Chapter 8

Sensitivity Analysis for Linear Programming

Finding the optimal solution to a linear programming model is important, but it is not the only information available. There is a tremendous amount of *sensitivity information*, or information about what happens when data values are changed.

Recall that in order to formulate a problem as a linear program, we had to invoke a *certainty assumption*: we had to know what value the data took on, and we made decisions based on that data. Often this assumption is somewhat dubious: the data might be unknown, or guessed at, or otherwise inaccurate. How can we determine the effect on the optimal decisions if the values change? Clearly some numbers in the data are more important than others. Can we find the "important" numbers? Can we determine the effect of misestimation?

Linear programming offers extensive capability for addressing these questions. We begin by showing how data changes show up in the optimal table. We then give two examples of how to interpret Solver's extensive output.

8.1 Tableau Sensitivity Analysis

Suppose we solve a linear program "by hand" ending up with an optimal table (or tableau to use the technical term). We know what an optimal tableau looks like: it has all non–negative values in Row 0 (which we will often refer to as the *cost row*), all non–negative right–hand–side values, and a basis (identity matrix) embedded. To determine the effect of a change in the data, we will try to determine how that change effected the final tableau, and try to reform the final tableau accordingly.

8.1.1 Cost Changes

The first change we will consider is changing a cost value by Δ in the original problem. We are given the original problem and an optimal tableau. If we had done exactly the same calculations beginning with the modified problem, we would have had the same final tableau except that the corresponding cost entry would be Δ lower (this is because we never do anything except add or subtract scalar multiples of Rows 1 through m to other rows; we never add or subtract Row 0 to other rows). For example, take the problem

```
Max 3x+2y
Subject to
   x+y <= 4
   2x+y <= 6
   x,y >= 0
```

The optimal tableau to this problem (after adding s_1 and s_2 as slacks to place in standard form) is:

z	x	y	s_1	s_2	RHS
1	0	0	1	1	10
0	0	1	2	-1	2
0	1	0	-1	1	2

Suppose the cost for x is changed to $3 + \Delta$ in the original formulation, from its previous value 3. After doing the same operations as before, that is the same pivots, we would end up with the tableau:

z	x	y	s_1	s_2	RHS
1	$-\Delta$	0	1	1	10
0	0	1	2	-1	2
0	1	0	-1	1	2

Now this is not the optimal tableau: it does not have a correct basis (look at the column of x). But we can make it correct in form while keeping the same basic variables by adding Δ times the last row to the cost row. This gives the tableau:

z	x	y	s_1	s_2	RHS
1	0	0	$1 - \Delta$	$1 + \Delta$	$10 + 2\Delta$
0	0	1	2	-1	2
0	1	0	-1	1	2

Note that this tableau has the same basic variables and the same variable values (except for z) that our previous solution had. Does this represent an optimal solution? It does only if the cost row is all non–negative. This is true only if

$$1 - \Delta \geq 0$$

$$1 + \Delta \geq 0$$

which holds for $-1 \leq \Delta \leq 1$. For any Δ in that range, our previous basis (and variable values) is optimal. The objective changes to $10 + 2\Delta$.

In the previous example, we changed the cost of a basic variable. Let's go through another example. This example will show what happens when the cost of a nonbasic variable changes.

```
Max 3x+2y + 2.5w
Subject to
   x+y +2w <= 4
   2x+y +2w <= 6
   x,y,w >= 0
```

Here, the optimal tableau is :

z	x	y	w	s_1	s_2	RHS
1	0	0	1.5	1	1	10
0	0	1	2	2	−1	2
0	1	0	0	−1	1	2

Now suppose we change the cost on w from 2.5 to $2.5 + \Delta$ in the formulation. Doing the same calculations as before will result in the tableau:

z	x	y	w	s_1	s_2	RHS
1	0	0	$1.5 - \Delta$	1	1	10
0	0	1	2	2	−1	2
0	1	0	0	−1	1	2

In this case, we already have a valid tableau. This will represent an optimal solution if $1.5 - \Delta \geq 0$, so $\Delta \leq 1.5$. As long as the objective coefficient of w is no more than 2.5+1.5=4 in the original formulation, our solution of $x = 2, y = 2$ will remain optimal.

The value in the cost row in the simplex tableau is called the *reduced cost*. It is zero for a basic variable and, in an optimal tableau, it is non–negative for all other variables (for a maximization problem).

Summary: Changing objective function values in the original formulation will result in a changed cost row in the final tableau. It might be necessary to add a multiple of a row to the cost row to keep the form of the basis. The resulting analysis depends only on keeping the cost row non–negative.

8.1.2 Right Hand Side Changes

For these types of changes, we concentrate on maximization problems with all $\leq$ constraints. Other cases are handled similarly.

Take the following problem:

$$
\begin{aligned}
\text{Max} \quad & 4x + 5y \\
\text{Subject to} \quad & \\
2x + 3y \quad & <= \quad 12 \\
x + y \quad & <= \quad 5 \\
x, y \quad & >= \quad 0
\end{aligned}
\tag{8.1}
$$

The optimal tableau, after adding slacks s_1 and s_2 is

z	x	y	s_1	s_2	RHS
1	0	0	1	2	22
0	0	1	1	−2	2
0	1	0	−1	3	3

Now suppose instead of 12 units in the first constraint, we only had 11. This is *equivalent* to forcing s_1 to take on value 1. Writing the constraints in the optimal tableau long–hand, we get

$$z + s_1 + 2s_2 = 22$$

$$y + s_1 - 2s_2 = 2$$

$$x - s_1 + 3s_2 = 3$$

If we force s_1 to 1 and keep s_2 at zero (as a nonbasic variable should be), the new solution would be $z = 21$, $y = 1$, $x = 4$. Since all variables are nonnegative, this is the optimal solution.

In general, changing the amount of the right–hand–side from 12 to $12 + \Delta$ in the first constraint changes the tableau to:

z	x	y	s_1	s_2	RHS
1	0	0	1	2	$22 + \Delta$
0	0	1	1	-2	$2 + \Delta$
0	1	0	-1	3	$3 - \Delta$

This represents an optimal tableau as long as the righthand side is all non–negative. In other words, we need Δ between -2 and 3 in order for the basis not to change. For any Δ in that range, the optimal objective will be $22 + \Delta$. For example, with Δ equals 2, the new objective is 24 with $y = 4$ and $x = 1$.

Similarly, if we change the right–hand–side of the second constraint from 5 to $5 + \Delta$ in the original formulation, we get an objective of $22 + 2\Delta$ in the final tableau, as long as $-1 \le \Delta \le 1$.

Perhaps the most important concept in sensitivity analysis is the *shadow price* λ_i^* of a constraint: *If the RHS of Constraint i changes by Δ in the original formulation, the optimal objective value changes by $\lambda_i^* \Delta$.* The shadow price λ_i^* can be found in the optimal tableau. It is the reduced cost of the slack variable s_i. So it is found in the cost row (Row 0) in the column corresponding the slack for Constraint i. In this case, $\lambda_1^* = 1$ (found in Row 0 in the column of s_1) and $\lambda_2^* = 2$ (found in Row 0 in the column of s_2). The value λ_i^* is really the marginal value of the resource associated with Constraint i. For example, the optimal objective value (currently 22) would increase by 2 if we could increase the RHS of the second constraint by $\Delta = 1$. In other words, the marginal value of that resource is 2, i.e. we are willing to pay up to 2 to increase the right hand side of the second constraint by 1 unit. You may have noticed the similarity of interpretation between shadow prices in linear programming and Lagrange multipliers in constrained optimization. Is this just a coincidence? Of course not. This parallel should not be too surprising since, after all, linear programming is a special case of constrained optimization. To derive this equivalence (between shadow prices and optimal Lagrange multipiers), one could write the KKT conditions for the linear program...but we will skip this in this course!

In summary, changing the right–hand–side of a constraint is identical to setting the corresponding slack variable to some value. This gives us the shadow price (which equals the reduced cost for the corresponding slack) and the ranges.

8.1.3   New Variable

The shadow prices can be used to determine the effect of a new variable (like a new product in a production linear program). Suppose that, in formulation (8.1), a new variable w has coefficient 4 in the first constraint and 3 in the second. What objective coefficient must it have to be considered for adding to the basis?

If we look at making w positive, then this is equivalent to decreasing the right hand side of the first constraint by $4w$ and the right hand side of the second constraint by $3w$ in the original formulation. We obtain the same effect by making $s_1 = 4w$ and $s_2 = 3w$. The overall effect of this is to decrease the objective by $\lambda_1^*(4w) + \lambda_2^*(3w) = 1(4w) + 2(3w) = 10w$. The objective value must

be sufficient to offset this, so the objective coefficient must be more than 10 (exactly 10 would lead to an alternative optimal solution with no change in objective).

Example 8.1.1 A factory can produce four products denoted by P_1, P_2, P_3 and P_4. Each product must be processed in each of two workshops. The processing times (in hours per unit produced) are given in the following table.

	P_1	P_2	P_3	P_4
Workshop 1	3	4	8	6
Workshop 2	6	2	5	8

400 hours of labour are available in each workshop. The profit margins are 4, 6, 10, and 9 dollars per unit of P_1, P_2, P_3 and P_4 produced, respectively. Everything that is produced can be sold. Thus, maximizing profits, the following linear program can be used.

```
MAX          4 X1 + 6 X2 + 10 X3 + 9 X4
SUBJECT TO
             3 X1 + 4 X2 +  8 X3 + 6 X4 <=   400        Row 1
             6 X1 + 2 X2 +  5 X3 + 8 X4 <=   400        Row 2
             X1, X2, X3, X4 >= 0
```

Introducing slack variables s_1 and s_2 in Rows 1 and 2, respectively, and applying the simplex method, we get the final tableau:

z	x_1	x_2	x_3	x_4	s_1	s_2	RHS
1	0.5	0	2	0	1.5	0	600
0	0.75	1	2	1.5	0.25	0	100
0	4.5	0	1	5	−0.5	1	200

(a) How many units of P_1, P_2, P_3 and P_4 should be produced in order to maximize profits?

(b) Assume that 20 units of P_3 have been produced by mistake. What is the resulting decrease in profit?

(c) In what range can the profit margin per unit of P_1 vary without changing the optimal basis?

(d) In what range can the profit margin per unit of P_2 vary without changing the optimal basis?

(e) What is the marginal value of increasing the production capacity of Workshop 1?

(f) In what range can the capacity of Workshop 1 vary without changing the optimal basis?

(g) Management is considering the production of a new product P_5 that would require 2 hours in Workshop 1 and ten hours in Workshop 2. What is the minimum profit margin needed on this new product to make it worth producing?

Answers:

(a) *From the final tableau, we read that $x_2 = 100$ is basic and $x_1 = x_3 = x_4 = 0$ are nonbasic. So 100 units of P_2 should be produced and none of P_1, P_3 and P_4. The resuting profit is $ 600 and that is the maximum possible, given the constraints.*

(b) *The reduced cost for x_3 is 2 (found in Row 0 of the final tableau). Thus, the effect on profit of producing x_3 units of P_3 is $-2x_3$. If 20 units of P_3 have been produced by mistake, then the profit will be $2 \times 20 = \$40$ lower than the maximum stated in* (a).

(c) *Let $4 + \Delta$ be the profit margin on P_1. The reduced cost remains nonnegative in the final tableau if $0.5 - \Delta \geq 0$. That is $\Delta \leq 0.5$. Therefore, as long as the profit margin on P_1 is less than 4.5, the optimal basis remains unchanged.*

(d) *Let $6 + \Delta$ be the profit margin on P_2. Since x_2 is basic, we need to restore a correct basis. This is done by adding Δ times Row 1 to Row 0. This effects the reduced costs of the nonbasic variables, namely x_1, x_3, x_4 and s_1. All these reduced costs must be nonnegative. This implies:*

$0.5 + 0.75\Delta \geq 0$

$2 + 2\Delta \geq 0$

$0 + 1.5\Delta \geq 0$

$1.5 + 0.25\Delta \geq 0.$

Combining all these inequalities, we get $\Delta \geq 0$. So, as long as the profit margin on P_2 is 6 or greater, the optimal basis remains unchanged.

(e) *The marginal value of increasing capacity in Workshop 1 is $\lambda_1^* = 1.5$.*

(f) *Let $400 + \Delta$ be the capacity of Workshop 1. The resulting RHS in the final tableau will be:*

$100 + 0.25\Delta$ *in Row 1, and*

$200 - 0.5\Delta$ *in Row 2.*

The optimal basis remains unchanged as long as these two quantities are nonnegative. Namely, $-400 \leq \Delta \leq 400$. So, the optimal basis remains unchanged as long as the capacity of Workshop 1 is in the range 0 to 800.

(g) *The effect on the optimum profit of producing x_5 units of P_5 would be $\lambda_1^*(2x_5) + \lambda_2^*(10x_5) = 1.5(2x_5) + 0(10x_5) = 3x_5$. If the profit margin on P_5 is sufficient to offset this, then P_5 should be produced. That is, we should produce P_5 if its profit margin is at least 3.*

Exercise 67 A paper mill converts pulpwood to low, medium and high grade newsprint. The pulpwood requirements for each newsprint, availability of each pulpwood, and selling price (per ton) are shown below:

	Low grade	Medium grade	High grade	Available (tons)
Virginia pine	2	2	1	180
White pine	1	2	3	120
Loblolly pine	1	1	2	160
Price	$900	$1000	$1200	

The associated linear program is

```
MAX        900 X1   + 1000 X2   + 1200 X3
SUBJECT TO
          2 X1   +    2 X2   +      X3   + S1                 =   180
            X1   +    2 X2   +    3 X3         + S2           =   120
            X1   +      X2   +    2 X3               + S3 =   160
            X1, X2, X3, S1, S2, S3 >= 0
```

with the optimal tableau

z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
1	0	200	0	300	300	0	90,000
0	1	0.8	0	0.6	−0.2	0	84
0	0	0.4	1	−0.2	0.4	0	12
0	0	−0.6	0	−0.2	−0.6	1	52

(a) In what range can the price of low grade paper vary without changing the optimal basis?

(b) What is the new optimal solution if the price of low grade paper changes to $800?

(c) At what price should medium grade paper be sold to make it profitable to produce?

(d) In what range can the availability of Virginia pine vary without changing the optimal basis?

(e) If 10 additional tons of Virginia pine are obtained, by how much will the optimal profit increase?

(f) If the pulpwood resources are increased as in (e), what is the new optimal solution (i.e. the optimal production levels of low, medium and high grade newsprint)?

(g) What would the plant manager be willing to pay for an additional ton of Loblolly pine?

Exercise 68 Snacks 'R Us is deciding what to produce in the upcoming month. Due to past purchases, they have 500 lb of walnuts, 1000 lb of peanuts, and 500 lb of chocolate. They currently sell three mixes: Trail Mix, which consists of 1 lb of each material; Nutty Crunch, which has two pounds of peanuts and 1 lb of walnut; and Choc-o-plenty, which has 2 pounds of chocolate and 1 lb of peanuts. They can sell an unlimited amount of these mixes, except they can sell no more than 100 units of Choc-o-plenty. The income on the three mixes is $2, $3, and $4, respectively. The problem of maximizing total income subject to these constraints is

Maximize $2x_1 + 3x_2 + 4x_3$

Subject to

$x_1 + x_2 \leq 500$

$x_1 + 2x_2 + x_3 \leq 1000$

$x_1 + 2x_3 \leq 500$

$x_3 \leq 100$

$x_i \geq 0$ for all i

The optimal tableau for this problem is (after adding slacks for the four constraints):

z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	RHS
1	0	0	0	1	1	0	3	1800
0	1	0	0	2	−1	0	1	100
0	0	1	0	−1	1	0	−1	400
0	0	0	0	−2	1	1	−3	200
0	0	0	1	0	0	0	1	100

Using the above tableau, determine the following:

(a) What is the solution represented by this tableau (both production quantities and total income)? How do we know it is the optimal tableau?

Production quantities: _____

Total income: _____

Optimal? _____ Reason: _____

(b) Suppose the income for Trail Mix (x_1) is only an estimate. For what range of values would the basis given by the above tableau still be optimal? What would be the solution (both production quantities and total income), if the income was only $2.75?

Range: Lower: _____ Upper: _____

Solution for $2.75: Total Income _____ Production: _____

(c) How much should Snacks 'R Us willing to spend to procure an extra pound of peanuts? How about a pound of walnuts? How about an extra pound of chocolate?

Peanuts: _____ Walnuts: _____ Chocolate: _____

(d) For what range of peanut–stock values is the basis given by the above tableau still optimal? What would be the solution (both production quantities and total objective) if there were only 900 lbs of peanuts available?

Range: Lower: _____ Upper: _____

Solution for 900 lbs: Total Income _____ Production: _____

(e) A new product, Extra-Walnutty Trail Booster, consists of 1 lb of each of peanuts and chocolate, and 2 lb of walnuts. What income do we need to make on this product in order to consider producing it?

Income: _____ Reason: _____

8.2 Solver Output

A large amount (but not all) of the previous analysis is available from the output of Solver. To access the information, simply ask for the sensitivity report after optimizing. Rather than simply give rules for reading the report, here are two reports, each with a set of questions that can be answered from the output.

8.2.1 Tucker Automobiles

Tucker Inc. needs to produce 1000 Tucker automobiles. The company has four production plants. Due to differing workforces, technological advances, and so on, the plants differ in the cost of producing each car. They also use a different amount of labor and raw material at each. This is summarized in the following table:

Plant	Cost ('000)	Labor	Material
1	15	2	3
2	10	3	4
3	9	4	5
4	7	5	6

The labor contract signed requires at least 400 cars to be produced at plant 3; there are 3300 hours of labor and 4000 units of material that can be allocated to the four plants.

This leads to the following formulation:

```
MIN     15 X1 + 10 X2 + 9 X3 + 7 X4
SUBJECT TO
        2 X1 + 3 X2 + 4 X3 + 5 X4 <=    3300
```

```
3 X1 + 4 X2 + 5 X3 + 6 X4 <=    4000
X3 >=    400
X1 + X2 + X3 + X4 =    1000
X1, X2, X3, X4 >= 0
```

Attached is the Solver output for this problem.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$11	X1	400	0	15	1E+30	3.5
$B$12	X2	200	0	10	2	1E+30
$B$13	X3	400	0	9	1E+30	4
$B$14	X4	0	7	7	1E+30	7

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$18	Labor	3000	0	3300	1E+30	300
$B$19	Material	4000	-5	4000	300	200
$B$20	Plant3Min	400	4	400	100	400
$B$21	Total	1000	30	1000	66.6667	100

Here are some questions to be answered:

1. What are the current production quantities? What is the current cost of production?

2. How much will it cost to produce one more vehicle? How much will we save by producing one less?

3. How would our solution change if it cost only $8,000 to produce at plant 2? For what ranges of costs is our solution (except for the objective value) valid for plant 2?

4. How much are we willing to pay for a labor hour?

5. How much is our union contract costing us? What would be the value of reducing the 400 car limit down to 200 cars? To 0 cars? What would be the cost of increasing it by 100 cars? by 200 cars?

6. How much is our raw material worth (to get one more unit)? How many units are we willing to buy at that price? What will happen if we want more?

7. A new plant is being designed that will use only one unit of workers and 4 units of raw material. What is the maximum cost it can have in order for us to consider using it?

8. By how much can the costs at plant 1 increase before we would not produce there?

8.2.2   Carla's Maps

Carla Lee, a current MBA student, decides to spend her summer designing and marketing bicycling maps of Western Pennsylvania. She has designed 4 maps, corresponding to four quadrants around Pittsburgh. The maps differ in size, colors used, and complexity of the topographical relief (the maps are actually 3-dimensional, showing hills and valleys). She has retained a printer to produce the maps. Each map must be printed, cut, and folded. The time (in minutes) to do this for the four types of maps is:

	Print	Cut	Fold
A	1	2	3
B	2	4	2
C	3	1	5
D	3	3	3
Avail	15000	20000	20000

The printer has a limited amount of time in his schedule, as noted in the table.

The profit per map, based on the projected selling price minus printers cost and other variable cost, comes out to approximately \$1 for A and B and \$2 for C and D. In order to have a sufficiently nice display, at least 1000 of each type must be produced.

This gives the formulation:

```
MAX      A + B + 2 C + 2 D
SUBJECT TO
         A + 2 B + 3 C + 3 D <=    15000
         2 A + 4 B + C + 3 D <=    20000
         3 A + 2 B + 5 C + 3 D <=    20000
         A >=    1000
         B >=    1000
         C >=    1000
         D >=    1000
```

Attached is the Solver output.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$11	XA	1500	0	1	1	0.333333
$B$12	XB	1000	0	1	0.333333	1E+30
$B$13	XC	1000	0	2	0.333333	1E+30
$B$14	XD	2833.333	0	2	1	0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$17	Print	15000	0.5	15000	100	366.6667
$B$18	Cut	16500	0	20000	1E+30	3500
$B$19	Fold	20000	0.166667	20000	7000	1000
$B$20	MinA	1500	0	1000	500	1E+30
$B$21	MinB	1000	-0.333333	1000	1750	1000
$B$22	MinC	1000	-0.333333	1000	500	1000
$B$23	MinD	2833.333	0	1000	1833.333	1E+30

Here are some questions to answer:

1. What are the production quantities and projected profit?

2. How much is Carla willing to pay for extra printing time? cutting time? folding time? For each, how many extra hours are we willing to buy at that price?

3. Suppose we reduced the 1000 limit on one item to 900. Which map should be decreased, and how much more would Carla make?

4. A fifth map is being thought about. It would take 2 minutes to print, 2 minutes to cut, and 3 minutes to fold. What is the least amount of profit necessary in order to consider producing this map? What is the effect of requiring 1000 of these also?

5. The marketing analysis on D is still incomplete, though it is known that the profit of $2 per item is within $.25 of the correct value. It will cost $500 to complete the analysis. Should Carla continue with the analysis?

Exercise 69 Recall the Red Dwarf toaster example (Exercise 61).

From Answer report: Final Cost: 7833.33

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$1	Manual	633.3333333	0	7	0.5	1E+30
$B$2	SemiAut	333.3333333	0	8	1E+30	0.25
$B$3	Robotic	33.33333333	0	8.5	0.5	2.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease

$B$5	Total	1000	10.83333333	1000	170	100
$B$6	Skilled	2233.333333	0	4500	1E+30	2266.666667
$B$7	Unskill	36000	-0.083333333	36000	1000	6800
$B$8	Assembly	2700	-0.166666667	2700	500	100

(a) What is the optimal allocation of production? What is the average cost/toaster of production.

(b) By how much can the cost of robots increase before we will change that production plan.

(c) How much is Red Dwarf willing to pay for more assembly room time? How many units is Red Dwarf willing to purchase at that price?

(d) How much will we save if we decide to produce only 950 toasters?

(e) A new production process is available that uses only 2 minutes of skilled labor, 10 minutes of unskilled labor, and an undetermined amount of assembly floor time. Its production cost is determined to be $10. What is the maximum assembly floor time that the process can take before it is deemed too expensive to use?

Answer:

(a) *633.3 should be produced manually, 333.3 should be produced semiautomatically, and 33.3 produced robotically, for an average cost of $7.383/toaster.*

(b) *It can increase by $0.50.*

(c) *Value is $0.16/minute, willing to purchase 500 at that price.*

(d) *Objective will go down by 50(10.833).*

(e) *Cost of $10 versus marginal cost of $10.833, leave 0.83. Unskilled labor costs $0.0833/unit. Therefore, if the new process takes any time at all, it will be deemed too expensive.*

Exercise 70 Kennytrail Amusement park is trying to divide its new 50 acre park into one of three categories: rides, food, and shops. Each acre used for rides generates $150/hour profit; each acre used for food generates $200/hour profit. Shops generate $300/hour profit. There are a number of restrictions on how the space can be divided.

1. Only 10 acres of land is suitable for shops.

2. Zoning regulations require at least 1000 trees in the park. A food acre has 30 trees; a ride acre has 20 trees; while a shop acre has no trees.

3. No more than 200 people can work in the park. It takes 3 people to work an acre of rides, 6 to work an acre of food, and 5 to work an acre of shops.

The resulting linear program and Solver output is attached:

```
MAX     150 RIDE + 200 FOOD + 300 SHOP
SUBJECT TO
      RIDE + FOOD + SHOP <=   50
      SHOP <=   10
      20 RIDE + 30 FOOD >=   1000
      3 RIDE + 6 FOOD + 5 SHOP <=   200
```

Answer report:

Target Cell (Max)

Cell	Name	Original Value	Final Value
$B$5	Cost	0	9062.5

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$1	Ride	0	31.25
$B$2	Food	0	12.5
$B$3	Shop	0	6.25

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$6	Land	50	$B$6<=$D$6	Binding	0
$B$7	ShopLim	6.25	$B$7<=$D$7	Not Binding	3.75
$B$8	Trees	1000	$B$8>=$D$8	Binding	0
$B$9	Workers	200	$B$9<=$D$9	Binding	0

Sensitivity report:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$1	Ride	31.25	0	150	83.33333333	76.6666667
$B$2	Food	12.5	0	200	115	125
$B$3	Shop	6.25	0	300	1E+30	116.666667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$6	Land	50	143.75	50	10	16.66666667
$B$7	ShopLim	6.25	0	10	1E+30	3.75
$B$8	Trees	1000	-4.375	1000	166.66667	100
$B$9	Workers	200	31.25	200	30	50

For each of the following changes, either find the answer or state that the information is not available from the Solver output. In the latter case, state why not.

(a) What is the optimal allocation of the space? What is the profit/hour of the park?

Optimal Allocation: _____Profit/hour: _____

(b) Suppose Food only made a profit of \$180/hour. What would be the optimal allocation of the park, and what would be the profit/hour of the park?

Optimal Allocation: _____Profit/hour: _____

(c) City Council wants to increase our tree requirement to 1020. How much will that cost us (in $/hour). What if they increased the tree requirement to 1200?

Increase to 1020: _____ Increase to 1200: _____

(d) A construction firm is willing to convert 5 acres of land to make it suitable for shops. How much should Kennytrail be willing to pay for this conversion (in $/hour).

Maximum payment: _____

(e) Kennytrail is considering putting in a waterslide. Each acre of waterslide can have 2 trees and requires 4 workers. What profit/hour will the waterslide have to generate for them to consider adding it?

Minimum Profit: _____ Reason: _____

(f) An adjacent parcel of land has become available. It is five acres in size. The owner wants to share in our profits. How much $/hour is Kennytrail willing to pay?

Maximum payment: _____

Exercise 71 Attached is the sensitivity report for the Diet Problem (see Section 5.4.1 in Chapter 5).

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$14	oatmeal	4	-3.1875	3	3.1875	1E+30
$B$15	chicken	0	12.46875	24	1E+30	12.46875
$B$16	eggs	0	4	13	1E+30	4
$B$17	milk	4.5	0	9	2.6923	1.38095
$B$18	pie	2	-3.625	20	3.625	1E+30
$B$19	pork&beans	0	4.375	19	1E+30	4.375

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$23	Energy	2000	0.05625	2000	560	100
$B$24	Protein	60	0	55	5	1E+30
$B$25	Calcium	1334.5	0	800	534.5	1E+30

Answer each of the following questions independently of the others.

1. What does the optimal diet consist of?

2. If the cost of oatmeal doubled to 6 cents/serving, should it be removed from the diet?

3. If the cost of chicken went down to half its current price, should it be added to the diet?

4. At what price would eggs start entering the diet?

5. In what range can the price of milk vary (rounding to the nearest tenth of a cent) while the current diet still remaining optimal?

6. During midterms, you need a daily diet with energy content increased from 2000 kcal to 2200 kcal. What is the resulting additional cost?

7. Your doctor recommends that you increase the calcium requirement in your diet from 800 mg to 1200 mg. What is the effect on total cost?

8. Potatoes cost 12 cents/serving and have energy content of 300 kcal per serving, but no protein nor calcium content. Should they be part of the diet?

Exercise 72 Attached is the sensitivity report for the Workforce Planning Problem (see Section 5.4.2 in Chapter 5).

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$14	Shift1	4	0	1	0.5	1
$B$15	Shift2	7	0	1	0	0.333333
$B$16	Shift3	1	0	1	0.5	0
$B$17	Shift4	4	0	1	0.5	0
$B$18	Shift5	3	0	1	0	0.333333
$B$19	Shift6	3	0	1	0.5	1
$B$20	Shift7	0	0.333333	1	1E+30	0.333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$24	Monday	14	0.333333	14	1.5	6
$B$25	Tuesday	17	0	13	4	1E+30
$B$26	Wednesday	15	0.333333	15	6	3
$B$27	Thursday	16	0	16	3	4
$B$28	Friday	19	0.333333	19	4.5	3
$B$29	Saturday	18	0.333333	18	1.5	6
$B$30	Sunday	11	0	11	4	1

Answer each of the following questions independently of the others.

1. What is the current total number of workers needed to staff the restaurant?

2. Due to a special offer, demand on thurdays increases. As a result, 18 workers are needed instead of 16. What is the effect on the total number of workers needed to staff the restaurant?

3. Assume that demand on mondays decreases: 11 workers are needed instead of 14. What is the effect on the total number of workers needed to staff the restaurant?

4. Currently, 15 workers are needed on wednesdays. In what range can this number vary without changing the optimal basis?

5. Currently, every worker in the restaurant is paid $1000 per month. So the objective function in the formulation can be viewed as total wage expenses (in thousand dollars). Workers have complained that Shift 4 is the least desirable shift. Management is considering increasing the wages of workers on Shift 4 to $1100. Would this change the optimal solution? What would be the effect on total wage expenses?

6. Shift 1, on the other hand, is very desirable (sundays off while on duty fridays and saturdays, which are the best days for tips). Management is considering reducing the wages of workers on Shift 1 to $ 900 per month. Would this change the optimal solution? What would be the effect on total wage expenses?

7. Management is considering introducing a new shift with the days off on tuesdays and sundays. Because these days are not consecutive, the wages will be $ 1200 per month. Will this increase or reduce the total wage expenses?