17) uma matriz é não singular

$\det(A) = 0$

$$\begin{bmatrix} \alpha-4 & 0 & 10 \\ 4 & \alpha+5 & 1 \\ 2 & 0 & \alpha-3 \end{bmatrix} = (\alpha-4)(\alpha+5)(\alpha-3)+0+0-20(\alpha+5)$$

$$\begin{matrix} \alpha-4 & 0 & 10 \\ 4 & \alpha+5 & 1 \end{matrix}$$

$((\alpha-4)(\alpha-3)-20)\cdot(\alpha+5)$

$=(\alpha^2-3\alpha-4\alpha+12-20)(\alpha+5)=$

$=(\alpha^2-7\alpha+6)(\alpha+5)$

$(\alpha^2-7\alpha+6)(\alpha+5)=0$

$\alpha^2-7\alpha+6=0 \quad \vee \quad \alpha+5=0$

$\alpha=-5$

$\frac{7\pm\sqrt{(7)^2-(4\times 6)}}{2}=0$

$\Leftrightarrow \frac{7\pm\sqrt{49-24}}{2} = \alpha\, \frac{7+\sqrt{25}}{2} \vee \frac{7+5}{2}$

$\alpha=6 \quad \alpha=1 \quad \wedge \quad \alpha=-5$

18) $$\begin{bmatrix} \beta & 0 & 1 \\ 0 & \beta-1 & 1 \\ 0 & 1 & \beta+5 \end{bmatrix} = \beta\cdot(\beta-1)\cdot(\beta+5) - \beta$$

$$\begin{matrix} \beta & 0 & 1 \\ 0 & \beta-1 & 1 \end{matrix}$$

$\beta((\beta-1)\cdot(\beta+5)-1)$

$\beta\cdot(\beta^2+5\beta-\beta-5-1)$

$\beta\cdot(\beta^2+4\beta-6)$

$\beta=0 \quad \vee \quad \beta=\frac{-4\pm\sqrt{16-(4.1.-6)}}{2}$

$\beta=\frac{-4+\sqrt{40}}{2}$

$\vee \beta=\frac{-4-\sqrt{40}}{2}$

?

$\Leftrightarrow \beta=0 \vee \beta=-2+\sqrt{10} \vee \beta=-2-\sqrt{10}$

19) $\det(A) \neq 0$

a matriz dos coeficientes é quadrada

20) $A = \begin{bmatrix} 1 & -1 & -1 \\ 4 & 2 & -4 \\ 3 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 6 \\ -1 \end{bmatrix}$

$$\det(A) = \begin{bmatrix} 1 & -1 & -1 \\ 4 & 2 & -4 \\ 3 & 2 & -1 \end{bmatrix} = -\cancel{2} - \cancel{8} + 12 + \cancel{6} + \cancel{8} - 4 = 12$$

$$\begin{matrix} 1 & -1 & -1 \\ 4 & 2 & -4 \end{matrix}$$

$$X = \frac{\begin{vmatrix} 0 & -1 & -1 \\ 6 & 2 & -4 \\ -1 & 2 & -1 \end{vmatrix}}{12} = \frac{\begin{vmatrix} 0 & -1 & -1 \\ 6 & 2 & -4 \\ -1 & 2 & -1 \end{vmatrix} \begin{matrix} 0 & -1 & -1 \\ 6 & 2 & -4 \end{matrix}}{12} = \frac{-12 - 4 - 2 + 0 - 6}{12}$$

$$= \frac{-24}{12} = -2$$

$$Y = \frac{\begin{vmatrix} 1 & 0 & -1 \\ 4 & 6 & -4 \\ 3 & -1 & -1 \end{vmatrix} \begin{matrix} 1 & 0 & -1 \\ 4 & 6 & -4 \end{matrix}}{12} = \frac{-6 + \cancel{4} + 18 - \cancel{4}}{12} = \frac{12}{12} = 1$$

$$Z = \frac{\begin{vmatrix} 1 & -1 & 0 \\ 4 & 2 & 6 \\ 3 & 2 & -1 \end{vmatrix} \begin{matrix} 1 & -1 & 0 \\ 4 & 2 & 6 \end{matrix}}{12} = \frac{-2 - 18 - 12 - 4}{12} = \frac{-18 - 18}{12} =$$

$$= \frac{-36}{12} = -3$$

$\{-2, 1, -3\}$ ✓

21) $AB = AC$

? $\det(AB) = \det(A) . \det(B)$

$\det(AC) = \det(A) . \det(C)$

22)

a) A é uma matriz $n \times n$

$\det(A . A^T) = \det(A) . \det(A^T) = \det(A) \det(A) =$

$\det(A) = \det(A^T)$

$= (\det(A))^2 \geq 0$

b) $\det(AB) = \det(I_n) = 1$

como $\det(AB) = \det(A) \times \det(B) = 1$

logo

$\det(A) \neq 0 \wedge \det(B) \neq 0$

c) Se A é uma matriz Singular logo A não é invertível logo $\det(A) = 0$

$\det(AB) = \det(A) \times \det(B) = 0 \times \det(B) = 0$

logo AB não é invertível logo a matriz AB é Singular

d) ~~det~~ $A^2 = A$

$\det(A^2) = \det(A)$?

e) $\det(A^{-1}) = \dfrac{1}{\det(A)}$

Como $A^{-1} = A$ então $\det(A^{-1}) = \det(A)$

$$\det(A) = \frac{1}{\det(A)}$$

$$\det(A) \times \det(A) = 1$$

$$(\det(A))^2 = 1$$

$$\det(A) = \pm\sqrt{1}$$

$$\det = \pm 1$$

f) $\det(\operatorname{adj} A) = \det(A^2)$

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj} A$$

$$\operatorname{adj} A = A^{-1} \cdot \det(A)$$

$$\det(\operatorname{adj} A) = \det((A^{-1}) \cdot \det(A)) =$$

$$= (\det(A))^3 \cdot \det(A^{-1}) = \frac{(\det(A))^3}{\det(A)} = (\det(A))^2 = \det(A^2)$$

(Exercício 24 com n em vez de 3)

23)

a) $\det(AA^T) = \det(A) \cdot \det(A^T) = \det(A) \times \det(A) = (\det(A))^2 = \det(A^2)$

$\det(A) = \det(A^T)$

Verdadeira

b) Falsa. Se A for uma matriz $n \times n$, $n = 2$

$$\det(-A) = (-1)^2 \cdot \det(A) = \det(A) \neq -\det(A)$$

c) $A^T = A^{-1}$

$$\det(A^{-1}) = \frac{1}{\det(A)} \Leftrightarrow \det(A^T) = \frac{1}{\det(A)} \Leftrightarrow \text{Falsa}$$

$$= \det(A) = \frac{1}{\det(A)} \Leftrightarrow \det(A)^2 = 1 \Leftrightarrow \det(A) = \pm 1$$

d) $\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix} = \underline{1} - 1 + 0 + 0 = 0$ falsa

$\begin{matrix}1&0&1\\0&1&0\end{matrix}$

e) ~~falsa~~ ~~eu não percebi isto~~

verdade. ~~no ppt~~

$AX = 0$

(18.?) → ? A solução não trivial $\Leftrightarrow \det(A) = 0$

f) $A^4 = I_n$

$\det(A^4) = \cancel{\det} I_n) \Leftrightarrow$

$\Leftrightarrow (\det(A))^4 = 1$

$\det(A) = 1$ verdade falsa

g) $A^2 = A$?

h) verdade. $\det(AB) = \det(A) \times \det(B) = 0$

i) ~~verdade~~ falsa $\det((AB) = \det(A) . \det B) = \det(B) \times \det(A)$

$= \det(BA)$

j) ~~verdade~~ falsa ???