2 a) $$\int \frac{e^{\arcsin x}}{(1-x^2)^{1/2}}\,dx = \int e^{\overbrace{\arcsin(x)}^{u(x)}} \cdot \underbrace{\frac{1}{\sqrt{1-x^2}}}_{u'(x)}\,dx =$$

$$e^{\arcsin(x)} + C,\ C \in \mathbb{R}$$

$\int f'(x) \cdot g(x) = f(x)g(x) - \int f(x)\,g'(x)\,dx$

b) $$\int tg^2(x)\,dx = \int tg^2(x) + 1 - 1\,dx = \int sec^2 - 1\,dx = \int sec^2\,dx - \int 1\,dx$$

$$tg\,x + C - x,\ C \in \mathbb{R}$$

$\int f'(x) \cdot g(x) = f(x)g(x) - \int f(x) g'(x)dx$

c) $$\int \underbrace{\frac{1}{x}}_{u'(x)} \cdot \cos(\underbrace{\ln(x)}_{u(x)}) = \sin(\ln(x)) + C, C \in \mathbb{R}$$

d) $$\int \frac{6}{x(\ln(4x))^3} dx = 6\int \frac{\cancel{6}}{\cancel{6}x} \cdot (\ln(4x))^{-3} dx = 6\int \underbrace{\frac{1}{x}}_{u'(x)} \cdot (\underbrace{\ln(4x)}_{u(x)})^{-3} = 6 \cdot \frac{(\ln(4x))^{-3+1}}{-3+1} + C$$

$$\boxed{\ln(4x)' = \frac{(4x)'}{4x} = \frac{4}{4x} = \frac{1}{x}}$$

$$= \frac{6}{-2}(\ln(4x))^{-2} + C, C \in \mathbb{R} = \frac{-3}{\ln^2(4x)} + C, C \in \mathbb{R}$$

e) $\int \frac{e^{3x}}{(e^{3x}-2)^6}\,dx = \frac{e^{3\left(\frac{\ln(t-2)}{3}\right)}}{t^6} \times \frac{1}{3(t-2)}\,dt = \frac{1}{3}\int \frac{(t-2)}{t^6 \cdot (t-2)} =$

$// \int f'(x) \cdot g(x) = f(x) g(x) - \int f(x) g'(x) dx$

Subs: $f(x) = t$

$t = e^{3x} - 2$

$-e^{3x} = -t - 2$

$e^{3x} = t - 2$ — função diferenciável e invertível em $\mathbb{R}^+$

$3x = \ln(t-2)$

$x = \frac{\ln(t-2)}{3}$

$$x' = \left(\frac{\ln(t-2)}{3}\right)' = \left(\frac{1}{3}\ln(t-2)\right)' = \frac{1}{3}'\ln(t-2) + \frac{1}{3}(\ln(t-2))' = \frac{1}{3} \cdot \frac{(t-2)'}{t-2} = \frac{1}{3(t-2)}$$

$$= \frac{1}{3}\int 1 \cdot t^{-6} = \frac{1}{-3 \times 5\, t^5} + C, C \in \mathbb{R}$$

$$= \frac{1}{-15(e^{3x}-2)^5}$$

g) $\displaystyle\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx = \int \underbrace{\frac{1}{x}}_{u'(x)} \cdot \frac{1}{\sqrt{1+(\underbrace{\ln(x)}_{u(x)})^2}} =$

$\int f'(x)\, g(x) = f(x)\, g(x) - \int f(x)\, g'(x)\, dx$

$$= \frac{x^{-1}}{\sqrt{1+(\ln(x))^2}}\,dx = \int \frac{\frac{1}{x}}{\sqrt{1+(\ln(x))^2}}\,dx = \operatorname{arcsen}(\ln(x)) + c,\ c \in \mathbb{R}$$

O(f) fica para depois.

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$$\int e^x \sqrt{e^x+1}\,dx = \int \cancel{u} \cdot \sqrt{u+1} \cdot \frac{1}{\cancel{u}}\,du =$$

Subst:

$u = e^x$

$x = \ln(u)$

$x' = (\ln(u))' = \frac{1}{u}$

$$= \int 1 \cdot \sqrt{u+1}\,du = \int \underbrace{1}_{f'(x)} \cdot \underbrace{(u+1)}_{f(x)}^{\frac{1}{2}}\,du = \frac{(u+1)^{\frac{3}{2}}}{\frac{3}{2}} \overset{x=e^x}{=} \frac{2\sqrt[2]{(u+1)^3}}{3}$$

$$= \frac{2\sqrt[2]{(e^x+1)^3}}{3} + C, C \in \mathbb{R}$$

i) $\displaystyle\int \frac{1}{x \cdot \ln(x)}\,dx = \int \frac{1}{\cancel{e^{u}}\, u} \cdot \cancel{e^{u}}\,du$

subst: $\ln(x) = u$

$x = e^{u}$

$x' = e^{u}$

$\varphi(u)$ e difeomorfismo e invertível em $\mathbb{R}^+$

$\underset{x \in \mathbb{R}}{=} \displaystyle\int \frac{1}{u}\,du = \ln|u| + C,\ C \in \mathbb{R}$

$= \ln|\ln|x|| + C,\ C \in \mathbb{R}$

j) $\displaystyle\int \frac{1}{\sqrt{x}}\, e^{\sqrt{x}}\,dx =$

$\int f'(x)\, g(x) = f(x)\,g(x) - \int f(x)\, g'(x)\,dx$

subs

$u = \sqrt{x}$

$x = u^2$

$x' = (u^2)' =$

$= 2u$

$\varphi(u)$ e difeomorfismo e invertível em $\mathbb{R}^+$

$\displaystyle\int \frac{1}{\cancel{u}}\, e^{u} \cdot 2\cancel{u}\,du =$

$= 2\displaystyle\int \frac{1}{f'(x)}\, e^{f(x)}\,dx =$

$= 2 \cdot e^{u} + C,\ C \in \mathbb{R}$

$= 2e^{\sqrt{x}} + C,\ C \in \mathbb{R}$