PS9

Monday, May 9, 2022 6:30 PM

Ex. 2

$R_t^{passive} = R_t^{stock\ index}$ $f^{passive} = 0,1\%$

$R_t^{active} = 2,2\% + R_t^{stock\ index} + \varepsilon_t$ $f^{active} = 1.2\%$; $\beta = 1$; $\sqrt{Var(\varepsilon_t)} = 3.5\%$

$R_t^{h.f.} = 1\% + 4(R_t^{active} - R_t^{passive})$ $r_f = 1\%$

ⓐ $R_t^{h.f.} = 1\% + 4(2.2\% + R_t^{stock\ index} + \varepsilon_t - R_t^{stock\ index}) = 1\% + 4(2.2\% + \varepsilon_t) = 1\% + 8.8\% + 4\varepsilon_t$

It can be seen that hedge fund's volatility comes only from the error term ε_t. Since the hedge applies 4 times leverage, the volatility will 4 times higher:

$\sqrt{Var(4\varepsilon_t)} = \sqrt{16\,Var(\varepsilon_t)} = 4 \cdot 3.5\% = 14\%$

ⓑ The hedge fund's beta = 0 since it buys and shorts the market at the same time, thus, the net position is 0 and $R_t^{stock\ index}$ is the only exposure to the market risk.

ⓒ $\alpha = E[R_t^{h.f}] - r_f = 4 \cdot 2.2\% = 8.8\%$

ⓓ Let's construct the investor's portfolio:

$40(2.2\% + R_t^{index} + \varepsilon_t) + 60 \cdot r_f = 40 \cdot 2,2\% + 40R_t^{index} + 40\varepsilon_t + 60 \cdot 1\%$

Let's find the replicating portfolio.

1) replicating the amount invested in the R_t^{index}: $40 \cdot R_t^{passive} = 40R_t^{index}$

2) replicating the exposure to ε_t gives: $10 \cdot R_t^{h.f.} = 10(1\% + 4(2.2\% + \varepsilon_t)) = 10 \cdot 1\% + 40 \cdot 2.2\% + 40\varepsilon_t$

3) as a result, the additional amount that should be invested in r_f: $50r_f$

Thus, we have the following replicating portfolio:

$50r_f + 10R_t^{h.t} + 40R_t^{passive} = 60 \cdot 1\% + 40 \cdot 2.2\% + 40\varepsilon_t + 40R_t^{index}$

Let's find the fair management fee for the hedge fund.

For the investor to be indeferent between 2 portfolios the following equation should hold:

$40 \cdot f^{active} = 10 \cdot f^{h.f} + 40 \cdot f^{passive} \Rightarrow$

$\Rightarrow 40 \cdot 1,2\% = 10 \cdot f^{h.f.} + 40 \cdot 0,1\%$

$f^{h.f} = \dfrac{40 \cdot 1,2\% - 40 \cdot 0,1\%}{10} = 4,4\%$

ⓔ $2\% = f_m^{h.f}$ $f_p = ?$

$4.4\% = f_p^{h.f.}(E[R_t^{h.f}] - r_f - f_m^{h.f}) + f_m^{h.f.} = f_p^{h.f}(8,8\% - 2\%) + 2\% = f_p\ 6,8\% + 2\% \Rightarrow$

$\Rightarrow f_p^{h.f.}\ 6.8\% = 2,4\% \Rightarrow f_p^{h.f} = \dfrac{2.4\%}{6.8\%} = 35,29\%$

(f) Actually, the hedge fund that charges 2-20 fee will be „cheaper" relative to the typical mutual fund. In this case the rational investor would prefer the hedge fund because the fees charged will be less and the return after fees will be higher than that of a mutual fund. It can also be inferred that α should determine the fees for active management. It has to be proportional to the α they earn: the higher the α the higher the fee should be (taking into account exposure and leverage).