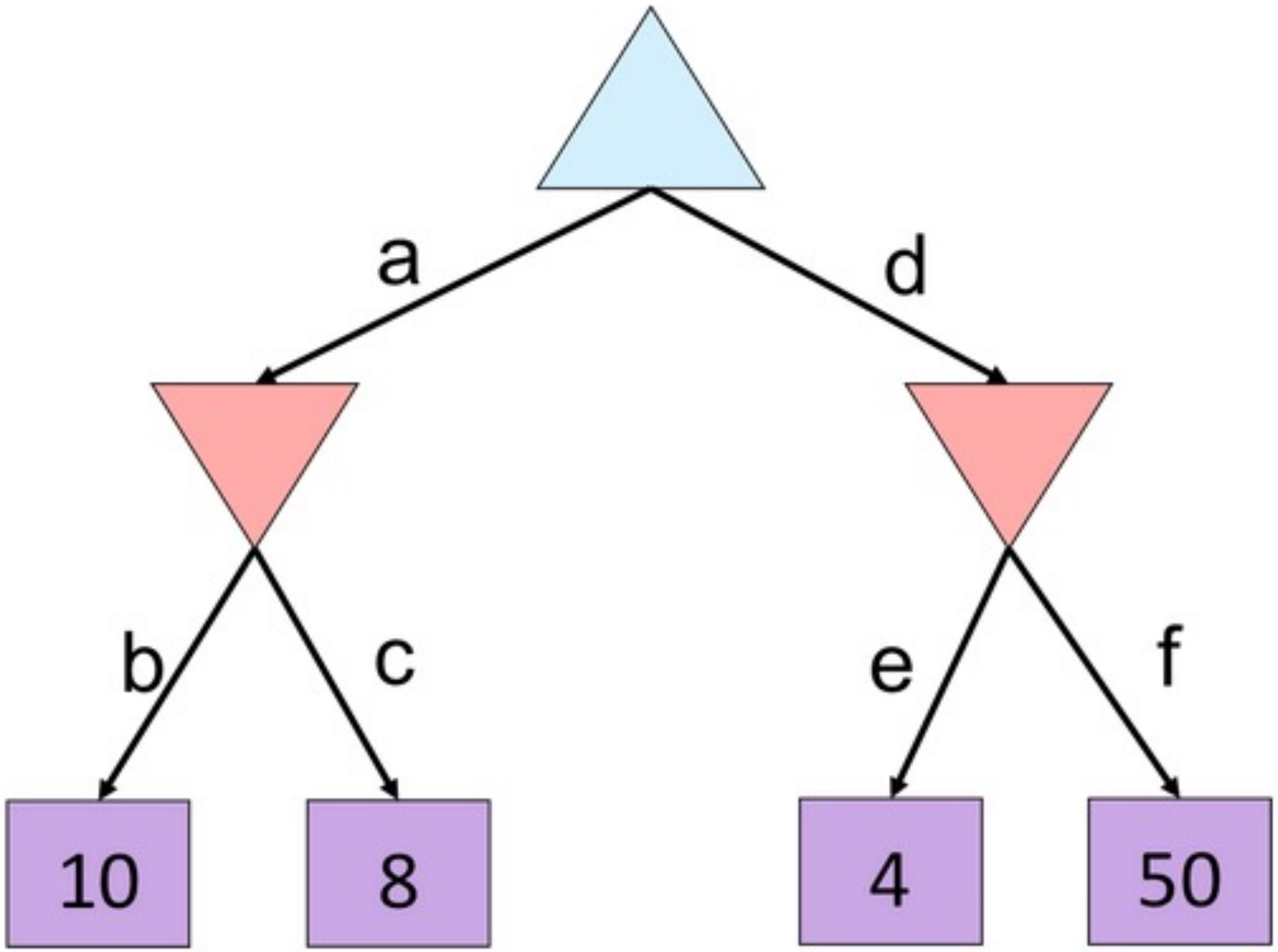
Alpha-Beta Example

α = best option so far from any MAX node on this path

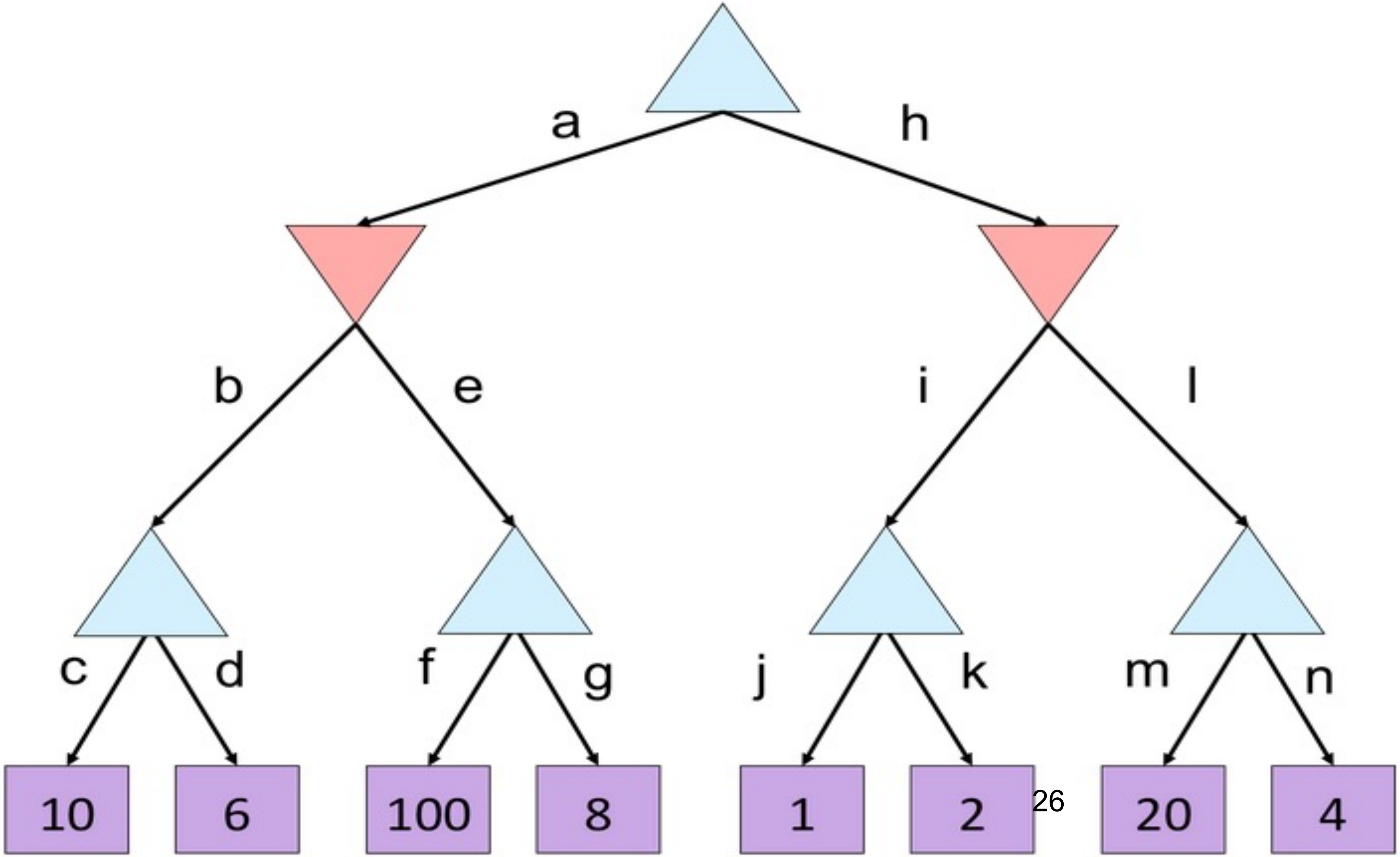


- ***The order of generation matters***: more pruning is possible if good moves come first

Alpha-Beta Quiz

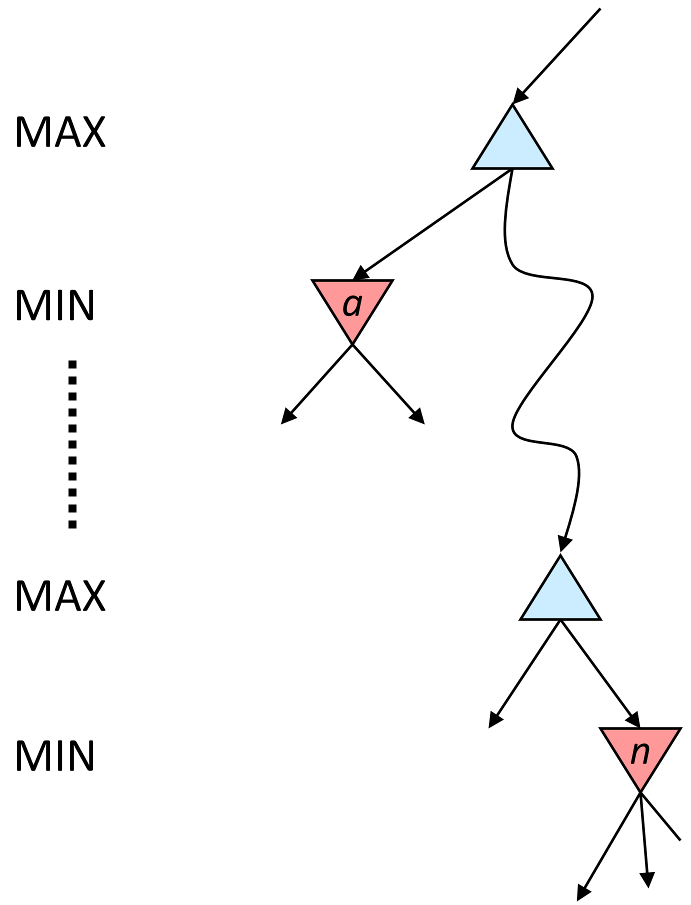


Alpha-Beta Quiz 2



Alpha-Beta Pruning

- General case (pruning children of MIN node)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let ***α*** be the best value that MAX can get so far at any choice point along the current path from the root
  - If *n* becomes worse than ***α***, MAX will avoid it, so we can prune *n*'s other children (it's already bad enough that it won't be played)
- Pruning children of MAX node is symmetric
  - Let **β** be the best value that MIN can get so far at any choice point along the current path from the root



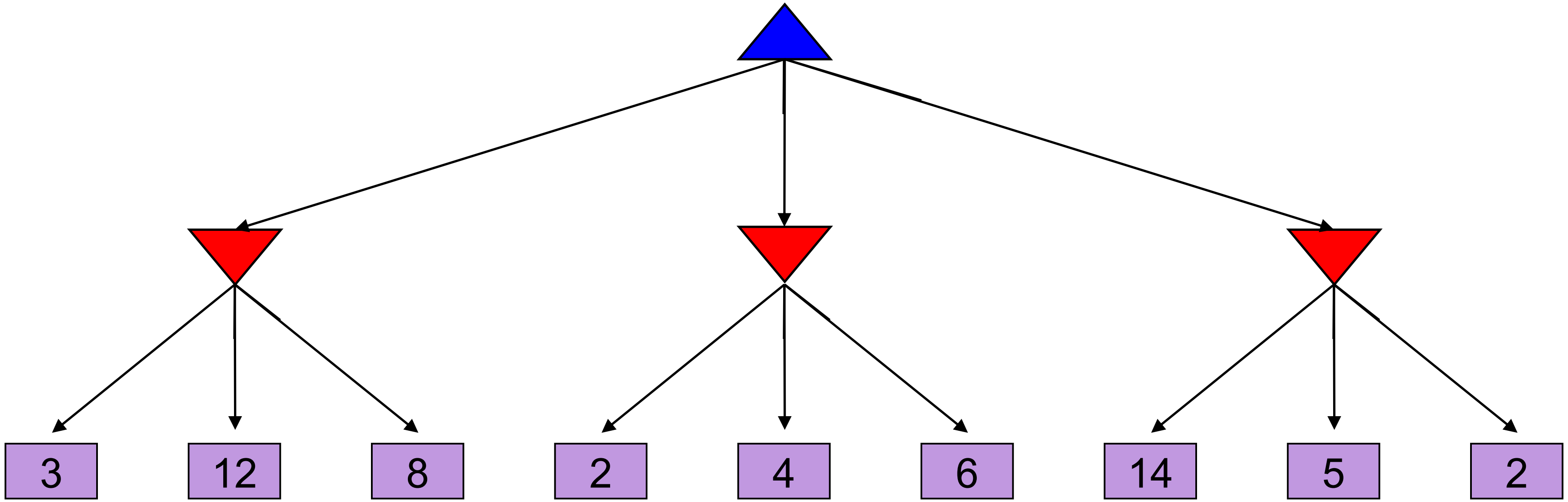
Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    if terminal(state) return Utility(state)
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor, α,β))
        if v ≥ β
            return v
        α = max(α, v)
    return v
```

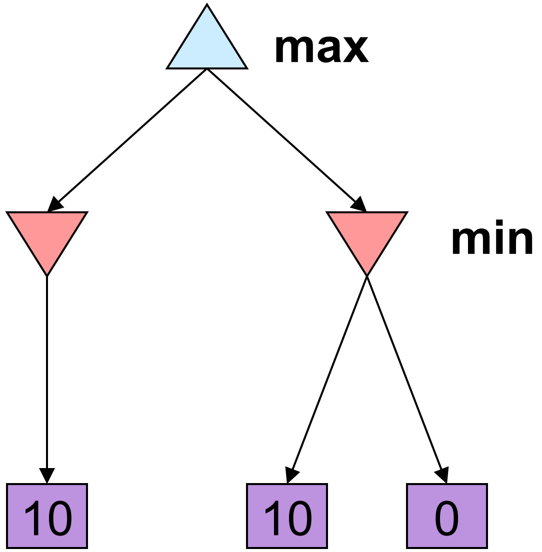
```
def min-value(state , α, β):
    if terminal(state) return Utility(state)
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

Alpha-beta code on example



Alpha-Beta Pruning Properties

- Theorem: This pruning has ***no effect*** on minimax value computed for the root!
- Good child ordering improves effectiveness of pruning
  - Iterative deepening helps with this
- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
- This is a simple example of metareasoning (reasoning about reasoning)
- For chess: only 35^{50} instead of 35^{100}!! Yaaay!!!!!



Summary

- Games are decision problems with $\geq$ 2 agents
  - Huge variety of issues and phenomena depending on details of interactions and payoffs
- For zero-sum games, optimal decisions defined by minimax
  - Simple extension to n-player “rotating” max with vectors of utilities
  - Implementable as a depth-first traversal of the game tree
  - Time complexity $O(b^m)$, space complexity $O(bm)$
- Alpha-beta pruning
  - Preserves optimal choice at the root
  - Alpha/beta values keep track of best obtainable values from any max/min nodes on path from root to current node
  - Time complexity drops to $O(b^{m/2})$ with ideal node ordering
- Exact solution is impossible even for “small” games like chess