→ 1ª Prova Escrita - ALGA - 1h30m E

1) a) $CA = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} = \cancel{\begin{bmatrix} \cancel{\ } & 0 & 0 \\ 0 & 0 \end{bmatrix}}$ ~~matriz nula não existe~~

$2 \times 3 \quad 2 \times 2$

não existe

$-2 -6 \quad +2+6$

$3+9$

$4 + \cancel{2} + \cancel{4} + \cancel{4} - \cancel{2} - \cancel{4}$

$CD = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 0 & -1 \end{bmatrix}$ ✓

b) $D = \begin{bmatrix} 2 & 0 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} \underset{L_2 \leftarrow L_2}{\xrightarrow{L_1 \to \frac{1}{2}L_1}} \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 1 & 1 \end{bmatrix} \xrightarrow{L_2 \leftarrow L_2 + 2L_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \underset{L_2 - L_3}{\xrightarrow{L_1 - L_3}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$car(D) = 2$ ✓

c) $E = CD$ $\quad E = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} =$

$= \begin{bmatrix} 5 & 3 \\ 0 & -1 \end{bmatrix}$ ✓

$\det(E) = -5+0 = -5 \neq 0$ logo E é

$car(E) = 2 =$ (nº de colunas) logo E é invertível

$$\left[\begin{array}{cc|cc} 5 & 3 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \underset{L_2 \leftarrow -L_2}{\overset{L_1 \leftarrow \frac{1}{5}L_1}{\sim}} \left[\begin{array}{cc|cc} 1 & 3/5 & 1/5 & 0 \\ 0 & 1 & 0 & -1 \end{array}\right] \overset{L_1 - \frac{3}{5}L_2}{\sim} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{5} & \frac{3}{5} \\ 0 & 1 & 0 & -1 \end{array}\right]$$

$$E^{-1} = \begin{bmatrix} 1/5 & 3/5 \\ 0 & -1 \end{bmatrix}$$

~~d) $X E^T + A = B \Leftrightarrow E^T X = (B - A)$~~

~~$X \begin{bmatrix} 5 & 3 \\ 0 & -1 \end{bmatrix}^T = B - A \Rightarrow \begin{bmatrix} 5 & 0 \\ 3 & -1 \end{bmatrix} X = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}$~~

~~$X \begin{bmatrix} 5 & 0 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \quad \left[\begin{array}{cc|cc} 5 & 0 & 1 & 0 \\ 3 & -1 & 0 & 3 \end{array}\right]$~~

~~$X \begin{bmatrix} 5 & 0 \\ 3 & -1 \end{bmatrix}$~~

d) $X E^T . (E^T)^{-1} = (B - A) . (E^T)^{-1}$

$X = (B - A) . (E^T)^{-1}$

$X = \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \right) (E^{-1})^T$

$X = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} . \begin{bmatrix} 1/5 & 0 \\ 3/5 & -1 \end{bmatrix}$

$X = \begin{bmatrix} 1/5 & 0 \\ 9/5 & -3 \end{bmatrix}$

1) ~~$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_3 & b_2 & c \end{vmatrix}$~~ ~~$| a_1$~~

2)

$$\begin{cases} 2x_1 + ax_2 - x_3 = 0 \\ x_2 + bx_3 = 1 \\ -2x_1 + \quad + x_3 = b \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & a & -1 & 0 \\ 0 & 1 & b & 1 \\ -2 & 0 & 1 & b \end{array}\right]$$

~~$1-b=0 \quad -b=-1 \quad$ logo $b=1$ ... sistema~~ ser impossível

$$\left[\begin{array}{ccc|c} 2 & a & -1 & 0 \\ 0 & 1 & b & 1 \\ -2 & 0 & 1 & b \end{array}\right] \overset{L_3 \leftarrow L_3 + L_1}{\sim} \left[\begin{array}{ccc|c} 2 & a & -1 & 0 \\ 0 & 1 & b & 1 \\ 0 & a & 0 & b \end{array}\right] \overset{L_3 \leftarrow L_3 - aL_2}{\sim}$$

$-ab \neq 0 \wedge b-a \neq 0$

$$\left[\begin{array}{ccc|c} 2 & a & -1 & 0 \\ 0 & 1 & b & 1 \\ 0 & 0 & -ab & b-a \end{array}\right]$$

* $Car([A|B]) = 3 = Car(a)$ $a \neq 0 \vee b \neq 0$
Sistema possível e determinado.

* Se $-ab = 0$, $b-a \neq 0$, $b=0 \vee a=0$
logo
$Car([A|B]) = 2 = Car(A) = 2$
logo o sistema é possível
indeterminado

* Se $car([A]) = 2 < Car([A|B]) = 3$
logo o sistema é ~~possível~~
impossível

$-ab = 0 \qquad b-a=0 \Rightarrow b=a=0$

3) Considera $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ e $B = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$

$AX = B$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 0 \end{array}\right] \overset{L_2 \leftarrow L_2 - L_1}{\sim} \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \overset{L_3 \leftarrow L_3 - L_2}{\sim} \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

a)

$$\begin{cases} 2x + z = 3 \\ y + t = 0 \end{cases} \Leftrightarrow \begin{cases} 2\times(1) + 1 = 3 \\ -1 + 1 = 0 \end{cases} \Rightarrow \begin{cases} 3 = 3 \\ 0 = 0 \end{cases}$$

P.V

b) $\{1, 2, 1\}$

$$\cancel{C(A) = \alpha_1 \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \alpha_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}$$

$\boxed{\begin{matrix} 1 \\ 2 \\ 1 \end{matrix}}$

$$\cancel{\begin{cases} 2\alpha_1 + \alpha_3 \\ 2\alpha_1 + \alpha_2 + 2\alpha_3 \\ \alpha_2 + \alpha_3 \end{cases}}$$

(c) $N(A)$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{L_2 \leftarrow L_2 - L_1} \left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

~~2 0 1 = 0~~

$$\begin{cases} 2x + z = 0 \\ y + z = 0 \\ z = t \end{cases}, t \in \mathbb{R} \Leftrightarrow \begin{cases} 2x + t = 0 \\ y = -t \\ z = t \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = -\frac{1}{2} t \\ y = -t \\ z = t \end{cases} \Leftrightarrow N(A) = \left\{\left(-\frac{1}{2} t, -t, t\right), t \in \mathbb{R}\right\}$$

d)

$$\begin{cases} 2x + z = 3 \\ y + z = 0 \\ z = t \end{cases}, t \in \mathbb{R} \Leftrightarrow \begin{cases} 2x + t = 3 \\ y + t = 0 \\ z = t \end{cases}, t \in \mathbb{R} \Leftrightarrow \begin{cases} x = \frac{3 - t}{2} \\ y = -t \\ z = t \end{cases}, t \in \mathbb{R}$$

$$X = \left\{\left(\frac{3 - t}{2}, -t, t\right); t \in \mathbb{R}\right\}$$

3)

1) $\cancel{\begin{vmatrix} a_1 & a_2 & a_3 \\ a_3 & b_2 & c \\ & & c \end{vmatrix}}$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$$

4) $$\begin{vmatrix} a_1+2c_1 & 3c_1 & b_1 \\ a_2+2c_2 & 3c_2 & b_2 \\ a_3 \quad 2c_3 & 3c_3 & b_3 \end{vmatrix} = 3\begin{vmatrix} a_1 & c_1 & b_1 \\ a_2 & c_2 & b_2 \\ a_3 & c_3 & b_3 \end{vmatrix} + 2\times 3\begin{vmatrix} c_1 & c_1 & b_1 \\ c_2 & c_2 & b_2 \\ c_3 & c_3 & b_3 \end{vmatrix}$$

$$= -3\begin{vmatrix} c_1 & b_1 & c_1 \\ c_2 & b_2 & c_2 \\ c_3 & b_3 & c_3 \end{vmatrix} + 5\times 0 =$$

$$= -3\times 5 = -15$$

5) $$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 3 \end{bmatrix} \qquad A^{31} =$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & 3 \end{bmatrix} \overset{L_2 \leftarrow L_2+L_4}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & 3 \end{bmatrix} \overset{L_4 \leftarrow L_4+L_1}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 4 \end{bmatrix} \overset{L_4 \leftarrow L_4-L_3}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix} \overset{L_2 \leftrightarrow L_3}{\sim}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$ $car(A) = 4 \neq 0$ logo A é invertível

~~det(A)~~

$$\det(A) = -A^{41} + 3A^{44} = (-1)^{4+1}\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + 3\left((-1)^{4+4}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}\right)$$

Teorema de Laplace linha 4

1 1 1
0 1 1

1 1 1
1 0 1

$(1+1-1-1) + 3(1-1-1) = 0 - 3 = -3$

$$A^{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3$$

1 1 1
0 1 1

o elemento é $\frac{3}{-3} = -1$

6) $X = (1, 0, 1, -2)$ $\quad Y = (1, 1, 0, -1)$

$\|X\| = \sqrt{1^2 + 0^2 + 1^2 + (-2)^2} = \sqrt{1+1+4} = \sqrt{6}$

$\|Y\| = \sqrt{1^2 + 1^2 + 0^2 + (-1)^2} = \sqrt{3}$

$\theta = \arccos \frac{(x \cdot y)}{\sqrt{3} \times \sqrt{6}} \Leftrightarrow \theta = \arccos \frac{1(1) + 0(1) + 1(0) + (-1)(-2)}{\sqrt{3 \times 6}} \Leftrightarrow$

$\Leftrightarrow \theta = \arccos \frac{1+2}{\sqrt{18}} \Leftrightarrow \theta = \arccos \frac{3}{\sqrt{18}} \Leftrightarrow$

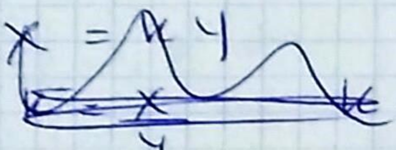
$\theta = \arccos \frac{3}{\sqrt{3^2 \times 2}} \Leftrightarrow \theta = \arccos \frac{3}{3\sqrt{2}} \Leftrightarrow \theta = \arccos \frac{\sqrt{2}}{2} \Leftrightarrow$

$$\begin{array}{r|l} 18 & 3 \\ 6 & 3 \\ 2 & 2 \\ 1 & \end{array}$$

$\Leftrightarrow \theta = \frac{\pi}{4}$

se X e Y são Colineares

$X = kY$



$(1, 0, 1, -2) = k(1, 1, 0, -1)$

$$\begin{cases} 1 = k \\ 0 = k \\ 1 = 0 \\ -2 = -k \end{cases}$$

Sistema Impossível

se X e Y são Ortogonais

$X \cdot Y = 0$

$(1, 0, 1, -2) \cdot (1, 1, 0, -1) = 1(1) + 0(1) + 1(0) + (-2)(-1) =$
$= 1 + 2 = 3 \neq 0$ logo não são ortogonais

7)
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

numa matriz quadrada a diagonal corresponde aos elementos

$a_{11}, a_{22}, a_{33}, a_{44}, \dots, a_{mn}$

sendo assim os seus elementos da transposta são

$a_{11}, a_{22}, a_{33}, a_{44}, \dots, a_{mn}$

logo o ~~...~~ $Tr(AA_T) =$ ~~...~~

$Tr(AA) \Rightarrow (Tr(A))^2 \geq 0$

→ Teste 1 2016 ←

a) $\det(G) = \frac{1}{2} \times \cancel{4\times2} \times \det(D)$

b)
$$F \begin{bmatrix} 1+2x & 2 & 3 \\ 4+5x & 5 & 6 \\ 7+8x & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + x \begin{bmatrix} 2 & 2 & 3 \\ 5 & 5 & 6 \\ 8 & 8 & 9 \end{bmatrix}$$

$\det(C) = \det(F)$
$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$
1 2 3
4 5 6

$= (1\times5\times9) + (4\times8\times3) + (7\times2\times6) - (7\times5\times3)$

$- (1\times8\times6) - (4\times2\times9) = 0$ logo como letícia

~~$= 45 + 96 + 14\times6 - 35\times3 - 48$~~

~~-73~~

C não é invertível

$= 45 + 96 + 84 - 105 - 48 - 13$

c)
$$\left[\begin{array}{ccccc|c} a, & b, & c, & c, & c, & 7 \\ a, & b, & c, & d, & e, & 8 \\ a, & a, & c, & d, & e, & 4 \end{array}\right]$$

Resposta (C)

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & 2 \\ 0 & 3 & 2 & 1 & 4 & 3 \\ 0 & 0 & 3 & 4 & 1 & 4 \end{array}\right]$$

$x + 2y + 3z + 4$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 3 & 3 \end{array}\right] = [\ ]$$

d) $A_{3\times3}$ $\quad C_1 \; C_2 \; C_3$

$$\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}$$

→ rag milho no pé

→ pedaço

~~então sai~~

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & 0 & 1 \end{vmatrix} = 1 - 3 = -2$$

$$\begin{matrix} 1 & 0 & 1 \\ 2 & 1 & 1 \end{matrix}$$

BAD

$$\left[\begin{matrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & 0 & 1 \end{matrix} \middle| \begin{matrix} 2 \\ 4 \\ 4 \end{matrix}\right] L_2 \leftarrow \sim$$

2) a $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ -1 & 1 \end{bmatrix}$ 3×2

possível

$B = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ 2×3

$$\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 5 & -1 \\ 0 & -3 & 1 \end{bmatrix} \quad \det \begin{vmatrix} 1 & 2 & 0 \\ 1 & 5 & -1 \end{vmatrix}$$

2b) $BA = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$

$2\times3 \quad 3\times2$ ~~det~~

$$\det(BA) = \begin{vmatrix} 5 & -2 \\ -2 & 2 \end{vmatrix} = 5\times2 - (-2\times-2) = 10 - 4 = 6 \neq 0$$

logo a matriz BA é invertível

3) $X = \begin{cases} 4 + 2a + 5b \\ a \\ b \end{cases}$

4) $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

$$\left[\begin{matrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{matrix} \middle| \begin{matrix} 1 \\ 3 \\ 0 \end{matrix}\right] \xrightarrow{L_2 \leftarrow L_2 - 3L_1} \left[\begin{matrix} 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 4 & 2 \end{matrix} \middle| \begin{matrix} 1 \\ 0 \\ 0 \end{matrix}\right] L_3 \leftarrow L_3 - 2L_2$$

$$\sim \left[\begin{matrix} 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{matrix} \middle| \begin{matrix} 1 \\ 0 \\ 0 \end{matrix}\right]$$

$$\begin{cases} 0x + 1y + 2z + 2d = 1 \\ 0x + 0y + 2z + 1d = 0 \\ x = t \\ y = s \\ z = b \end{cases} \Leftrightarrow \begin{cases} s + 2b + -2b = 1 \\ 0t + 0s + 2b + d = 0 \end{cases}$$

Teste 1 - 2015

1a) $x = (1,2,3)$ e $y = (1,0,2)$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 0 & 2 \\ i & j & k \\ 1 & 2 & 3 \end{vmatrix} = 4i + 0k + 3j - 2k - 0i - 2j =$$

$$= 4i - 2k + j$$

$$(4, 1, -2)$$

$$(1,2,3).(1,0,2) = 1 + 0 + \cancel{7}6 = 7 \neq 9$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 0 & 2 \end{vmatrix} \quad 0 + 6 + 4 - 0 - 4 - 6 = 0,$$

$$\bar{X} \times Y . \bar{X} = 0$$

b) $\left[\begin{array}{c|c} a_1\, b_1\, c_1\, d_1 & e \\ a_2\, b_2\, c_2\, d_2 & f \\ a_3\, b_3\, c_3\, d_3 & g \end{array}\right]$ $Car([A|B]) = Car(A)$

mas o sistema não é possível determinado

é esta

c) $(A^T \times B)^T = B^T \times (A^T)^T = B^T A \neq AB^T$ $\boxed{X}$

$$(2(AB^{-1}))^{-1} = 2^{-1} \cdot (A^{-1} \cdot (B^{-1})^{-1}) = \boxed{\frac{1}{2} \cdot B \cdot A^{-1}}$$

$$(A^{-1} B)^T = B^T . (A^{-1})^T =$$

d) $\det(A) = -2$ e $\det(B) = 3$

$$\det(B^{-1}) = \frac{1}{\det(B)} = \frac{1}{3} \quad X$$

$$\det(AB^T) = \det(A) \times \det(B^T) = \det(A) \times \det(B) = -2 \times 3 = -6$$

$$\boxed{\det(2A^{-1}) = 2^3 \det(A^{-1}) = 8 \times \frac{1}{-2} = -4} \checkmark$$

e) ~~$X = (0,1,0)$~~ ~~$Y = (1,0,2)$~~ ~~$Z = (1,1,2)$~~ não sai

2)
a) $\begin{vmatrix} 2 & 0 & 0 & 3 \\ 0 & 2 & 3 & 0 \\ 0 & 3 & 2 & 0 \\ 3 & 0 & 0 & 2 \end{vmatrix} = 2A^{11} + \cancel{\ldots} 0A^{21} + 0A^{31} + 3A^{41}$

usando o teorema de Laplace na primeira coluna

$= \cancel{2(-1)} \; 2\left((-1)^{1+1}\begin{vmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}\right) + 3\left((-1)^{4+1}\begin{vmatrix} 0 & 0 & 3 \\ 2 & 3 & 0 \\ 3 & 2 & 0 \end{vmatrix}\right)$

$\begin{matrix} 2 & 3 & 0 \\ 3 & 2 & 0 \end{matrix}$ $\qquad$ $\begin{matrix} 0 & 0 & 3 \\ 2 & 3 & 0 \end{matrix}$

$= 2(8+0+0-0-0\cancel{m}-18) - 3(12-27) =$

$= 2(-10) - 3(-15) = -20 + 45 = 25$

b) Como o $\det(A) = 25 \neq 0$ logo é invertível

calculado na alínea (2a)

c) $N(A)$

$\left[\begin{array}{cccc|c} 2 & 0 & 0 & 3 & 0 \\ 0 & 2 & 3 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 \\ 3 & 0 & 0 & 2 & 0 \end{array}\right] \begin{matrix} L_4 := L_4 - \frac{3}{2}L_1 \\ \sim \\ L_3 := L_3 - \frac{3}{2}L_2 \end{matrix} \left[\begin{array}{cccc|c} 2 & 0 & 0 & 3 & 0 \\ 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & -\frac{5}{2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{5}{2} & 0 \end{array}\right]$

$\frac{4}{2} - \frac{9}{2} = -\frac{5}{2}$

$\begin{cases} 2x + 3d = 0 \\ 2y + 3z = 0 \\ -\frac{5}{2}z = 0 \\ -\frac{5}{2}d = 0 \end{cases}$ $\quad N(A) \begin{cases} x = 0 \\ y = 0 \\ z = 0 \\ d = 0 \end{cases}$ $\quad \begin{bmatrix} x \\ y \\ z \\ d \end{bmatrix} \in \mathbb{R}^4, \cancel{x, y, z,}\; x = 0 \wedge y = 0 \wedge z = 0 \wedge d = 0 \}$

$N(A) = 0$

)

$\left[\begin{array}{cccc|c} 2 & 0 & 0 & 3 & 5 \\ 0 & 2 & 3 & 0 & 5 \\ 0 & 3 & 2 & 0 & 5 \\ 3 & 0 & 0 & 2 & 5 \end{array}\right] \cancel{L_4 := L_4 \ldots \sim \left[\begin{array}{cccc|c} 2 & 0 & 0 & 3 & 5 \end{array}\right]}$

Pertence

$\begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} = 1C_1 + 1C_2 + 1C_3 + 1C_4 =$

$= \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 2+3 \\ 3+2 \\ 3+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$

$-3t = 2 \qquad , \; = -3 \qquad 3 \qquad - \; 3$