Using the sprintr package

Guo Yu

2020-02-28

The `sprintr` package contains the implementations of a computationally efficient method, called sprinter, to fit large interaction models based on the reluctant interaction selection principle. The details of the method can be found in Yu, Bien, and Tibshirani (2019) *Reluctant interaction modeling*. In particular, `sprinter` is a multi-stage method that fits the following pairwise interaction model:

$$y = \sum_{j=1}^{p} X_j \beta_j^* + \sum_{\ell \le k} X_\ell X_k \gamma_{\ell k}^* + \varepsilon.$$

This document serves as an introduction of using the package with a simple simulated data example.

Data simulation

We consider the following simple simulation setting, where $X \sim N(\mathbf{0}, \mathbf{I}_p)$. There are two non-trivial main effects $\beta_1 = 1$, $\beta_2 = -2$, and $\beta_j = 0$ for $j > 2$. The two important interactions are $X_1 * X_3$ with $\gamma_{13} = 3$, and $X_4 * X_5$ with $\gamma_{45} = -4$. With $\varepsilon \sim N(0, 1)$, the following code simulates $n = 100$ observation from the model above with $p = 100$.

```
library(sprintr)
set.seed(123)
n <- 100
p <- 100
x <- matrix(data = rnorm(n * p), nrow = n, ncol = p)
y <- x[, 1] - 2 * x[, 2] + 3 * x[, 1] * x[, 3] - 4 * x[, 4] * x[, 5] + rnorm(100)
```

Using `sprinter` function

The function `sprinter` implements the sprinter method (please note that the function name `sprinter` is different from the package name `sprintr`), which involves the following three main steps:

- Fit a lasso (with cross-validation) of the response y only on main effects X (if `square = FALSE` by default) or with both main effects and squared effects (X, X^2) (if `square = TRUE`).
- Carry out a screening procedure based on the residual from the previous step. The number of the selected candidate interactions can be specified by a path of `num_keep` values.
- With a path of tuning parameter `lambda`, fit a lasso of the response on main effects, squared effects (if `square = TRUE`), and selected interactions from the previous step.

There are two tuning parameters: `num_keep` (used in Step 2) and `lambda` (used in Step 3). If `num_keep` is not specified, it will then be automatically compuated as a list of decreasing values, starting from $n/\lceil \log n \rceil$ (see, e.g., Fan & Lv (2008)), based on the number of tuning parameters (`n_num_keep`, default to be 5). If `lambda` is not specified, then `sprinter` would compute its own path of tuning parameter values based on the number of tuning parameters (`nlam`) and the range of the path (`lam_min_ratio`). Finally, a `verbose` option (default to be `TRUE`) can be turned on to see the progress of the computation.

```
fit <- sprinter(x = x, y = y, square = FALSE, nlam = 100, lam_min_ratio = 0.01)
#>
  |
```

```
  |                                                                      |   0%
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```

sprinter output

The output of `sprinter` is a `S3` object including several useful components. In particular, the component `step3` is a list of length `n_num_keep`, with `step3[j]` containing information from Step 3 fit when the tuning parameter in Step 2 is `num_keep[j]`.

Within each `step3` component, there exists a `idx` object representing all variables considered in Step 3:

```
fit1 <- fit$step3[[1]]
fit1$idx[(p + 1) : nrow(fit1$idx), ]
#>       index_1 index_2    score
#>  [1,]       4       5 336.5337
#>  [2,]       4      96 250.2720
#>  [3,]       1       3 248.2150
#>  [4,]      29      95 181.0657
#>  [5,]       5      25 178.7409
#>  [6,]       5      97 163.1376
#>  [7,]      13      30 161.3963
#>  [8,]       5      79 158.3629
#>  [9,]      17      77 157.7678
#> [10,]      95      96 152.6769
#> [11,]       4      29 152.2820
#> [12,]       4      72 150.7680
#> [13,]      76      77 147.6554
#> [14,]      43      49 144.7388
#> [15,]       4      78 144.2851
#> [16,]      30      86 143.1823
#> [17,]       1      76 142.8303
#> [18,]       4      57 139.6105
#> [19,]       5      10 139.5537
#> [20,]       2       6 136.9426
#> [21,]      51      71 136.6593
#> [22,]      13      99 136.0234
```

In particular, `fit1$idx[, 1:2]` contains the indices of all the variables indices, and the last column represents their corresponding scores used to select candidate interactions in Step 2. The two columns of `idx` represents the index pair (ℓ, k) of a selected interaction $X_\ell * X_k$, where $\ell \leq k$. If the first entry of an index pair is zero, i.e., $(\ell = 0, k)$, then it represents a main effect X_k (with zero score).

The output `fit1$coef` is a `nrow(fit1$idx)`-by-`length(fit1$lambda)` matrix. Each column of `fit1$coef` is a vector of estimates of all variable coefficients considered in Step 3 corresponding to one value of the lasso tuning parameter `lambda`. For example, for the 30-th tuning parameter, we have the corresponding coefficient estiamte:

```
estimate <- fit1$coef[, 30]
cb <- cbind(fit1$idx, estimate)
cb[cb[, 3] != 0, ]
```

```
#>       index_1 index_2    score  estimate
#>  [1,]       4       5 336.5337 -3.162528
#>  [2,]       4      96 250.2720  0.000000
#>  [3,]       1       3 248.2150  1.618273
#>  [4,]      29      95 181.0657  0.000000
#>  [5,]       5      25 178.7409  0.000000
#>  [6,]       5      97 163.1376  0.000000
#>  [7,]      13      30 161.3963  0.000000
#>  [8,]       5      79 158.3629  0.000000
#>  [9,]      17      77 157.7678  0.000000
#> [10,]      95      96 152.6769  0.000000
#> [11,]       4      29 152.2820  0.000000
#> [12,]       4      72 150.7680  0.000000
#> [13,]      76      77 147.6554  0.000000
#> [14,]      43      49 144.7388  0.000000
#> [15,]       4      78 144.2851  0.000000
#> [16,]      30      86 143.1823  0.000000
#> [17,]       1      76 142.8303  0.000000
#> [18,]       4      57 139.6105  0.000000
#> [19,]       5      10 139.5537  0.000000
#> [20,]       2       6 136.9426  0.000000
#> [21,]      51      71 136.6593  0.000000
#> [22,]      13      99 136.0234  0.000000
```

Summarizing `sprinter` output by `print`, `plot`, and `summary`

The output of `sprinter` has an associated `print` function, that prints information (the number of nonzero main effects and nonzero interactions) of Step 3 fits along a path of Step 3 tuning parameters, for a given value of Step 2 tuning parameter (specified by `which`). For example, the following codes prints the output when the 2nd value of Step-2 tuning parameter is used:

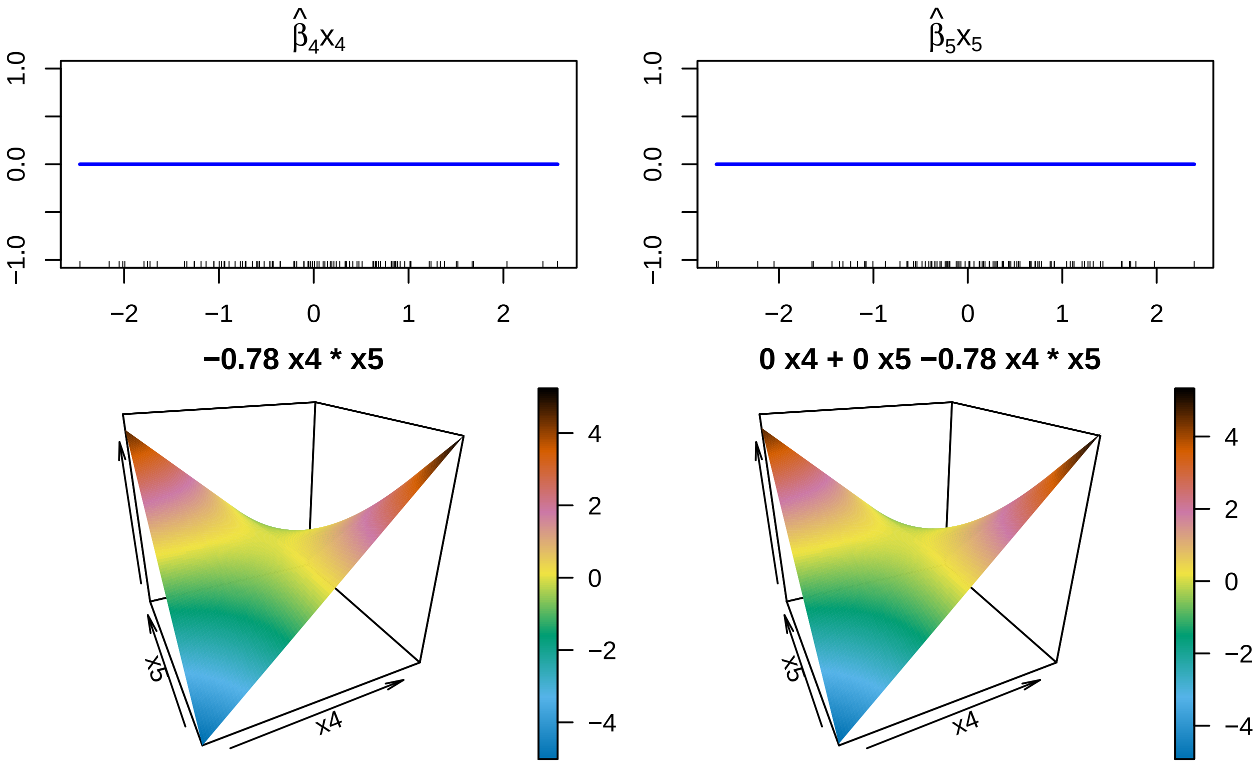
```
print(fit, which = 2)
#>
#> Call:  sprinter(x = x, y = y, square = FALSE, nlam = 100, lam_min_ratio = 0.01)
#>
#>         lambda #nz main #nz interaction
#>   [1,] 4.21300        0               0
#>   [2,] 4.02200        0               1
#>   [3,] 3.83900        0               1
#>   [4,] 3.66500        0               1
#>   [5,] 3.49800        0               1
#>   [6,] 3.33900        0               1
#>   [7,] 3.18700        0               1
#>   [8,] 3.04200        0               1
#>   [9,] 2.90400        0               2
#>  [10,] 2.77200        0               2
#>  [11,] 2.64600        0               2
#>  [12,] 2.52600        0               2
#>  [13,] 2.41100        0               2
#>  [14,] 2.30100        0               2
#>  [15,] 2.19700        0               2
#>  [16,] 2.09700        0               2
#>  [17,] 2.00200        0               2
```

```
#>  [18,] 1.91100         0                2
#>  [19,] 1.82400         0                2
#>  [20,] 1.74100         2                2
#>  [21,] 1.66200         2                2
#>  [22,] 1.58600         2                2
#>  [23,] 1.51400         2                2
#>  [24,] 1.44500         2                2
#>  [25,] 1.38000         2                2
#>  [26,] 1.31700         2                2
#>  [27,] 1.25700         2                2
#>  [28,] 1.20000         2                2
#>  [29,] 1.14500         2                2
#>  [30,] 1.09300         2                2
#>  [31,] 1.04400         2                2
#>  [32,] 0.99630         2                2
#>  [33,] 0.95100         2                2
#>  [34,] 0.90780         2                2
#>  [35,] 0.86650         2                2
#>  [36,] 0.82710         2                2
#>  [37,] 0.78950         2                2
#>  [38,] 0.75360         2                2
#>  [39,] 0.71940         2                3
#>  [40,] 0.68670         2                3
#>  [41,] 0.65550         2                3
#>  [42,] 0.62570         3                3
#>  [43,] 0.59720         3                3
#>  [44,] 0.57010         3                3
#>  [45,] 0.54420         3                3
#>  [46,] 0.51950         3                3
#>  [47,] 0.49580         3                3
#>  [48,] 0.47330         3                3
#>  [49,] 0.45180         3                3
#>  [50,] 0.43130         3                4
#>  [51,] 0.41170         3                4
#>  [52,] 0.39290         3                4
#>  [53,] 0.37510         3                4
#>  [54,] 0.35800         3                4
#>  [55,] 0.34180         4                4
#>  [56,] 0.32620         4                4
#>  [57,] 0.31140         4                7
#>  [58,] 0.29720         5                7
#>  [59,] 0.28370         5                8
#>  [60,] 0.27080         5                8
#>  [61,] 0.25850         5                8
#>  [62,] 0.24680         5                8
#>  [63,] 0.23560         5                8
#>  [64,] 0.22490         5                8
#>  [65,] 0.21460         6                8
#>  [66,] 0.20490         7                9
#>  [67,] 0.19560         9                9
#>  [68,] 0.18670        11                9
#>  [69,] 0.17820        11                9
#>  [70,] 0.17010        11                9
```

```
#>  [71,] 0.16240       12              9
#>  [72,] 0.15500       14              9
#>  [73,] 0.14790       15              9
#>  [74,] 0.14120       16             10
#>  [75,] 0.13480       16             10
#>  [76,] 0.12870       16             11
#>  [77,] 0.12280       16             11
#>  [78,] 0.11720       17             12
#>  [79,] 0.11190       17             11
#>  [80,] 0.10680       17             11
#>  [81,] 0.10200       18             11
#>  [82,] 0.09734       18             11
#>  [83,] 0.09291       21             12
#>  [84,] 0.08869       23             12
#>  [85,] 0.08466       24             12
#>  [86,] 0.08081       26             13
#>  [87,] 0.07714       27             12
#>  [88,] 0.07363       29             11
#>  [89,] 0.07028       33             11
#>  [90,] 0.06709       33             12
#>  [91,] 0.06404       34             12
#>  [92,] 0.06113       34             12
#>  [93,] 0.05835       34             12
#>  [94,] 0.05570       34             12
#>  [95,] 0.05317       34             12
#>  [96,] 0.05075       37             12
#>  [97,] 0.04844       40             12
#>  [98,] 0.04624       41             12
#>  [99,] 0.04414       41             12
#> [100,] 0.04213       42             12
```

The `plot` function is also defined for `sprinter` output to look at the effects from a certain interaction (specified by `which`). For example, by examining the effect of interaction between X_4 and X_5, we run the following `plot` function that produces 4 panels:

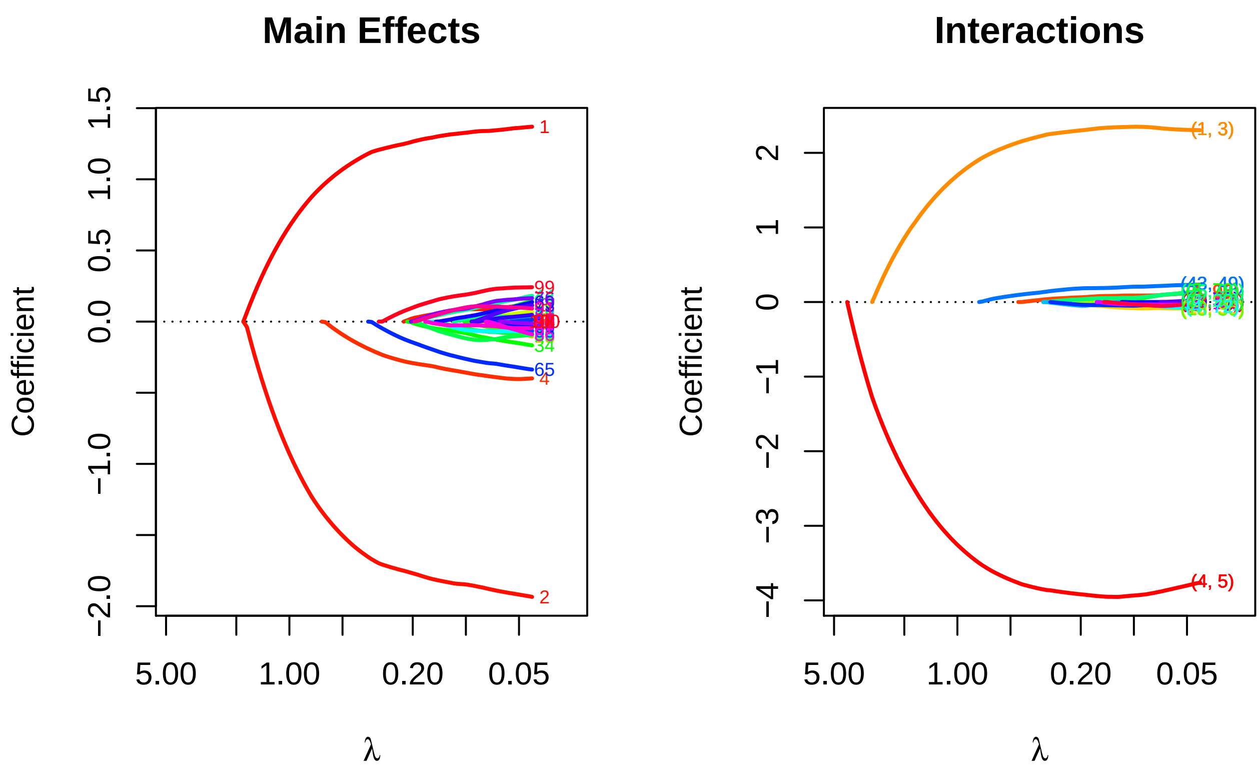
```
plot(fit, newdata = x, which = c(4, 5))
#> Warning in plot.sprinter(fit, newdata = x, which = c(4, 5)): Tuning
#> parameter pair indices not provided. Plot the pair (1,100) by default.
```



The top two panels show the marginal relationship of the predicted response on the two main effects, i.e., $\hat{\beta}_4 X_4$ and $\hat{\beta}_5 X_5$ respectively. The lower-left panel shows the dependence of the predicted response on the interaction alone, i.e., $\hat{\gamma}_{45} X_4 * X_5$, and the lower-right panel shows $\hat{\beta}_4 X_4 + \hat{\beta}_5 X_5 + \hat{\gamma}_{45} X_4 * X_5$. From the plot we see that the predicted response does not depend on either of the two main effects, but depends on their interaction (with coefficient -0.78)

Finally, `summary` function shows the dependence of coefficient estimates for each main effects (left panel) and interactions(right panel) on the Step-3 tuning parameters:

```
summary(fit)
```



Using cross-validation with `cv.sprinter`

The function `cv.sprinter()` performs cross-validation to select the best value pairs of Step-2 and Step-3 tuning parameters.

```
fit_cv <- cv.sprinter(x = x, y = y, square = FALSE, n_num_keep = 5, nlam = 100, lam_min_ratio = 0.01)
#> cv initial:
#>
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#> cv fold  1 :
#>
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#> cv fold  2 :
#>
```

```
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#> cv fold  3 :
#>
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#> cv fold  4 :
#>
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#> cv fold  5 :
#>
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  |                                                                      |   0%
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  |=============================================                         |  67%
  |
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```

`cv.sprinter` output

The output of `cv.sprinter` is a `S3` object. The most intersting information is `fit_cv$compact`, which is a matrix of three columns. The first two columns show the indices pairs of all variables finally selected by cross-validation, and the last column is the coefficient estimate corresponding to those selected variables.

```
fit_cv$compact
#>      index_1 index_2   coefficient
#> [1,]       0       1  1.3627135262
#> [2,]       0       2 -1.8941582052
#> [3,]       0       3  0.1050959648
#> [4,]       0       4 -0.4092385412
#> [5,]       0      21  0.0007039331
#> [6,]       0      25 -0.0838433007
```

```
#>  [7,]       0      26 -0.0499478617
#>  [8,]       0      31 -0.0297285465
#>  [9,]       0      34 -0.0878596104
#> [10,]       0      35  0.0412542636
#> [11,]       0      38 -0.0415217706
#> [12,]       0      39 -0.1453470347
#> [13,]       0      43  0.0642984181
#> [14,]       0      44  0.1203193907
#> [15,]       0      51 -0.0126056316
#> [16,]       0      64  0.0196750028
#> [17,]       0      65 -0.2296652640
#> [18,]       0      66 -0.0175774607
#> [19,]       0      69  0.0073273880
#> [20,]       0      76  0.1071258137
#> [21,]       0      78 -0.0136436397
#> [22,]       0      87 -0.0550568118
#> [23,]       0      91  0.0885291261
#> [24,]       0      99  0.2078193532
#> [25,]       0     100  0.0225545973
#> [26,]       4       5 -4.0211027225
#> [27,]       1       3  2.4013600490
#> [28,]       4      96  0.0890808049
#> [29,]      29      95 -0.1008666063
```

We see (from the first two rows and the last two rows) that the fit selected by cross-validation includes all the four important variables ($X_1, X_2, X_4 * X_5, X_1 * X_3$) in the model, with relatively accurate estimates of their coefficients.

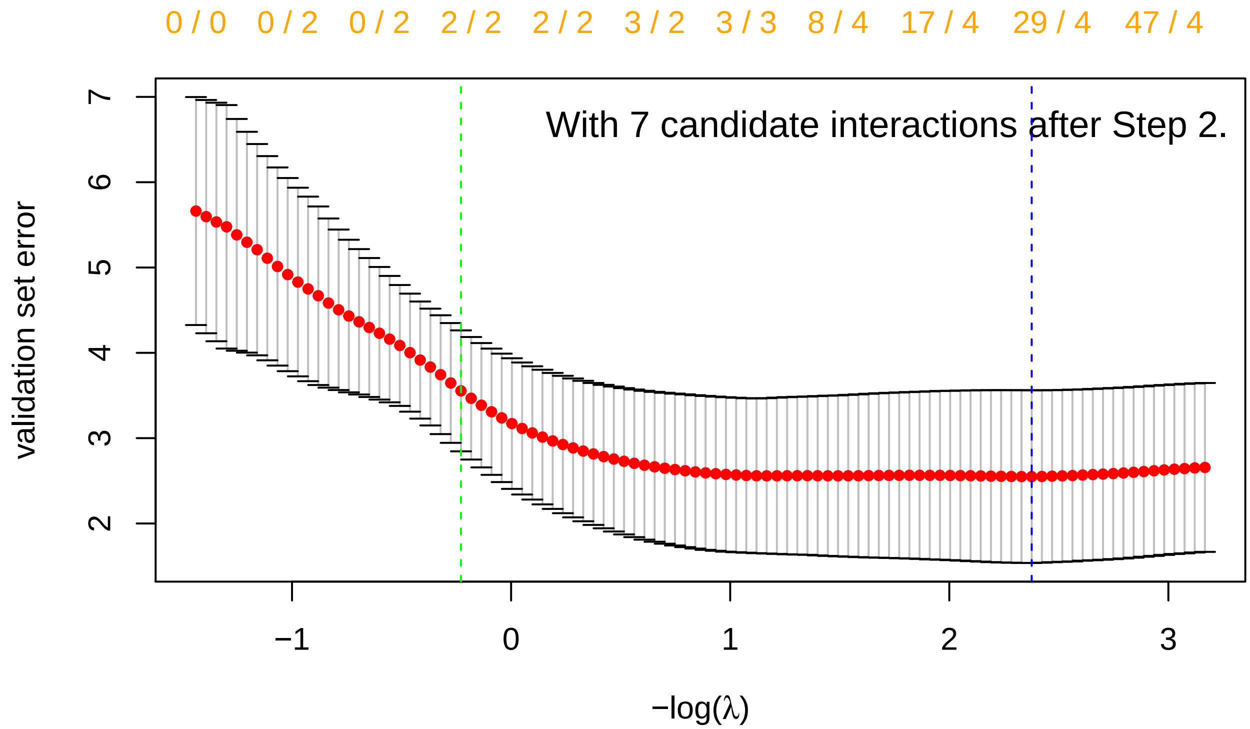
Summarizing `cv.sprinter` output by `print` and `plot`

Associated with the output of `cv.sprinter` are the `print` and `plot` functions. `print` functions can be used to the summary of the cross-validation process, indicating information such as the best number of candidate interactions in Step 2, and the validation error mean/standard errors, number of non-zero main effects/interactions for the Step-3 tuning parameter selected by `min` rule and `1se` rule.

```
print(fit_cv)
#>
#> Call:  cv.sprinter(x = x, y = y, square = FALSE, n_num_keep = 5, nlam = 100,      lam_min_ratio = 0.
#>
#> Best number of candidate interactions in Step 2: 7
#>
#>        lambda mean(vali-err) se(vali-err) #nonzero-main #nonzero-inter
#> min 0.09291          4.364       0.8519            25              4
#> 1se 1.25700          5.296       1.2940             2              2
```

The `plot` function for the output of `cv.sprinter` shows the validation error across different folds as a function of Step-3 tuning parameters (for a fixed value of Step-2 tuning parameter chosen by cross-validation). The top of the plot shows the number of nonzero main effects / nonzero interactions corresponding (in orange) to a value of Step-3 tuning parameters.

```
plot(fit_cv)
```



The blue vertical line shows the Step-3 tuning parameter selected by `min` rule, and the green vertical line shows the Step-3 tuning parameter selected by `1se` rule.

Prediction

The `predict` function is defined for both the object returned by `sprinter` and `cv.sprinter` that computes the prediction for a new data matrix of main effects:

```
newdata <- matrix(rnorm(20 * p), nrow = 20, ncol = p)
pred <- predict(fit, newdata = newdata)
```

The prediction for `sprinter` object computes the prediction at `newdata` for all the (Step-2, Step-3) tuning parameter pairs, and the prediction for `cv.sprinter` object just computes the prediction at `newdata` for the best tuning parameter pairs selected by cross-validation.

```
pred_cv <- predict(fit_cv, newdata = newdata)
```

Support for other response families

Under construction