PS5

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Problem 3:

(a) In order to find the equation that describes expected returns in equilibrium, we need find the values of α_0; F_1 and F_2. To do this let's build and solve a system of equations:

$$\begin{cases} 0.12 = \alpha_0 + 1F_1 + 0.5F_2 \\ 0.134 = \alpha_0 + 3F_1 + 0.2F_2 \\ 0.12 = \alpha_0 + 3F_1 + (-0.5)F_2 \end{cases}$$

Solving the system of equations gives the following values:

$\alpha_0 = 0,1$ $\quad F_1 = 0,01$ $\quad F_2 = 0,02 = \mu_{F_2}$

Thus, the equation that describes expected returns in equilibrium is given by:

$R_j = 0,1 + \beta_{j1} \cdot 0,01 + \beta_{j2} \cdot 0,02$

(b) $\mu_M - R_0 = 0,04$;

Portfolio 4: $\beta_{41} = 1$; $\beta_{42} = 0$

Portfolio 5: $\beta_{51} = 0$ $\quad \beta_{52} = 1$

R_0 - ? β_{5M} - ? β_{4M} - ?

$0,1 + 1 \cdot 0,01 = 0,11$ - expected return of the Portfolio 4

$0,1 + 0,02 = 0,12$ - expected return of the Portfolio 5

$\boxed{\mu - R_0 = \beta(\mu_{R_F} - R_0)}$ ⇒ we can use APT to find β_{4M} and β_{5M} and use $(R_M - R_0)$ as F_1

Since $\alpha = 0.1 \Rightarrow R_0 = 0,1$

Portfolio 4: $0,11 - 0,1 = \beta_{4M} \cdot 0,04 \Rightarrow \beta_{4M} = 0,25$

Portfolio 5: $0,12 - 0,1 = \beta_{5M} \cdot 0,04 \Rightarrow \beta_{5M} = 0,5$

Problem 4:

(a) False. CAPM implies that stocks with the same expected returns <u>should</u> have the same beta because if CAPM holds the following equation should hold: $\mu_i = R_0 + \beta_i(\mu_M - R_0)$. Since in our case $\mu_i = \mu_j$ and R_0, μ_M are the same for all the assets on the market, betas of the assets i and j has to be the same for CAPM to hold.

(b) False. CAPM doesn't consider idiosyncratic risk. It considers only systematic risk. Idiosyncratic risk is supposed to be diversifiable and is not rewarded with the excess return. It can also be observed from the following equation:

$R_i - R_0 = \alpha_i + \beta_i(R_M - R_0) + \varepsilon_i$, where $E[\varepsilon_i] = 0$, which represent the return for idiosyncratic risk.

(c) False. Since the tangency portfolio is also market portfolio ⇒ Market portfolio is mean-variance efficient and any risky asset's premium is proportional to it's covariance with the market portfolio: $\mu_i - R_0 = a_M \, Cov(R_i; R_M) \; \forall i$. It also holds for the market portfolio : $\mu_m - R_0 = a_m V(R_m)$. Thus, all risky assets' means and standard deviations are on the SML: $\mu_i = R_0 + \beta_i (R_m - R_0)$, where $\beta_i = \frac{Cov(R_i; R_m)}{Var(R_m)}$. It can be concluded that beta is relevant measure of risk for all risky assets capturing the risk of assets in relation to the market risk, but standard diviation in not an appropriate risk measure under CAPM since it doesn't capture the relation with the market risk, but it's still appropriate for the market portfolio under CAPM because the portfolio is MVE.

(d) True. Beta is appropriat risk measure for all individual risky aesets under CAPM (The argumentation is the same as in point c.)

(e) If $\alpha > 0 \Rightarrow$ the asset is undervalued because it's expected return is lower than the actual one. If you are a mean-variance efficient investor than you should invest in this asset unless it increases the Sharpe Ratio : max $SR = \sqrt{SR_{14}^2 + IR_i^2}$, where information ratio allows to take into account idiosyncratic risk.

(f) Yes, it has to be uncorrelated, otherwise it would be market risk. We can decompose the the risk of each asset: $\underbrace{Var[R_i]}_{\text{total risk}} = \underbrace{\beta^2 Var[R_m]}_{\text{systematic risk}} + \underbrace{Var[\varepsilon_i]}_{\text{idiosyncratic risk}}$. and under CAPM we have the following equation $R_j = \alpha_j + \beta_j (R_m - R_0) + \varepsilon_j$. where $E[\varepsilon_j] = 0$ and that also implies uncorrelated idiosyncratic risk since $\varepsilon \sim \mathcal{N}(0, \sigma^2)$ with $\sigma^2 = \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ \vdots & & \ddots \\ 0 & 0 & \sigma^2 \end{pmatrix}$