ECE4634
Digital Communications
Fall 2007

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Lecture #17: Bandpass Modulation – BFSK

Analog and Digital Communications

Overview

Analog and Digital Communications

- We have been studying bandpass digital modulation techiques
- To date we have looked at two *linear* binary modulation schemes
  - BASK
  - BPSK
- Today we look at a third binary modulation scheme that is *non-linear*
  - Binary Frequency Shift Keying

Lecture Objective

- The objectives of today's lecture are
  - to introduce Binary Frequency Shift Keying (BFSK)
  - to discuss the impact of coherent frequencies and continuous phase
  - to examine the power spectral density

Three Ways of Representing Bandpass Signals

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- We will need some additional analytical tools to handle bandpass signals
- Magnitude and Phase

$$v(t) = R(t)\cos[\omega_c t + \theta(t)]$$

- In Phase and Quadrature

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- Complex Envelope

$$v(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$$

Bandpass Modulation - FSK

- Binary Frequency Shift Keying (BFSK)
- We modulate or change the frequency depending on the data bit to be sent
- Basic Idea:
  - Send one tone f_1 for a 1
  - Send another tone f_2 for a 0
  - Then we transmit the signal $s(t)$:

$$1 \Rightarrow s(t) = \cos\left(2\pi f_1 t + \theta_1\right)\Big|_0^{T_s}$$

$$0 \Rightarrow s(t) = \cos\left(2\pi f_2 t + \theta_2\right)\Big|_0^{T_s}$$

θ_1 and θ_2 are aribtrary constants that simply reflect the fact that the two oscillators are not phase locked.

FSK - Magnitude and Phase Representation

- $s(t) = R(t)\cos\left[\omega_c t + \theta(t)\right]$ where
  - $R(t) = 1\Big|_0^T$
  - $1 \Rightarrow \theta(t) = \theta_1 + 2\pi\Delta f t\Big|_0^{T_s}$

    $0 \Rightarrow \theta(t) = \theta_2 - 2\pi\Delta f t\Big|_0^{T_s}$

$$\boxed{\begin{aligned} f_c &= \frac{f_1 + f_2}{2} \\ \Delta f &= \frac{f_1 - f_2}{2} \end{aligned}}$$

- I/Q and complex envelopes are not as easy to interpret
- FSK is widely used for robust communications
  - Like ASK, it can be non-coherently received (i.e., we don't need phase reference)
  - Like BPSK, it is a constant envelope signal

I/Q and Complex Envelope

- I/Q Representation

$$x(t) = \cos\left[\theta(t)\right] = \begin{cases} \cos\left[\theta_1 + 2\pi\Delta f t\right]\Big|_0^{T_s} & b = 1 \\ \cos\left[\theta_2 - 2\pi\Delta f t\right]\Big|_0^{T_s} & b = 0 \end{cases}$$

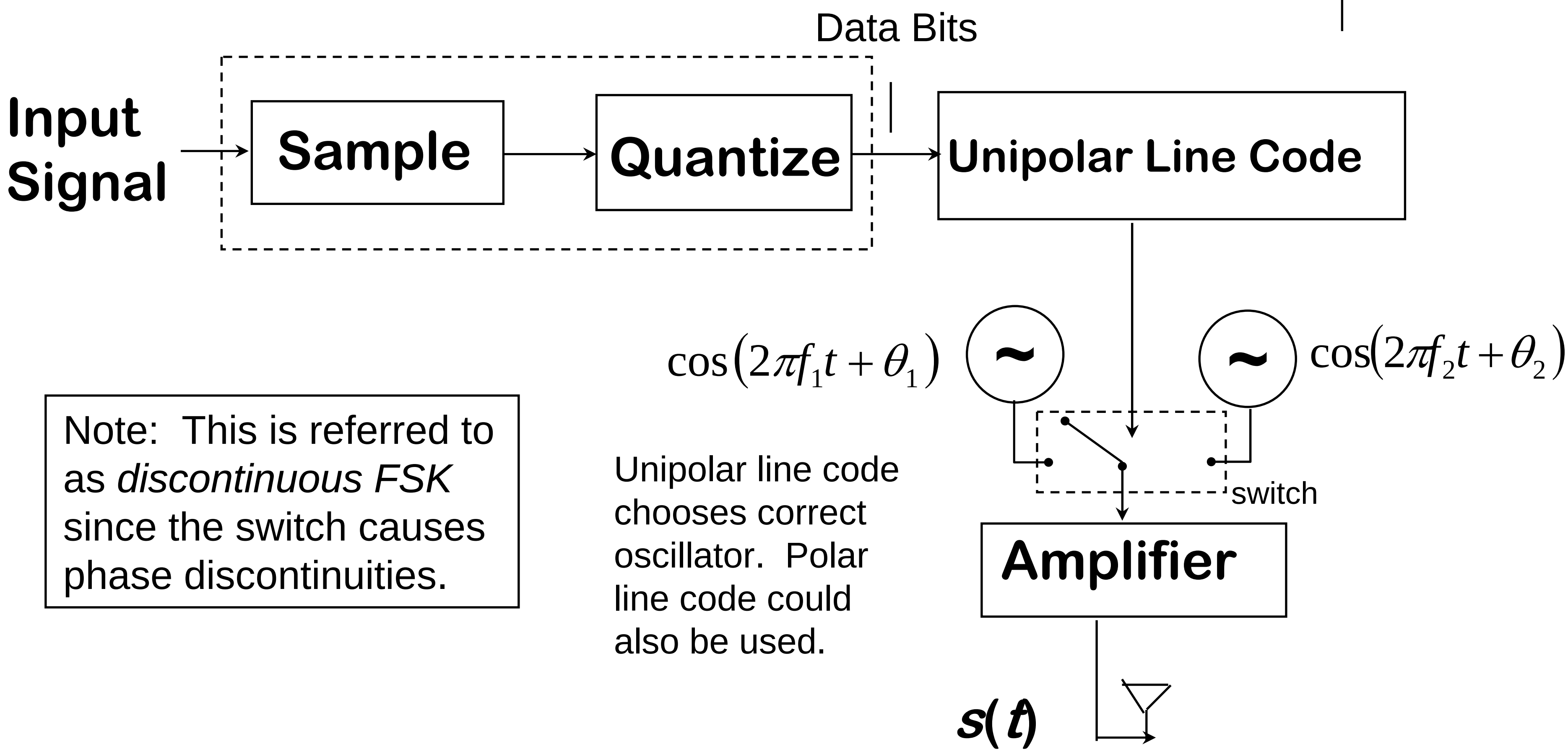
$$y(t) = \sin\left[\theta(t)\right] = \begin{cases} \sin\left[\theta_1 + 2\pi\Delta f t\right]\Big|_0^{T_s} & b = 1 \\ \sin\left[\theta_2 - 2\pi\Delta f t\right]\Big|_0^{T_s} & b = 0 \end{cases}$$

- Complex envelope

$$g(t) = \cos\left[\theta(t)\right] + j\sin\left[\theta(t)\right]$$

$$= \begin{cases} \cos\left[\theta_1 + 2\pi\Delta f t\right] + j\sin\left[\theta_1 + 2\pi\Delta f t\right]\Big|_0^{T_s} & b = 1 \\ \cos\left[\theta_2 - 2\pi\Delta f t\right] + j\sin\left[\theta_2 - 2\pi\Delta f t\right]\Big|_0^{T_s} & b = 0 \end{cases}$$

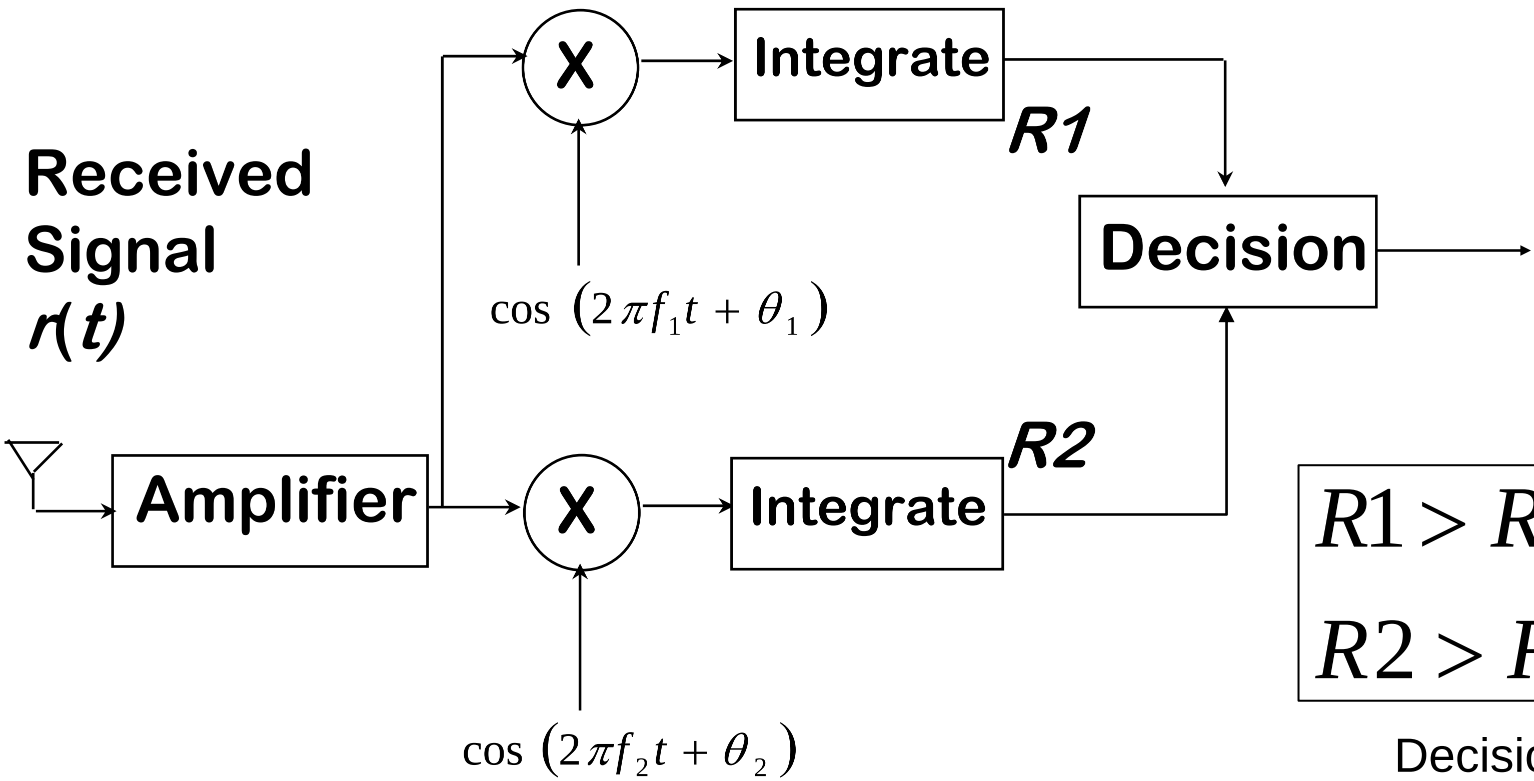
Transmitter for BFSK



Note: This is referred to as *discontinuous FSK* since the switch causes phase discontinuities.

Unipolar line code chooses correct oscillator. Polar line code could also be used.

Coherent Receiver for FSK



Output Data Bits

$$R1 > R2 \rightarrow 1$$
$$R2 > R1 \rightarrow 0$$

Decision Rule

Note: Phases of internally generated sinusoids are matched to incoming sinusoids.

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Continuous Phase BFSK

- We can also create a BFSK signal using a frequency modulator. Such a scheme keeps the phase a continuous function (i.e., there are no phase jumps).
- This type of FSK has better spectral properties and is called *continuous phase* FSK

$$s(t) = A_c \cos\left[\omega_c t + D_f \int_{-\infty}^{t} m(\lambda)d\lambda\right]$$

$$= \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$

$$g(t) = A_c e^{j\theta(t)}$$

$$\theta(t) = D_f \int_{-\infty}^{t} m(\lambda)d\lambda$$

m(t) = polar NRZ line code
$D_f = 2\pi\Delta f$

Coherent Carriers vs. Continuous Phase

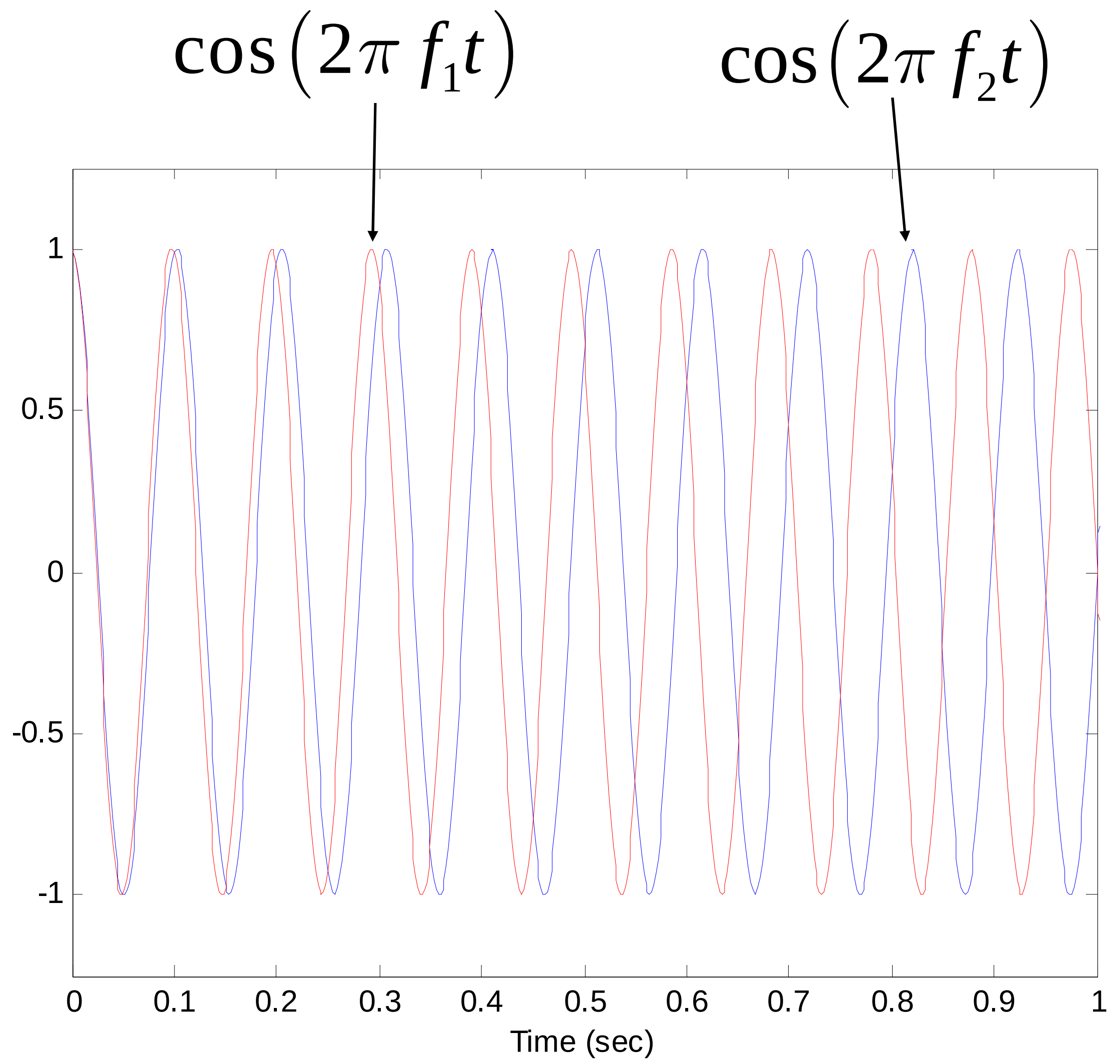
- The two signals sent in BFSK are

$$\cos\left(2\pi f_1 t + \theta_1\right)$$
$$\cos\left(2\pi f_2 t + \theta_2\right)$$

- If $\theta_1 = \theta_2$ we say that the carriers are *coherent*
- If in addition to being coherent, there are no phase changes in the carrier, we say that the modulation is *continuous phase.*
- Both will have an impact on the bandwidth of the transmit signal
- Coherent carriers are a necessary but not a sufficient condition for continuous phase

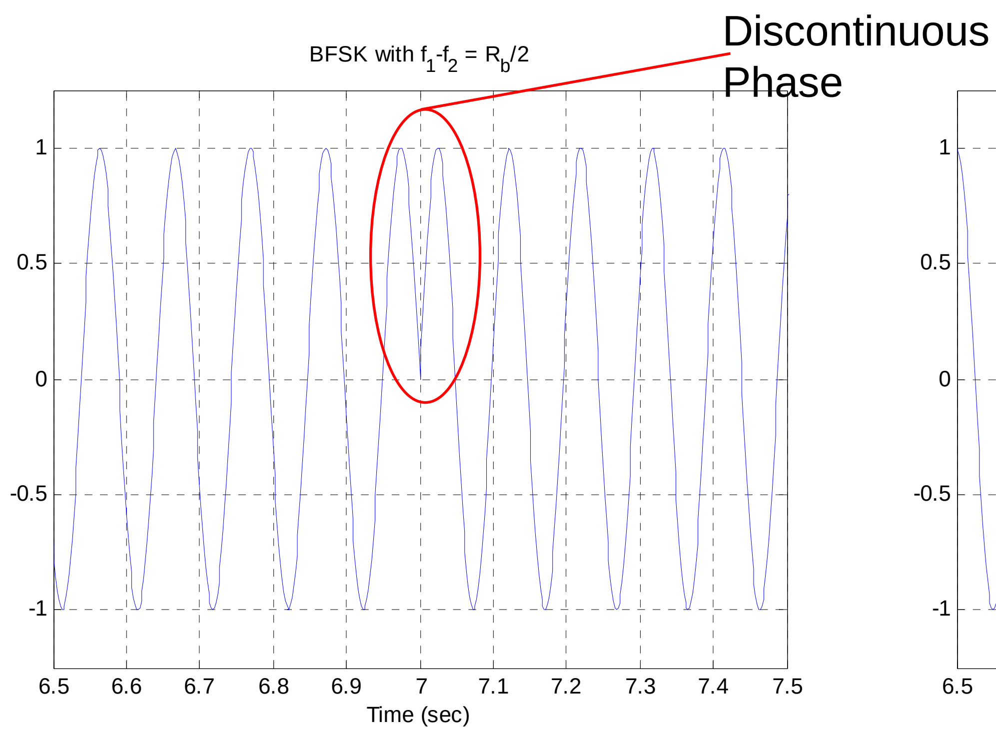
Coherent Carriers

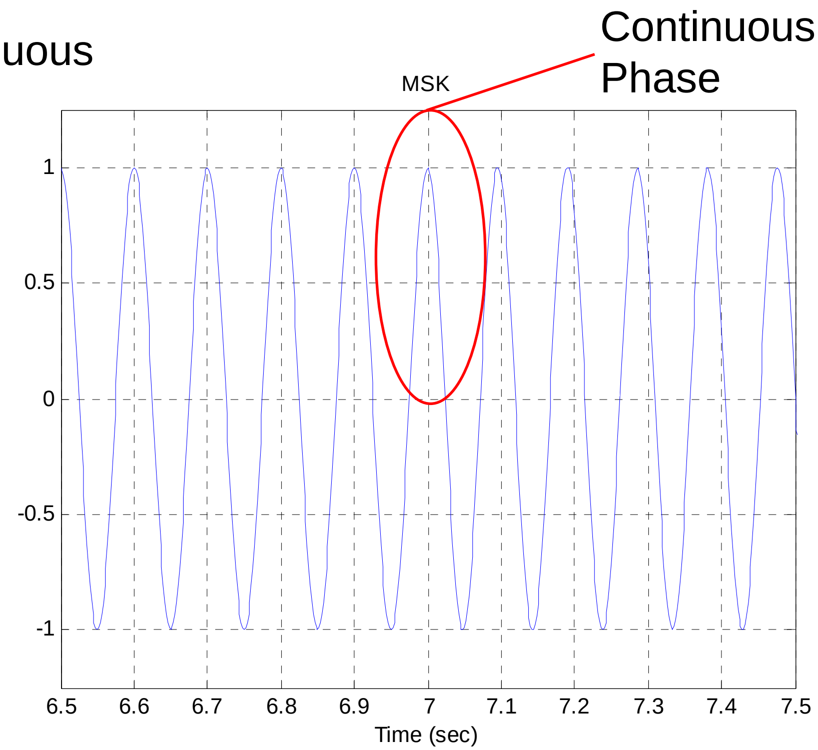
- Two Carriers have the same phase



Discontinuous Phase

- The modulated signal exhibits phase jumps





Frequency Separation 2Δf - Non-coherent Carriers

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$$\int_0^{T_b} \cos\left(2\pi f_1 + \theta_1\right)\cos\left(2\pi f_2 + \theta_2\right)dt = 0$$

$$\underbrace{\int_0^{T_b} \cos\left(2\pi\left(f_1 + f_2\right) + \theta_1 + \theta_2\right)dt}_{\text{negligible}} + \int_0^{T_b} \cos\left(2\pi\left(f_1 - f_2\right) + \theta_1 - \theta_2\right)dt = 0$$

$$\int_0^{T_b} \cos\left(2\pi\left(f_1 - f_2\right) + \theta_1 - \theta_2\right)dt = 0$$

$$\int_0^{T_b} \cos\left(2\pi f' + \theta'\right)dt = 0$$

Thus, the signal *cos(2πf'+θ')* must go through an integer number of cycles. For this to be satisfied:

$$2\pi(f_1 - f_2)T_b = 2\pi k$$

$$f_1 - f_2 = \frac{k}{T_b}$$

The minimum frequency separation is then $1/T_b = R_b$

Frequency Separation 2Δf - Coherent Carriers

Analog and Digital Communications

$$\int_0^{T_b} \cos\left(2\pi f_1 + \theta_1\right)\cos\left(2\pi f_2 + \theta_1\right) dt = 0$$

$$\underbrace{\int_0^{T_b} \cos\left(2\pi\left(f_1 + f_2\right) + 2\theta_1\right) dt}_{\text{negligible}} + \int_0^{T_b} \cos\left(2\pi\left(f_1 - f_2\right) + \theta_1 - \theta_1\right) dt = 0$$

$$\int_0^{T_b} \cos\left(2\pi\left(f_1 - f_2\right)\right) dt = 0$$

$$\int_0^{T_b} \cos\left(2\pi 2\Delta f t\right) dt = 0$$

$$\frac{1}{2\pi f}\sin\left(4\pi\Delta f t\right)\Big|_0^T = 0$$

$$\frac{1}{2\pi f}\sin\left(4\pi\Delta f T\right) = 0$$

$$4\pi\Delta f T = k\pi$$

$$\Delta f_{\min} = \frac{1}{4T}$$

Thus, the minimum frequency separation $(f_1\text{-}f_2)$ is $1/2T_b = R_b/2$

Power Spectral Density of FSK

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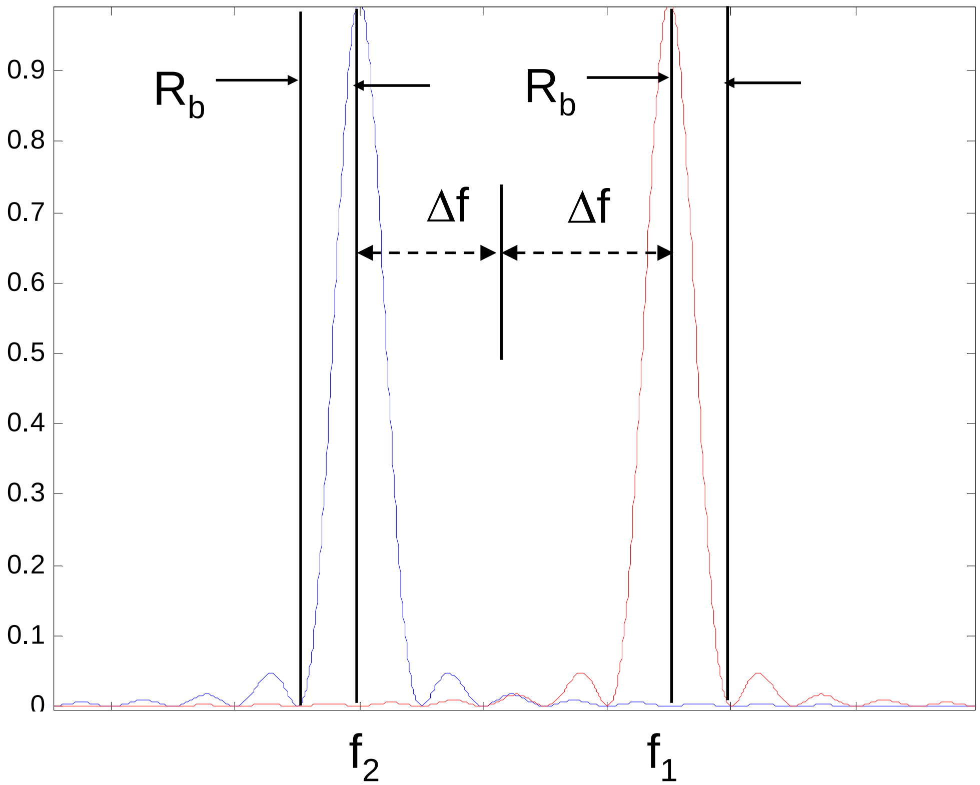
- Each of two tones can be thought of as an ASK signal
- In other words, we are modulating two separate carriers by unipolar NRZ baseband waveforms.



Null-to-Null Bandwidth:

$$B = R + (f_1 - f_2) + R$$
$$= 2R + 2\Delta f$$

For orthogonality (coherent FSK):

$$\Delta f_{\min} = \frac{R}{2}$$

$$(f_2 - f_1)_{\min} = R$$

Power Spectral Density

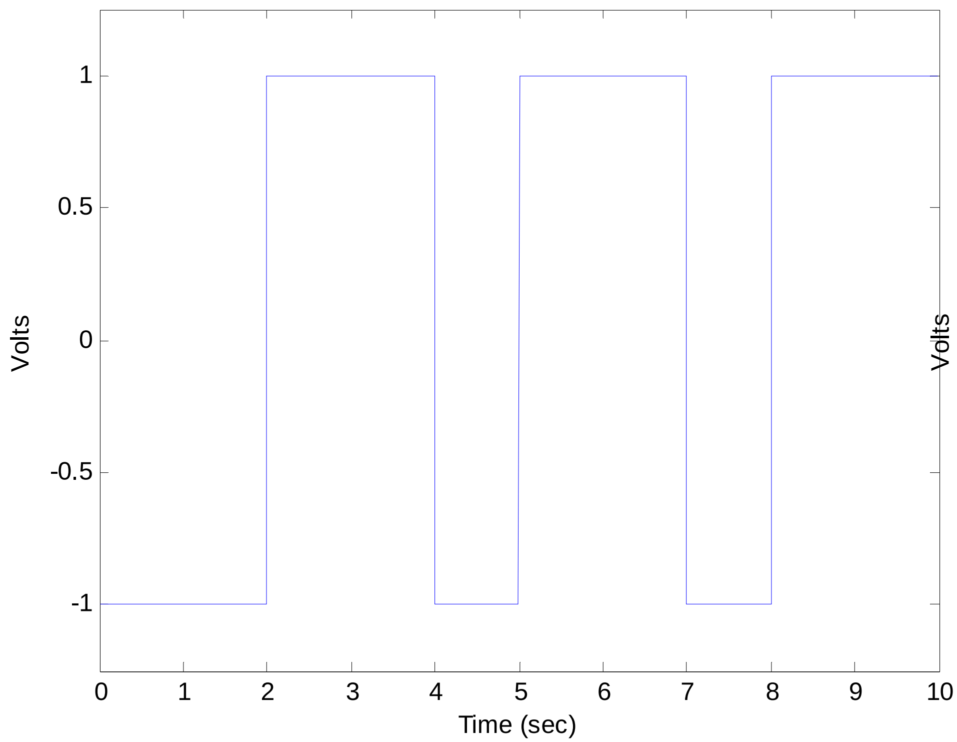
- Let's consider the BFSK signal the sum of two BASK signals:

$$P_v(f) = \frac{A^2 T_b}{8}\left[\operatorname{sinc}^2\big((f-f_1)T_b\big) + \operatorname{sinc}^2\big((f+f_1)T_b\big) + \frac{1}{T_b}\delta(f-f_1) + \frac{1}{T_b}\delta(f+f_1)\right]$$
$$+ \frac{A^2 T_b}{8}\left[\operatorname{sinc}^2\big((f-f_2)T_b\big) + \operatorname{sinc}^2\big((f+f_2)T_b\big) + \frac{1}{T_b}\delta(f-f_2) + \frac{1}{T_b}\delta(f+f_2)\right]$$

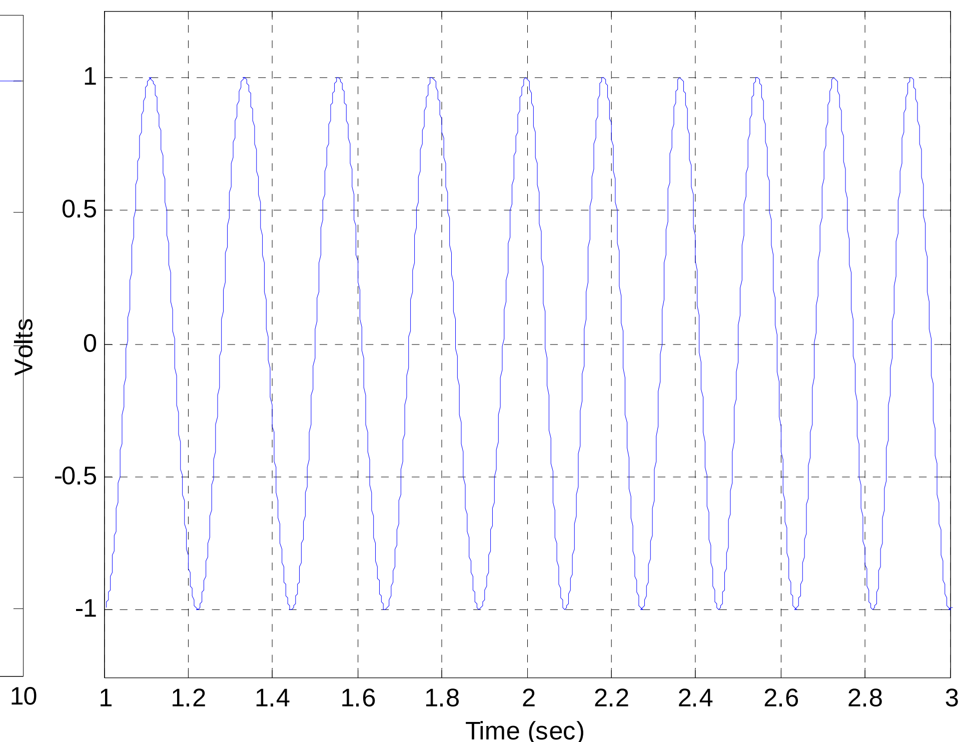
- Since $f_1 - f_2 = R_s$ and the first nulls occur at $f_2 - R_s$ and $f_1 + R_s$, the null-to-null BW is $3R_s$
- Four tones exist in the spectrum at +/- f_1 and +/- f_2

Example

$R_b = R_s = 1\text{bps}$



$f_c = 5\text{Hz} \rightarrow f_1 = 4.5\text{Hz},\ f_2 = 5.5\text{Hz}$



$f_1 = 4.5\text{Hz}$, $f_2 = 5.5\text{Hz}$

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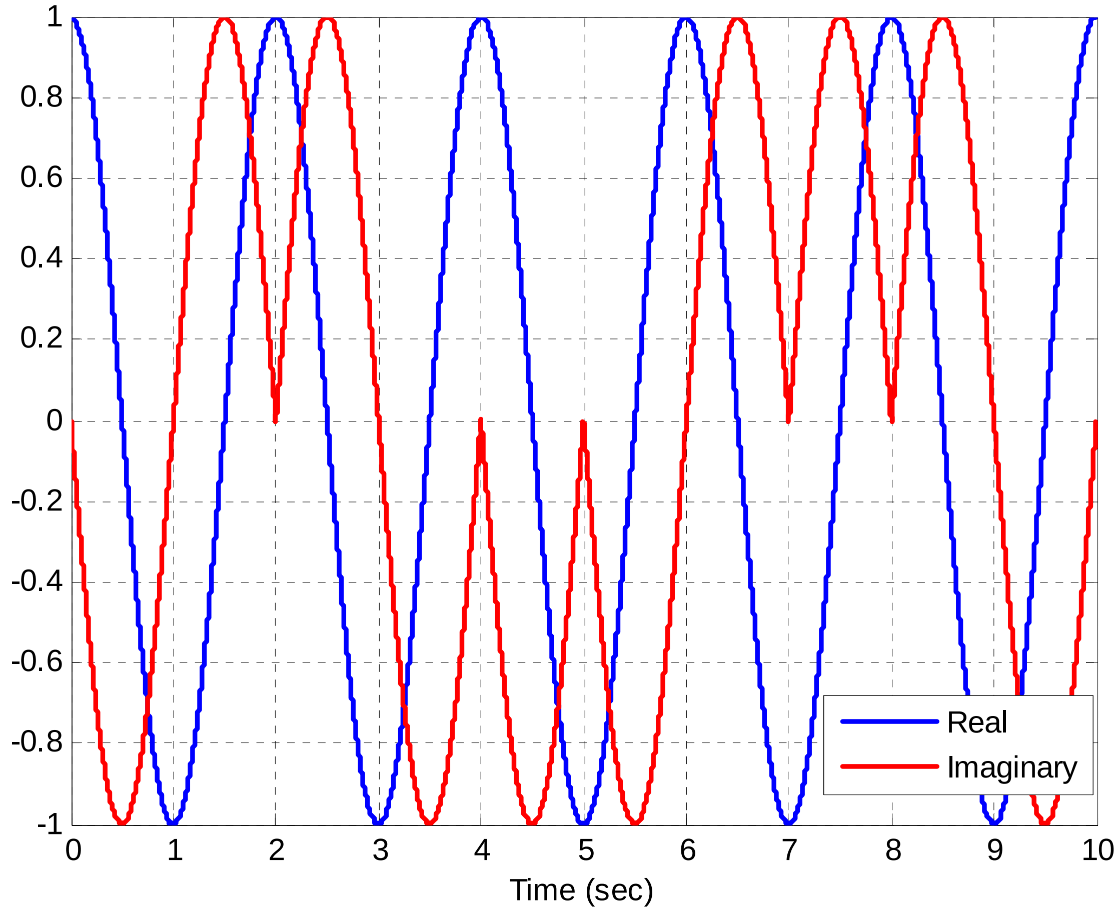
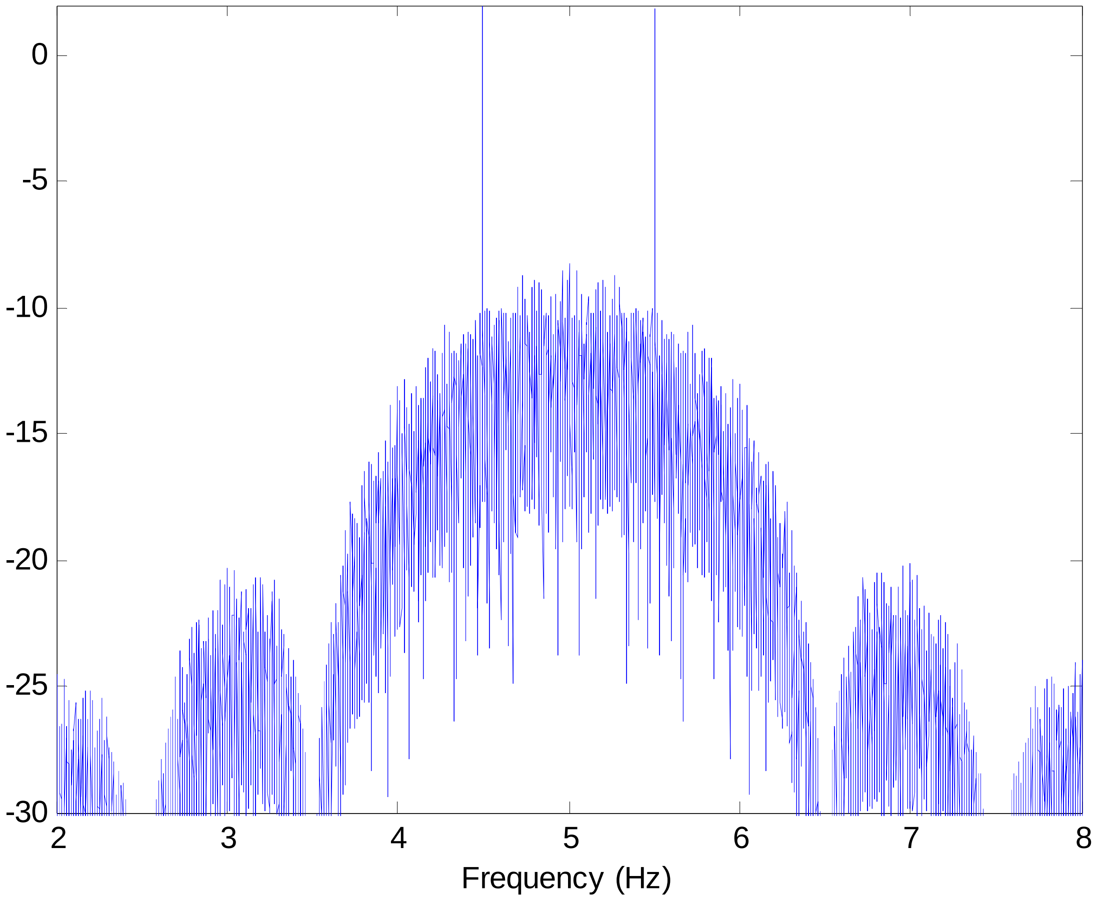
Example – cont.



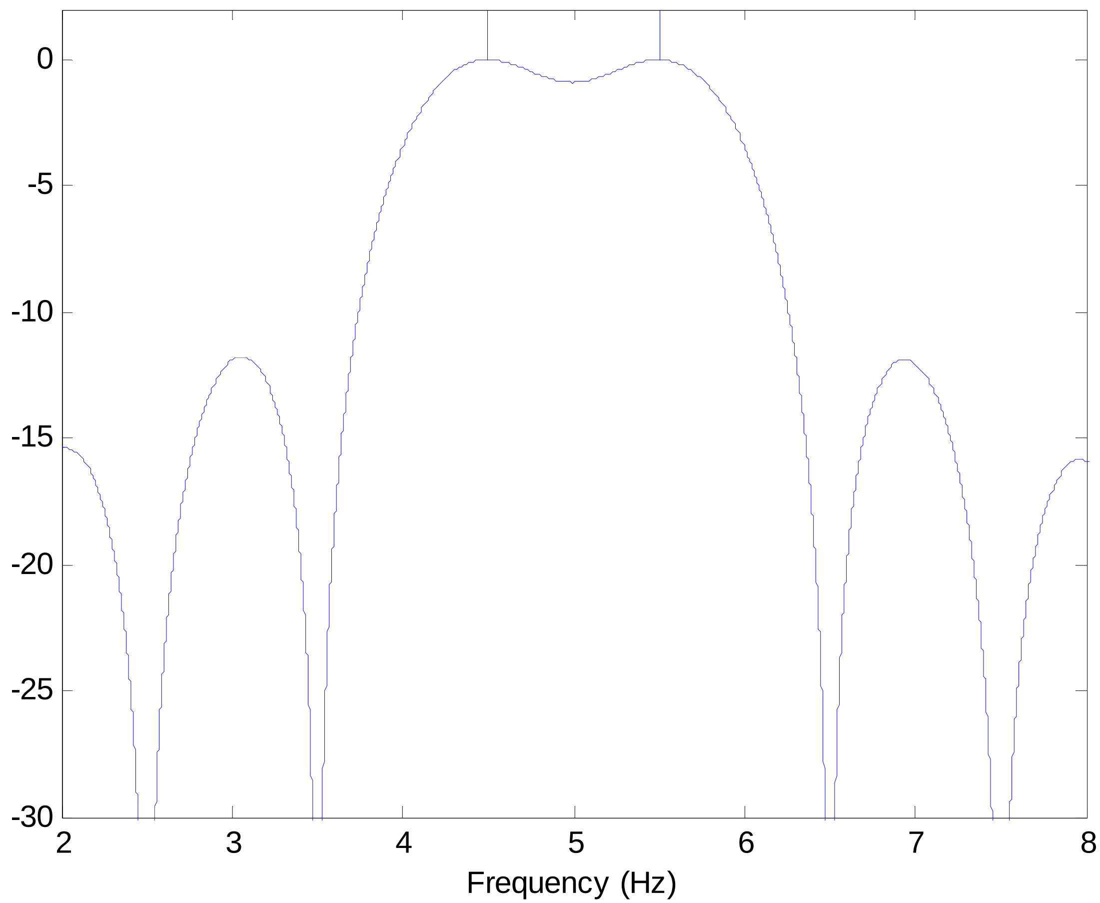
- I & Q components
- Real and Imaginary components

Example - ESD



- f_c = 5Hz
- f_1 = 4.5Hz
- f_2 = 5.5Hz
- R_b = 1bps

Theoretical PSD



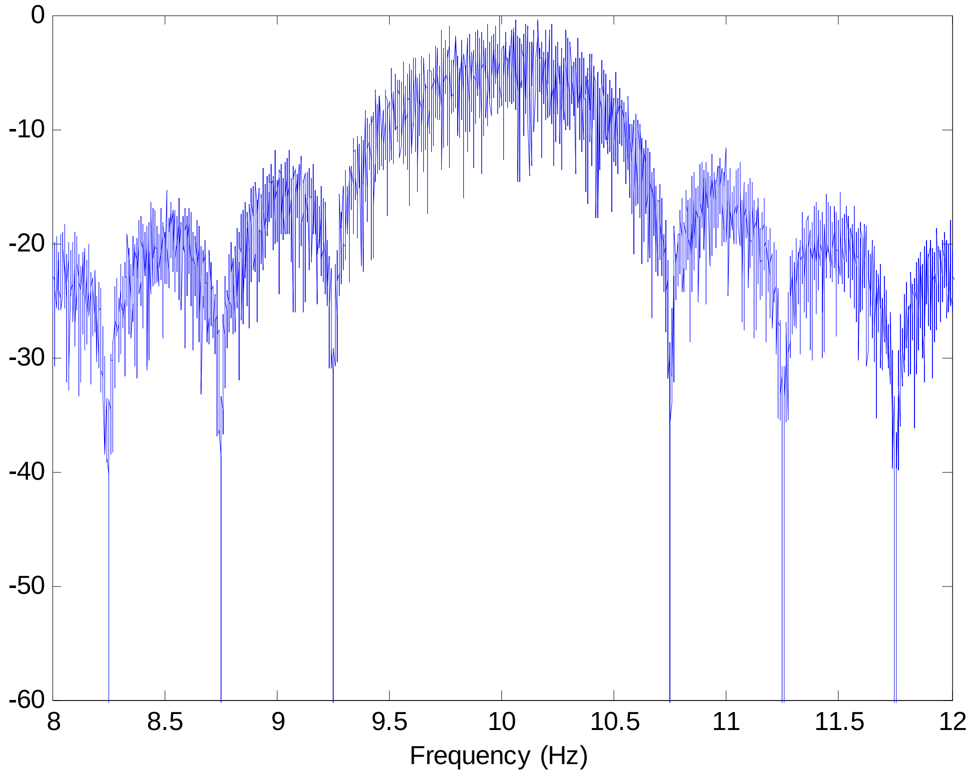
- f_c = 5Hz
- f_1 = 4.5Hz
- f_2 = 5.5Hz
- R_b = 1bps

Minimum Shift Keying

- Minimum Shift Keying is a form of BFSK that has
  - Minimum frequency separation $2\Delta f = R_b/2$
    - Thus requires coherent carriers
  - Continuous Phase
- Results in substantially reduced bandwidth as compared to standard BFSK
- Can also be seen as a form of QPSK with pulse shaping
  - We will see this in the next lecture

Minimum Shift Keying

- Example – f_c = 10Hz, f_1 = 9.75Hz, f_2 = 10.25Hz, R_b = 1bps



Bandwidth Requirements

- For non-coherent carriers, the minimum null-to-null bandwidth is

$$W_{null} = 3R_b$$

- For coherent carriers, the minimum null-to-null bandwidth is

$$W_{null} = 2.5R_b$$

- For coherent carriers AND continuous phase, the minimum null-to-null bandwidth is

$$W_{null} = 1.5R_b$$

Summary

- We have now examined three *binary* digital modulation schemes
  - BPSK
  - BASK
  - BFSK
- How can be reduce the bandwidth requirements of these schemes without reducing the bit rate?
  - Pulse shaping – we have discussed some about this already
  - *M*-ary modulation – we will consider this next

Appendix A

An alternate view of the required frequency separation for orthogonal symbols

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Another View

Calculate the correlation coefficient and set to zero

$$\rho(\Delta f, \Delta\theta) = \frac{1}{C}\int_0^T \cos(2\pi f_1 t + \theta_1)\cos(2\pi f_2 t + \theta_2)\,dt$$

$$= \frac{1}{T}\int_0^T \left[\cos\left(2\pi(f_1 + f_2)t + \theta_1 + \theta_2\right) + \cos\left(2\pi\Delta f t + \Delta\theta\right)\right]dt$$

$$\approx \frac{1}{T}\int_0^T \cos(2\pi 2\Delta f t + \Delta\theta)\,dt$$

$$= \frac{1}{2\pi 2\Delta f T}\left(\sin(2\pi 2\Delta f t + \Delta\theta) - \sin(\Delta\theta)\right)$$

$$C = \sqrt{\int_0^T \cos^2(2\pi f_1 t + \theta_1)\,dt \int_0^T \cos^2(2\pi f_2 t + \theta_2)\,dt}$$

$$= \sqrt{\frac{T}{2}\frac{T}{2}}$$

$$= \frac{T}{2}$$

Correlation Coefficient (cont.)

- If $\Delta\theta = 0$

$$\rho(\Delta f, \Delta\theta) = \frac{1}{2\pi\Delta f T}\left(\sin\left(2\pi 2\Delta f T + \Delta\theta\right) - \sin\left(\Delta\theta\right)\right)$$
$$= \mathrm{sinc}\left(4\Delta f T\right)$$

- Thus, the correlation goes to zero for

$$2\Delta f = \frac{1}{2T}$$
$$\Delta f = \frac{1}{4T} = \frac{R}{4} \quad \Rightarrow \quad \left|f_1 - f_2\right| = \frac{R}{2}$$

Correlation Coefficient (cont.)

- If $\Delta\theta \neq 0$

$$\rho(\Delta f, \Delta\theta) = \frac{1}{2\pi 2\Delta f T}\left(\sin\left(2\pi 2\Delta f T + \Delta\theta\right) - \sin\left(\Delta\theta\right)\right)$$

$$= \frac{1}{\pi 2\Delta f T}\sin\left(\pi 2\Delta f T\right)\cos\left(\pi 2\Delta f T + \Delta\theta\right)$$

- Thus, since $\Delta\theta$ is unknown the correlation goes to zero for

$$2\Delta f = \frac{1}{T}$$

$$\Delta f = \frac{1}{2T} = \frac{R}{2} \quad \Rightarrow \quad \left|f_1 - f_2\right| = R$$

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