Problem 1

a. Let V_t be the NAV of fund at time t.

$$V_{t+1} = V_t(1+R_{t+1}) - (\delta+f)V_t(1+R_{t+1})$$
$$= V_t(1-\delta-f)(1+R_{t+1})$$

b. $V_{t+1} = V_0(1-\delta-f)^{t+1}\prod_{k=0}^{t}(1+R_{k+1})$

$E(R_t) = R_f + \beta(u_m - R_f) + \varepsilon_t$.

devidend at $t+1$: $d_{t+1} = V_t(1+R_{t+1})\cdot\delta = \frac{V_{t+1}\cdot\delta}{1-\delta-f}$

① $\therefore$ at time t, the value of payout P to investor of future n period is

$$P_t = E\left[\sum_{i=1}^{n}\frac{V_{t+i}\delta}{1-\delta-f}\cdot\frac{1}{(1+k)^i}\right] = E\left[\frac{\delta}{1-\delta-f}\sum_{i=1}^{n}\frac{V_{t+i}}{(1+k)^i}\right] = E\left[\frac{\delta}{1-\delta-f}\sum_{i=1}^{n}\frac{V_t(1-\delta-f)^i\prod_{k=2}^{i}(1+R_{t+k})}{(1+k)^i}\right]$$

for $\frac{E_t(1+R_{t+1})}{1+k} = 1$

$$P_t = E\left(\frac{\delta}{1-\delta-f}\sum_{i=1}^{n}V_t(1-\delta-f)^i\right) = E\left(\delta V_t\sum_{i=0}^{n-1}(1-\delta-f)^i\right) = \delta V_t\cdot\frac{1-(1-\delta-f)^n}{1-(1-\delta-f)} = \delta V_t\frac{1-(1-\delta-f)^n}{\delta+f}$$

if $n\to\infty$, $P_t = \frac{\delta V_t}{\delta+f}$

② similarly for the value of management fee F of future n perod is

$$F_t = E\left(\sum_{i=1}^{n}\frac{V_{t+i}f}{1-\delta-f}\frac{1}{(1+k)^i}\right) = fV_t\frac{1-(1-\delta-f)^n}{\delta+f}$$

if $n\to\infty$ $F_t = \frac{fV_t}{\delta+f}$

③ Therefore, it has nothing to do with systematic or idiosyncratic risk, if $E_t(1+R_{t+1})/(1+k) = 1$ always holds.

c.

$$\text{discount} = \frac{V_t - V_t\frac{\delta}{\delta+f}}{V_t} = 1-\frac{\delta}{\delta+f} = \frac{f}{\delta+f} = \frac{0.44\%}{2.27\%+0.44\%} = 16.24\%$$

d.

No, we can't. In our analysis here, we have an important assumption that $\frac{E_t(1+R_{t+1})}{1+k} = 1$, which is not always true in real world.

However, it still provides some explanation in qunatitative way. The explanation that discount rate is dorminated by dividend and manage fee is somehow reasonable and it is quite similiar to real value (16.24% vs 14.4%). Additionally, this explanation comes from CAPM, simple but efficient.