Fiche 2

1a) $\begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} = 3 \times 7 - 4 \times 5 = 21 - 20 = 1$ ✓

b) $\begin{vmatrix} 0 & 3 \\ 1 & 2 \end{vmatrix} = 0 \times 2 - 3 \times 1 = 0 - 3 = -3$ ✓

c) $\begin{vmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{vmatrix} = \sin^2\alpha - ((-\cos\alpha) . \cos(\alpha)) = \sin^2\alpha + \cos$

d)
$$
\begin{vmatrix} 0 & 1 & 4 & 5 \\ -1 & -2 & -4 & 6 \\ 0 & 0 & -1 & 0 \\ 1 & 2 & 7 & 2 \end{vmatrix} = -A_{33} = -8
$$

$$
A_{33} = (-1)^{3+3} \det(M_{33}) = \begin{vmatrix} 0 & 1 & 5 \\ -1 & -2 & 6 \\ 1 & 2 & 2 \end{vmatrix}
$$

$$
\begin{array}{ccc} 0 & 1 & 5 \\ -1 & -2 & 6 \end{array}
$$

$$
-10 + 6 + 10 + 2 = 8
$$

Foclie 21

3) $\begin{vmatrix} \lambda+2 & -1 & 3 \\ 2 & \lambda-1 & 2 \\ 0 & 0 & \lambda+4 \end{vmatrix} = 0$ $\Rightarrow (\lambda+2)(\lambda-1)(\lambda+4)-(2\times-1\times(\lambda+4)) = 0$

$\lambda+2 \quad -1 \quad 3$

$2 \quad \lambda-1 \quad 2$

$\Rightarrow (\lambda+2)(\lambda-1)(\lambda+4)+2(\lambda+4)=0$

$((\lambda+2)(\lambda-1)+2)(\lambda+4)=0$

$(\lambda+2)(\lambda-1)+2=0 \quad \wedge \quad \lambda+4=0$

$\lambda^2+2\lambda+2\lambda-2+2=0 \qquad \lambda=-4$

$\lambda^2+\lambda=0 \qquad \wedge \quad \lambda=-4$

$\lambda(\lambda+1)=0$

$\lambda=0 \wedge \lambda=-1 \wedge \lambda=-4$

4) $\det(cA) = c^n \det(A)$

$$\det(A) = \begin{vmatrix} c\alpha_{11} & c\alpha_{12} & c\alpha_{13} & \alpha_{1m} \\ c\alpha_{21} & c\alpha_{22} & c\alpha_{32} \\ c\alpha_{31} & c\alpha_{32} & c\alpha_{23} \\ c\alpha_{m1} & c\alpha_{m2} & c\alpha_{m3} & c\alpha_{mm} \end{vmatrix} = c^n \begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{1m} \\ \alpha_{21} & \alpha_{22} & \alpha_{32} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{m1} & \alpha_{m2} & \alpha_{m3} & \alpha_{mm} \end{vmatrix} = c^n \det(A)$$

5)

$|A| = 3 \quad |B| = -5$

a) $|A^T| = |A| = 3$ ✓

b) $|AB| = |A| \times |B| = 3 \times -5 = -15$ ✓

c) $|A^4| = |A| \times |A| \times |A| \times |A| = 3^4 = 17 \times 3 \times 8 \cdot 1$
   $= 3^4 = 81$

d) $|B^{-1}| = -\frac{1}{5}$