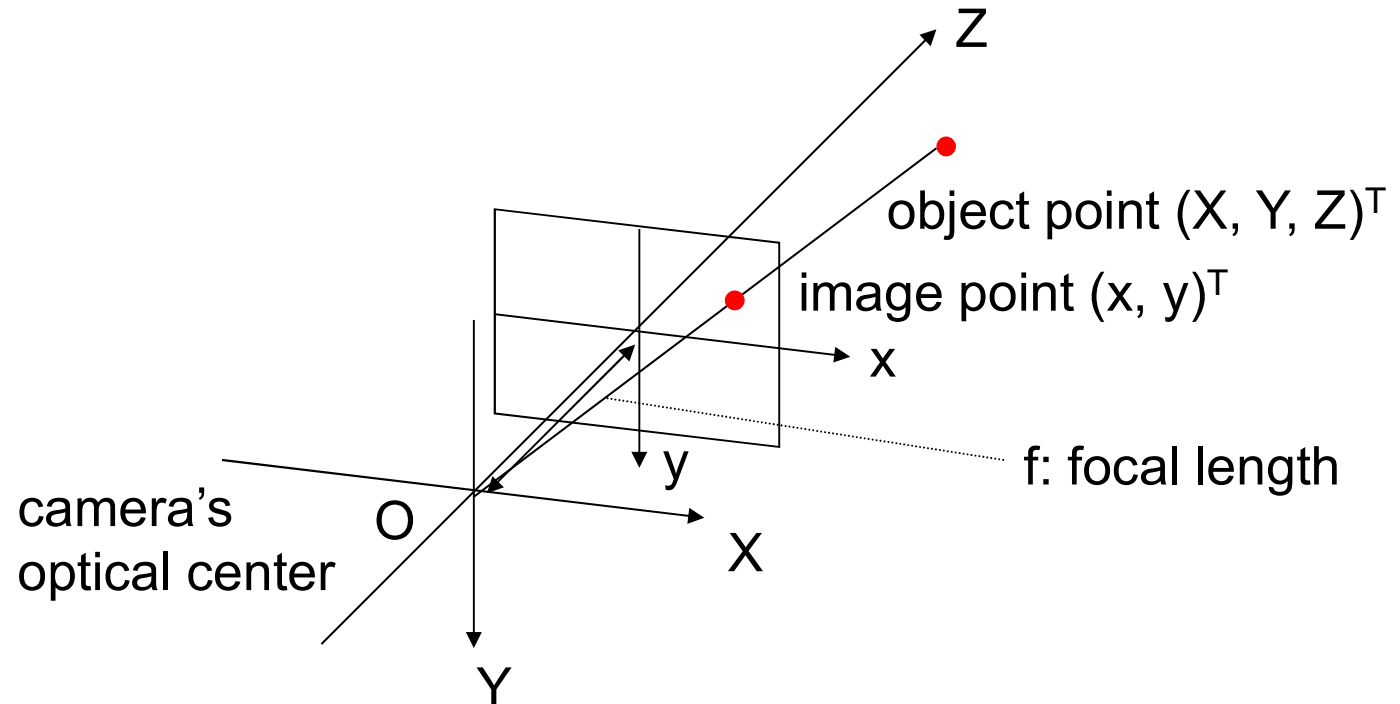


• Outline

- Introduction
- Theory
- Software

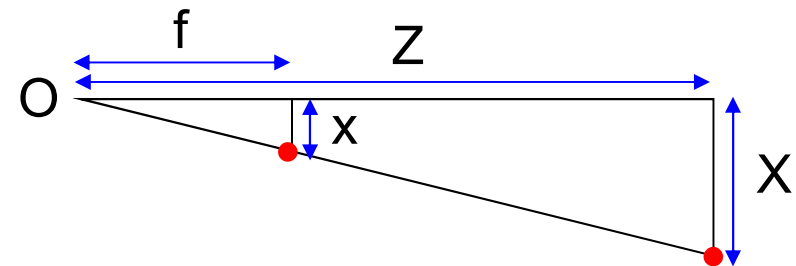
Pinhole Camera Model



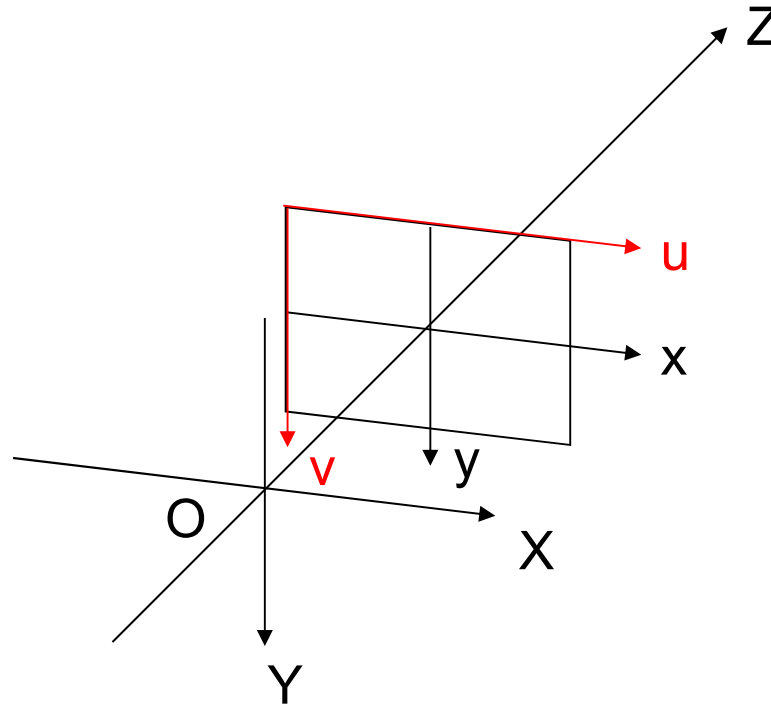
$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{f}{Z} \begin{pmatrix} X \\ Y \end{pmatrix}$$

X-Y-Z: camera coordinate frame

x-y: (normalized) image coordinate frame



Pixel Coordinates



$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} k_u x + c_u \\ k_v y + c_v \end{pmatrix}$$

u-v: image coordinate (pixel coordinate) frame

• Camera Intrinsic Matrix

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{f}{Z} \begin{pmatrix} X \\ Y \end{pmatrix} \qquad \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} k_u x + c_u \\ k_v y + c_v \end{pmatrix}$$

$$\begin{aligned} Z \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} &= \begin{pmatrix} f k_u & 0 & c_x \\ 0 & f k_v & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \\ &= \boxed{A} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \end{aligned}$$

3x3 camera intrinsic matrix

• Coordinate Transformation of the Object Point

- Say, the object point coordinates are expressed in a different coordinate frame X' - Y' - Z'

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} + t$$

R: 3x3 rotation matrix

t: 3D translation vector

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \left(R \mid t \right) \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix}$$

homogeneous coordinate of $(X', Y', Z')^T$

Camera Projection Matrix

$$Z \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = A \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = (R \mid t) \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix}$$

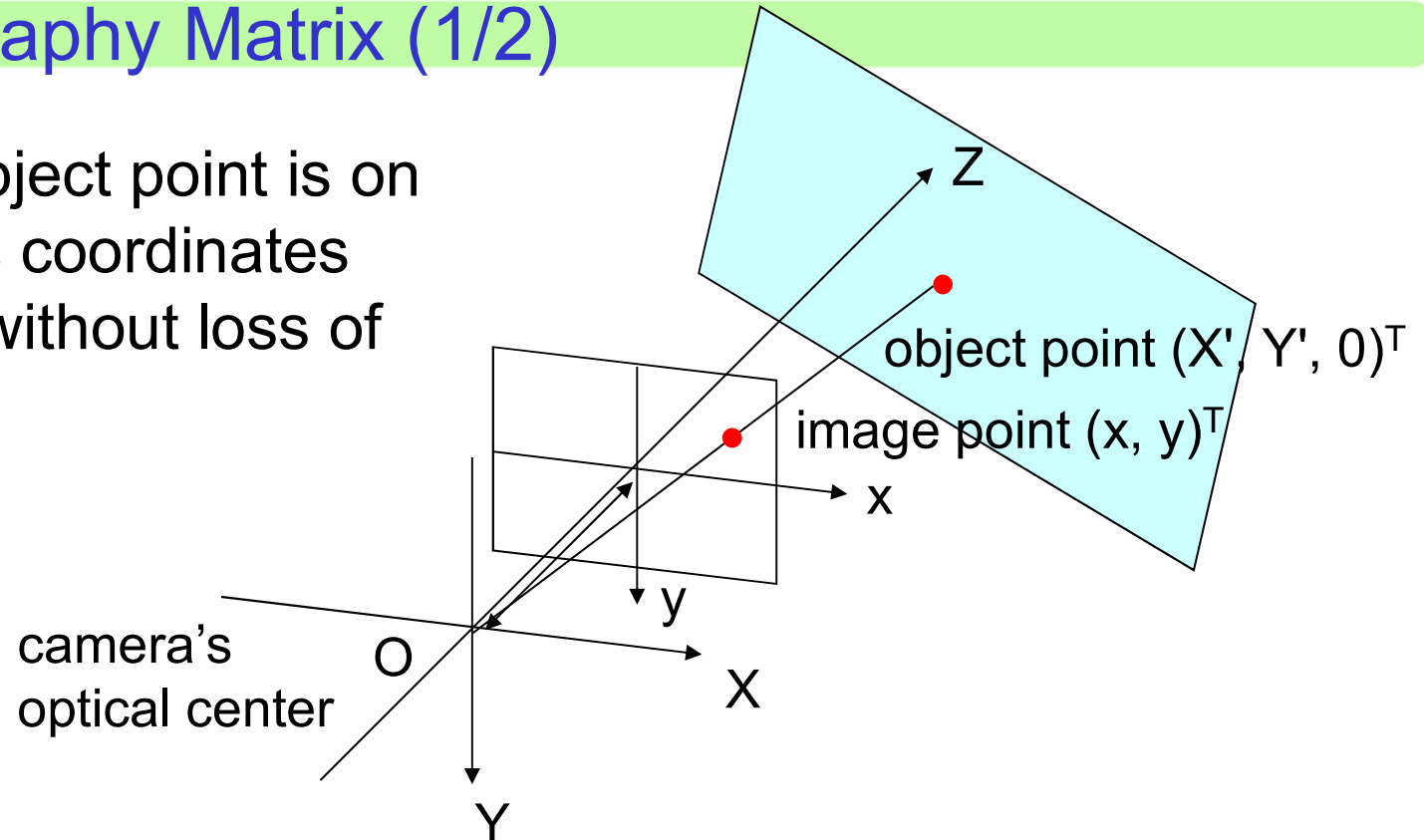
$$Z \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = A (R \mid t) \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix}$$

$$= P \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix}$$

3x4 camera projection matrix

Homography Matrix (1/2)

- Assume the object point is on a plane and its coordinates are $(X', Y', 0)$ without loss of generality.



$$Z \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = P \begin{pmatrix} X' \\ Y' \\ 0 \\ 1 \end{pmatrix} = \boxed{H} \begin{pmatrix} X' \\ Y' \\ 1 \end{pmatrix}$$

3x3 homography matrix

Homography Matrix (2/2)

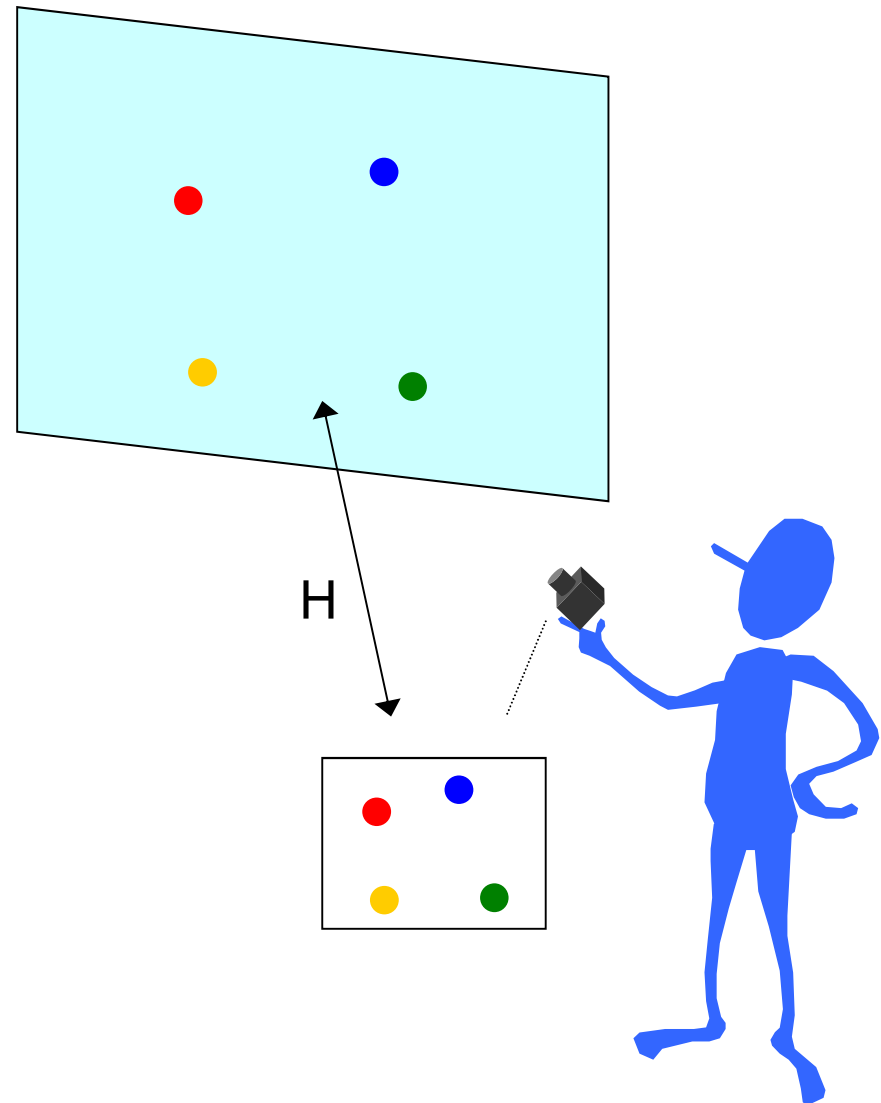
$$Z \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H \begin{pmatrix} X' \\ Y' \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \simeq H \begin{pmatrix} X' \\ Y' \\ 1 \end{pmatrix} \quad \begin{pmatrix} X' \\ Y' \\ 1 \end{pmatrix} \simeq H^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

- When the homography matrix is given, the mapping **from/to** (X', Y') and (u, v) are uniquely determined
 - The scale factor Z is determined from the 3rd row eqn.
 - Then the other rows eqns are solved
- When n (≥ 4) known points on a plane are observed by a camera, H can be computed

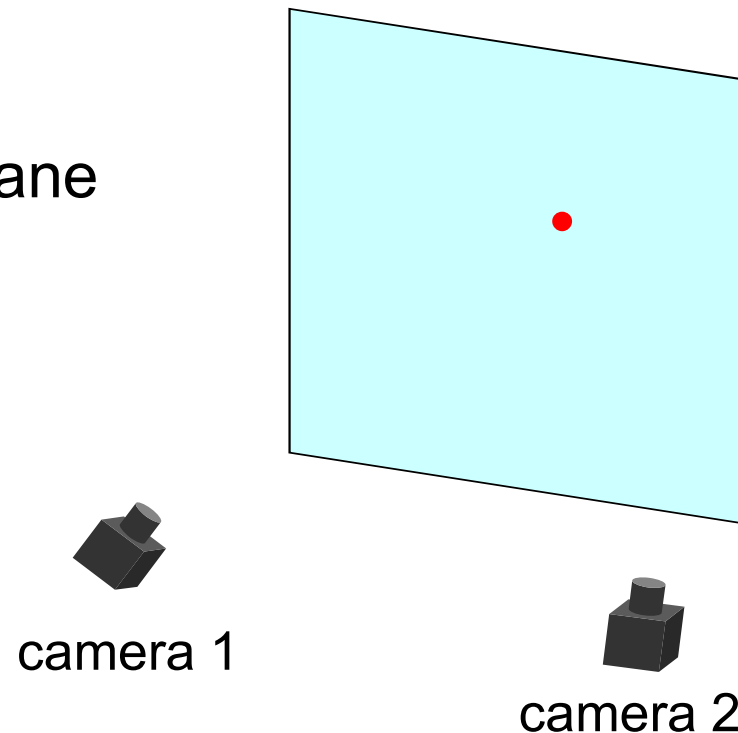
Summary so far

- If you have four or more corresponding points between the screen and the camera image, homography matrix between them can be computed
- Once homography matrix is obtained, any point coordinates can be transformed from screen to camera, and vice versa



• Homography Between Two Views (1/2)

- Think of two cameras observing a point on a plane



$$\begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} \simeq H_1 \begin{pmatrix} X' \\ Y' \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} \simeq H_2 \begin{pmatrix} X' \\ Y' \\ 1 \end{pmatrix}$$

Homography Between Two Views (1/2)

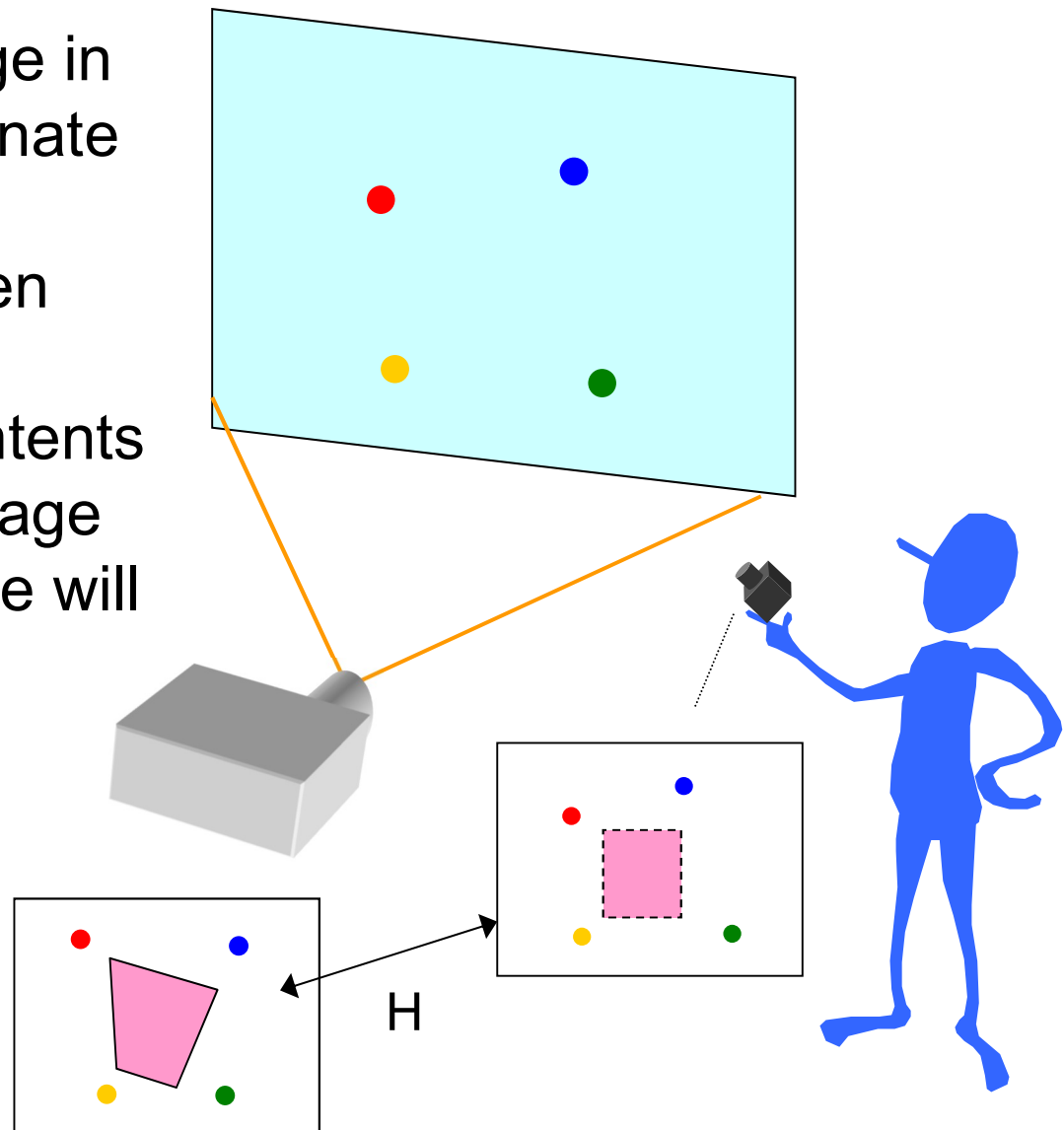
$$\begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} \simeq \boxed{H_2 H_1^{-1}} \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}$$

3x3 homography matrix that gives mapping between the pixel coordinates of camera 1 / camera 2

- When $n \geq 4$ corresponding points on a plane are observed by the two cameras, $H = H_2 H_1^{-1}$ can be computed
- Once H is obtained, the mapping from/to camera 1 and camera 2 pixel coordinates is straightforward
- One of the camera can be **a projector**

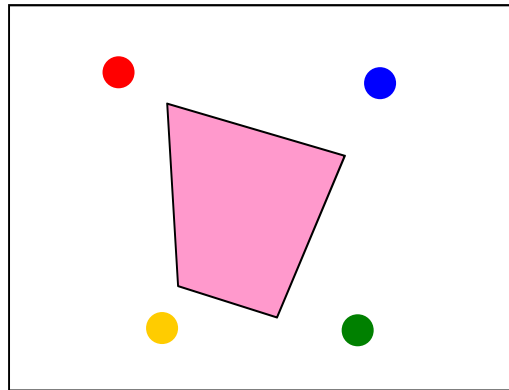
So, what to do is...

- (Prepare a contents image in the camera image coordinate frame)
- Find homography between projector and camera
- Transform (warp) the contents image to the projector image
- Then, the projected image will look as if the camera is a handheld projector

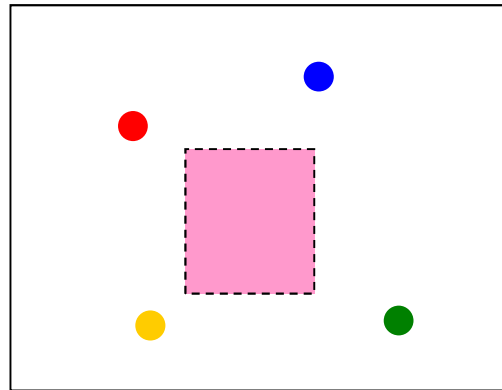


Transitive Application of Homography Matrices

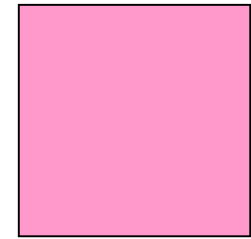
projector image plane



camera image plane



content image



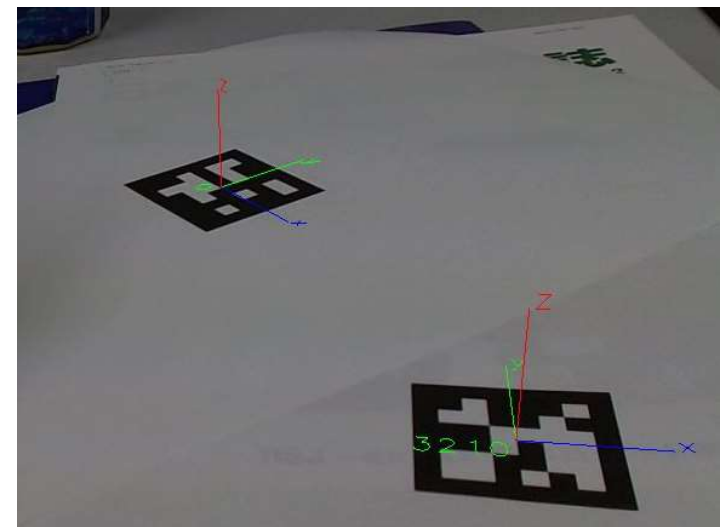
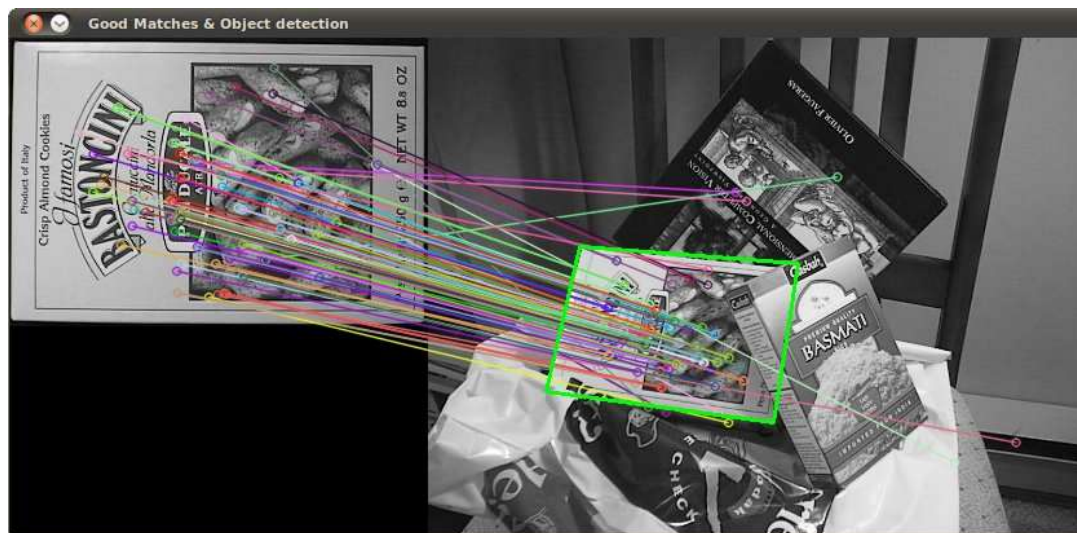
H_1

H_2

H_1H_2

Corresponding Points Problem

- In principle, because we know what kind of image we display from the projector, making correspondence between the points in the projector image and those in the camera image
- But how to do it practically?
 - Local image feature descriptors (Image keypoints)
 - Fiducial markers



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Python Install – OpenCV 3 pre-built for Windows

Install python 2.x or 3.x

- <https://www.python.org/downloads/>

Windows (pre-built binaries)

- Install pip:
<http://stackoverflow.com/questions/4750806/how-do-i-install-pip-on-windows>
- Pre-built binaries (OpenCV 2.4 & 3.1, python 2.7 & 3.4-3.5):
<http://www.lfd.uci.edu/~gohlke/pythonlibs/#opencv>
- Run in cmd or powershell:
`pip install [filename_of_pre-built_binaries]`
- Open python IDLE, write these lines to check that it works:
`import cv2`
`print(cv2.__version__) # in python 3.x`
`# print cv2.__version__ in python 2.x`

Python Install – OpenCV 3 for OSX and Ubuntu

OSX (for Python 3.4 or 2.7)

- <http://www.pyimagesearch.com/2015/06/29/install-opencv-3-0-and-python-3-4-on-osx/>
- <http://www.pyimagesearch.com/2015/06/15/install-opencv-3-0-and-python-2-7-on-osx/>

Ubuntu (for Python 3.4 or 2.7)

- <http://www.pyimagesearch.com/2015/07/20/install-opencv-3-0-and-python-3-4-on-ubuntu/>
- <http://www.pyimagesearch.com/2015/06/22/install-opencv-3-0-and-python-2-7-on-ubuntu/>

• Sample Code for python with OpenCV 3

Inside folder python_sample

- background_demo.py – shows an image
- camcapture_demo.py – shows captured webcam frames

• OpenCV information

- Official Site
 - <http://opencv.org/>
- Python-OpenCV Tutorial
 - http://docs.opencv.org/3.2.0/d6/d00/tutorial_py_root.html
 - Geometric Transformations of Images
 - http://docs.opencv.org/3.2.0/da/d6e/tutorial_py_geometric_transformations.html
 - Feature Matching + Homography to find Objects
 - http://docs.opencv.org/3.2.0/d1/de0/tutorial_py_feature_homography.html