CHAPTER 15

UNIPROCESSOR SCHEDULING EXERCISES

15.1 Rate Monotonic

Consider System 108.

System 108

τ_i	C_i	$D_i = T_i$
τ_1	1	5
τ_2	2	8
τ_3	1	10
τ_4	5	20

Exercise 1. Identify the properties of the system.

- Is this system periodic?
- Is the system synchronous or asynchronous?
- Are the deadlines implicit, constrained or arbitrary?

Answer

- Yes, the system is periodic because each tasks repeats at a specific interval.
- Every task starts at $t = 0$ so the system is indeed synchronous.
- Deadlines are implicit because they are all equal to the period.

Exercise 2. Verify that this system is schedulable using with RM using the processor *utilisation* technique described in Theorem 33, page 24.

Answer

As stated in Theorem 33, we need to have $U(\tau) \leqslant n(\sqrt[n]{2}-1)$ for the system to be schedulable. The taskset utilisation is the sum of the individual task utilisations.

$$U(\tau) = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

$$U(\tau) = \frac{1}{5} + \frac{2}{8} + \frac{1}{10} + \frac{5}{20} = 0.2 + 0.25 + 0.1 + 0.25 = 0.8$$

$$U \leqslant n(\sqrt[n]{2} - 1) \Leftrightarrow U \leqslant 4(\sqrt[4]{2} - 1) \approx 0.7568$$

We have $75.68\% < 80\% \leqslant 100\%$, so we can not claim anything about the feasibility of this system.

Exercise 3. Verify that this system can be scheduled using the *worst case response time* technique (Theorem 29, page 22).

Answer

For the worst response time technique, we first need to order the tasks by decreasing order of priority. With RM, we have $p_i = \frac{1}{T_i}$, i.e. $\tau_1 \succ \tau_2 \succ \tau_3 \succ \tau_4$.

Then, we iteratively find the worst case response time for the first job of each task. For this system, we have:

- τ_1:
  1. $r_1^1 = 1$
  2. $r_1^2 = 1 + 0 = 1$
  3. Fixed point found, $r_1 \leqslant D_1 \rightarrow$ OK
- τ_2:
  1. $r_2^1 = 2$
  2. $r_2^2 = 2 + \left\lceil \frac{2}{5} \right\rceil \cdot 1 = 2 + 1 = 3$
  3. $r_2^3 = 2 + \left\lceil \frac{3}{5} \right\rceil \cdot 1 = 2 + 1 = 3$
  4. Fixed point found, $r_2 \leqslant D_2 \rightarrow$ OK
- τ_3:
  1. $r_3^1 = 1$
  2. $r_3^2 = 1 + \left\lceil \frac{1}{5} \right\rceil \cdot 1 + \left\lceil \frac{1}{8} \right\rceil \cdot 2 = 1 + 1 + 2 = 4$
  3. $r_3^3 = 1 + \left\lceil \frac{4}{5} \right\rceil \cdot 1 + \left\lceil \frac{4}{8} \right\rceil \cdot 2 = 1 + 1 + 2 = 4$
  4. Fixed point found, $r_3 \leqslant D_3 \rightarrow$ OK

- τ_4:

  1. $r_4^1 = 5$
  2. $r_4^2 = 5 + \left\lceil \frac{5}{5} \right\rceil \cdot 1 + \left\lceil \frac{5}{8} \right\rceil \cdot 2 + \left\lceil \frac{5}{10} \right\rceil \cdot 1 = 5 + 1 + 2 + 1 = 9$
  3. $r_4^3 = 5 + \left\lceil \frac{9}{5} \right\rceil \cdot 1 + \left\lceil \frac{9}{8} \right\rceil \cdot 2 + \left\lceil \frac{9}{10} \right\rceil \cdot 1 = 5 + 2 + 4 + 1 = 12$
  4. $r_4^4 = 5 + \left\lceil \frac{12}{5} \right\rceil \cdot 1 + \left\lceil \frac{12}{8} \right\rceil \cdot 2 + \left\lceil \frac{12}{10} \right\rceil \cdot 1 = 5 + 3 + 4 + 2 = 14$
  5. $r_4^5 = 5 + \left\lceil \frac{14}{5} \right\rceil \cdot 1 + \left\lceil \frac{14}{8} \right\rceil \cdot 2 + \left\lceil \frac{14}{10} \right\rceil \cdot 1 = 5 + 3 + 4 + 2 = 14$
  6. Fixed point found, $r_4 \leqslant D_4 \rightarrow$ OK

We have found that the worst case response times of each task are lower than their respective deadlines, so the system is schedulable.

Exercise 4. Plot the scheduling of System 108 using Rate Monotonic.

Answer

See Figure 15.1. All deadlines were met between 0 and 20, thus we know that this system is feasible (thanks to the feasibility interval $[0, \max\{D_i | i = 1, \dots, n\})$ from Theorem 32, page 23).

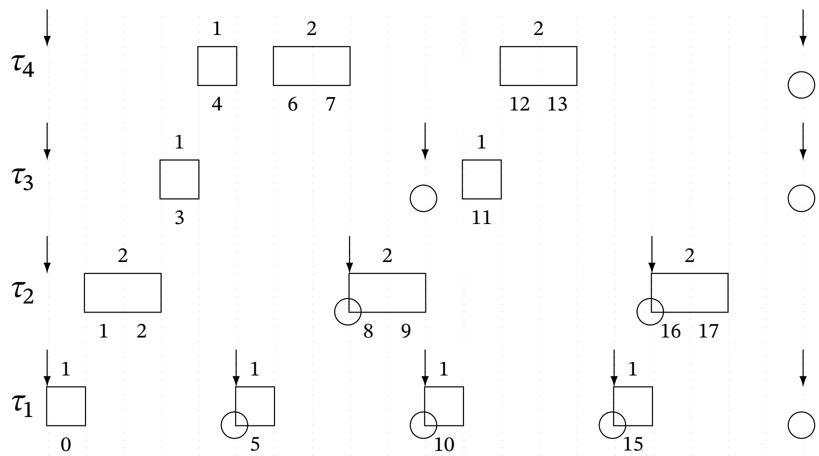

Figure 15.1: Scheduling of System 108 in the interval $[0, 20)$.

Exercise 5. Find a periodic system with $U(\tau) \leqslant 1$ that can not be scheduled with RM.

Answer

See System 109 where the task set has $U(\tau) = 1$. The schedule is shown in Figure 15.2.

System 109

τ_i	C_i	$D_i = T_i$
τ_1	2	4
τ_2	3	6

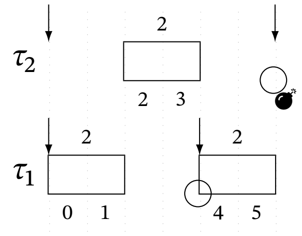

Figure 15.2: Scheduling of System 109 fails with RM

Consider System 110.

System 110

τ_i	C_i	$D_i = T_i$
τ_1	4	10
τ_2	3	15
τ_3	4	20

Exercise 6. Verify that System 110 could be scheduled using RM algorithm with the following techniques:

1. Use Theorem 33, page 24 that is based on the processor *utilisation* $U(\tau)$.
2. Use Theorem 29, page 22 that is based on the *worst case response time*.

Answer

1. First we compute the utilisation $U(\tau) = \frac{4}{10} + \frac{3}{15} + \frac{4}{20} = 0.8$.
   From Theorem 33, we know that $U(\tau) \leqslant n(\sqrt[n]{2} - 1)$ is a sufficient property for the system to be schedulable. For $n = 3$, we have $3(\sqrt[3]{2} - 1) \approx 0.78$.
   Since $U(\tau) > 0.78$, the sufficient condition does not hold and we can not deduce anything about the schedulability of the system with Theorem 33.

2. (a) τ_1:
      i. $w_0 = 4$
      ii. $w_1 = 4 + 0 = 4$
      iii. Fixed point found, $w_1 \leqslant D_1 \rightarrow$ OK

   (b) τ_2:
      i. $w_0 = 3$
      ii. $w_1 = 3 + \left\lceil \frac{3}{10} \right\rceil \cdot 4 = 3 + 4 = 7$
      iii. $w_2 = 3 + \left\lceil \frac{7}{10} \right\rceil \cdot 4 = 3 + 4 = 7$
      iv. Fixed point found, $w_2 \leqslant D_2 \rightarrow$ OK

   (c) τ_3:

i. $w_0 = 4$

ii. $w_1 = 4 + \left\lceil \frac{4}{10} \right\rceil \cdot 4 + \left\lceil \frac{4}{15} \right\rceil \cdot 3 = 4 + 4 + 3 = 11$

iii. $w_2 = 4 + \left\lceil \frac{11}{10} \right\rceil \cdot 4 + \left\lceil \frac{11}{15} \right\rceil \cdot 3 = 4 + 8 + 3 = 15$

iv. $w_3 = 4 + \left\lceil \frac{15}{10} \right\rceil \cdot 4 + \left\lceil \frac{15}{15} \right\rceil \cdot 3 = 4 + 8 + 3 = 15$

v. Fixed point found, $w_3 \leqslant D_3 \rightarrow$ OK

15.2 Deadline Monotonic

Consider the System 111. These are all *periodic*, *synchronous* tasks with *constrained deadlines.*

System 111

τ_i	C_i	D_i	T_i
τ_1	*1*	*5*	*5*
τ_2	*2*	*4*	*8*
τ_3	*1*	*3*	*10*
τ_4	*5*	*15*	*20*

Exercise 7. Verify that this system could be scheduled using the Deadline Monotonic algorithm for priority assignment.

Answer

Since we have a system with constrained deadlines, we can not use the method based on the utilisation factor (Theorem 33, page 24). We can however check the schedulability of the system by finding a suitable feasibility interval and then simulate the system.

Because this is a synchronous system, we know that the worst-case response time will be achieved at $t = 0$ (Theorem 29, page 22) since all tasks release a job simultaneously.

After sorting tasks in decreasing order of priority, we can compute this worst-case response time by fixed-point iteration as shown in Equation 3.2, page 23.

1. Sort tasks by priority (assigned by DM, $p_i = \frac{1}{D_i}$) in decreasing order $\tau_3 \succ \tau_2 \succ \tau_1 \succ \tau_4$.

2. Find the worst response time of the first job of each task:

   (a) τ_3:

      i. $w_0 = 1$

      ii. $w_1 = 1 + 0 = 1$

      iii. Fixed point found, $w_1 \leqslant D_3 \rightarrow$ OK

   (b) τ_2:

      i. $w_0 = 2$

ii. $w_1 = 2 + \left\lceil \frac{2}{10} \right\rceil \cdot 1 = 2 + 1 = 3$

iii. $w_2 = 2 + \left\lceil \frac{3}{10} \right\rceil \cdot 1 = 2 + 1 = 3$

iv. Fixed point found, $w_2 \leqslant D_2 \rightarrow$ OK

(c) τ_1:

i. $w_0 = 1$

ii. $w_1 = 1 + \left\lceil \frac{1}{10} \right\rceil \cdot 1 + \left\lceil \frac{1}{8} \right\rceil \cdot 2 = 1 + 1 + 2 = 4$

iii. $w_2 = 1 + \left\lceil \frac{4}{10} \right\rceil \cdot 1 + \left\lceil \frac{4}{8} \right\rceil \cdot 2 = 1 + 1 + 2 = 4$

iv. Fixed point found, $w_2 \leqslant D_1 \rightarrow$ OK

(d) τ_4:

i. $w_0 = 5$

ii. $w_1 = 5 + \left\lceil \frac{50}{100} \right\rceil \cdot 1 + \left\lceil \frac{5}{8} \right\rceil \cdot 2 + \left\lceil \frac{5}{5} \right\rceil \cdot 1 = 5 + 1 + 2 + 1 = 9$

iii. $w_2 = 5 + \left\lceil \frac{9}{10} \right\rceil \cdot 1 + \left\lceil \frac{9}{8} \right\rceil \cdot 2 + \left\lceil \frac{9}{5} \right\rceil \cdot 1 = 5 + 1 + 4 + 1 = 11$

iv. $w_3 = 5 + \left\lceil \frac{11}{10} \right\rceil \cdot 1 + \left\lceil \frac{11}{8} \right\rceil \cdot 2 + \left\lceil \frac{11}{5} \right\rceil \cdot 1 = 5 + 2 + 4 + 3 = 14$

v. $w_4 = 5 + \left\lceil \frac{14}{10} \right\rceil \cdot 1 + \left\lceil \frac{14}{8} \right\rceil \cdot 2 + \left\lceil \frac{14}{5} \right\rceil \cdot 1 = 5 + 2 + 4 + 3 = 14$

vi. Fixed point found, $w_4 \leqslant D_4 \rightarrow$ OK

Since a worst case response time lower or equal to the deadline has been found for each task, we know that the system is schedulable with DM.

Exercise 8. Find a suitable feasibility interval and plot the scheduling of System 111 using DM.

Answer

Since this is a synchronous constrained deadline system, we can take $[0, D^{max})$ as the feasibility interval (Theorem 32, page 23), which in this case is $[0, 150)$.

The resulting plot is show in Figure 15.3.

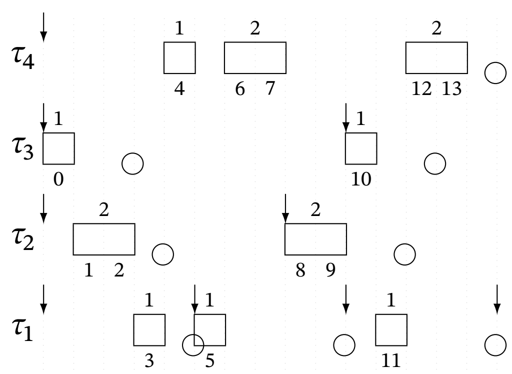

Figure 15.3: Scheduling of System 111

Consider System 112 made of periodic, synchronous tasks with constrained deadlines.

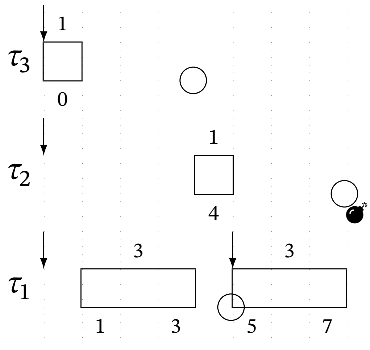

Figure 15.4: Scheduling of System 112 misses a deadline at $t = 8$.

System 112

τ_i	C_i	D_i	T_i
τ_1	3	5	5
τ_2	2	8	9
τ_3	1	4	11

Exercise 9. Verify whether this system can be scheduled using DM as priority assignment.

Answer

1. We first order the tasks by priority ($p_i = \frac{1}{D_i}$) in decreasing order: $\tau_3 \succ \tau_1 \succ \tau_2$.

2. After that we have to find the worst case response time of the first job of each task. We can do so by finding the fixed point:

   τ_3
   i. $w_0 = 1$
   ii. $w_1 = 1 + 0 = 1$
   iii. Fixed point found, $w_1 \leqslant D_1 \to$ OK

   τ_1
   i. $w_0 = 3$
   ii. $w_1 = 3 + \left\lceil \frac{3}{11} \right\rceil \cdot 1 = 3 + 1 = 4$
   iii. $w_2 = 3 + \left\lceil \frac{4}{11} \right\rceil \cdot 1 = 3 + 1 = 4$
   iv. Fixed point found, $w_2 \leqslant D_1 \to$ OK

   τ_2
   i. $w_0 = 2$
   ii. $w_1 = 2 + \left\lceil \frac{2}{11} \right\rceil \cdot 1 + \left\lceil \frac{2}{5} \right\rceil \cdot 3 = 2 + 1 + 3 = 6$
   iii. $w_2 = 2 + \left\lceil \frac{6}{11} \right\rceil \cdot 1 + \left\lceil \frac{6}{5} \right\rceil \cdot 3 = 2 + 1 + 6 = 9$
   iv. $w_2 > D_2$: a deadline is missed for τ_2.

This shows that this system is not DM-schedulable, as illustrated in Figure 15.4

Exercise 10. Are there other algorithms that can successfully schedule τ by assigning fixed priorities to each task?

Answer

No because DM fails and DM is FTP-optimal (see Definition 35, page 25).

15.3 Systems with arbitrary deadlines

System 113

τ_i	C_i	D_i	T_i
τ_1	1	5	5
τ_2	2	4	8
τ_3	1	15	10
τ_4	5	22	20

Exercise 11. Find a feasibility interval for System 113.

Answer

Since this is a system with arbitrary deadlines, we can no longer use test methods based on the utilisation nor on the worst response time because these methods only apply to constrained deadlines system.

Instead, we have to simulate the execution of System 113 within an appropriate feasibility interval.

Theorem 40, page 27 states that $[0, \lambda_n]$ is a feasibility interval for synchronous systems with arbitrary deadlines. Theorem 3.5.1, page 27 shows how to compute λ_n through fixed-point iteration.

1. $\lambda_0 = 1 + 2 + 1 + 5 = 9$
2. $\lambda_1 = \left\lceil \frac{9}{5} \right\rceil \cdot 1 + \left\lceil \frac{9}{8} \right\rceil \cdot 2 + \left\lceil \frac{9}{10} \right\rceil \cdot 1 + \left\lceil \frac{9}{20} \right\rceil \cdot 5 = 2 + 4 + 1 + 5 = 12$
3. $\lambda_2 = \left\lceil \frac{12}{5} \right\rceil \cdot 1 + \left\lceil \frac{12}{8} \right\rceil \cdot 2 + \left\lceil \frac{12}{10} \right\rceil \cdot 1 + \left\lceil \frac{12}{20} \right\rceil \cdot 5 = 3 + 4 + 2 + 5 = 14$
4. $\lambda_3 = \left\lceil \frac{14}{5} \right\rceil \cdot 1 + \left\lceil \frac{14}{8} \right\rceil \cdot 2 + \left\lceil \frac{14}{10} \right\rceil \cdot 1 + \left\lceil \frac{14}{20} \right\rceil \cdot 5 = 3 + 4 + 2 + 5 = 14$
5. The fixed-point is found, the feasibility interval is: $[0, \lambda_3) = [0, 14)$

For your information, here is the resulting scheduling.

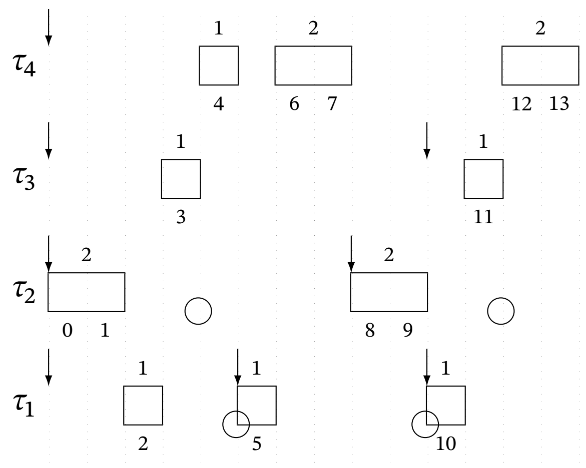

Figure 15.5: Scheduling of System 113 with DM

15.4 Audsley

Consider System 114. This system is asynchronous and has constrained deadlines.

System 114

τ_i	O_i	C_i	D_i	T_i
τ_1	*10*	*1*	*2*	*3*
τ_2	*5*	*2*	*5*	*5*
τ_3	*0*	*3*	*10*	*15*

Exercise 12. Which feasibility interval can we use for such system? Compute it.

Answer

As stated by Theorem 44, page 29, the interval $[O_{\max}, O_{\max} + 2P)$ can be used for such systems.

1. $P = \operatorname{lcm}\{T_i \mid i = 1, \dots, n\} = \operatorname{lcm}(3, 5, 15) = 15$
2. $O_{\max} = \max\{O_i \mid i = 1, \dots, n\} = \max(10, 5, 0) = 10$
3. The feasibility interval $[O_{\max}, O_{\max} + 2 \cdot P)$ is $[10, 10 + 2 \cdot 15) = [10, 40)$

Theorem 45, page 29 suggests the better feasibility interval $[0, S_n + P)$. However, we need a fixed task priority assignment to use it, which is not yet the case since it is the purpose of Audsley's algorithm.

Exercise 13. Using Audsley's algorithm (Algorithm 1, page 33), find a feasible fixed task priority assignment.

Answer

To find a feasible priority assignment, we have to find a Lowest Priority Viable task (Definition 49, page 32). To do so, we must consider a task τ_i as having the lowest priority and run the system until the end of its feasibility interval $[O_{\max}, O_{\max} + 2 \cdot P)$. If τ_i has not missed any deadline in this interval (other tasks can miss deadlines regardless), then it is Lowest Priority Viable.

- Try with τ_1 as lowest priority viable task. We consider the following order : $\tau_3 \succ \tau_2 \succ \tau_1$
  τ_1 is not lowest priority viable because it misses its first deadline at $t = 12$.
- Try with τ_2 as lowest priority viable task. We consider the following order : $\tau_3 \succ \tau_1 \succ \tau_2$
  τ_2 is not lowest priority viable because it misses a deadline at $t = 20$.
- Try with τ_3 as lowest priority viable task. We consider the following order : $\tau_2 \succ \tau_1 \succ \tau_3$
  τ_3 is lowest priority viable because it meets all its deadlines in $[0, 40]$. We now seek for the lowest priority viable task in $\tau \setminus \{\tau_3\}$.
  - Try with τ_1 as lowest priority viable task. We consider the following order : $\tau_2 \succ \tau_1$.
    τ_1 is not a lowest priority viable task in $\tau \setminus \{\tau_3\}$ because it misses its first deadline at $t = 12$.
  - Try with τ_2 as lowest priority viable task. We consider the following order : $\tau_1 \succ \tau_2$.
    Because τ_2 misses no deadline, it is lowest priority viable in $\tau \setminus \{\tau_3\}$.
    We now seek for the lowest priority viable task in $\tau \setminus \{\tau_2, \tau_3\}$. Because only τ_1 is left, and $U(\tau_1) < 1$, it is lowest priority viable task in $\tau \setminus \{\tau_2, \tau_3\}$.

We therefore have found a priority order: $\tau_1 \succ \tau_2 \succ \tau_3$.

Exercise 14. Plot the scheduling of these 3 tasks using the feasibility interval (Theorem 44, page 29)

$$[O_{\max}, O_{\max} + 2P)$$

Answer
Let us calculate $[O_{\max}, O_{\max} + 2P)$:

- $O_{\max} = 10$
- $P = \mathrm{lcm}(3, 5, 15)$
- Feasibility interval: $[O_{\max}, O_{\max} + 2P) = [0, 40)$

The schedule is shown in Figure 15.6.

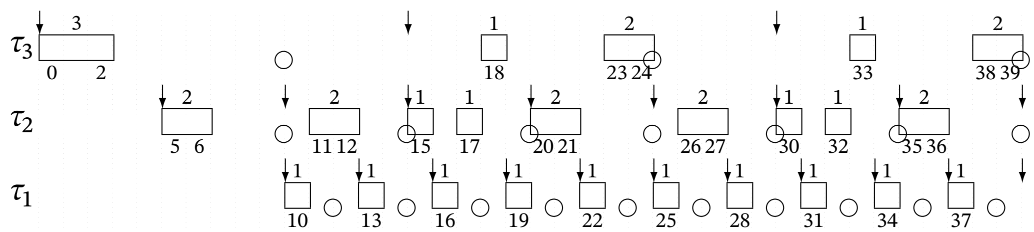

Figure 15.6: Simulation of System 114 in $[0, 40)$

Exercise 15. Why can we use $[0, S_n + P)$ from Theorem 45, page 29 as feasibility interval in this case? Calculate this interval.

Answer

Theorem 45 is applicable to systems for which a fixed task priority has already been assigned. With Audsley's algorithm, we have a feasible FTP assignment.

In order to find the interval $[0, S_n + P)$ we have to find S_n. We already know that $P = 15$ from a previous exercise. S_n can be calculated using Theorem 45, page 29.

In our case we have $\tau_1 \succ \tau_2 \succ \tau_3$.

1. $S_1 = O_1 = 10.$
2. $S_2 = O_2 + \left\lceil \frac{S_1 - O_2}{T_2} \right\rceil \cdot T_2 = 5 + 1 \cdot 5 = 10.$
3. $S_3 = O_3 + \left\lceil \frac{S_2 - O_3}{T_3} \right\rceil \cdot T_3 = 0 + 1 \cdot 15 = 15.$

The interval is $[0, 15 + 15) = [0, 30)$, which is lower than $[0, O_{\max} + 2P)$.

15.5 Earliest Deadline First

System 115

τ_i	C_i	D_i	T_i
τ_1	1	5	5
τ_2	2	4	8
τ_3	1	3	10
τ_4	5	15	20

Exercise 16. Find the feasibility interval for System 115 when using EDF to priorise jobs.

Answer

The feasibility interval for EDF is $[0, L)$ from Corollary 59, page 37, where L is the first idle point given by 4.1.2, page 37.

In our case we have:

1. $L_0 = 1 + 2 + 1 + 5 = 9,$
2. $L_1 = \left\lceil \frac{9}{5} \right\rceil \cdot 1 + \left\lceil \frac{9}{80} \right\rceil \cdot 2 + \left\lceil \frac{9}{10} \right\rceil \cdot 1 + \left\lceil \frac{9}{20} \right\rceil \cdot 5 = 12,$
3. $L_2 = \left\lceil \frac{12}{5} \right\rceil \cdot 1 + \left\lceil \frac{12}{8} \right\rceil \cdot 2 + \left\lceil \frac{12}{10} \right\rceil \cdot 1 + \left\lceil \frac{12}{20} \right\rceil \cdot 5 = 14,$
4. $L_3 = \left\lceil \frac{14}{5} \right\rceil \cdot 1 + \left\lceil \frac{14}{8} \right\rceil \cdot 2 + \left\lceil \frac{14}{10} \right\rceil \cdot 1 + \left\lceil \frac{14}{20} \right\rceil \cdot 5 = 14$

The fixed point is found, therefore a feasibility interval is $[0, 14)$

Exercise 17. Plot the scheduling of these tasks within the feasibility interval using EDF.

Answer
The simulation can be seen in Figure 15.7.

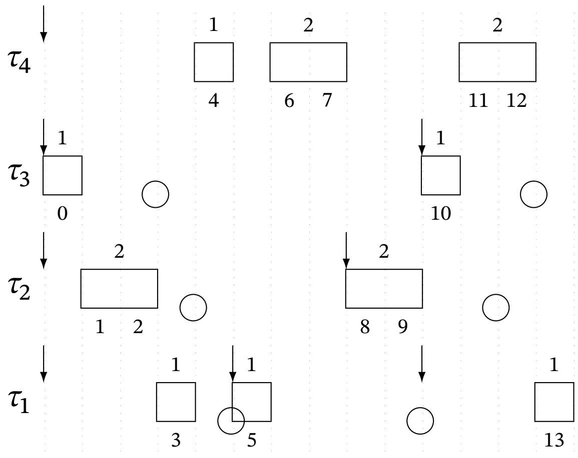

Figure 15.7: Simulation of System 115 with EDF

Exercise 18. Find a system of periodic tasks that could be scheduled using EDF, but not using DM.

Answer
See System 116

System 116

τ_i	O_i	C_i	D_i	T_i
τ_1	0	5	10	10
τ_2	0	5	13	20
τ_3	0	1 to 5	15	20

If DM is used, the task τ_3 misses its deadline at $t = 15$ as shown in Figure 15.8. The same system is schedulable by EDF.

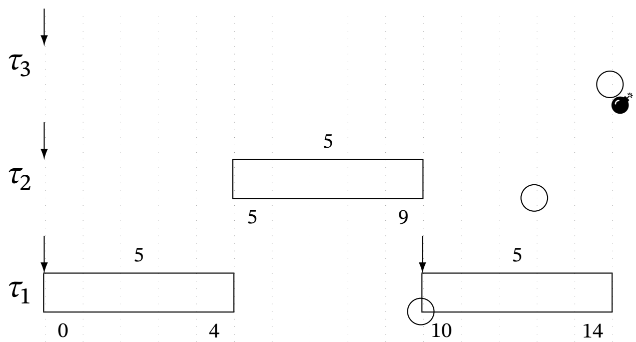

Figure 15.8: Deadline Monotonic misses a deadline while scheduling System 116

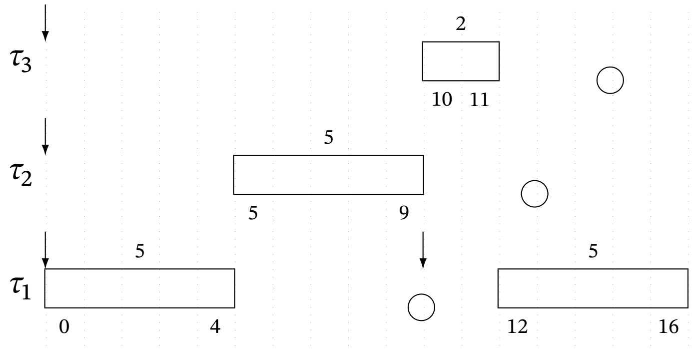

Figure 15.9: EDF is able to schedule System 116

15.6 Least Laxity First

Consider System 117.

System 117

τ_i	O_i	C_i	D_i	T_i
τ_1	0	1	5	5
τ_2	0	2	4	8
τ_3	0	1	3	10
τ_4	0	5	15	20

Exercise 19. Plot the scheduling of System 117 in the interval $[0, 20)$ using LLF. Consider a situation where priorities are recalculated every time unit.

Answer

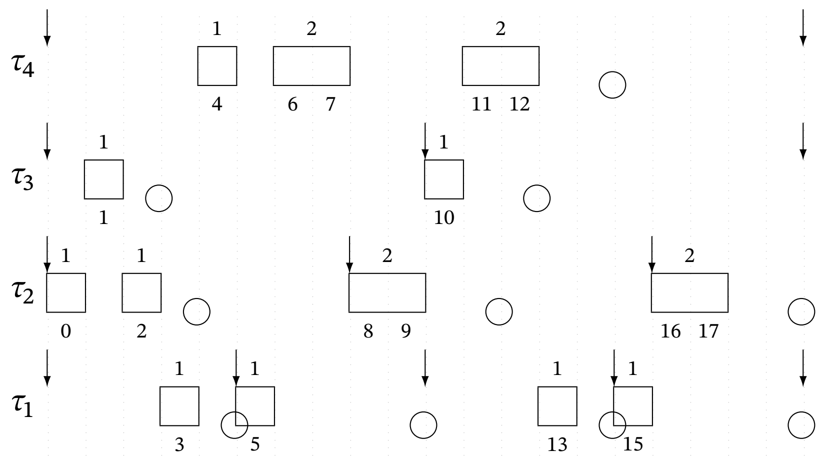

Figure 15.10: LLF scheduling