3)

a) $\left[\begin{array}{ccc|c} 1 & c & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k^2-4 & k-2 \end{array}\right]$ $Car(A) = Car([A|B]) = n = 3$

$k^2 - 4 \neq 0 \quad \wedge \; k - 2 \neq 0$

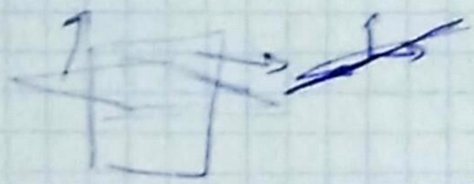
$k^2 \neq 4 \quad \wedge \; k \neq 2$

$k \neq \pm\sqrt{4} \quad \wedge \; k \neq 2$

$k \neq \pm 2 \quad \wedge \; k \neq 2$

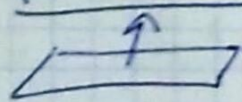
$k \neq 2 \wedge k \neq -2 \wedge k \neq 2$

$k \in \mathbb{R} \setminus \{-2, 2\}$



---

b) produto escalar dos vetores da reta ⊥ pl.



$(1, 1, (k^2-5)) \cdot (1, \frac{2}{3}, \frac{1}{3}) = 0$

$1 + \frac{2}{3} + \frac{1}{3}(k^2 - 5) = 0$

$\frac{3}{3} + \frac{2}{3} + \frac{1}{3}k^2 - \frac{5}{3} = 0$

$\frac{1}{3}k^2 = 0$

$k = 0$

vetor da reta

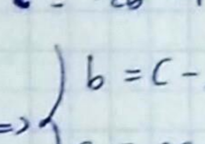
$\begin{cases} (1, 1, -1) \cdot (a, b, c) = 0 \\ (1, 2, 1) \cdot (a, b, c) = 0 \end{cases} \Leftrightarrow$

$\begin{cases} a + b - c = 0 \\ a + 2b + c = 0 \end{cases}$ Se $a = 1$

$\begin{cases} 1 + b - c = 0 \\ 1 + 2b + c = 0 \end{cases} \Leftrightarrow \begin{cases} b = c - 1 \\ c = -2b - 1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} b = c - 1 \\ c = -2(c-1) - 1 \end{cases} \Leftrightarrow \begin{cases} b = c - 1 \\ c = -2c + 2 - 1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} b = c - 1 \\ 3c = 1 \end{cases} \Leftrightarrow \begin{cases} b = \frac{1}{3} - 1 \\ c = \frac{1}{3} \end{cases} \Leftrightarrow \begin{cases} b = \frac{2}{3} \\ c = \frac{1}{3} \end{cases}$



5) a) Reta que passa $Q(1, 0, 1)$

$\vec{u}(-3, 2, 1)$

$(x, y, z) = (1, 0, 1) + k(-3, 2, 1), \; k \in \mathbb{R}$

8) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right) \underset{L_3 \leftarrow L_3 - 2L_1}{\overset{L_2 \leftarrow L_2 - L_1}{\sim}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & -1 & -1 & 0 \end{array}\right] \overset{L_2 \leftrightarrow L_3}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -3 & 2 \end{array}\right]$

$\begin{cases} x + y + z = 1 \\ -y - z = 0 \\ -3z = 2 \end{cases} \Leftrightarrow \begin{cases} -y = z \\ z = \frac{2}{-3} \end{cases} \Leftrightarrow \begin{cases} y = -z \\ z = -\frac{2}{3} \end{cases} \Leftrightarrow \begin{cases} y = \frac{2}{3} \\ z = -\frac{2}{3} \end{cases} \Leftrightarrow \begin{cases} x + \frac{2}{3} - \frac{2}{3} = 1 \\ = \end{cases}$

~~T. A1 - 2012~~

$$\begin{cases} x = 1 \\ y = \frac{2}{3} \\ z = \frac{2}{3} \end{cases}$$

~~x, y, t~~

$$C.S = \left\{1, \frac{2}{3}, -\frac{2}{3}\right\}$$

Teste 1 - 2014

a) $\det(2M^{-1} . M^T) = 2^4 . \det(M^{-1}) . \det(M^T) =$

$= 2^4 \times \frac{1}{\det(M)} \times \det(M) = 2^4 = 16$

$2 \times 2 \times 2 \times 2$: 4, 8, 16

b) $\begin{vmatrix} 5a_2 - 3b_2 & a_2 & -9 & -2 \\ 5a_1 - 3b_1 & a_1 & -5 & 2 \\ 5a_3 - 3b_3 & a_3 & -1 & 5 \\ 5a_4 - 3b_4 & a_4 & -4 & 6 \end{vmatrix} = - \begin{vmatrix} 5a_1 - 3b_1 & a_1 & -5 & 2 \\ 5a_2 - 3b_2 & a_2 & -9 & -2 \\ 5a_3 - 3b_3 & a_3 & -1 & 5 \\ 5a_4 - 3b_4 & a_4 & -4 & 6 \end{vmatrix} =$

$= - \left(\cancel{5 \begin{vmatrix} a_1 & a_1 & -5 & 2 \\ a_2 & a_2 & -9 & -2 \\ a_3 & a_3 & -1 & 5 \\ a_4 & a_4 & -4 & 6 \end{vmatrix}}^{\,0} - 3 \begin{vmatrix} b_1 & a_1 & -5 & 2 \\ b_2 & a_2 & -9 & -2 \\ b_3 & a_3 & -1 & 5 \\ b_4 & a_4 & -4 & 6 \end{vmatrix}^T \right) =$

$= 3 \begin{vmatrix} b_1 & b_2 & b_3 & b_4 \\ a_1 & a_2 & a_3 & a_4 \\ -5 & -9 & -1 & -4 \\ 2 & -2 & 5 & 6 \end{vmatrix} = -3 \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ -5 & -9 & -1 & -4 \\ 2 & -2 & 5 & 6 \end{vmatrix} =$

$= (-1)(-3) \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ -5 & -9 & -1 & -4 \\ -2 & 2 & -5 & -6 \end{vmatrix} = (-1).(-3).(-2) = 3.(-2) = -6$

2) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -2 & 0 & 3 \\ 3 & -6 & 2 & 9 \end{bmatrix} \underset{L_2 (\leftrightarrow) L_1}{\sim} \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 3 & -6 & 2 & 9 \end{bmatrix} \underset{L_3 \leftarrow L_3 - 3L_1}{\sim} \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

$a + 3 = \underline{1} \ \wedge \ b - a = 2 \Leftrightarrow a = 1 - 3 \ \wedge \ b - a = 2 \Leftrightarrow$

$\Leftrightarrow a = -2 \ \wedge \ b + 2 = 2 \Leftrightarrow a = -2 \wedge b = 0$

a) ... A_{4-r} → TAI - 2013

3) $$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 5 \\ 0 & a+3 & 1 & 0 & -b-1 \\ 0 & 0 & b-a & 0 & -2(1+b) \end{array}\right]$$

no caso em que o sistema é impossível

$car(A) = 2$, $b - a = 0$

$car([A|B]) = 3$, $-2(1+b) \neq 0$

$nul(A) = 4 - 2 = 2$

~~no caso em que o sistema~~

O sistema não é possível e determinado

$car(A) = 3 \wedge car([A|B]) = 3 \neq 4$ portanto o sistema não é possível determinado

~~O sistema é possível e indeterminado~~

~~$car(A) = \dots \wedge car([A|B]$~~

$b - a = 0 \quad \wedge -2(1+b) = 0 \Leftrightarrow b - a = 0 \wedge -2 - 2b = 0$

$b = -1$

$-1 - a = 0$

$-a = 1$

$a = -1 \quad \wedge b = -1$

$D \in \mathcal{C}(C)$

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 0 & -1 \\ 1 & -2 & 0 & 3 & 5 \\ 3 & -6 & 2 & 9 & 13 \end{array}\right] \xrightarrow{L_3 \leftarrow L_3 - 3L_2} \left[\begin{array}{cccc|c} 0 & 1 & 1 & 0 & -1 \\ 1 & -2 & 0 & 3 & 5 \\ 0 & 0 & 2 & 0 & -2 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 5 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 2 & 0 & -2 \end{array}\right]$$

$car(A) = car([A|B])$ o sistema é possível logo $D \in \mathcal{C}(C)$

$$Q = \begin{bmatrix} -2 & -3 & 1 \\ 2 & 1 & 0 \\ 4 & 3 & -1 \end{bmatrix} \quad det(Q) = \begin{vmatrix} -2 & -3 & 1 \\ 2 & 1 & 0 \\ 4 & 3 & -1 \end{vmatrix} = 2 + 6 - 3k$$

$\begin{matrix} -2 & -3 & 1 \\ 2 & 1 & 0 \end{matrix}$

$= 14 - 4k$

$14 - 4 \neq 0$

$-4k \neq -14 \quad k \neq \frac{14}{4}$

$\mathbb{R} \setminus \{\frac{14}{4}\}$

6) $\cancel{Q^{-1} = \frac{Adj\, Q}{\det(Q)}}$ Não é preciso

$$\left[\begin{array}{ccc|ccc} -2 & -3 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & -1 & 0 & 0 & 1 \end{array}\right] \underset{L_2 \leftarrow L_2 + L_1}{\sim} \left[\begin{array}{ccc|ccc} -2 & -3 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 1 & 0 \\ 0 & 3 & -1 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\underset{L_3 \leftarrow L_3 + \frac{3}{2}L_2}{\sim} \left[\begin{array}{ccc|ccc} -2 & -3 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{3}{2} & \frac{3}{2} & 1 \end{array}\right] \underset{\substack{L_2 \leftarrow -\frac{1}{2}L_2 \\ L_1 \leftarrow -\frac{1}{2}L_1}}{\overset{L_3 \leftarrow -2L_3}{\sim}} \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -3 & -3 & -2 \end{array}\right]$$

$3 - 1(-2x) = 0$

$2x = -3$

$x = \frac{-3}{2}$

$$\underset{L_2 \leftarrow L_2 + \frac{1}{2}L_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -2 & -2 & -\frac{3}{2} \\ 0 & 0 & 1 & -3 & -3 & -2 \end{array}\right] \underset{L_1 \leftarrow L_1 + \frac{1}{2}L_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 0 & -2 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 1 & 0 & -2 & -2 & -\frac{3}{2} \\ 0 & 0 & 1 & -3 & -3 & -2 \end{array}\right]$$

$$\underset{L_1 \leftarrow L_1 - \frac{3}{2}L_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{4} \\ 0 & 1 & 0 & -2 & -2 & -\frac{3}{2} \\ 0 & 0 & 1 & -3 & -3 & -2 \end{array}\right]$$

$$Q^{-1} = \begin{bmatrix} 1 & \frac{3}{2} & -\frac{3}{4} \\ -2 & -2 & -\frac{3}{2} \\ -3 & -3 & -2 \end{bmatrix}$$

$-2 + \frac{6}{2}$

$\frac{-4+6}{2}$

$\frac{2}{2}$

$-\frac{3}{2} + \frac{6}{2}$

$\frac{3}{2}$

$-\frac{3}{2} - \frac{3}{2}\left(\frac{3}{2}\right)$

$-3 + \frac{9}{4}$

$\frac{-12+9}{4}$

$$Q^{-1} X^T = C$$

$$Q\, Q^{-1} X^T = Q\, C$$

$$X^T = Q \cdot C$$

$$X^T = \underset{(3\times 3)}{\begin{bmatrix} -2 & -3 & 1 \\ 2 & 1 & 0 \\ 0 & 3 & -1 \end{bmatrix}} \underset{(3\times 4)}{\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & -2 & 0 & 3 \\ 3 & -6 & 2 & 1 \end{bmatrix}}$$

$$X^T = \underset{(3\times 4)}{\begin{bmatrix} 0 & -2 & 0 & 0 \\ 1 & 0 & 2 & 3 \\ 0 & 0 & -2 & 0 \end{bmatrix}}$$

$$X = \begin{bmatrix} 0 & -2 & 0 & 0 \\ 1 & 0 & 2 & 3 \\ 0 & 0 & -2 & 0 \end{bmatrix}^T \Leftrightarrow X = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & 3 & 0 \end{bmatrix} //$$

$\rightarrow$ T.A1 - 2013 $\leftarrow$

a) $M = \begin{bmatrix} 1 & k & -8 \\ 0 & k & -9 \\ 0 & 1 & 5 \end{bmatrix} = 5k + 9$

$\begin{matrix} 1 & k & -8 \\ 0 & k & -9 \end{matrix}$

$5k + 9 \neq 0 \Leftrightarrow 5k \neq -9 \Leftrightarrow k \neq \frac{-9}{5}$

$\mathbb{R} \setminus \{-\frac{9}{5}\}$

b) ~~não existe~~

$\begin{bmatrix} 1 & k & -8 \\ 0 & k & -9 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 & -2 \\ 0 & -5 & -9 \\ 0 & 1 & -k \end{bmatrix} = I_m$

$\begin{bmatrix} 1 & -2-5k-8 & -2-9k+8k \\ 0 & -5k-9 & -9\cancel{k+9k} \\ 0 & 0 & -9-5k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$-9 - 5k = 1$

$-5k = 10$

$k = \frac{10}{-5} = -2$

$-2 - 5 \times (-2) - 8 = -2 - 8 + 10 = 0$

2a) $\begin{cases} x + 8y + \beta z = 3(\alpha - 3) \\ 0x + 4\alpha y + z = -4 \\ 0 + 0 - 4z = -2\beta \end{cases}$

P. D.

~~4α~~ $-2\beta \neq 0$ $\beta \neq 0$

$\wedge$ $4\alpha \neq 0$ $\alpha \neq 0$

$3(\alpha - 3) \neq 0$ $\alpha - 3 \neq 0$

$\alpha \neq 3$

~~$\mathbb{R}$~~

$\left[\begin{array}{ccc|c} 1 & 8 & \beta & 3(\alpha-3) \\ 0 & 4\alpha & 1 & -4 \\ 0 & 0 & -4 & -2\beta \end{array}\right]$

3a) $N(A)$ $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 4 & 0 \\ -1 & -3 & -9 & 0 \end{array}\right] \underset{L_3 \leftarrow L_3 + L_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & -3 & -12 & 0 \end{array}\right] \underset{L_3 \leftarrow L_3 + 3L_2}{\sim}$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\begin{cases} x - 3z = 0 \\ y + 4z = 0 \\ z = t \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x = 3t \\ y = -4t \\ z = t \end{cases}$

$N(A) = \{(3t, -4t, t), t \in \mathbb{R}\}$

b)

$B = 6C_1 + 5C_2 - 9C_3$

c) $B = \begin{bmatrix} 0 \\ 5 \\ -15 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ -6 \end{bmatrix} + \begin{bmatrix} 27 \\ -36 \\ 81 \end{bmatrix} = \begin{bmatrix} 33 \\ -31 \\ 60 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 33 \\ 0 & 1 & 4 & -31 \\ -1 & -3 & 9 & 60 \end{array}\right] \underset{L_3 \leftarrow L_3 + L_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 33 \\ 0 & 1 & 4 & -31 \\ 0 & -3 & 6 & 93 \end{array}\right] \underset{L}{\sim}$