



University  
of Glasgow | School of  
Computing Science

Honours Individual Project Dissertation

# 3D ANIMATION OF BARENDREGT'S LAMBDA CUBE

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# Abstract

As of this April, it will have been thirty-three years since the Lambda Cube was first described by Henk Barendregt . However, it has been an existence marked by obscurity; having little presence on the internet, and rarely being fully depicted. The goal of this project was to create a website with a three dimensional representation of the Every abstract follows a similar pattern. Motivate; set aims; describe work; explain results.

“XYZ is bad. This project investigated ABC to determine if it was better. ABC used XXX and YYY to implement ZZZ. This is particularly interesting as XXX and YYY have never been used together. It was found that ABC was 20% better than XYZ, though it caused rabies in half of subjects.”

# Education Use Consent

Consent for educational reuse withheld. Do not distribute.

# Contents

# 1 | Introduction

Why should the reader care about what are you doing and what are you actually doing?

## 1.1 Guidance

**Motivate** first, then state the general problem clearly.

## 1.2 Writing guidance

### 1.2.1 Who is the reader?

This is the key question for any writing. Your reader:

- is a trained computer scientist: *don't explain basics.*
- has limited time: *keep on topic.*
- has no idea why anyone would want to do this: *motivate clearly*
- might not know *anything* about your project in particular: *explain your project.*
- but might know precise details and check them: *be precise and strive for accuracy.*
- doesn't know or care about you: *personal discussions are irrelevant.*

Remember, you will be marked by your supervisor and one or more members of staff. You might also have your project read by a prize-awarding committee or possibly a future employer. Bear that in mind.

### 1.2.2 References and style guides

There are many style guides on good English writing. You don't need to read these, but they will improve how you write.

- *How to write a great research paper* (**recommended**, even though you aren't writing a research paper)
- *How to Write with Style* . Short and easy to read. Available online.
- *Style: The Basics of Clarity and Grace* A very popular modern English style guide.
- *Politics and the English Language* A famous essay on effective, clear writing in English.
- *The Elements of Style* Outdated, and American, but a classic.
- *The Sense of Style* Excellent, though quite in-depth.

#### Citation styles

- If you are referring to a reference as a noun, then cite it as: “ discusses the role of language in political thought.”
- If you are referring implicitly to references, use: “There are many good books on writing.”

There is a complete guide on good citation practice by Peter Coxhead available here: <http://www.cs.bham.ac.uk/~pxc/refs/index.html>. If you are unsure about how to cite on-line sources, please see this guide: <https://student.unsw.edu.au/how-do-i-cite-electronic-sources>

### 1.2.3 Plagiarism warning

**WARNING**

If you include material from other sources without full and correct attribution, you are committing plagiarism. The penalties for plagiarism are severe. Quote any included text and cite it correctly. Cite all images, figures, etc. clearly in the caption of the figure.

## 2 | Background

What did other people do, and how is it relevant to what you want to do?

### 2.1 Guidance

- Don't give a laundry list of references.
- Tie everything you say to your problem.
- Present an argument.
- Think critically; weigh up the contribution of the background and put it in context.
- **Don't write a tutorial**; provide background and cite references for further information.

## 3 | Analysis/Requirements

What is the problem that you want to solve, and how did you arrive at it?

### 3.1 Guidance

Make it clear how you derived the constrained form of your problem via a clear and logical process.



## 4 | Design

How is this problem to be approached, without reference to specific implementation details?

### 4.1 Guidance

Design should cover the abstract design in such a way that someone else might be able to do what you did, but with a different language or library or tool.

## 5 | Implementation

What did you do to implement this idea, and what technical achievements did you make?

### 5.1 Guidance

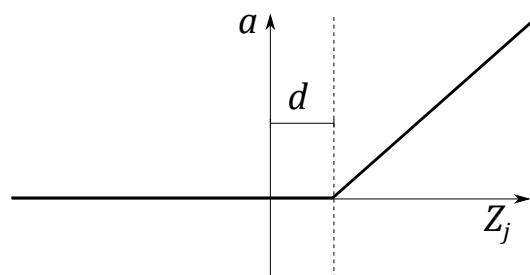
You can't talk about everything. Cover the high level first, then cover important, relevant or impressive details.

### 5.2 General points

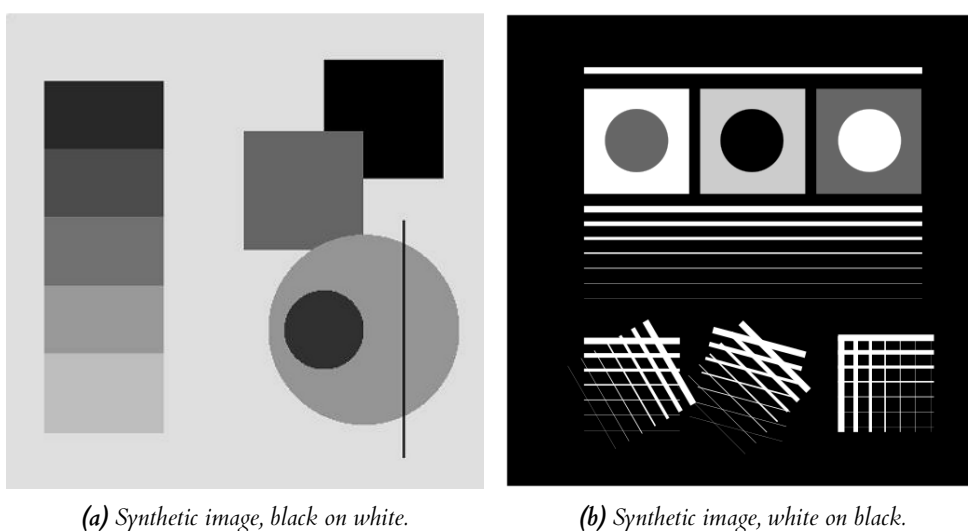
These points apply to the whole dissertation, not just this chapter.

#### 5.2.1 Figures

*Always* refer to figures included, like Figure ??, in the body of the text. Include full, explanatory captions and make sure the figures look good on the page. You may include multiple figures in one float, as in Figure ??, using **subcaption**, which is enabled in the template.



**Figure 5.1:** In figure captions, explain what the reader is looking at: “A schematic of the rectifying linear unit, where  $a$  is the output amplitude,  $d$  is a configurable dead-zone, and  $Z_j$  is the input signal”, as well as why the reader is looking at this: “It is notable that there is no activation at all below 0, which explains our initial results.” Use **vector image formats (.pdf)** where possible. Size figures appropriately, and do not make them over-large or too small to read.



(a) Synthetic image, black on white.

(b) Synthetic image, white on black.

**Figure 5.2:** Synthetic test images for edge detection algorithms. (a) shows various gray levels that require an adaptive algorithm. (b) shows more challenging edge detection tests that have crossing lines. Fusing these into full segments typically requires algorithms like the Hough transform. This is an example of using subfigures, with **subrefs** in the caption.

### 5.2.2 Equations

Equations should be typeset correctly and precisely. Make sure you get parenthesis sizing correct, and punctuate equations correctly (the comma is important and goes *inside* the equation block). Explain any symbols used clearly if not defined earlier.

For example, we might define:

$$\hat{f}(\xi) = \frac{1}{2} \left[ \int_{-\infty}^{\infty} f(x) e^{2\pi i x \xi} \right], \quad (5.1)$$

where  $\hat{f}(\xi)$  is the Fourier transform of the time domain signal  $f(x)$ .

### 5.2.3 Algorithms

Algorithms can be set using `algorithm2e`, as in Algorithm ??.

**Data:**  $f_X(x)$ , a probability density function returning the density at  $x$ .

$\sigma$  a standard deviation specifying the spread of the proposal distribution.

$x_0$ , an initial starting condition.

**Result:**  $s = [x_1, x_2, \dots, x_n]$ ,  $n$  samples approximately drawn from a distribution with PDF  $f_X(x)$ .

**begin**

$s \leftarrow []$

$p \leftarrow f_X(x)$

$i \leftarrow 0$

**while**  $i < n$  **do**

$x' \leftarrow \mathcal{N}(x, \sigma^2)$

$p' \leftarrow f_X(x')$

$a \leftarrow \frac{p'}{p}$

$r \leftarrow U(0, 1)$

**if**  $r < a$  **then**

$x \leftarrow x'$

$p \leftarrow f_X(x)$

$i \leftarrow i + 1$

append  $x$  to  $s$

**end**

**end**

**end**

**Algorithm 1:** The Metropolis-Hastings MCMC algorithm for drawing samples from arbitrary probability distributions, specialised for normal proposal distributions  $q(x'|x) = \mathcal{N}(x, \sigma^2)$ . The symmetry of the normal distribution means the acceptance rule takes the simplified form.

### 5.2.4 Tables

If you need to include tables, like Table ??, use a tool like <https://www.tablesgenerator.com/> to generate the table as it is extremely tedious otherwise.

### 5.2.5 Code

Avoid putting large blocks of code in the report (more than a page in one block, for example). Use syntax highlighting if possible, as in Listing ??.

**Table 5.1:** The standard table of operators in Python, along with their functional equivalents from the `operator` package. Note that table captions go above the table, not below. Do not add additional rules/lines to tables.

Operation	Syntax	Function
Addition	<code>a + b</code>	<code>add(a, b)</code>
Concatenation	<code>seq1 + seq2</code>	<code>concat(seq1, seq2)</code>
Containment Test	<code>obj in seq</code>	<code>contains(seq, obj)</code>
Division	<code>a / b</code>	<code>div(a, b)</code>
Division	<code>a / b</code>	<code>truediv(a, b)</code>
Division	<code>a // b</code>	<code>floordiv(a, b)</code>
Bitwise And	<code>a &amp; b</code>	<code>and_(a, b)</code>
Bitwise Exclusive Or	<code>a ^ b</code>	<code>xor(a, b)</code>
Bitwise Inversion	<code>~a</code>	<code>invert(a)</code>
Bitwise Or	<code>a   b</code>	<code>or_(a, b)</code>
Exponentiation	<code>a ** b</code>	<code>pow(a, b)</code>
Identity	<code>a is b</code>	<code>is_(a, b)</code>
Identity	<code>a is not b</code>	<code>is_not(a, b)</code>
Indexed Assignment	<code>obj[k] = v</code>	<code>setitem(obj, k, v)</code>
Indexed Deletion	<code>del obj[k]</code>	<code>delitem(obj, k)</code>
Indexing	<code>obj[k]</code>	<code>getitem(obj, k)</code>
Left Shift	<code>a &lt;&lt; b</code>	<code>lshift(a, b)</code>
Modulo	<code>a % b</code>	<code>mod(a, b)</code>
Multiplication	<code>a * b</code>	<code>mul(a, b)</code>
Negation (Arithmetic)	<code>- a</code>	<code>neg(a)</code>
Negation (Logical)	<code>not a</code>	<code>not_(a)</code>
Positive	<code>+ a</code>	<code>pos(a)</code>
Right Shift	<code>a &gt;&gt; b</code>	<code>rshift(a, b)</code>
Sequence Repetition	<code>seq * i</code>	<code>repeat(seq, i)</code>
Slice Assignment	<code>seq[i:j] = values</code>	<code>setitem(seq, slice(i, j), values)</code>
Slice Deletion	<code>del seq[i:j]</code>	<code>delitem(seq, slice(i, j))</code>
Slicing	<code>seq[i:j]</code>	<code>getitem(seq, slice(i, j))</code>
String Formatting	<code>s % obj</code>	<code>mod(s, obj)</code>
Subtraction	<code>a - b</code>	<code>sub(a, b)</code>
Truth Test	<code>obj</code>	<code>truth(obj)</code>
Ordering	<code>a &lt; b</code>	<code>lt(a, b)</code>
Ordering	<code>a &lt;= b</code>	<code>le(a, b)</code>

```

def create_callahan_table(rule="b3s23"):
    """Generate the lookup table for the cells."""
    s_table = np.zeros((16, 16, 16, 16), dtype=np.uint8)
    birth, survive = parse_rule(rule)

    # generate all 16 bit strings
    for iv in range(65536):
        bv = [(iv >> z) & 1 for z in range(16)]
        a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p = bv

        # compute next state of the inner 2x2
        nw = apply_rule(f, a, b, c, e, g, i, j, k)
        ne = apply_rule(g, b, c, d, f, h, j, k, l)
        sw = apply_rule(j, e, f, g, i, k, m, n, o)
        se = apply_rule(k, f, g, h, j, l, n, o, p)

        # compute the index of this 4x4
        nw_code = a | (b << 1) | (e << 2) | (f << 3)
        ne_code = c | (d << 1) | (g << 2) | (h << 3)
        sw_code = i | (j << 1) | (m << 2) | (n << 3)
        se_code = k | (l << 1) | (o << 2) | (p << 3)

        # compute the state for the 2x2
        next_code = nw | (ne << 1) | (sw << 2) | (se << 3)

        # get the 4x4 index, and write into the table
        s_table[nw_code, ne_code, sw_code, se_code] = next_code

    return s_table

```

**Listing 5.1:** The algorithm for packing the  $3 \times 3$  outer-totalistic binary CA successor rule into a  $16 \times 16 \times 16 \times 16$  4 bit lookup table, running an equivalent, notionally 16-state  $2 \times 2$  CA.

## 6 | Evaluation

How good is your solution? How well did you solve the general problem, and what evidence do you have to support that?

### 6.1 Guidance

- Ask specific questions that address the general problem.
- Answer them with precise evidence (graphs, numbers, statistical analysis, qualitative analysis).
- Be fair and be scientific.
- The key thing is to show that you know how to evaluate your work, not that your work is the most amazing product ever.

### 6.2 Evidence

Make sure you present your evidence well. Use appropriate visualisations, reporting techniques and statistical analysis, as appropriate.

If you visualise, follow the basic rules, as illustrated in Figure ??:

- Label everything correctly (axis, title, units).
- Caption thoroughly.
- Reference in text.
- **Include appropriate display of uncertainty (e.g. error bars, Box plot)**
- Minimize clutter.

See the file `guide_to_visualising.pdf` for further information and guidance.



**Figure 6.1:** Average number of fingers detected by the touch sensor at different heights above the surface, averaged over all gestures. Dashed lines indicate the true number of fingers present. The Box plots include bootstrapped uncertainty notches for the median. It is clear that the device is biased toward undercounting fingers, particularly at higher  $z$  distances.



## 7 | Conclusion

Summarise the whole project for a lazy reader who didn't read the rest (e.g. a prize-awarding committee).

### 7.1 Guidance

- Summarise briefly and fairly.
- You should be addressing the general problem you introduced in the Introduction.
- Include summary of concrete results (“the new compiler ran 2x faster”)
- Indicate what future work could be done, but remember: **you won't get credit for things you haven't done.**

# A | Appendices

Typical inclusions in the appendices are:

- Copies of ethics approvals (required if obtained)
- Copies of questionnaires etc. used to gather data from subjects.
- Extensive tables or figures that are too bulky to fit in the main body of the report, particularly ones that are repetitive and summarised in the body.
- Outline of the source code (e.g. directory structure), or other architecture documentation like class diagrams.
- User manuals, and any guides to starting/running the software.

**Don't include your source code in the appendices.** It will be submitted separately.