1. a) $y \sim N(X\beta, I)$ WHERE $\beta \in \mathbb{R}^d$ UNKNOWN PAR. VECTOR

$X \in \mathbb{R}^{n \times d}$ DESIGN MATRIX

LIKELYHOOD : $p(y|\beta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i - \beta)^2\right)$

$x_i \rightarrow$ i-th row of X

$y_i \rightarrow$ i-th el. of y

PRIOR: $\pi_\lambda(\beta) \propto \prod_{j=1}^{p} \exp(-\lambda|\beta_j|)$

POSTERIOR: $p(\beta|y) = p(y|\beta) \cdot \pi_\lambda(\beta) =$

$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i-\beta)^2\right) \cdot \prod_{j=1} \exp(-\lambda|\beta_j|)$

$\hat{\beta}$ = MAXIMIZER OF POSTERIOR

$\hat{\beta} = \underset{\beta}{\arg\max}\, P(\beta|y) = \underset{\beta}{\arg\max}\left(\prod_{i=1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i-\beta)^2\right) \cdot \prod_{j=1}^{n} \exp(-\lambda|\beta_j|)\right)$

$= \underset{\beta}{\arg\max}\left(\exp\left(-\frac{1}{2}\sum_{i=1}^{n}(x_i-\beta)^2 - \lambda\sum_{j=1}^{n}|\beta_j|\right)\right) / \ln$

$= \underset{\beta}{\arg\max}\left(-\frac{1}{2}\sum_{i=1}^{n}(x_i-\beta)^2 - \lambda\|\beta_j\|_1\right)$

$\arg\max(f) = \arg\min(-f)$

$= \underset{\beta}{\arg\min}\left(\frac{1}{2}\sum_{i=1}^{n}(x_i-\beta)^2 + \lambda\|\beta\|_1\right)$

= LASSO ✓