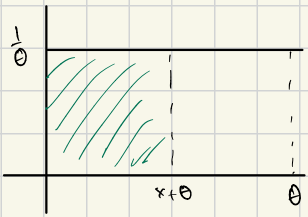
STAT - HW 3

(3) $X_1, \dots, X_n \sim$ iid UNIF $[0, \theta]$, $\theta \in \Theta := (0, \infty)$

SAMPLING DISTRIBUTION OF $\hat{\theta}_n - \theta$, WHERE $\hat{\theta}_n = \max\{X_1, \dots X_n\}$

$$P_\theta(\hat{\theta}_n - \theta \le x) = P_\theta(\max\{X_1, \dots X_n\} \le x + \theta)$$
$$= P_\theta(X_1 \le x + \theta \text{ AND } X_2 \le x + \theta \dots \text{ AND } X_n \le x + \theta)$$

THEY ARE IID $\Rightarrow \prod_{i=1}^{n} P_\theta(X_i \le x + \theta)$



$$\begin{cases} 0, & \text{IF } x + \theta < 0 \text{ OR } x < -\theta \\ \dfrac{x+\theta}{\theta} & \text{IF } 0 \le x + \theta < \theta \text{ OR } -\theta \le x \le 0 \\ 1 & \text{ELSE} \end{cases}$$

$\Rightarrow$ CDF $\quad F_\theta(x) := \begin{cases} 0 & \text{IF } x < -\theta \\ \left(1 + \frac{x}{\theta}\right)^n & -\theta \le x \le 0 \\ 1 & \text{ELSE} \end{cases}$

I CALCULATE THE q_p

$$P_\theta(\hat{\theta}_n - \theta \le q_p) = p \qquad \left(1 + \frac{q_p}{\theta}\right)^n = p$$

$$1 + \frac{q_p}{\theta} = p^{\frac{1}{n}}$$

$$q_p = \theta\left(p^{\frac{1}{n}} - 1\right)$$
$$= \boxed{p^{\frac{1}{n}}\theta - \theta}$$

BY THE HINT $\Rightarrow P_\theta\left(q_{\frac{\alpha}{2}} \le \hat{\theta}_n - \theta \le q_{1-\frac{\alpha}{2}}\right) = 1 - \alpha$

$$\left(\tfrac{\alpha}{2}\right)^{\frac{1}{n}}\theta - \cancel{\theta} \le \hat{\theta}_n - \cancel{\theta} \le \left(1 - \tfrac{\alpha}{2}\right)^{\frac{1}{n}}\theta - \cancel{\theta} \qquad / \cdot \frac{1}{\theta \hat{\theta}_n}$$

$$\left(\tfrac{\alpha}{2}\right)^{\frac{1}{n}} \cdot \frac{1}{\hat{\theta}_n} \le \frac{1}{\theta} \le \left(1 - \tfrac{\alpha}{2}\right)^{\frac{1}{n}} \cdot \frac{1}{\hat{\theta}_n} \qquad / \cdot (\,)^{-1}$$

$$\hat{\theta}_n \left(\tfrac{\alpha}{2}\right)^{-\frac{1}{n}} \ge \theta \ge \hat{\theta}_n \left(1 - \tfrac{\alpha}{2}\right)^{-\frac{1}{n}}$$

$$\Rightarrow CI_\alpha = \left[\hat{\theta}_n \left(1 - \tfrac{\alpha}{2}\right)^{-\frac{1}{n}}, \hat{\theta}_n \left(\tfrac{\alpha}{2}\right)^{-\frac{1}{n}}\right]$$