
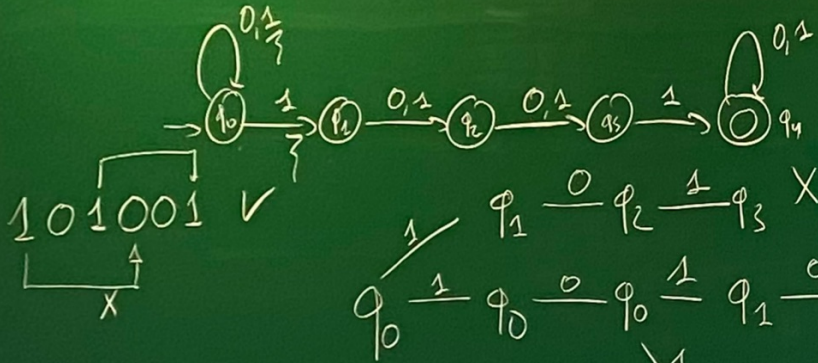


Sept, 23 ad



Non-determinism in FA's

Finite automata. non-determinism.



Two 's' separated by two
bits

w is accepted
iff \exists accepting
run

$$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \quad X$$

$$q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_3 \xrightarrow{1} q_4 \quad \checkmark$$

$$\phantom{q_0 \xrightarrow{1}} \phantom{q_0 \xrightarrow{0}} \phantom{q_0 \xrightarrow{1}} \phantom{q_1 \xrightarrow{0}} \phantom{q_2 \xrightarrow{0}} \phantom{q_3 \xrightarrow{1}} $$

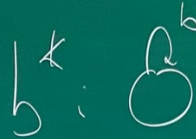
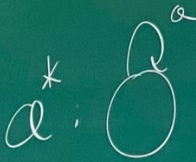
$$q_0 \xrightarrow{} q_0 \xrightarrow{} q_0 \xrightarrow{\frac{1}{1}} q_0 \quad X$$

$$\phantom{q_0 \xrightarrow{}} \phantom{q_0 \xrightarrow{}} \phantom{q_0 \xrightarrow{\frac{1}{1}}} \phantom{q_0 \xrightarrow{}} \phantom{q_1 \xrightarrow{}} \phantom{q_2 \xrightarrow{}} \phantom{q_3 \xrightarrow{}} \phantom{q_4 \xrightarrow{}}$$

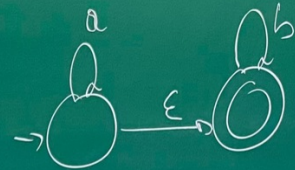
Spontaneous transitions

$S: Q \times \Sigma^* \rightarrow 2^Q$
 \downarrow current state
 \hookrightarrow possible next states

I want an automaton for a^*b^*



$q_1 \xrightarrow{\epsilon} q_2$ Spontaneous transition.



Convenient way of writing automata.

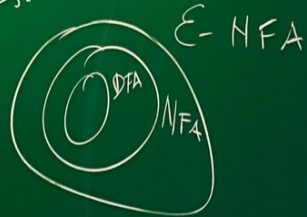
3 classes of F.A.

Classes of finite automata

Most general ϵ -NFA non-det. finite automata with ϵ -transitions.
allows non-det and ϵ -trans.

NFA: ϵ -trans. are not allowed

$$\delta(q, \epsilon) = \emptyset \text{ for all } q$$



DFA: Deterministic F.A

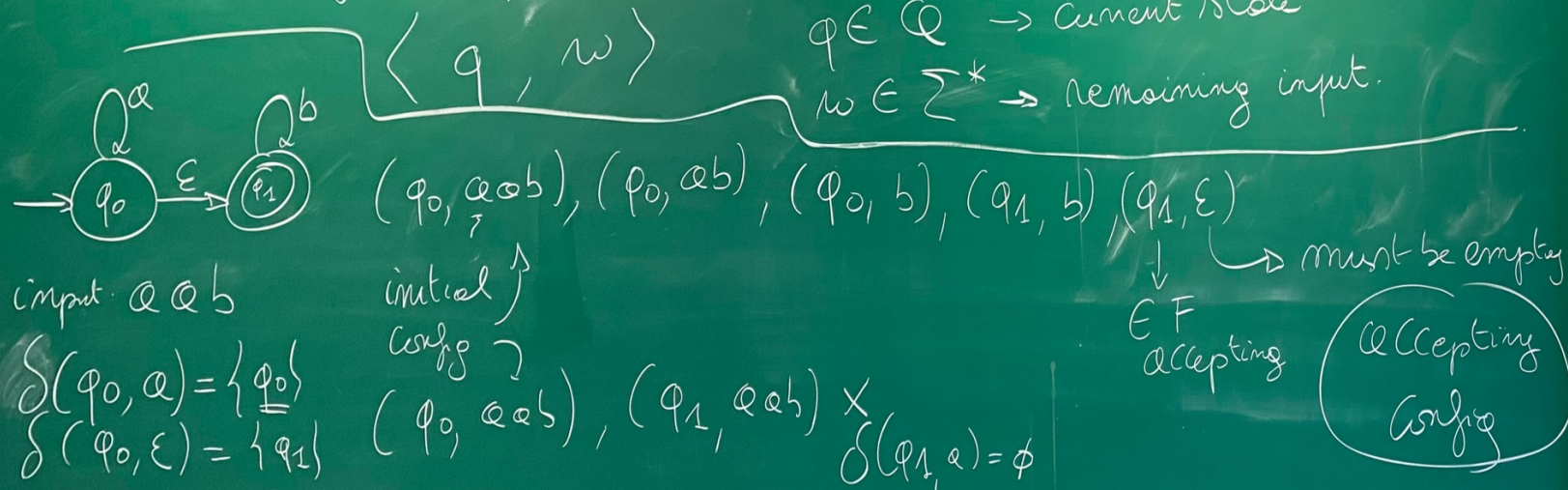
$$|\delta(q, a)| = 1 \quad \forall q \in Q, \forall a \in \Sigma$$

must be an NFA

Semantics of FA

How to define "accepted language"?

We need a notion of "Configuration" = snapshot of the current state of the FA reading a word.

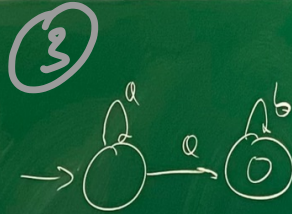


Getting rid of non-determinism

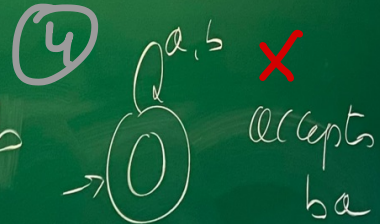
Some intuitions: let's try to remove non-determinism in automaton ①



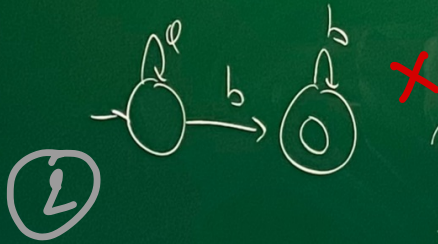
rejects ba
accepts aa
accepts b



rejects b



accepts ba



rejects aa



✓

From Reg. Expressions to ϵ -NFA

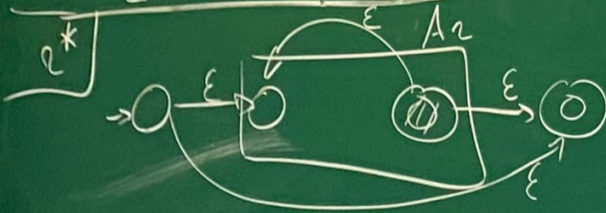
Reg Exp $\rightarrow \epsilon$ -NFA

Base

$\phi \rightarrow \text{start} \rightarrow \text{end}$

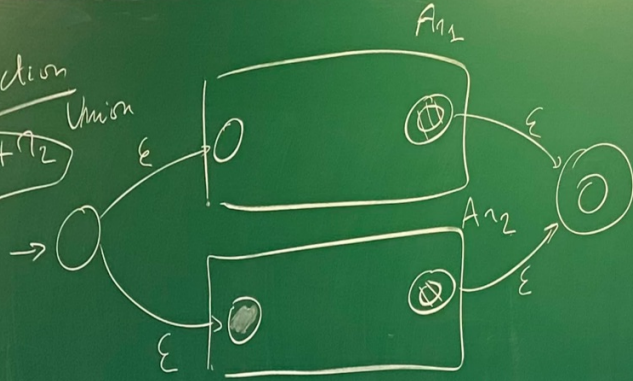
$\epsilon \rightarrow \text{start} \xrightarrow{\epsilon} \text{end}$

$a \in \Sigma \rightarrow \text{start} \xrightarrow{a} \text{end}$

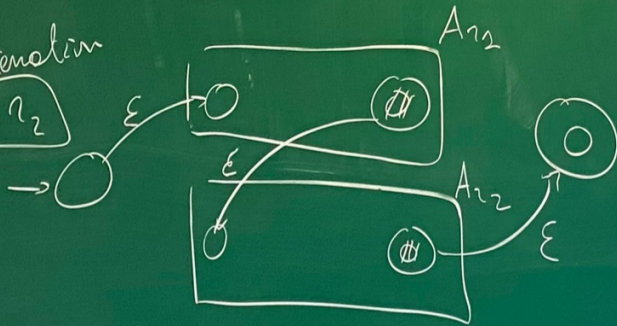


Induction

Union $r_1 + r_2$

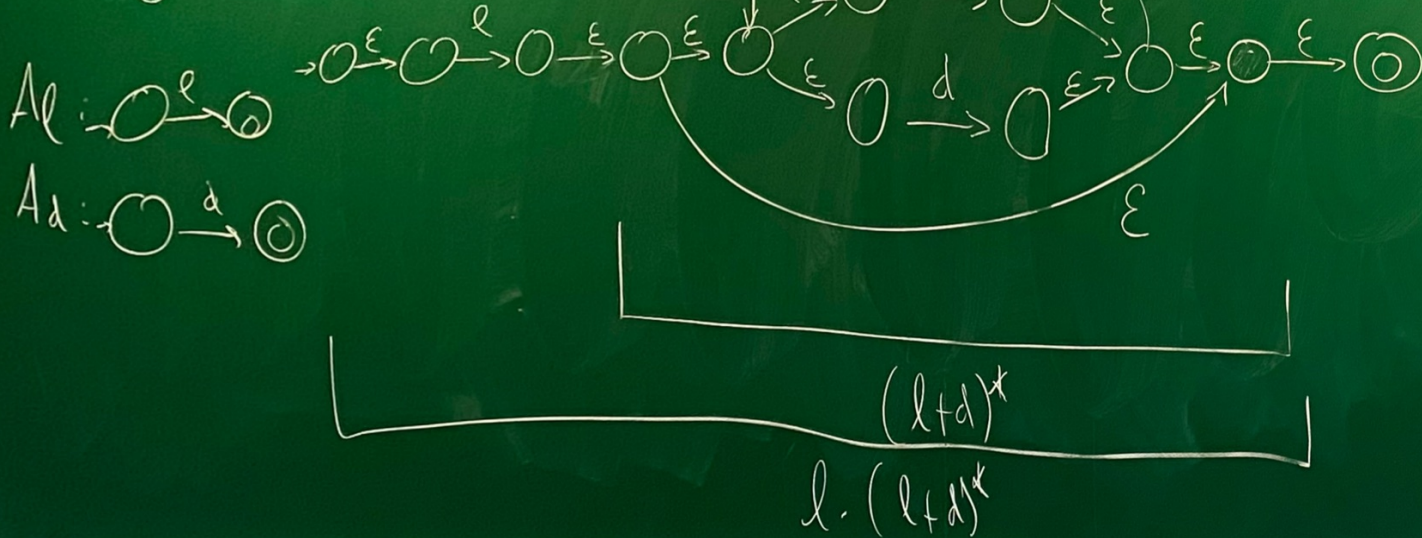


Concatenation $r_1 \cdot r_2$



Example

$$l(l+d)^{*} \textcircled{2}$$



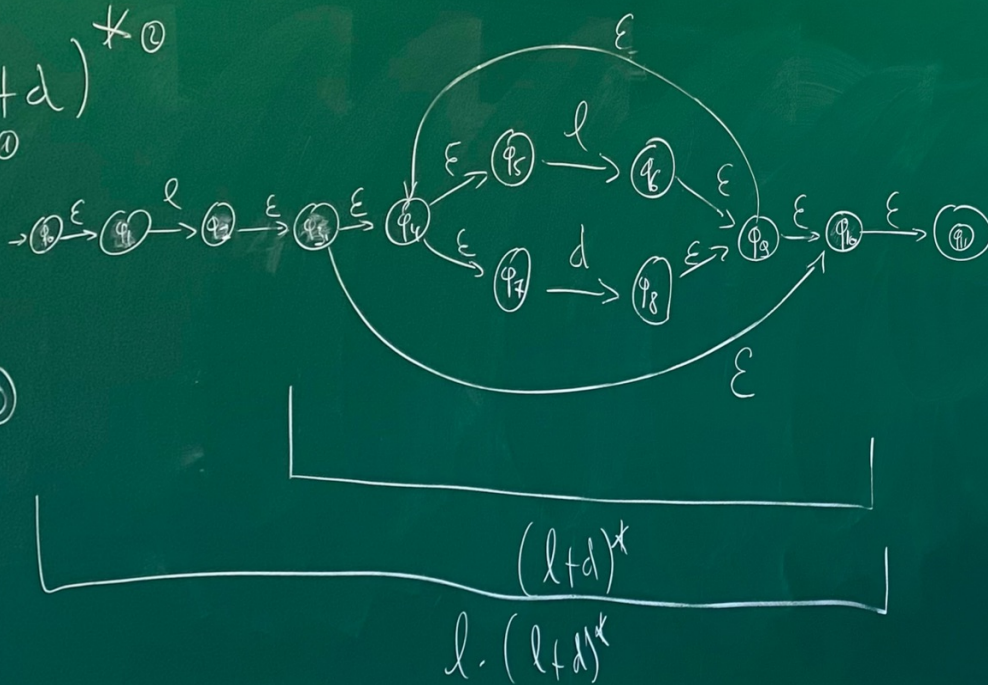
The ϵ -closure

ϵ -closure(q) = all the states that we can reach from q by taking ϵ 's only. (0 or more)

$$l.(l+d)^{*}$$

$$A_l: \text{ } \circ \xrightarrow{l} \circ$$

$$A_d: \text{ } \circ \xrightarrow{d} \circ$$



Example

$$\epsilon\text{-closure}(q_6) = \{q_3, q_{10}, q_4, q_5, q_7, q_{11}, q_6\}$$

Removing non-determinism

Let's get an intuition of the construction

States of the DFA = sets
of states

of the NFA

getting rid of non-determinism?

