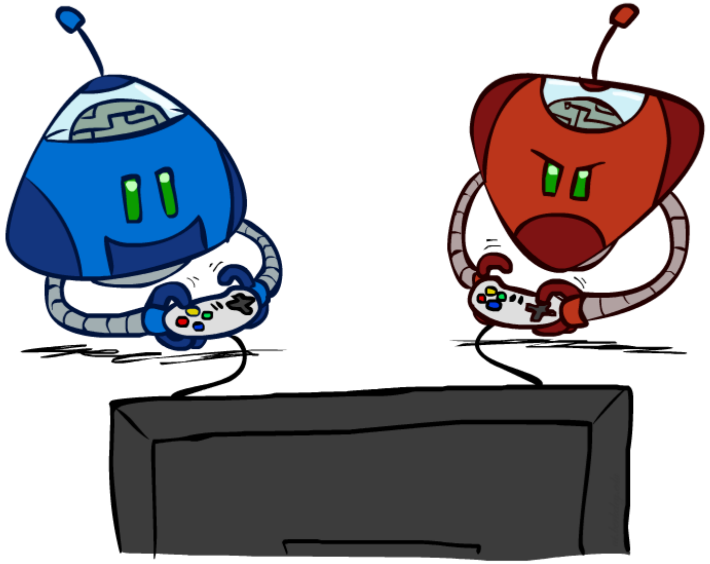
# Artificial Intelligence - INFOF311

## Games and adversarial search II

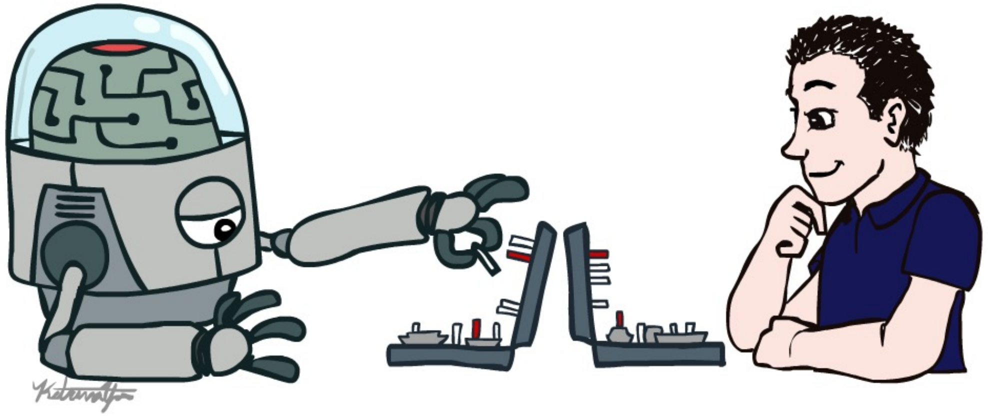


Instructor : Tom Lenaerts

# Acknowledgement

We thank Stuart Russell for his generosity in allowing us to use the slide set of the UC Berkeley Course CS188, Introduction to Artificial Intelligence. These slides were created by Dan Klein, Pieter Abbeel and Anca Dragan for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.

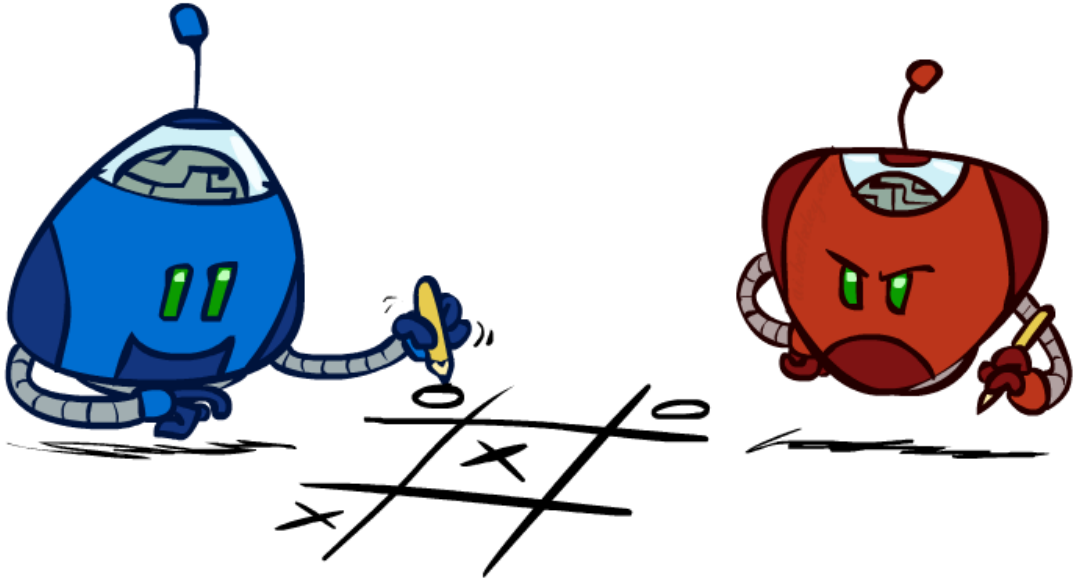
CHAI Center for Human-Compatible Artificial Intelligence



The slides for INFOF311 are slightly modified versions of the slides of the spring and summer CS188 sessions in 2021 and 2022

# Outline

- Finite lookahead and evaluation
- Games with chance elements
- Monte Carlo tree search



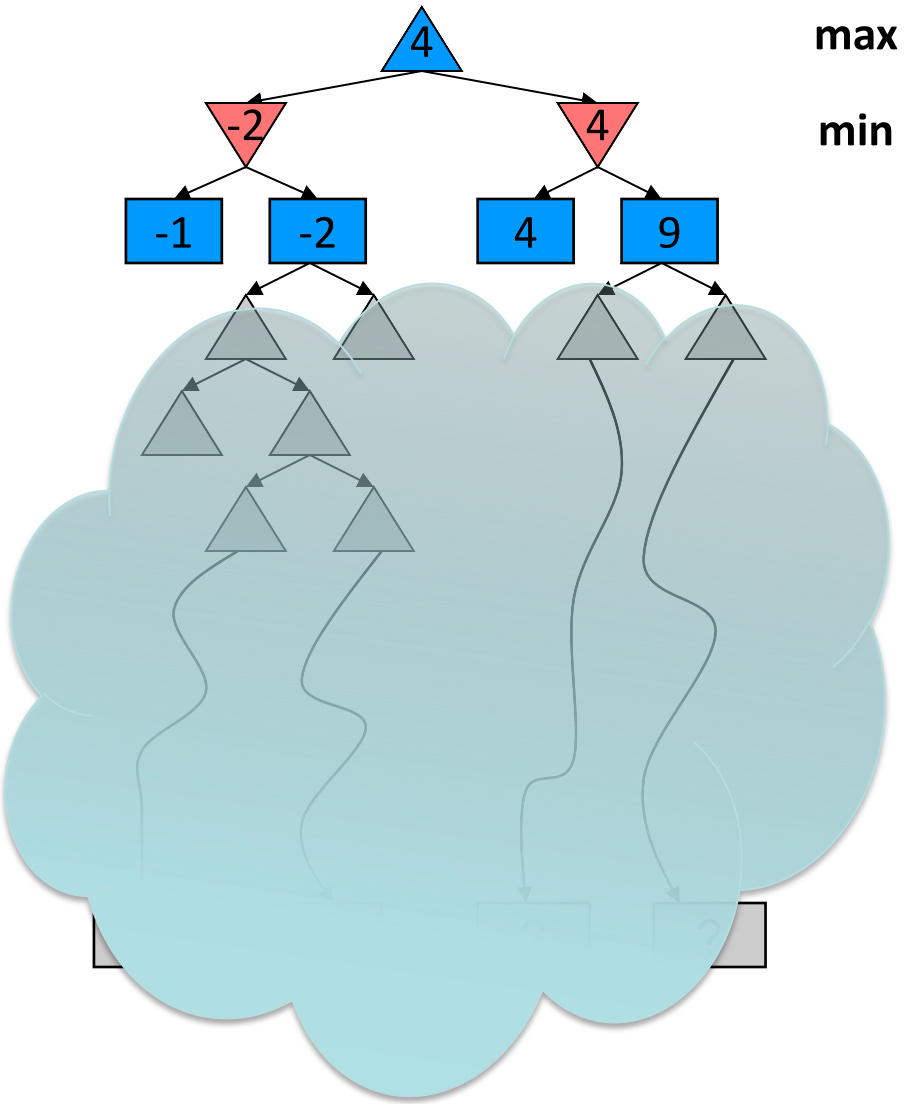
# The story so far…

- Focus on two-player, zero-sum, deterministic, observable, turn-taking games
- Minimax defines rational behavior
- Recursive DFS implementation: space complexity $O(bm)$, time complexity $O(b^m)$
- Alpha-beta pruning with good node ordering reduces time complexity to $O(b^{m/2})$
- Still nowhere close to solving chess, let alone Go or StarCraft

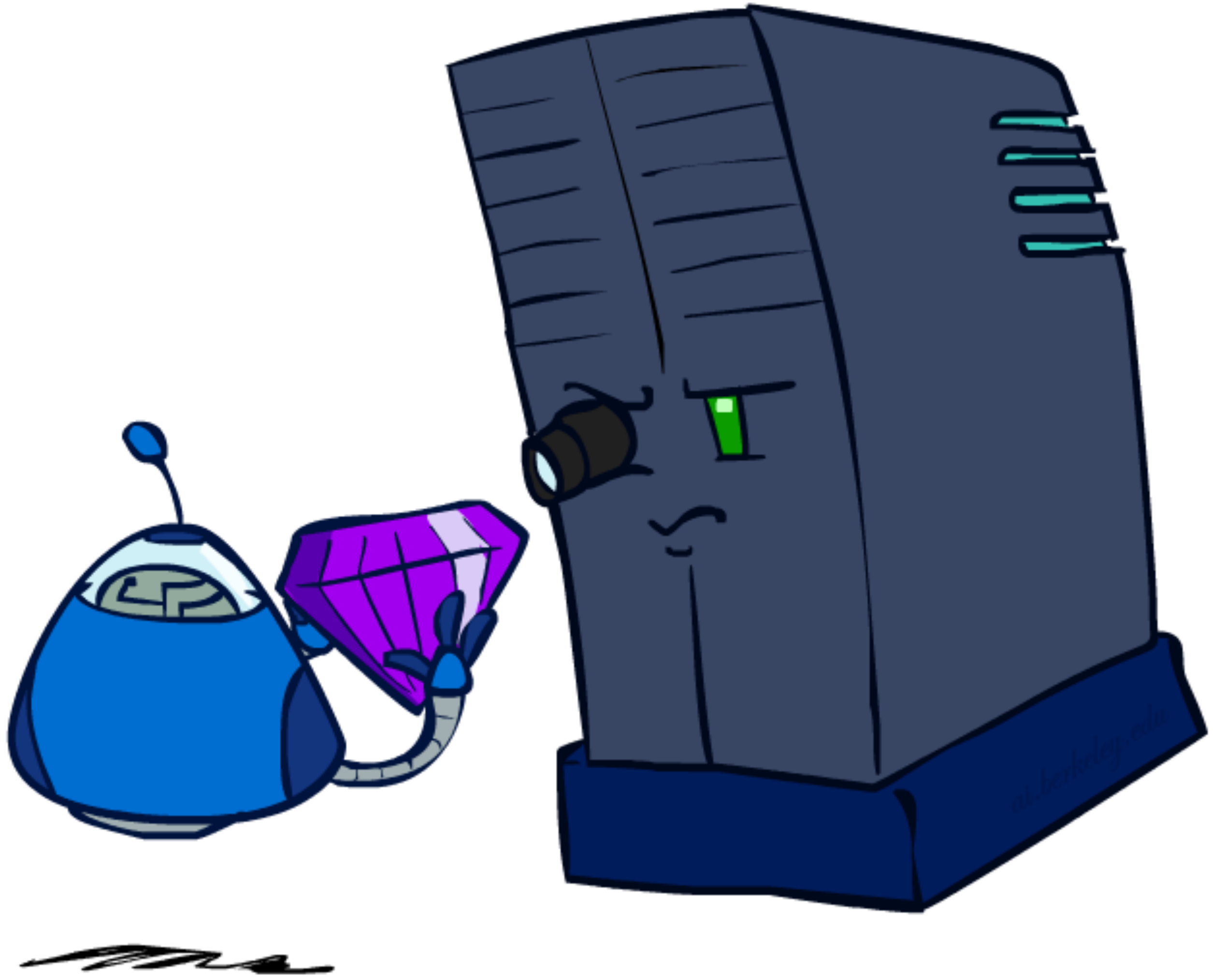
# Resource Limits

# Resource Limits

- Problem: In realistic games, cannot search to leaves!
- Solution (Shannon, 1950): Bounded lookahead/Depth-limited search
  - Search only to a preset ***depth limit*** or ***horizon***
  - Use an ***evaluation function*** for non-terminal positions
- Guarantee of optimal play is gone
- Example:
  - Suppose we can explore 1M nodes per move
  - Chess with alpha-beta, $35^{(8/2)}$ =~ 1M; depth 8 is quite good
- Use iterative deepening for an anytime algorithm

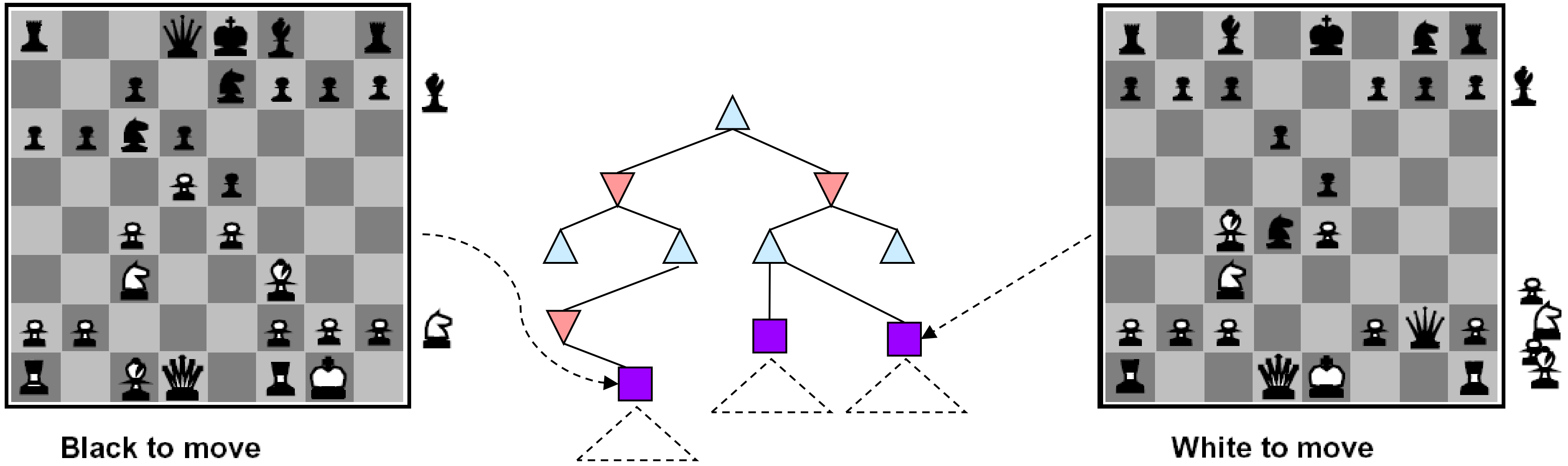


# Evaluation Functions



# Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



Black to move

White slightly better

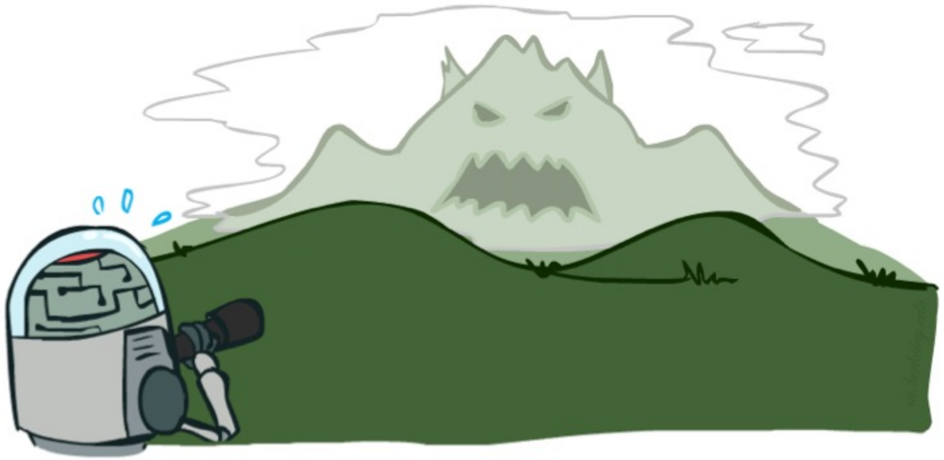
White to move

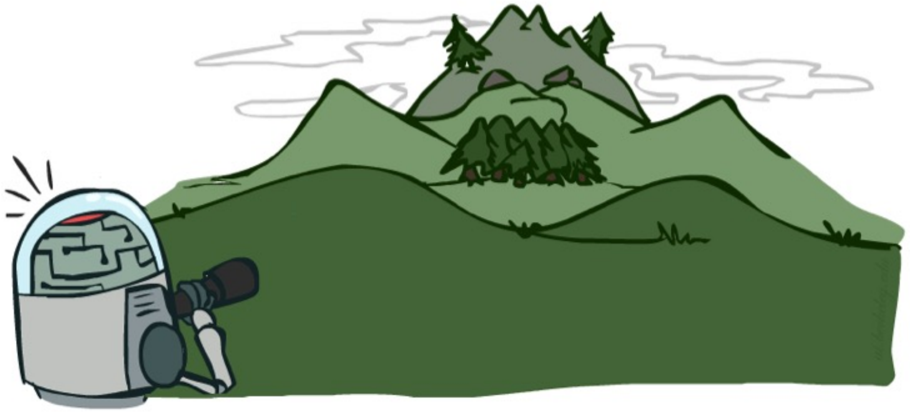
Black winning

- Typically weighted linear sum of features:
  - EVAL(s) = $w_1 f_1(s) + w_2 f_2(s) + \ldots. + w_n f_n(s)$
  - E.g., $w_1 = 9$, $f_1(s)$ = (num white queens – num black queens), etc.
- Or a more complex nonlinear function (e.g., NN) trained by self-play RL
- Terminate search only in ***quiescent*** positions, i.e., no major changes expected in feature values

# Depth Matters

- Evaluation functions are always imperfect
- Deeper search => better play (usually)
- Or, deeper search gives same quality of play with a less accurate evaluation function
- An important example of the tradeoff between complexity of features and complexity of computation



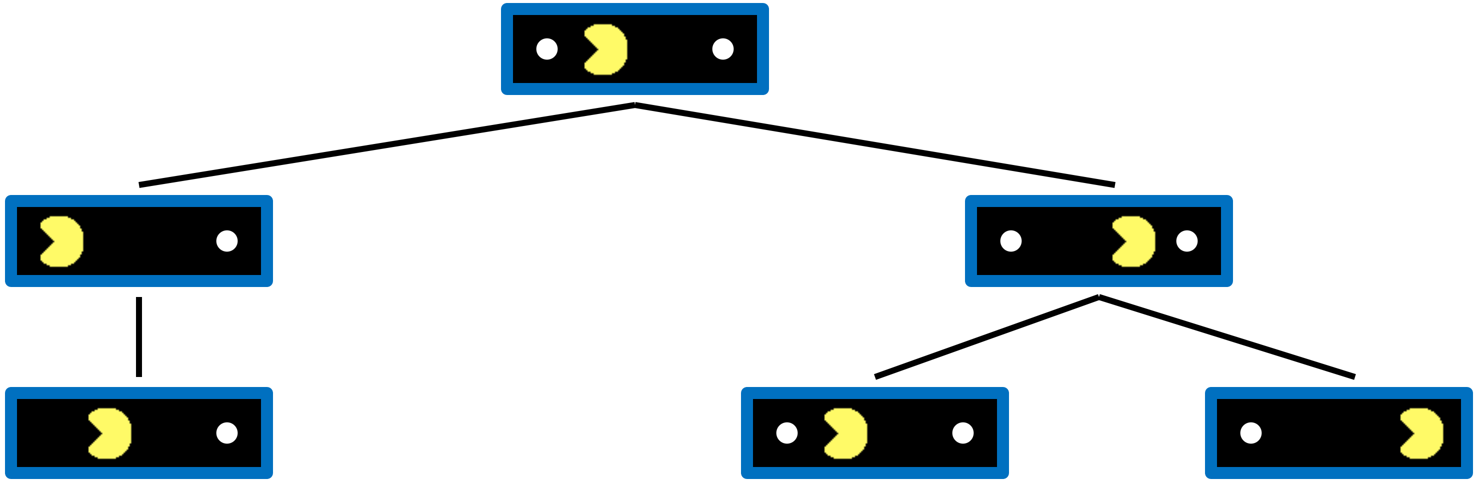


[Demo: depth limited (L6D4, L6D5)]

# Video of Demo Thrashing (d=2)

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function) (L6D6)]

# Why Pacman Starves



- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

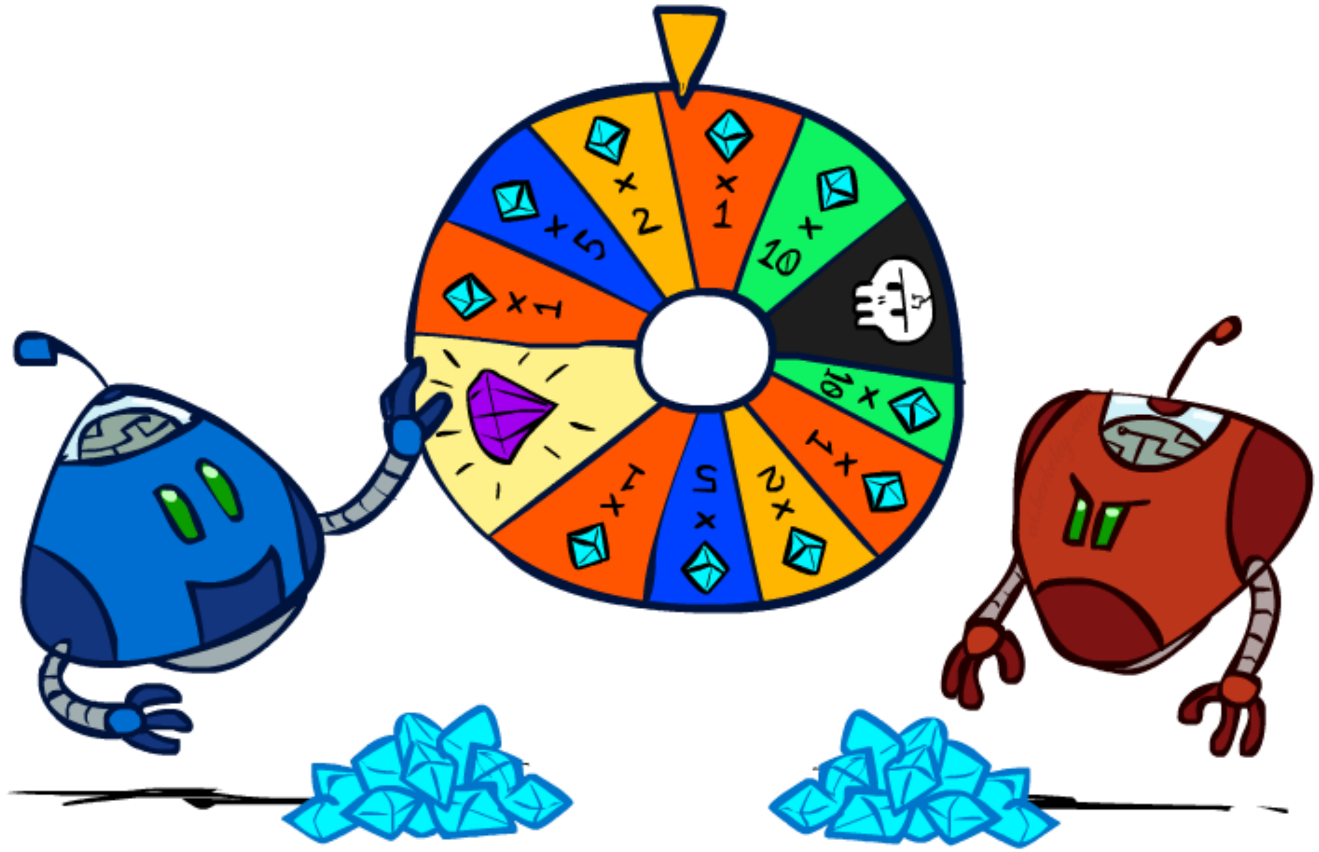
# Video of Demo Thrashing -- Fixed (d=2)

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function) (L6D7)]

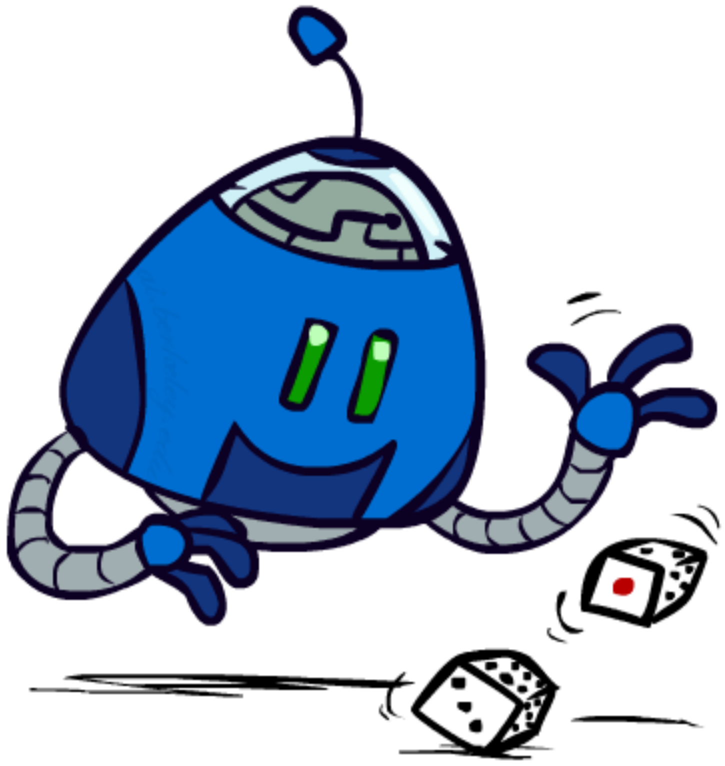
# Pacman with Depth-2 Lookahead

# Pacman with Depth-10 Lookahead

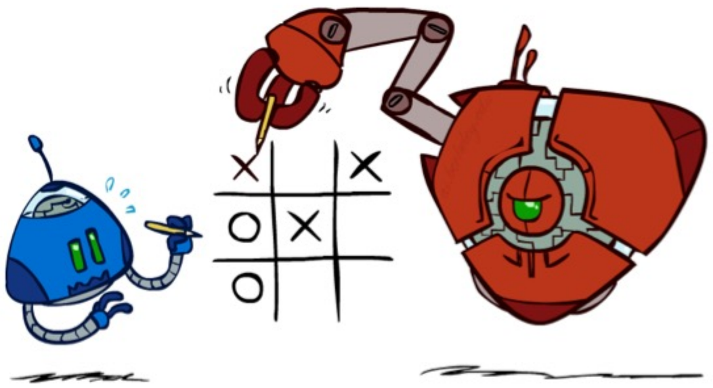
# Games with uncertain outcomes

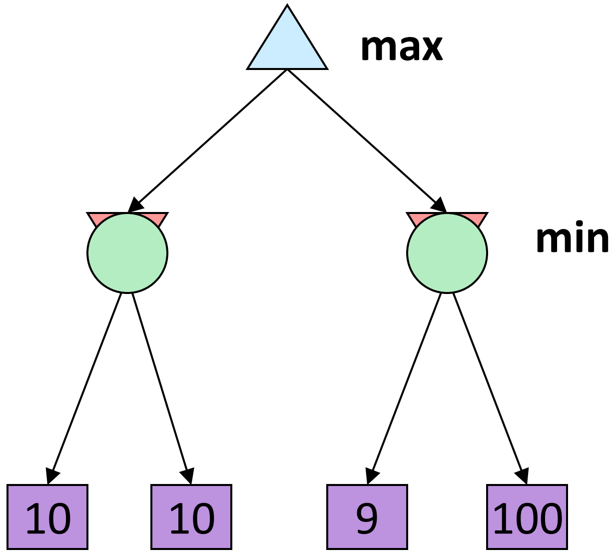


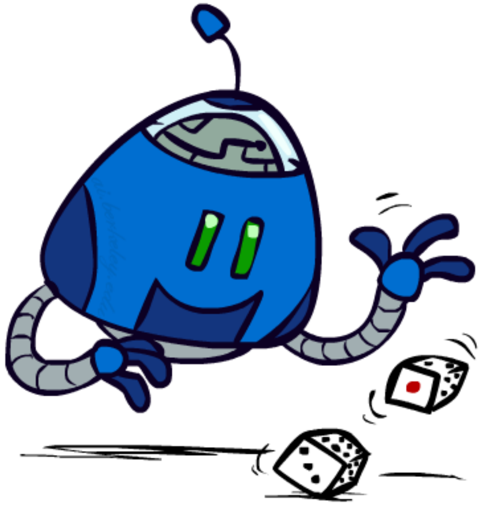
# Uncertain Outcomes



# Worst-Case vs. Average Case



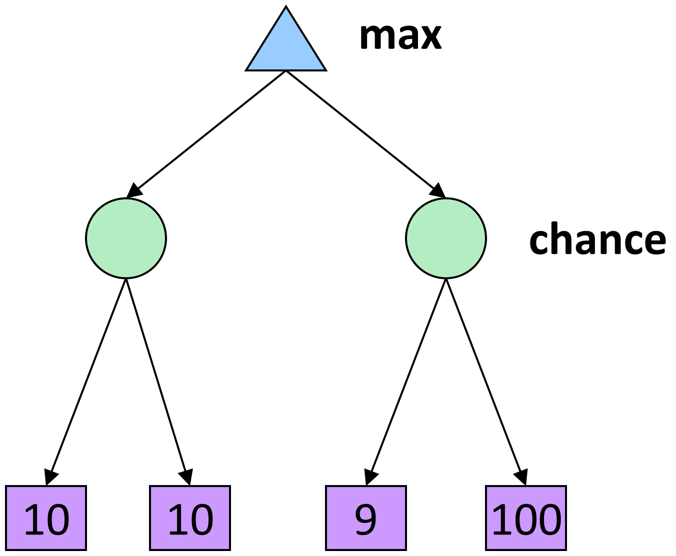


Idea: Uncertain outcomes controlled by chance, not an adversary!

# Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes
- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children
- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes




[Demo: min vs exp (L7D1,2)]

# Expectimax Pseudocode

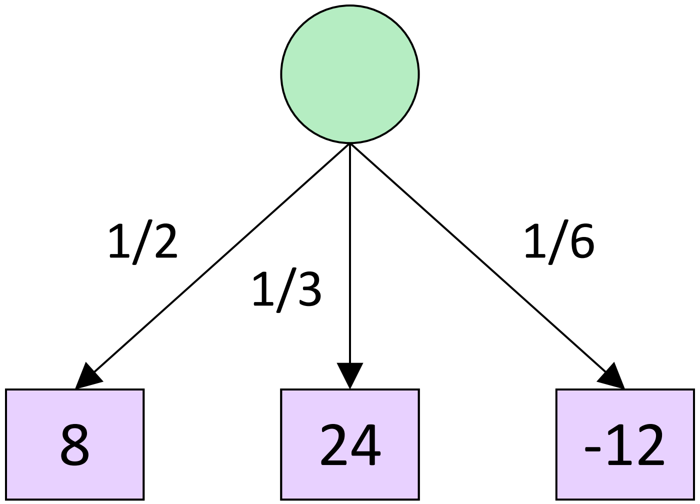
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

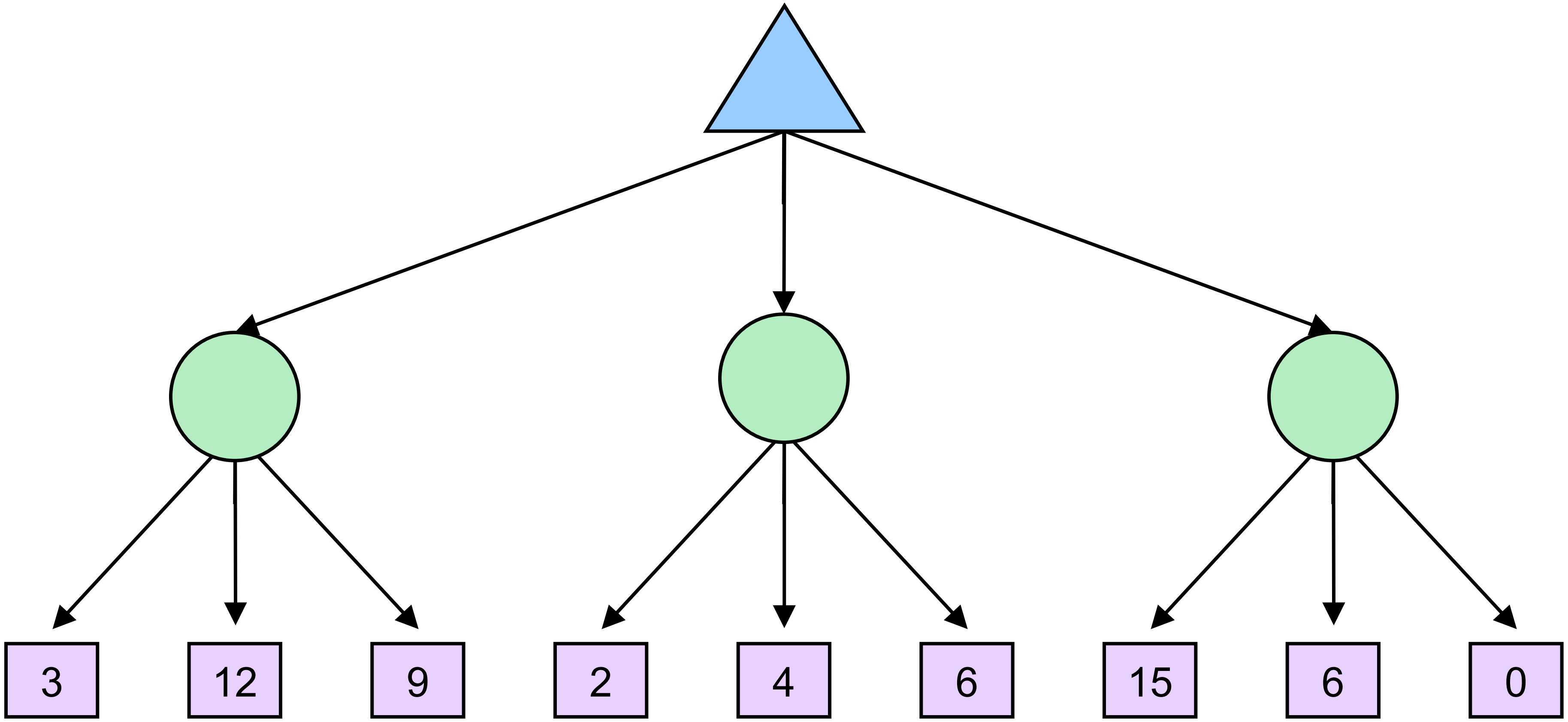
# Expectimax Pseudocode

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

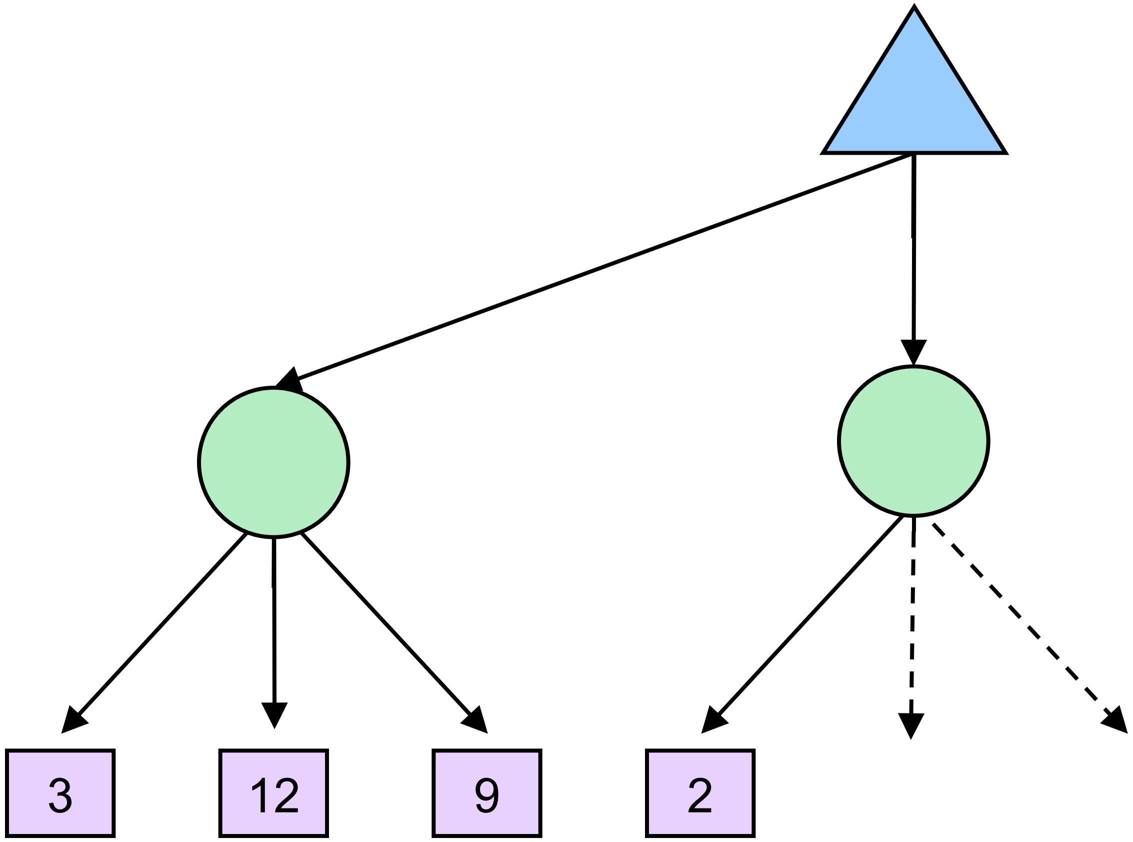


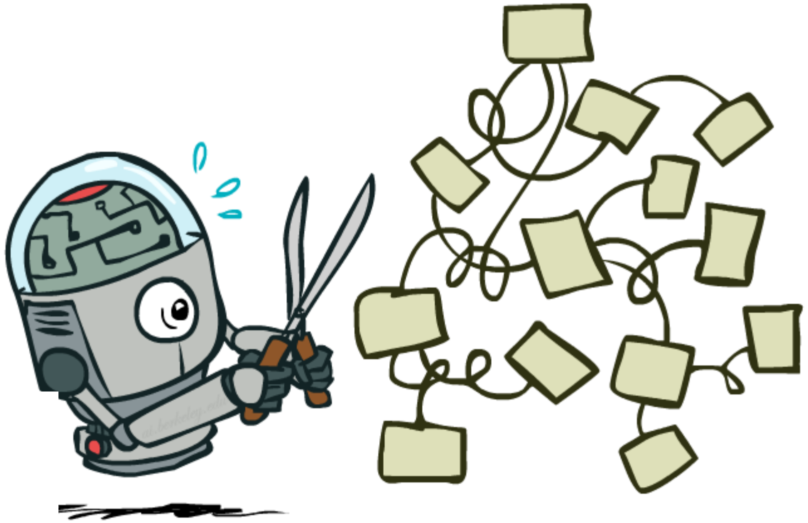

$$v = (1/2)\ (8) + (1/3)\ (24) + (1/6)\ (-12) = 10$$

# Expectimax Example

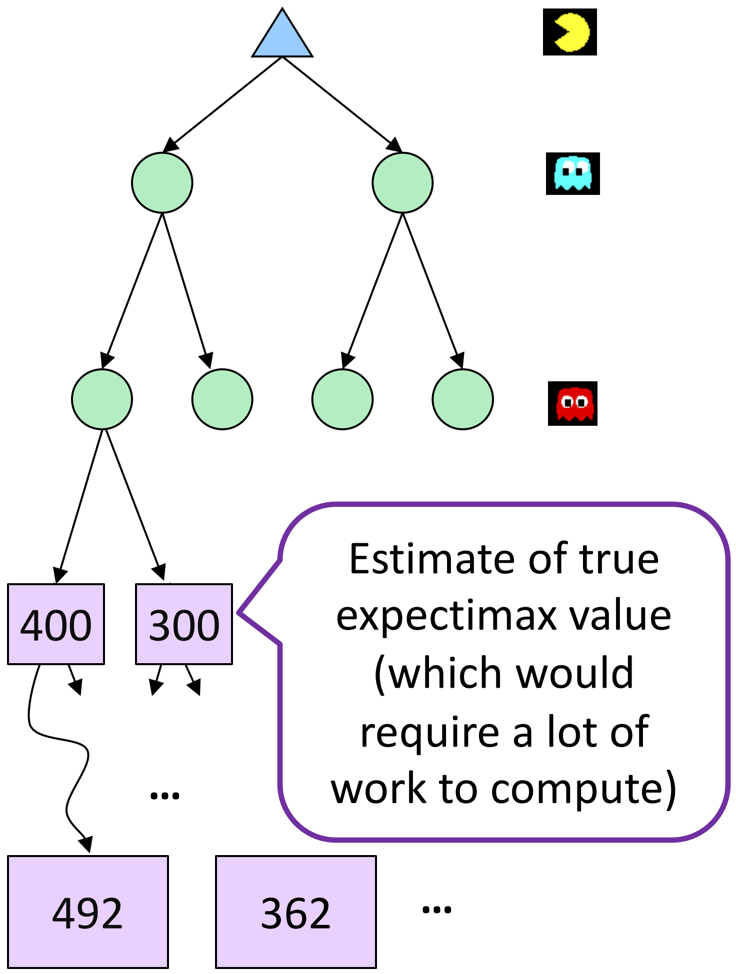


# Expectimax Pruning?

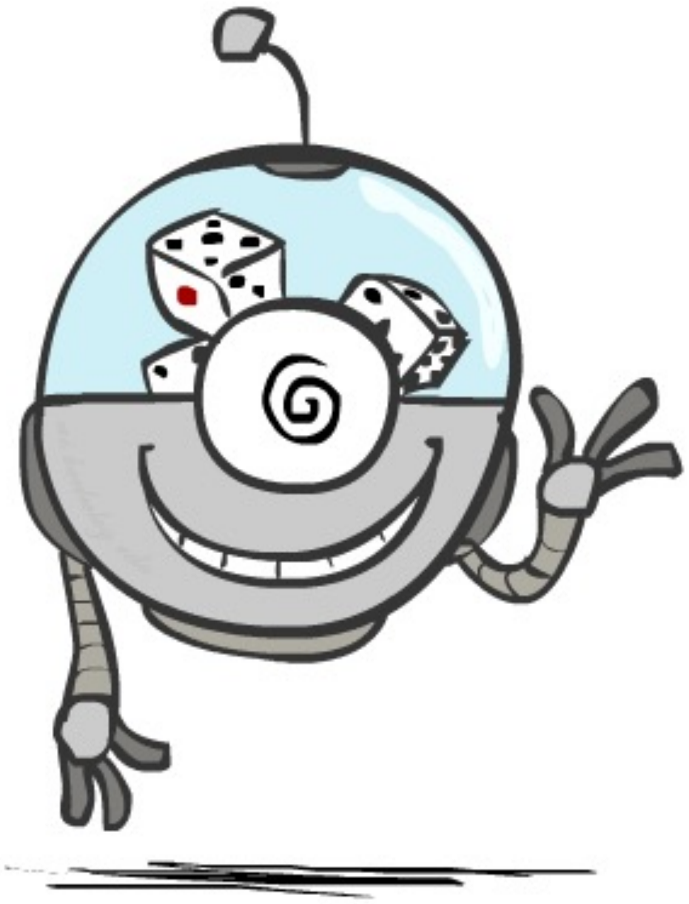


# Depth-Limited Expectimax



# Probabilities



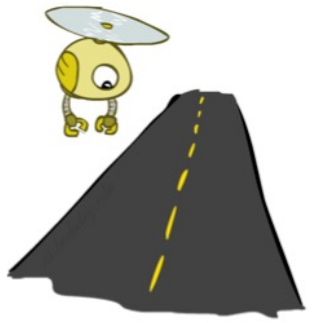
# Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

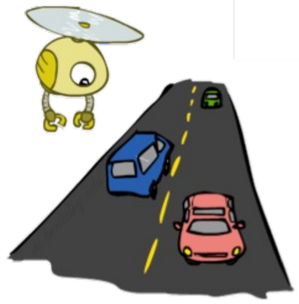
- Example: Traffic on freeway
  - Random variable: T = whether there’s traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We’ll talk about methods for reasoning and updating probabilities later



0.25

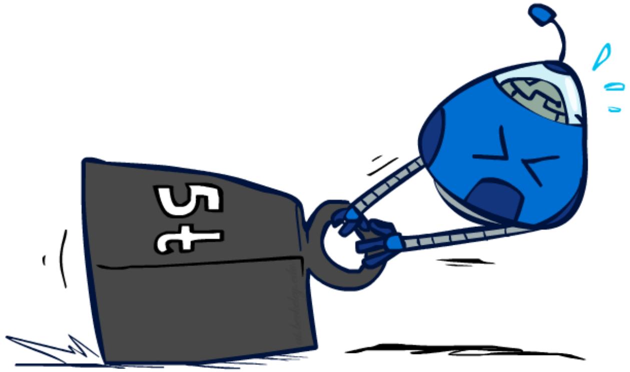


0.50

0.25

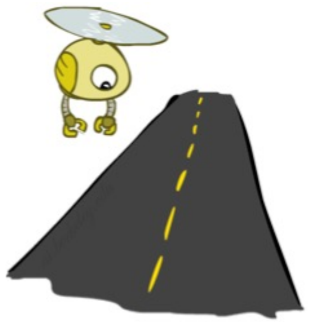
# Reminder: Expectations

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes



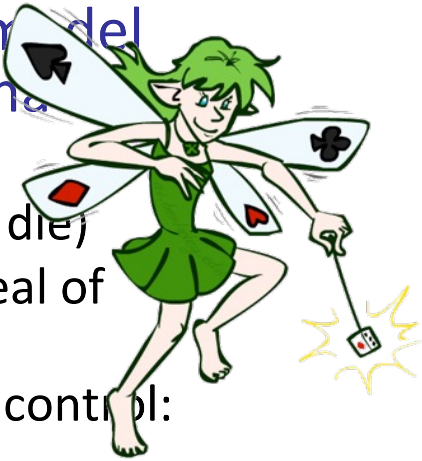

- Example: How long to get to the airport?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Time: | 20 min | | 30 min | | 60 min | | |
| | x | + | x | + | x | ⇒ | 35 min |
| Probability: | 0.25 | | 0.50 | | 0.25 | | |

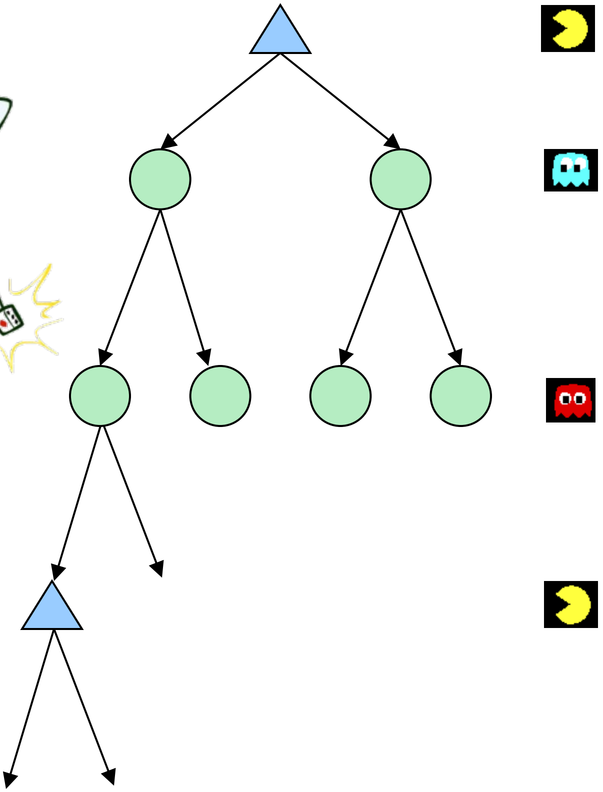


# What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!



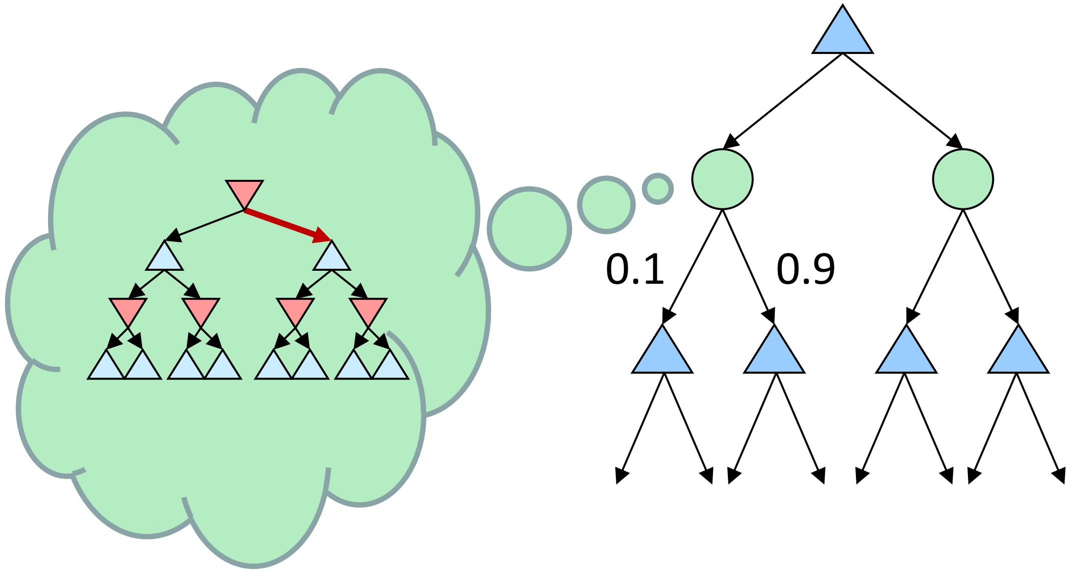
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes



*Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!*

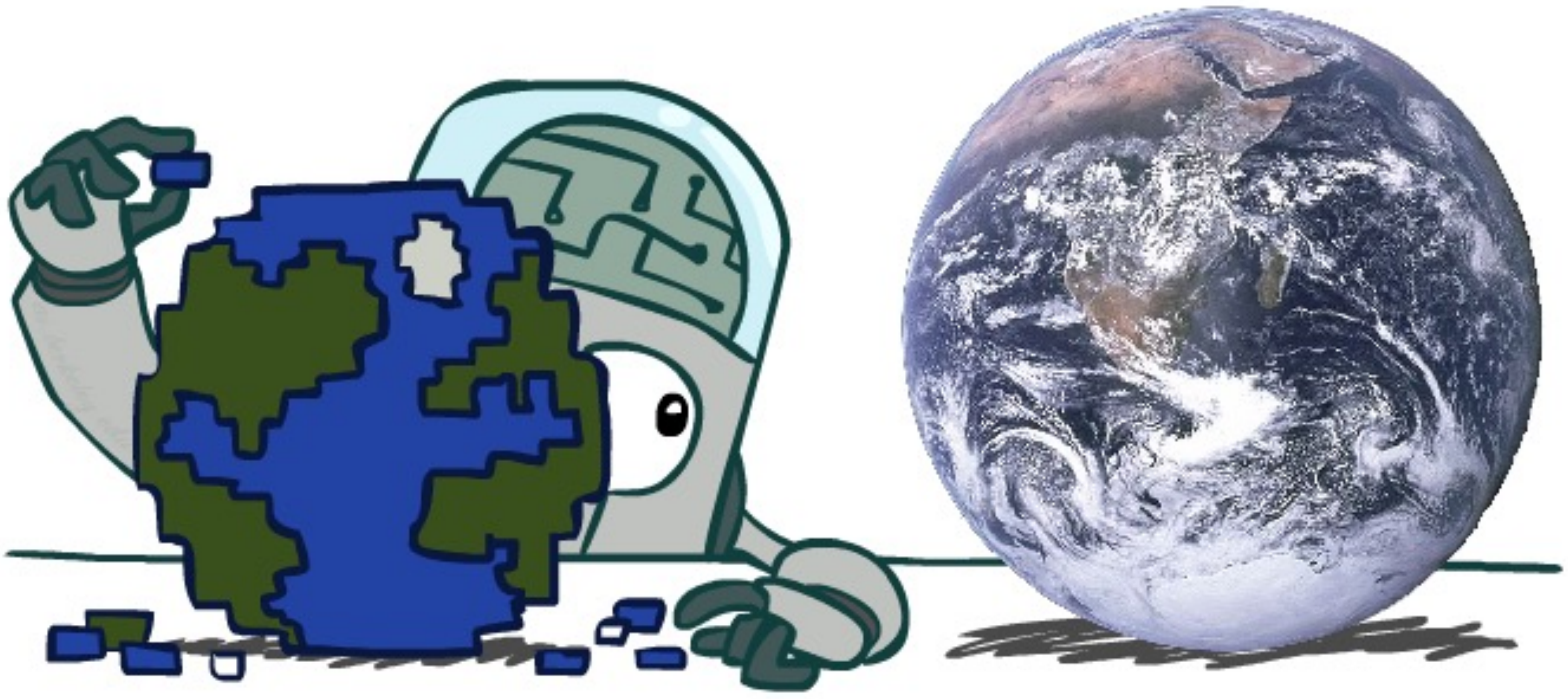
# Quiz: Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?




- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - … except for minimax, which has the nice property that it all collapses into one game tree

# Modeling Assumptions



# The Dangers of Optimism and Pessimism

## Dangerous Optimism

Assuming chance when the world is adversarial

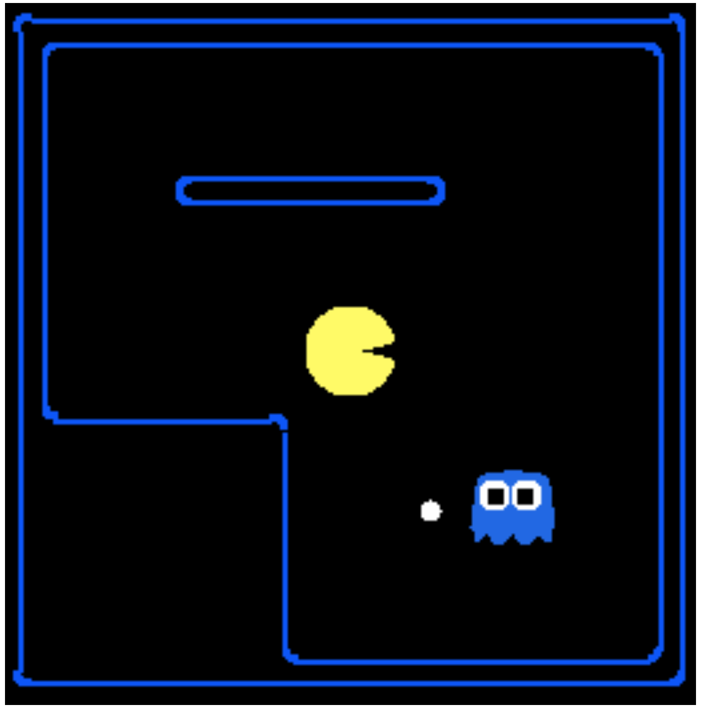

## Dangerous Pessimism

Assuming the worst case when it's not likely

# Video of Demo Minimax vs Expectimax (Min)

# Video of Demo Minimax vs Expectimax (Exp)

# Assumptions vs. Reality



| | Adversarial Ghost | Random Ghost |
|---|---|---|
| Minimax Pacman | | |
| Expectimax Pacman | | |

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

[Demos: world assumptions (L7D3,4,5,6)]

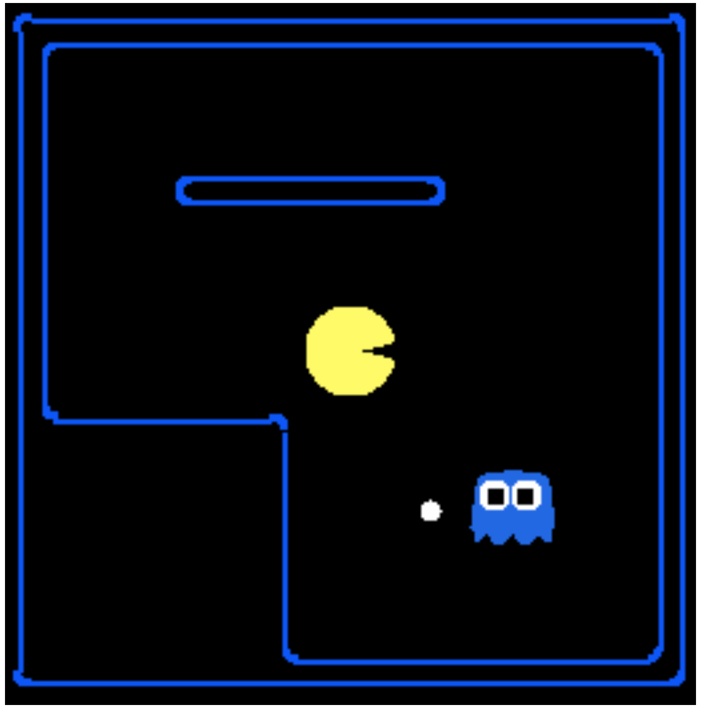
# Video of Demo World Assumptions
# Adversarial Ghost – Minimax Pacman

# Video of Demo World Assumptions
# Random Ghost – Expectimax Pacman

# Video of Demo World Assumptions
## Adversarial Ghost – Expectimax Pacman

# Video of Demo World Assumptions
# Random Ghost – Minimax Pacman

# Assumptions vs. Reality



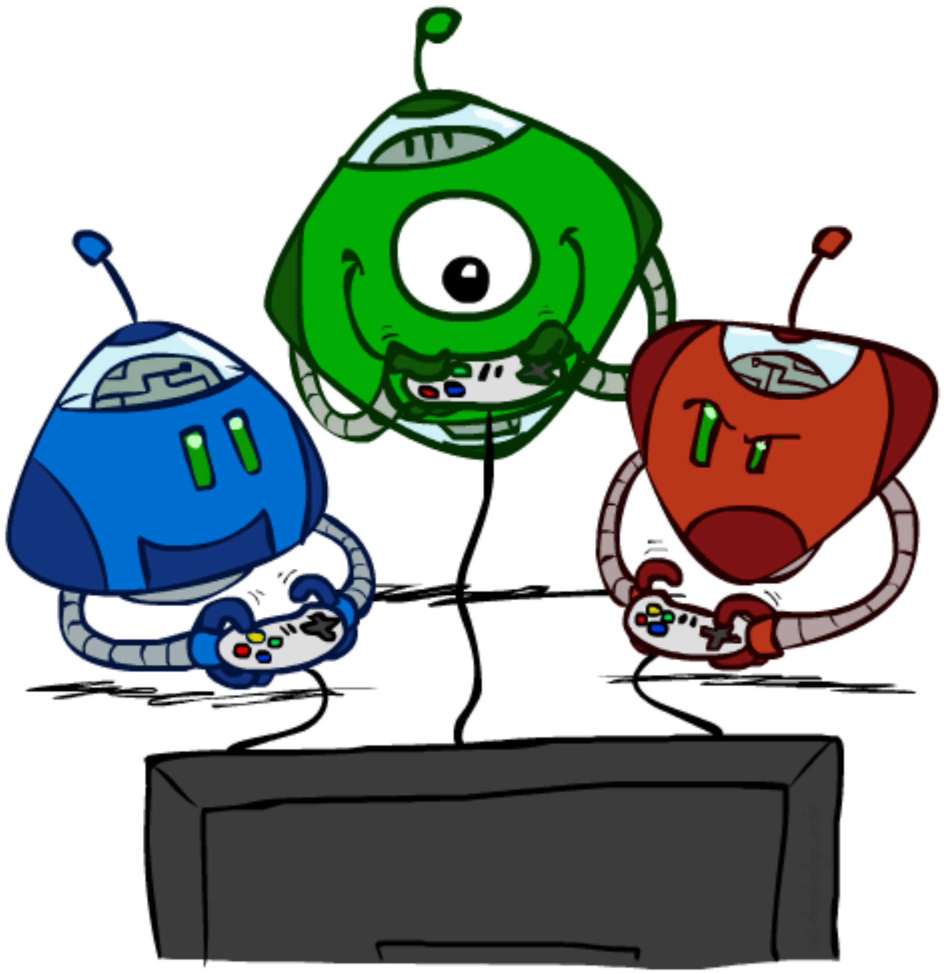
| | Adversarial Ghost | Random Ghost |
|---|---|---|
| Minimax Pacman | Won 5/5<br>Avg. Score: 483 | Won 5/5<br>Avg. Score: 493 |
| Expectimax Pacman | Won 1/5<br>Avg. Score: -303 | Won 5/5<br>Avg. Score: 503 |

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

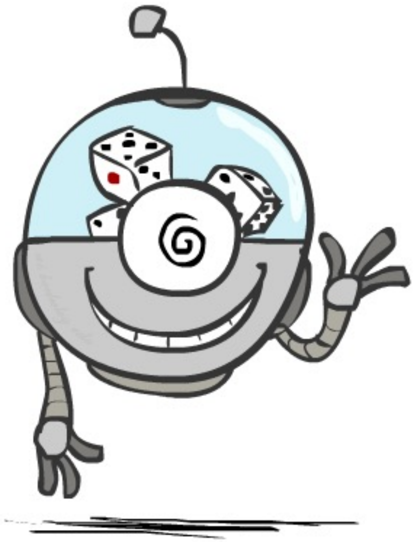
[Demos: world assumptions (L7D3,4,5,6)]

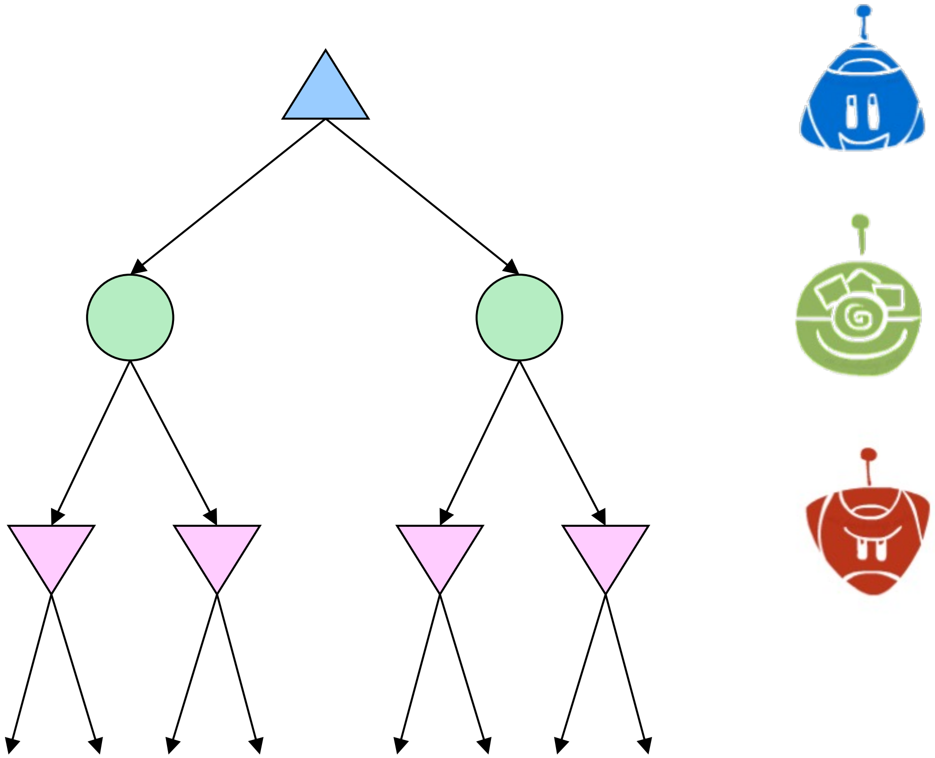
# Other Game Types



# Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children



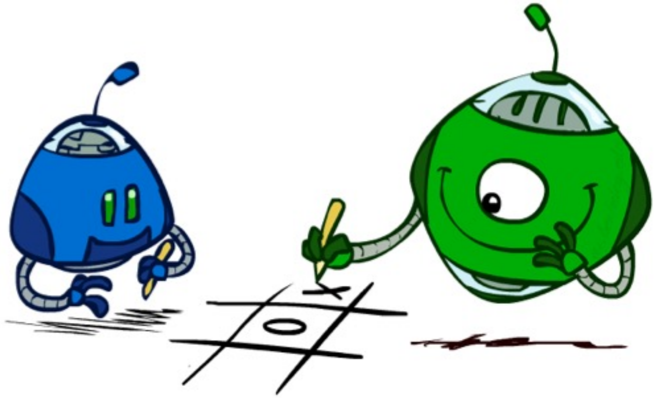


# Example: Backgammon

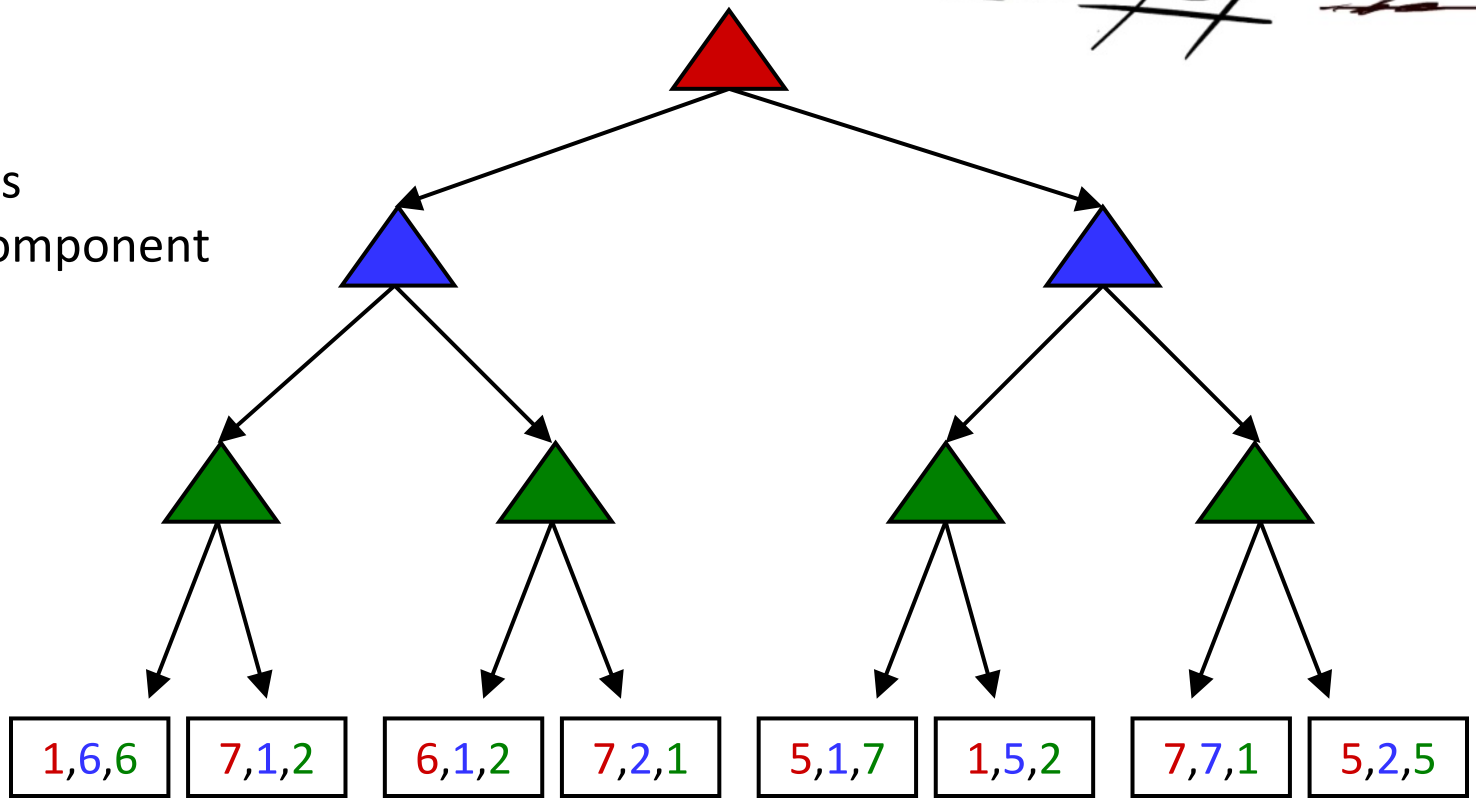
- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon $\approx$ 20 legal moves
  - Depth 2 = $20 \times (21 \times 20)^3 = 1.2 \times 10^9$
- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1st AI world champion in any game!

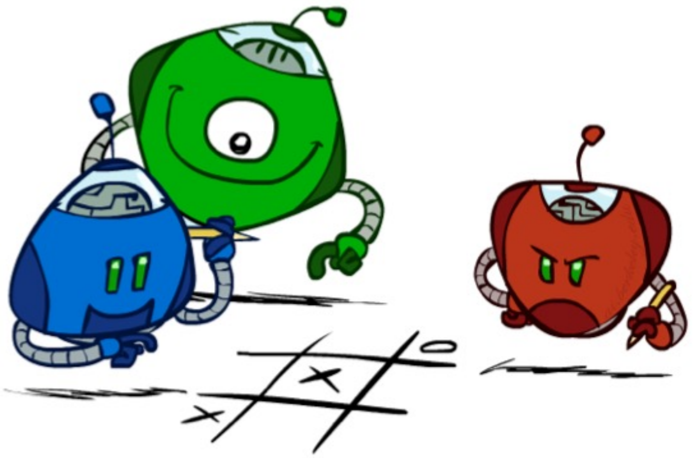
Image: Wikipedia

# Multi-Agent Utilities



- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…



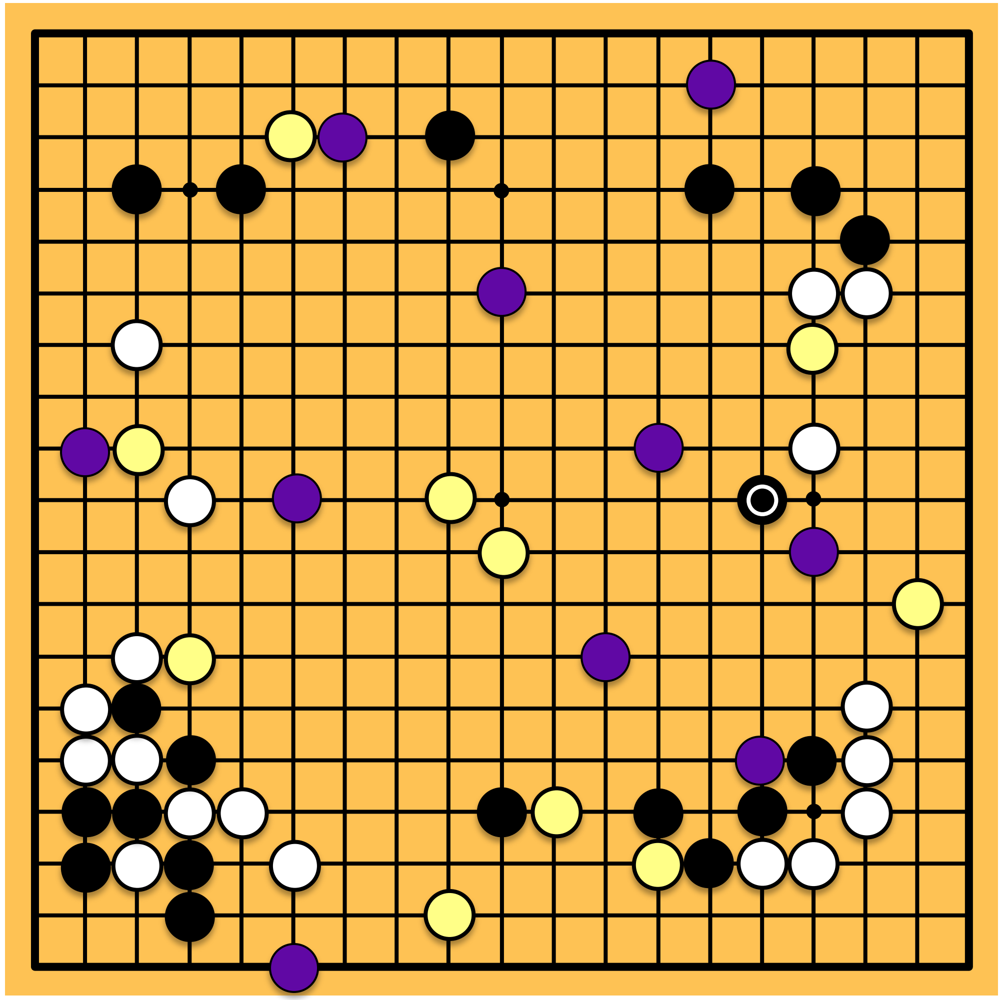
# Monte Carlo Tree Search

- Methods based on alpha-beta search assume a fixed horizon
  - Pretty hopeless for Go, with $b > 300$
- MCTS combines two important ideas:
  - ***Evaluation by rollouts*** – play multiple games to termination from a state $s$ (using a simple, fast rollout policy) and count wins and losses
  - ***Selective search*** – explore parts of the tree that will help improve the decision at the root, regardless of depth

# Rollouts

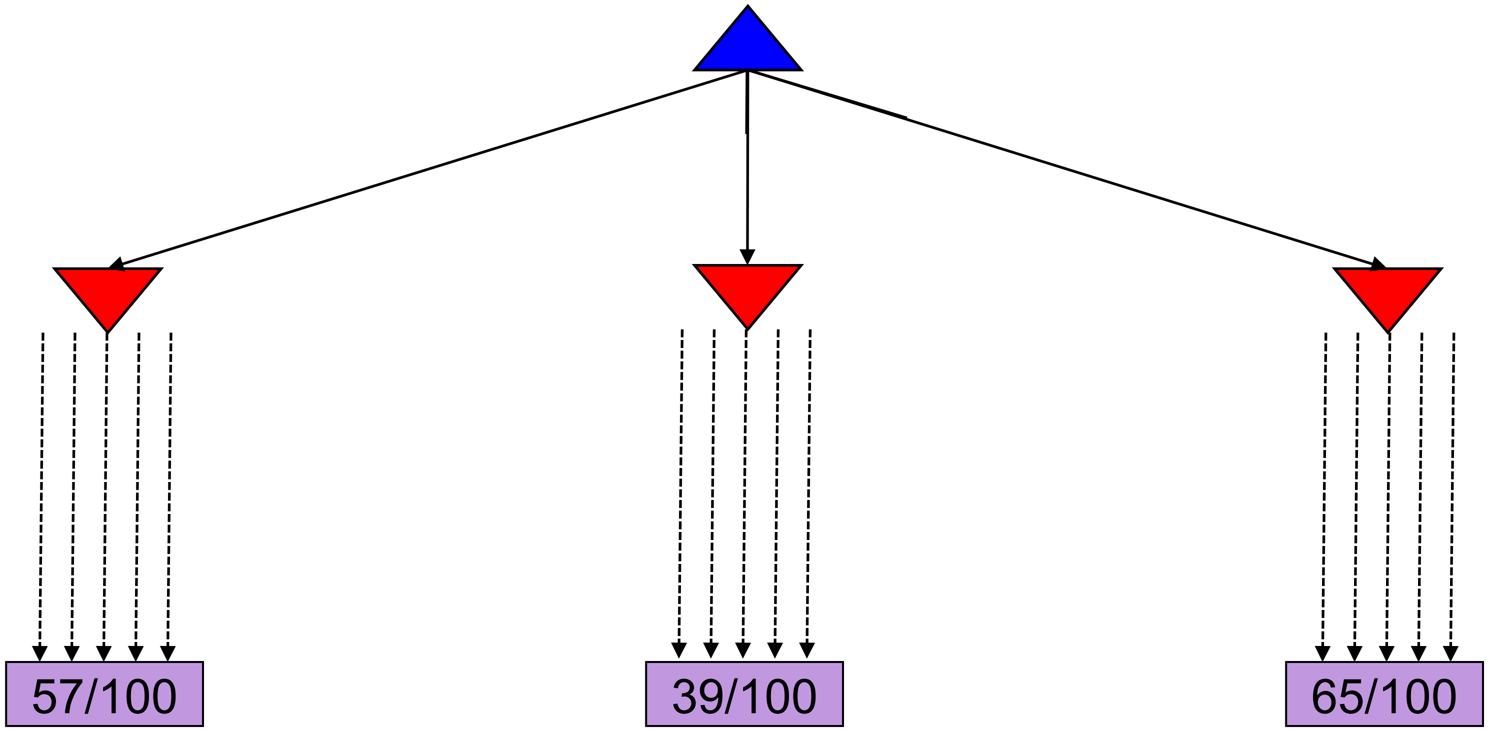
- For each rollout:
  - Repeat until terminal:
    - Play a move according to a fixed, fast rollout policy
  - Record the result
- Fraction of wins correlates with the true value of the position!
- Having a "better" rollout policy helps

“Move 37”



# MCTS Version 0

- Do *N* rollouts from each child of the root, record fraction of wins
- Pick the move that gives the best outcome by this metric



# MCTS Version 0

- Do *N* rollouts from each child of the root, record fraction of wins
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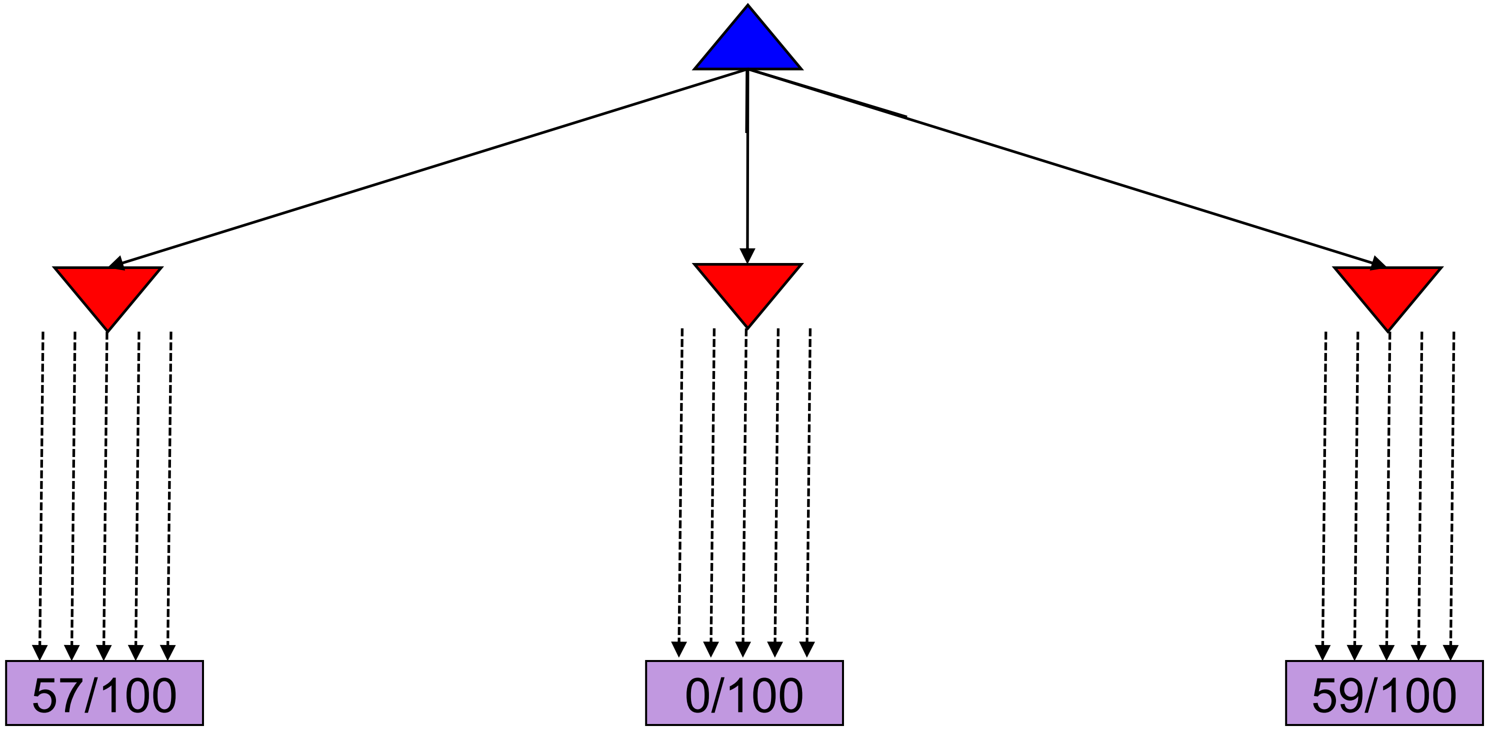
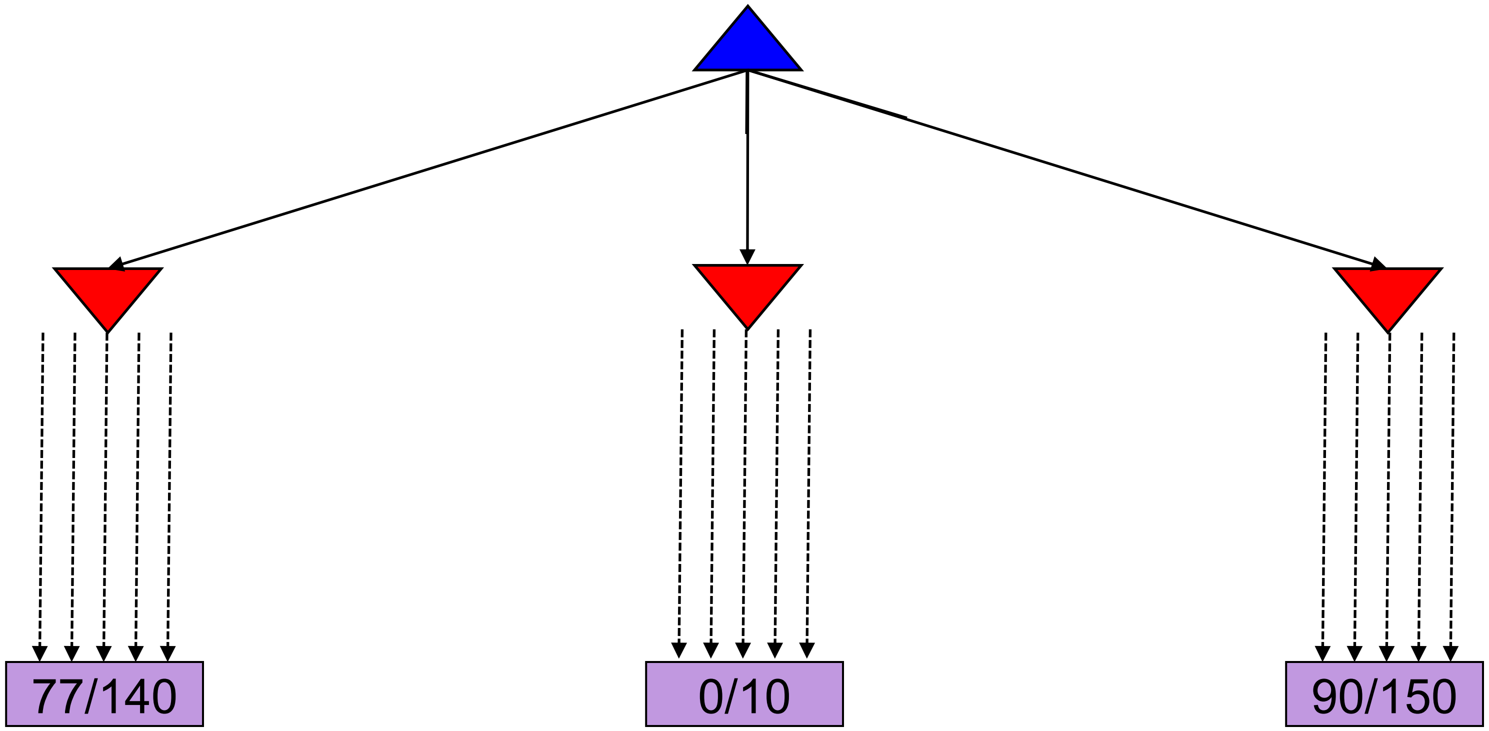


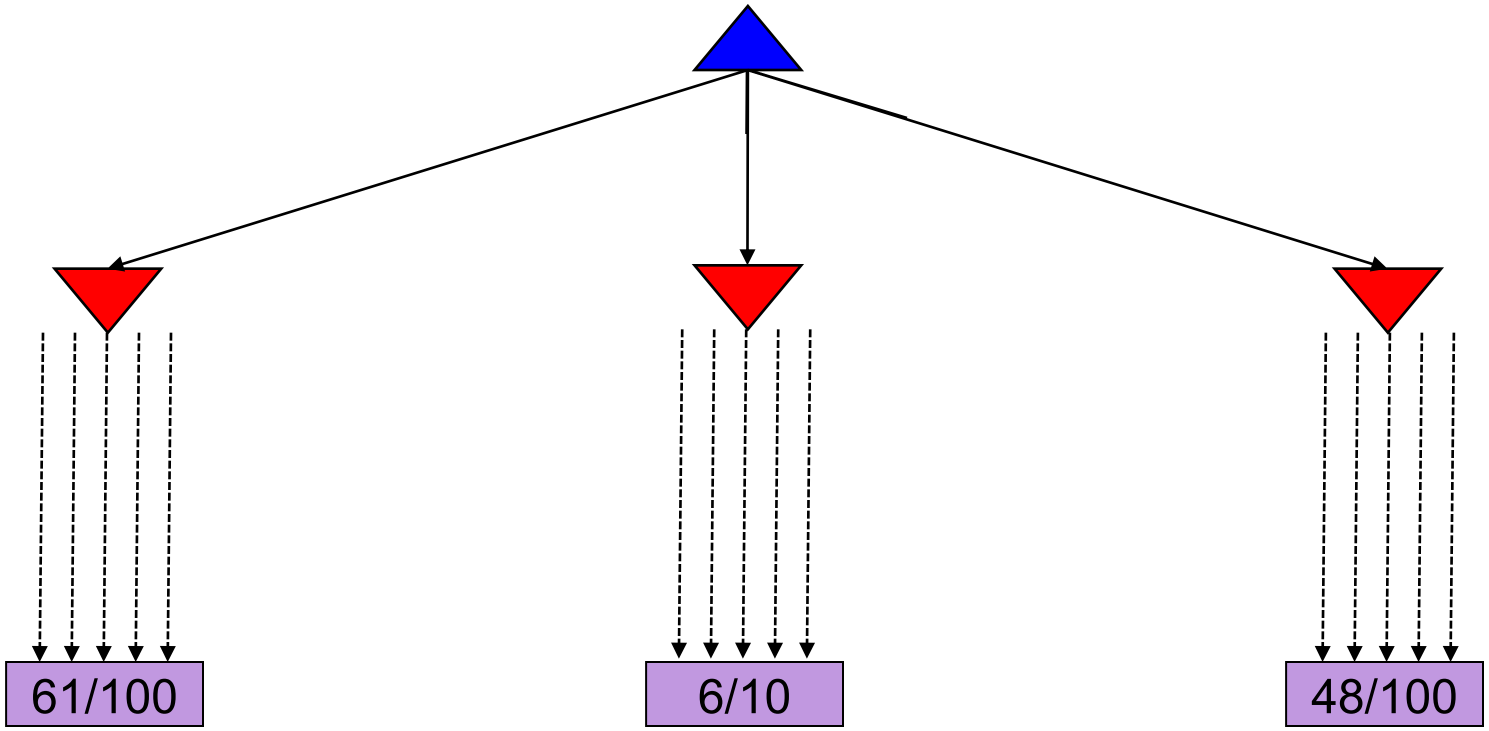
# MCTS Version 0.9

- Allocate rollouts to more promising nodes



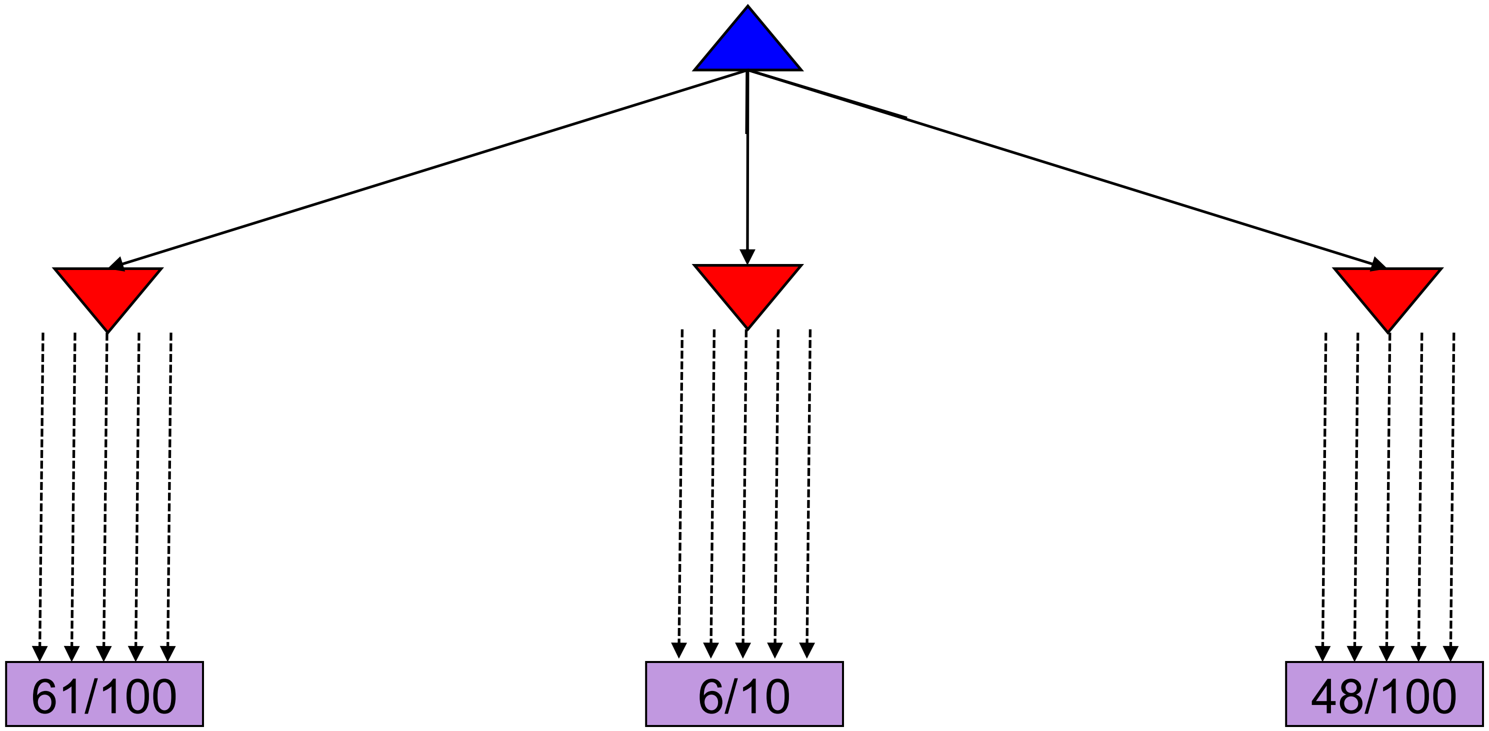
# MCTS Version 0.9

- Allocate rollouts to more promising nodes



# MCTS Version 1.0

- Allocate rollouts to more promising nodes
- Allocate rollouts to more uncertain nodes



# UCB heuristics

- UCB1 formula combines "promising" and "uncertain":

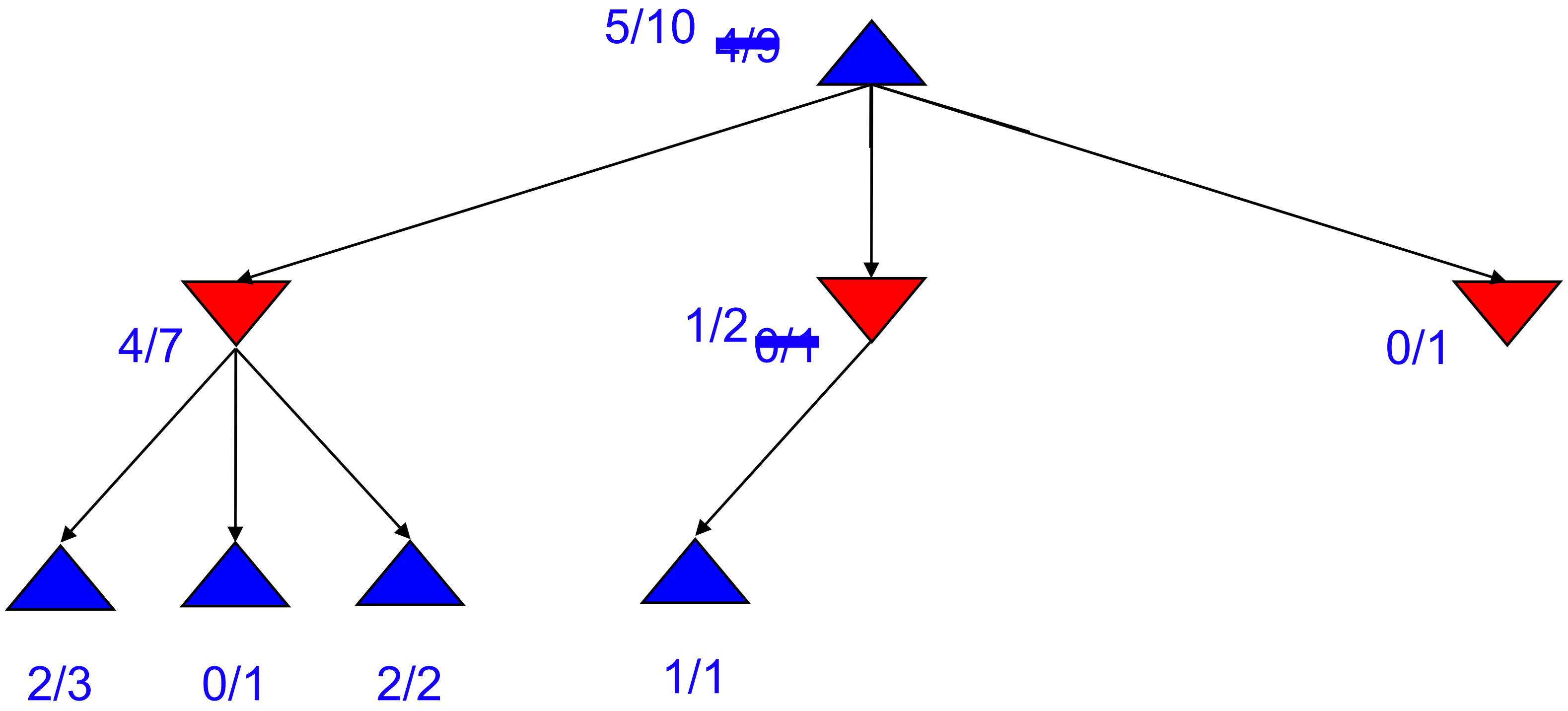
$$UCB1(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\textsc{Parent}(n))}{N(n)}}$$

- $N(n)$ = number of rollouts from node $n$
- $U(n)$ = total utility of rollouts (e.g., # wins) for Player(Parent($n$))
- A provably not terrible heuristic for ***bandit problems***
  - (which are not the same as the problem we face here!)

# MCTS Version 2.0: UCT

- Repeat until out of time:
  - Given the current search tree, recursively apply UCB to choose a path down to a leaf (not fully expanded) node *n*
  - Add a new child *c* to *n* and run a rollout from *c*
  - Update the win counts from *c* back up to the root
- Choose the action leading to the child with highest *N*

# UCT Example



# Why is there no min or max?????

- "Value" of a node, $U(n)/N(n)$, is a weighted ***sum*** of child values!
- Idea: as $N \rightarrow \infty$ , the vast majority of rollouts are concentrated in the best child(ren), so weighted average $\rightarrow$ max/min
- Theorem: as $N \rightarrow \infty$ UCT selects the minimax move
  - (but $N$ never approaches infinity!)

# Summary

- Games require decisions when optimality is impossible
  - Bounded-depth search and approximate evaluation functions
- Games force efficient use of computation
  - Alpha-beta pruning, MCTS
- Game playing has produced important research ideas
  - Reinforcement learning (checkers)
  - Iterative deepening (chess)
  - Rational metareasoning (Othello)
  - Monte Carlo tree search (chess, Go)
  - Solution methods for partial-information games in economics (poker)
- Video games present much greater challenges – lots to do!
  - $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$, partially observable, often > 2 players