

$$1. a) \quad y \sim N(X\beta, I)$$

WHERE $\beta \in \mathbb{R}^d$ UNKNOWN PAR. VECTOR
 $X \in \mathbb{R}^{n \times d}$ DESIGN MATRIX

$$\text{LIKELIHOOD: } p(y|\beta) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i - \beta)^2\right)$$

$x_i \rightarrow i\text{-th row of } X$

$y_i \rightarrow i\text{-th el. of } y$

$$\text{PRIOR: } \pi_\lambda(\beta) \propto \prod_{j=1}^d \exp(-\lambda|\beta_j|)$$

$$\begin{aligned} \text{POSTERIOR: } p(\beta|y) &= p(y|\beta) \cdot \pi_\lambda(\beta) = \\ &= \prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i - \beta)^2\right) \cdot \prod_{j=1}^d \exp(-\lambda|\beta_j|) \end{aligned}$$

$\hat{\beta}$ = MAXIMIZED OF POSTERIOR

$$\begin{aligned} \hat{\beta} &= \underset{\beta}{\operatorname{argmax}} p(\beta|y) = \underset{\beta}{\operatorname{argmax}} \left(\prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i - \beta)^2\right) \cdot \prod_{j=1}^d \exp(-\lambda|\beta_j|) \right) \\ &= \underset{\beta}{\operatorname{argmax}} \left(\exp\left(-\frac{1}{2} \sum_{i=1}^n (x_i - \beta)^2 - \lambda \sum_{j=1}^d |\beta_j|\right) \right) / \ln \\ &= \underset{\beta}{\operatorname{argmax}} \left(-\frac{1}{2} \sum_{i=1}^n (x_i - \beta)^2 - \lambda \|\beta\|_1 \right) \end{aligned}$$

$$\operatorname{argmax}(f) = \operatorname{argmin}(-f)$$

$$\begin{aligned} &\rightarrow = \underset{\beta}{\operatorname{argmin}} \left(\frac{1}{2} \sum_{i=1}^n (x_i - \beta)^2 + \lambda \|\beta\|_1 \right) \\ &= \text{LASSO} \checkmark \end{aligned}$$