# ECE4634
# Digital Communications
# Fall 2007

Instructor: R. Michael Buehrer

Lecture #19: *M*-ary FSK

Pulse shaping

Analog and Digital Communications

# Overview

- Previously we examined two methods of multilevel (*M*-ary) modulation *M*-PSK and QAM
  - Allows for improved bandwidth efficiency but degraded energy efficiency
- Today we extend our discussion to *M*-FSK modulation
  - Does not improve bandwidth efficiency (in fact degrades bandwidth efficiency) but improves energy efficiency
- What to read – Sections 7.5, 7.7

# Bandpass Modulation - FSK

- Binary Frequency Shift Keying (BFSK)
- We modulate or change the frequency depending on the data bit to be sent
- Basic Idea:
  - Send one tone $f_1$ for a 1
  - Send another tone $f_2$ for a 0
  - Then we transmit the signal $s(t)$:

$$1 \Rightarrow s(t) = \cos\left(2\pi f_1 t + \theta_1\right)\Big|_0^{T_s}$$

$$0 \Rightarrow s(t) = \cos\left(2\pi f_2 t + \theta_2\right)\Big|_0^{T_s}$$

$\theta_1$ and $\theta_2$ are arbitrary constants that simply reflect the fact that the two oscillators are not phase locked. Better spectral properties are achieved if $\theta_1 = \theta_2$ and if the phase is continuous.



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# Continuous Phase BFSK



- We can also create a BFSK signal using a frequency modulator. Such a scheme keeps the phase a continuous function (i.e., there are no phase jumps).
- This type of FSK has better spectral properties and is called *continuous phase* FSK

$$s(t) = A_c \cos\left[\omega_c t + D_f \int_{-\infty}^{t} m(\lambda)d\lambda\right]$$

$$= \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$

$$g(t) = A_c e^{j\theta(t)}$$

$$\theta(t) = D_f \int_{-\infty}^{t} m(\lambda)d\lambda$$

*m(t)* = polar NRZ line code
$D_f = 2\pi\Delta f$

# Minimum Shift Keying

- Can be viewed as a pulse shaped version of Offset QPSK with Sinusoidal pulse shaping

$$s(t) = A\left\{\left[\sum_{n=-\infty}^{\infty} I_{2n} g\left(t - nT_s\right)\right]\cos\left(2\pi f_c t\right) + \left[\sum_{n=-\infty}^{\infty} I_{2n+1} g\left(t - nT_s - \frac{T_s}{2}\right)\right]\sin\left(2\pi f_c t\right)\right\}$$

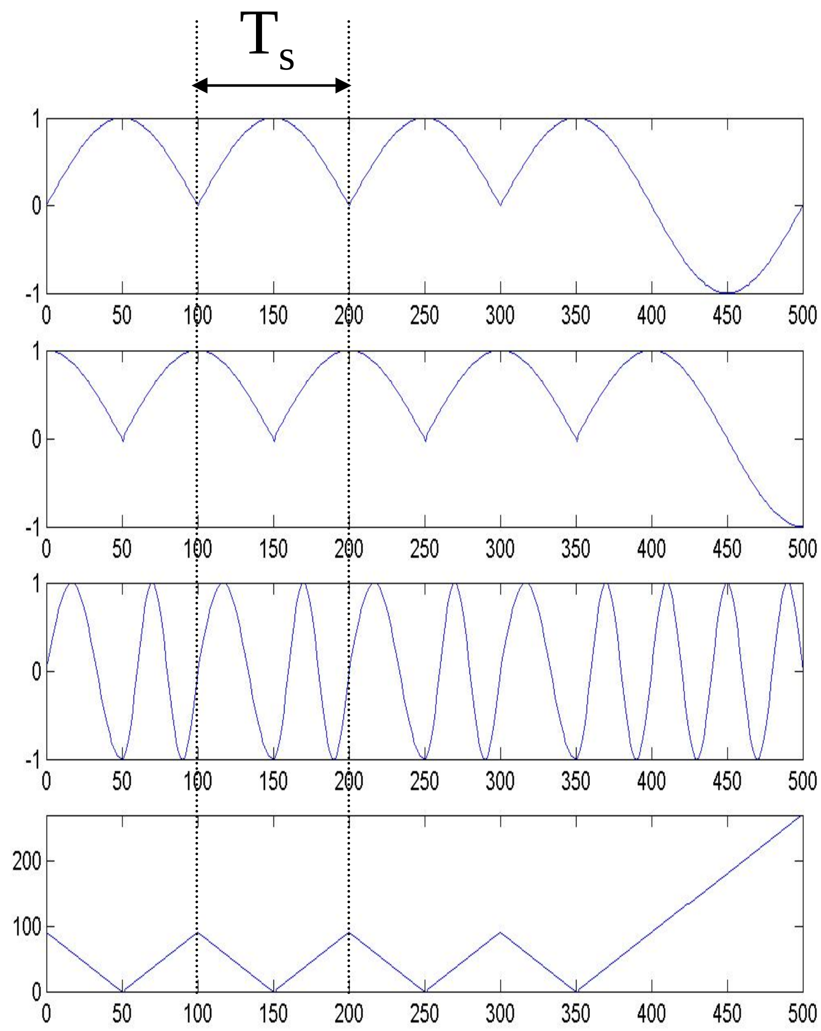
$$I_n \in \{+1, -1\}$$

$$I_n = \left(-1\right)^{b_n}$$

$$g(t) = \begin{cases} \sin\left(\dfrac{\pi t}{T_s}\right) & 0 \le t \le T_s \\ 0 & else \end{cases}$$

- Can also be viewed as a special case of FSK
  - With minimum coherent frequency separation (thus the name *Minimum* Shift Keying)

# Minimum Shift Keying (MSK)




**I channel bits - pulse shaped with $\sin(\pi t/T_s)$**

**Q channel bits - pulse shaped with $\sin(\pi t/T_s)$ and staggered by $T_s/2$**

**Tx Signal**

**Phase**

***Note: Phase is continuous***

# MSK vs BFSK

- When viewed/demodulated bit by bit, we have an BFSK signal. However, because of the phase constraints imposed, we can view the signal over a two bit interval as offset QPSK with sinusoidal pulse shaping.

**Each bit changes the direction of the phase rotation**

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Binary Data** | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ | $b_{10}$ | $b_{11}$ | $b_{12}$ | $b_{13}$ | $b_{14}$ |

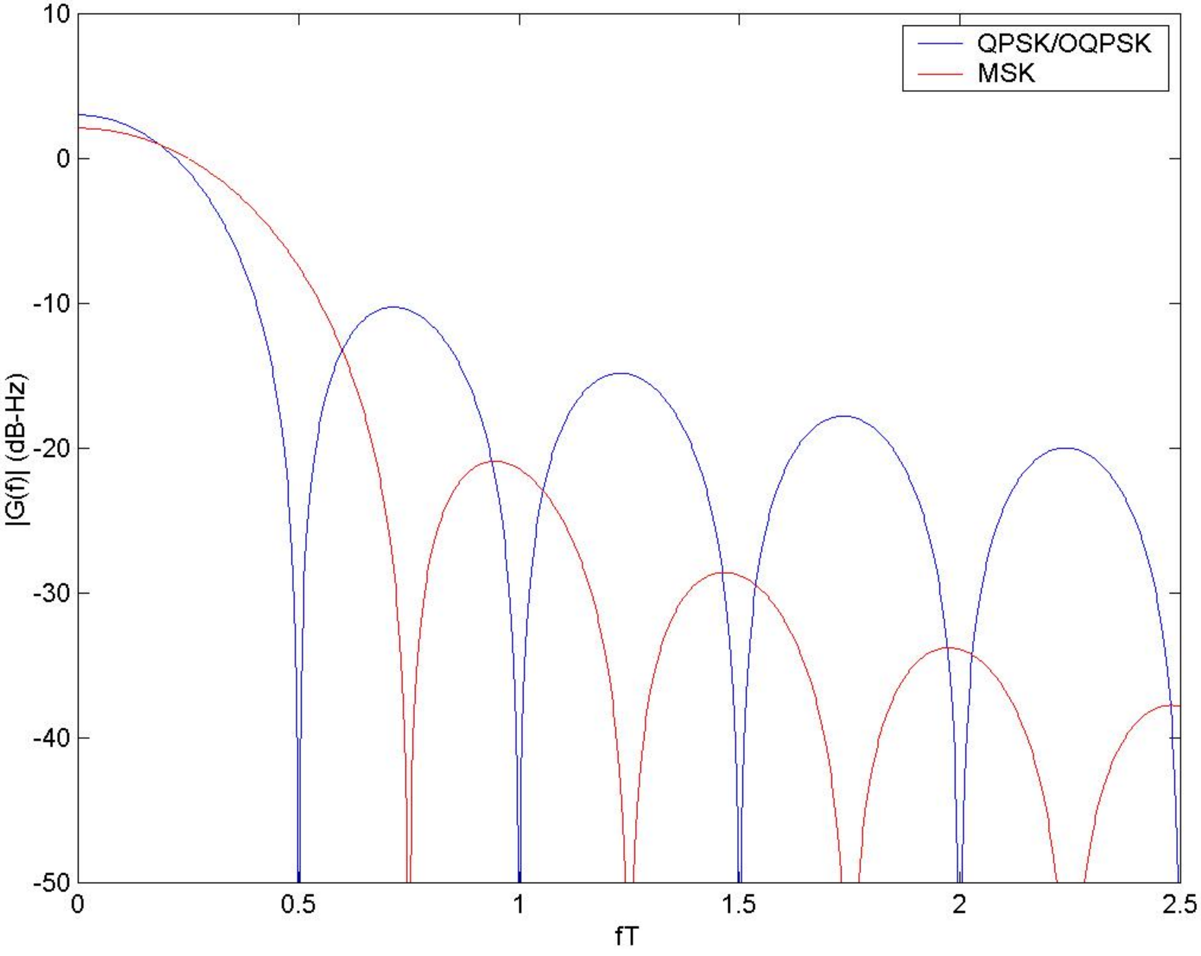
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **I-channel** | $b_1$ | $b_3$ | $b_5$ | $b_7$ | $b_9$ | $b_{11}$ | $b_{13}$ |
| **Q-channel** | $b_2$ | $b_4$ | $b_6$ | $b_8$ | $b_{10}$ | $b_{12}$ | $b_{14}$ |

**I/Q functions stay constant over a 2 bit interval**

# Spectrum of MSK and O/QPSK

Main lobe is wider

Side-lobes fall off faster



# *M*-ary FSK

- To extend FSK to *M* > 2 we simply add frequencies
- Example 4-FSK

$$s_1(t) = A\cos(\omega_1 t)\Big|_0^T$$

$$s_2(t) = A\cos(\omega_2 t)\Big|_0^T$$

$$s_3(t) = A\cos(\omega_3 t)\Big|_0^T$$

$$s_4(t) = A\cos(\omega_4 t)\Big|_0^T$$

Four orthogonal carriers
Four symbols
Two bits per symbol

# Bandwidth Efficiency

- Non-coherent carriers
  - Recall that the minimum frequency spacing to maintain orthogonality between carriers is $2\Delta f_{min} = R_s$
  - Further, if we assume square pulses, the first null bandwidth on either side of the first and last carrier is $R_s$.
  - Thus, the total bandwidth (null-to-null) is thus

$$\boxed{\begin{aligned} B &= R_s + (M-1)(f_1 - f_2) + R_s \\ &= 2R_s + (M-1)2\Delta f \\ &= 2R_s + (M-1)R_s \\ &= (M+1)R_s \end{aligned}}$$

# Bandwidth Efficiency

- Coherent Carriers
  - Recall that the minimum frequency spacing to maintain orthogonality between carriers is $2\Delta f_{min} = R_s/2$
  - Further, if we assume square pulses, the first null bandwidth on either side of the first and last carrier is $R_s$.
  - Thus, the total bandwidth (null-to-null) is

$$\boxed{\begin{aligned} B &= R_s + (M-1)(f_1 - f_2) + R_s \\ &= 2R_s + (M-1)2\Delta f \\ &= 2R_s + (M-1)R_s/2 \\ &= \left(\frac{M+3}{2}\right)R_s \end{aligned}}$$

# Bandwidth Efficiency

- We typically refer to bandwidth efficiency $\eta_{BW}$ as the bit rate over the bandwidth
- Thus for non-coherent carriers:

$$B = (M+1)R_s$$

$$R_b = (\log_2 M)R_s$$

$$\eta_{BW} = \frac{R_b}{B} = \frac{(\log_2 M)R_s}{(M+1)R_s}$$

$$= \frac{\log_2 M}{M+1}$$

Thus, the bandwidth efficiency of FSK decreases with M. This is in contrast to PSK, QAM, ASK.

# Bandwidth Efficiency

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| Modulation Scheme | First null-to-null bandwidth efficiency |
|---|---|
| *M*-PSK | $\eta_{BW} = \frac{\log_2 M}{2}$ |
| *M*-ASK | $\eta_{BW} = \frac{\log_2 M}{2}$ |
| QAM | $\eta_{BW} = \frac{\log_2 M}{2}$ |
| *M*-FSK (non-coherent carriers) | $\eta_{BW} = \frac{\log_2 M}{M+1}$ |

# Orthogonal Signaling

- FSK is one form of a family of modulation schemes known as orthogonal modulation schemes
- With orthogonal modulation schemes all *M* symbols are orthogonal
  - Requires *M* “dimensions” – we will discuss this more in a week or so
  - Increasing the number of dimensions improves performance but drastically reduces bandwidth efficiency

# Conclusions - MFSK

- Frequency Shift Keying can be analyzed just as any other modulation scheme
- It is slightly different due to the fact that as *M* increases, the number of dimensions (i.e., the number of orthogonal components) increases.
  - This is in contrast to PSK, ASK and QAM which have a constant number of dimensions.
- The result is that bandwidth efficiency goes down with *M* and energy efficiency goes up with *M*

# Pulse shaping

- So far we have discussed controlling bandwidth by using multi-level modulation
  - Reducing the symbol rate without reducing the bit rate reduces the bandwidth
  - Has implications on performance
- We can also reduce the bandwidth of bandpass modulation schemes through pulse shaping
  - Analogous to baseband case

# Pulse shaping

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- Recall that both *M*-PSK, *M*-ASK and QAM can be represented as

$$s(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- where

Square pulse

$$x(t) = \sum_{i=-\infty}^{\infty} a_i p(t - iT_s)$$

$$y(t) = \sum_{i=-\infty}^{\infty} b_i p(t - iT_s)$$

PSK → $a_i = \cos\left(\frac{2\pi}{M} i\right) \quad b_i = \sin\left(\frac{2\pi}{M} i\right)$

QAM→ $a_i, b_i \in \left\{\pm 1, \pm 2, .. \pm \sqrt{M}\right\}$

ASK→ $a_i \in \{0, 1, 2, .. M - 1\}$

$b_i = 0$

# Power Spectral Density

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- Thus, the complex baseband version is

$$g(t) = x(t) + jy(t)$$

$$= \sum_{i=-\infty}^{\infty} (a_i + jb_i)\, p(t - iT)$$

- We know from the modulation principle that

$$P_v(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c)\right]$$

- Further, for a digitally modulated pulse stream

$$\boxed{P_x(f) = \underbrace{\frac{\sigma_a^2}{T_s}\left|F(f)\right|^2}_{continuous} + \underbrace{\frac{m_a^2}{T_s}\sum_{k=-\infty}^{\infty}\left|F\left(\frac{n}{T_s}\right)\right|^2 \delta\left(f - \frac{n}{T_s}\right)}_{discrete}}$$

Pulse spectrum

Data properties

# Power Spectral Density

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- Assume equal *average* power for all schemes

$$E\left\{\left|a_i + jb_i\right|^2\right\} = P_{avg}$$

$$= \begin{cases} \sigma^2 & PSK, QAM \\ \sigma^2 + A_{avg}^2 & ASK \end{cases}$$

$$\boxed{P_g(f) = \frac{P_{avg}}{T_s}\left|F(f)\right|^2} \qquad \text{PSK, QAM}$$

$$\boxed{P_g(f) = \frac{P_{avg} - A_{avg}^2}{T_s}\left|F(f)\right|^2 + \frac{A_{avg}^2}{T_s}\sum_{k=-\infty}^{\infty}\left|F\left(\frac{n}{T_s}\right)\right|^2 \delta\left(f - \frac{n}{T_s}\right)} \qquad \text{ASK}$$

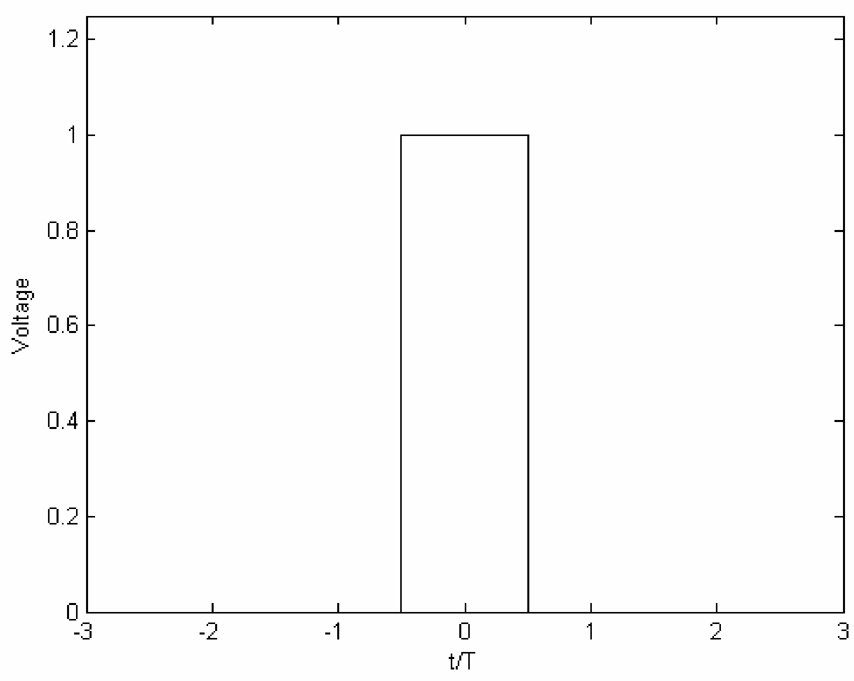
- Thus we can control the power spectral density through the pulse shape *F(f)*

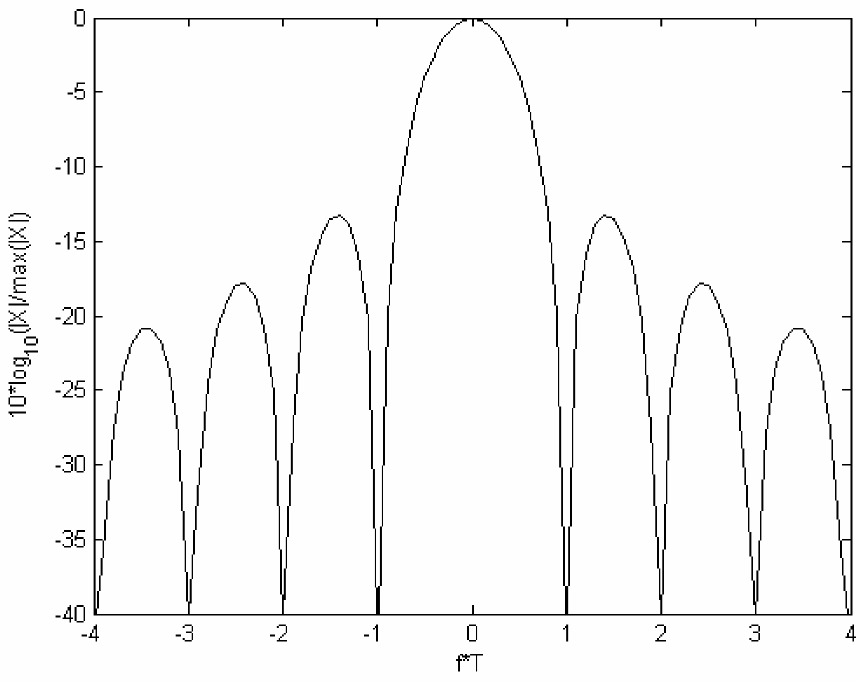
# Pulse Shaping

- Pulse shaping for bandpass systems (when PSK, QAM, ASK are used) is identical to pulse shaping for baseband systems
- Raised cosine (and sinc) pulses are preferable since they have controlled bandwidth properties and have zero ISI
- Since a matched filter is desired at the receiver, we actually use *square root raised cosine* pulses

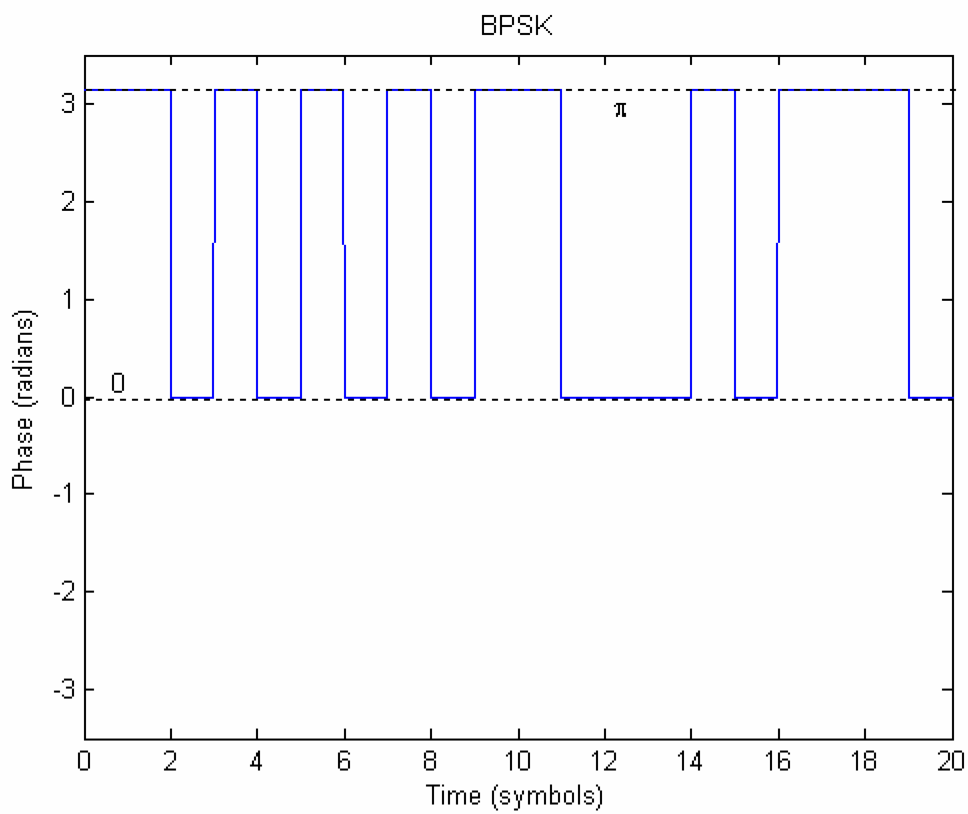
# Square Pulses

- Pulse shape                                   Spectrum



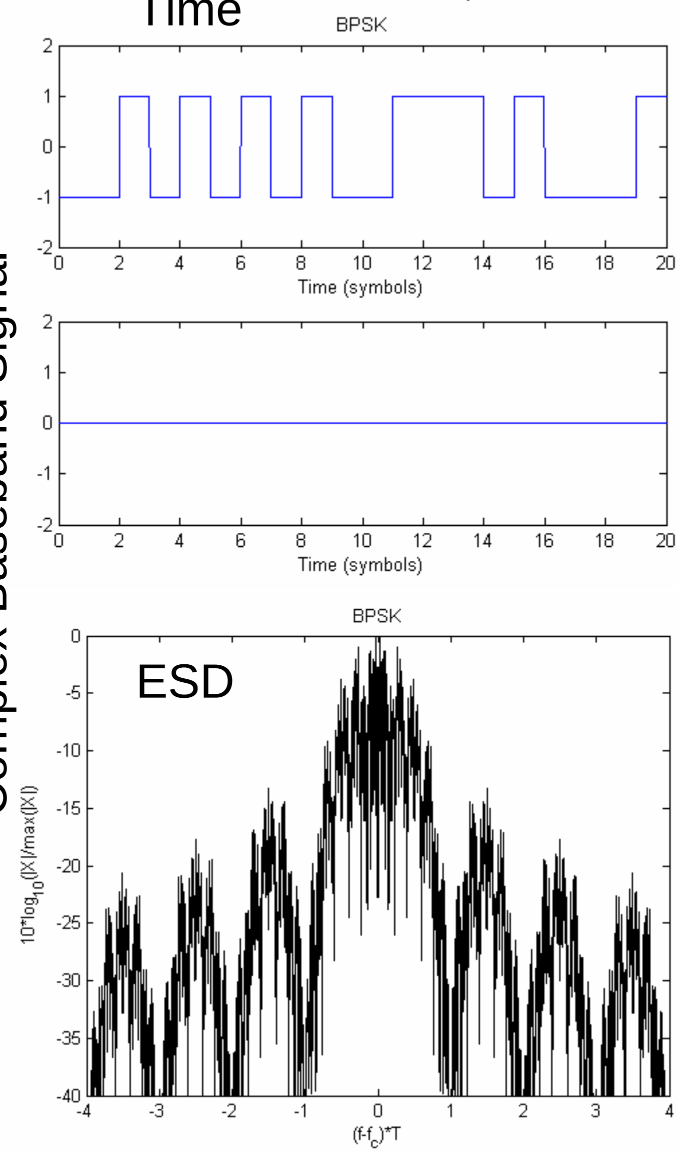
# Square Pulses – BPSK




Phase of Carrier

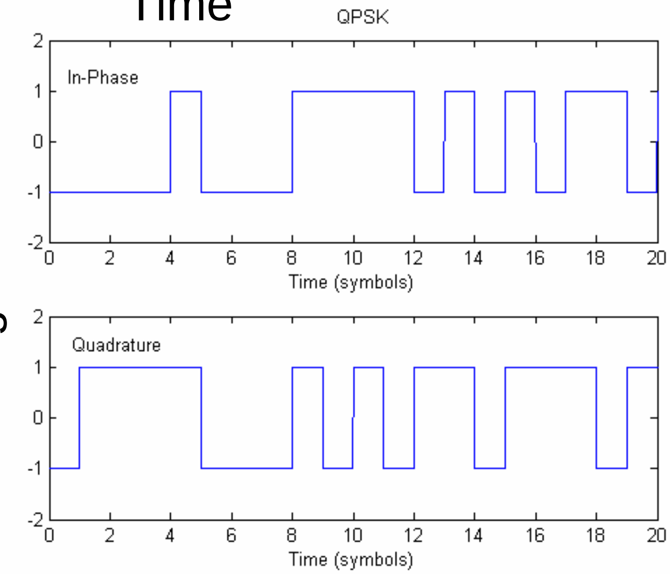
Time

Complex Baseband Signal

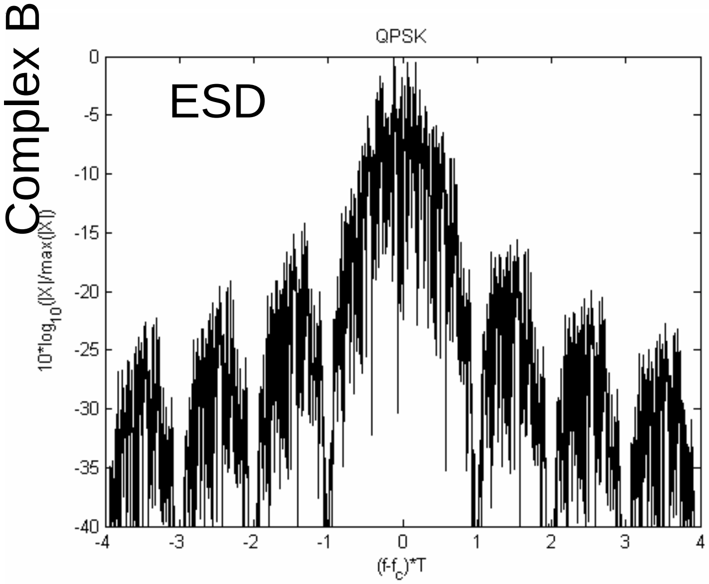


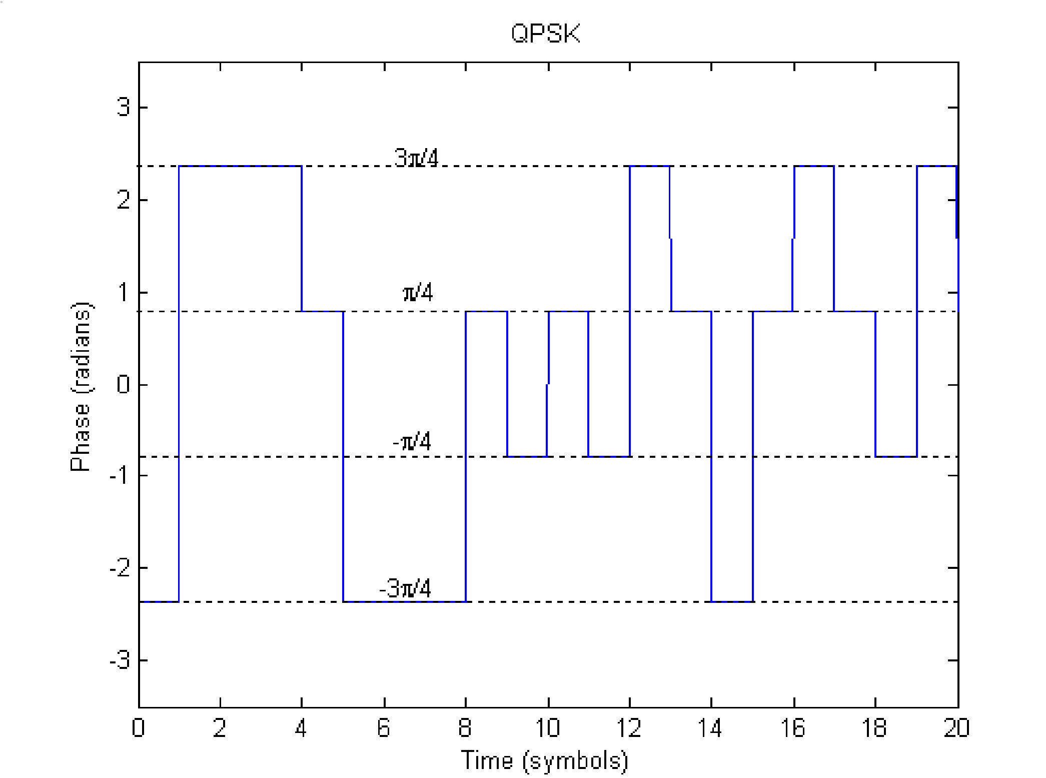
# Square Pulses – QPSK

Time




Complex Baseband Signal




Phase of Carrier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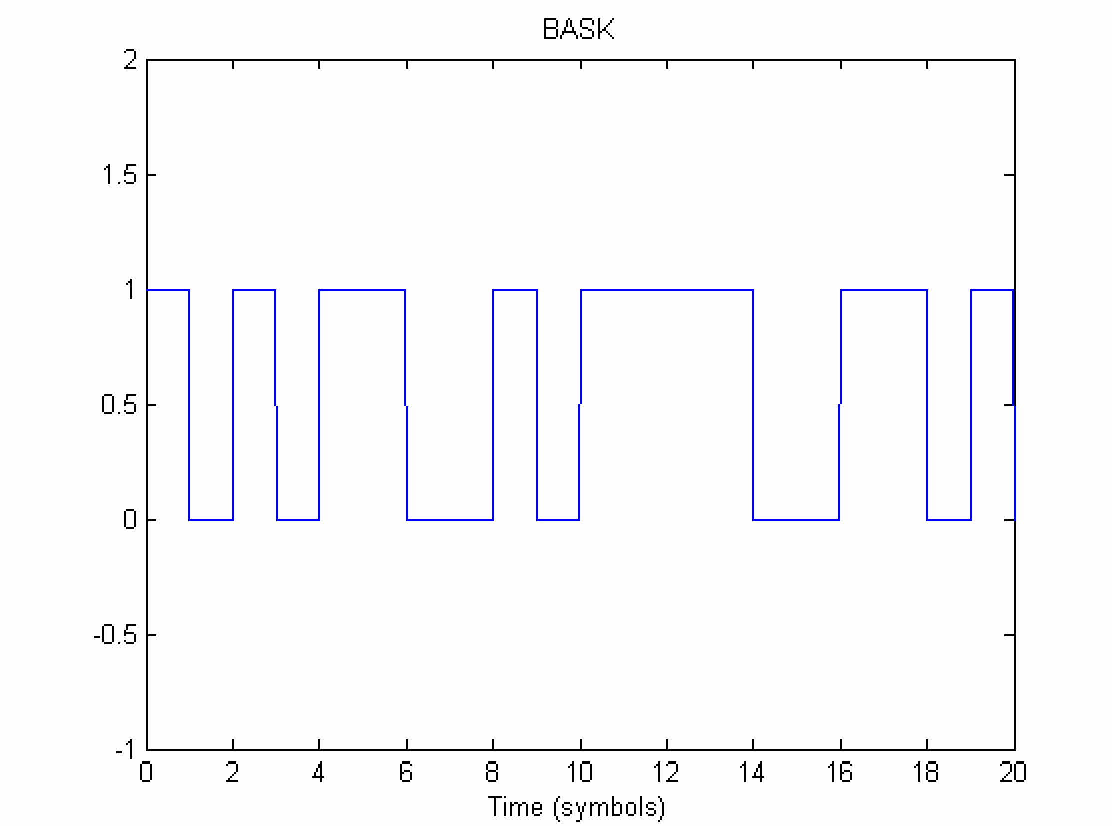
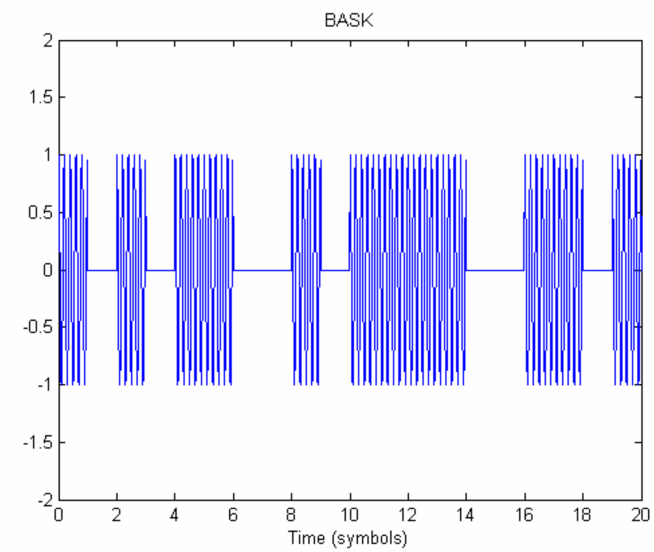
# Square Pulses – BASK



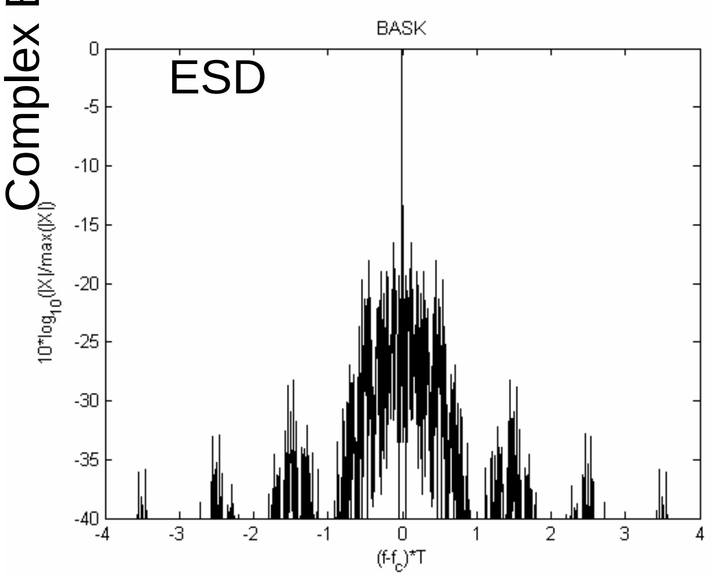

Amplitude of Carrier

Time

Complex Baseband Signal



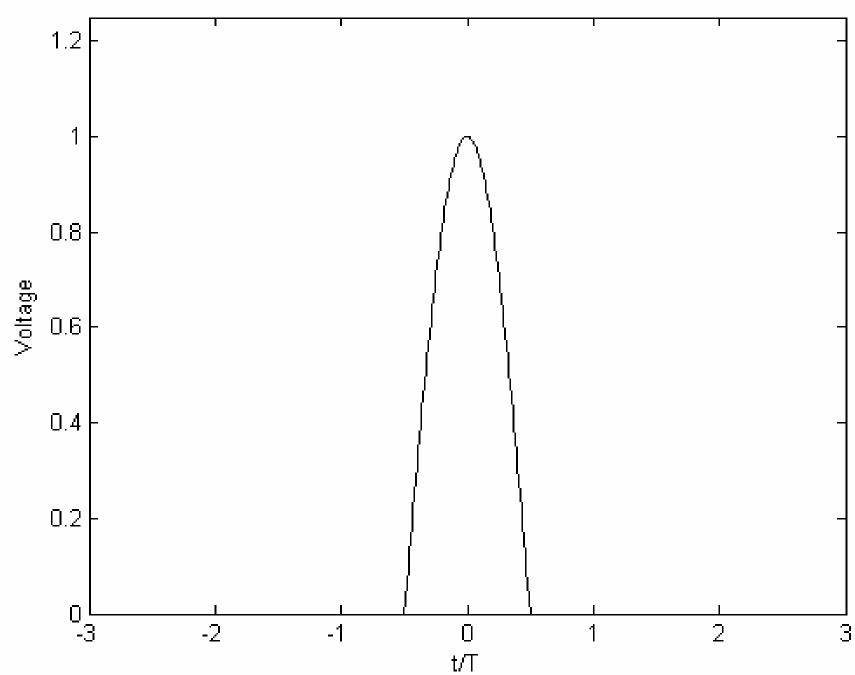

ESD

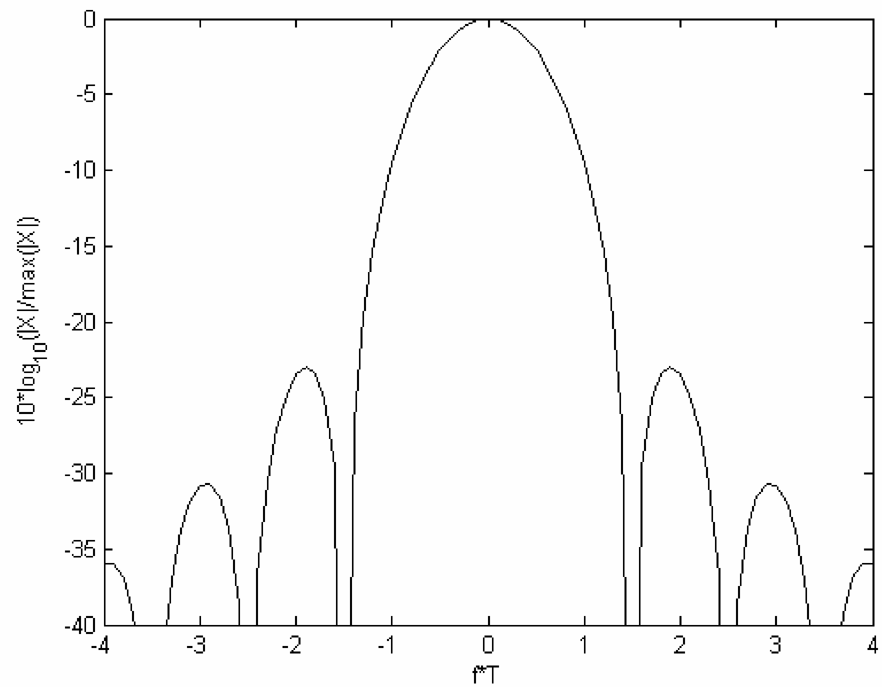


# Cosine Pulses

- Pulse shape

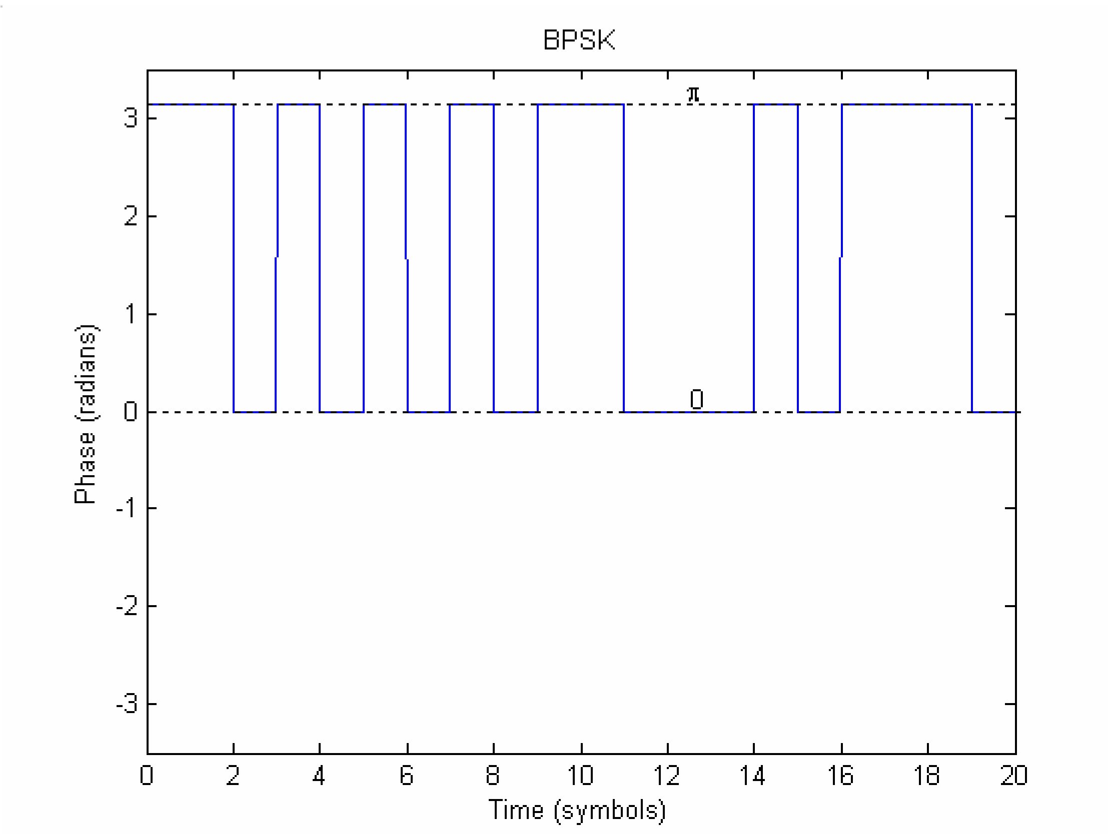
Spectrum



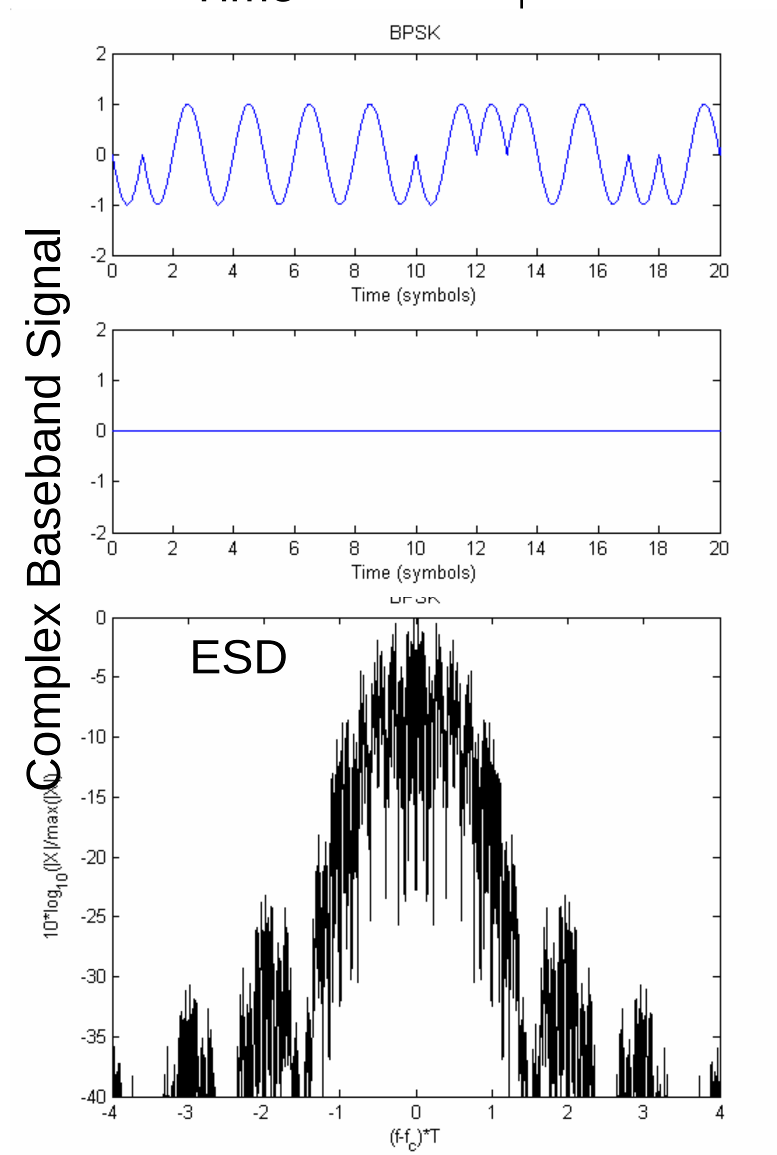



# Cosine Pulses – BPSK

Time




Phase of Carrier

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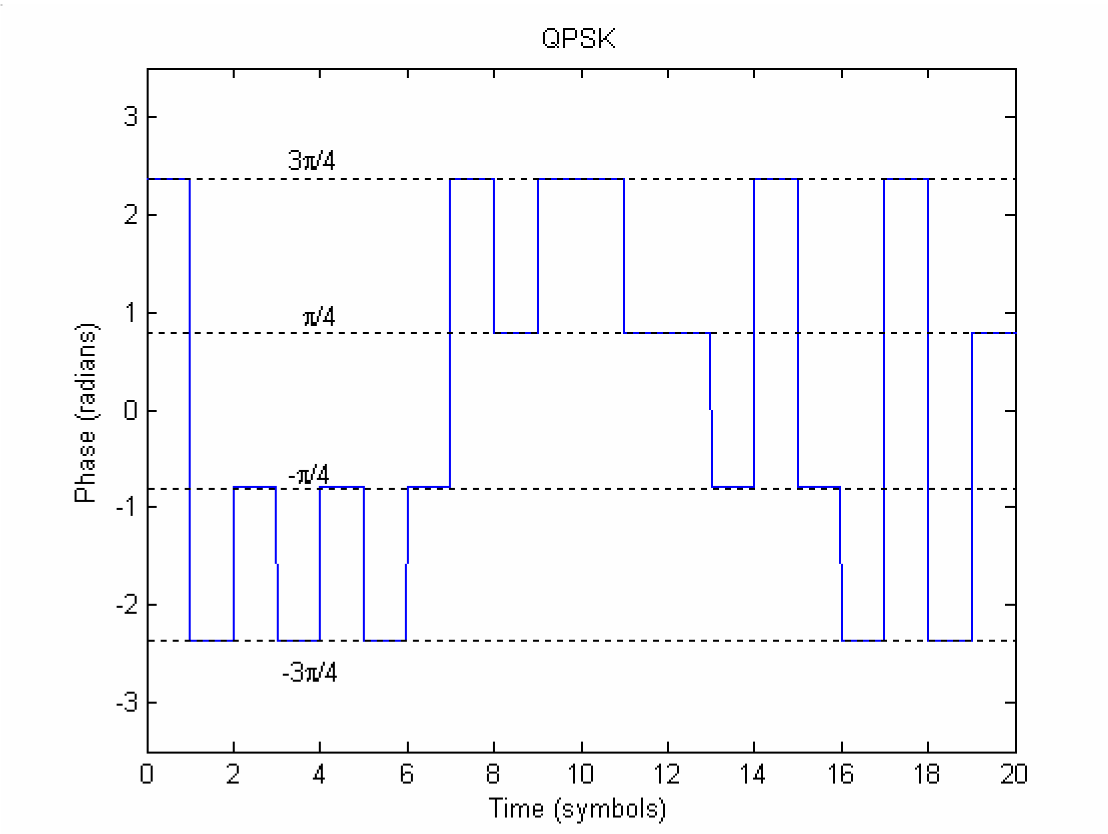
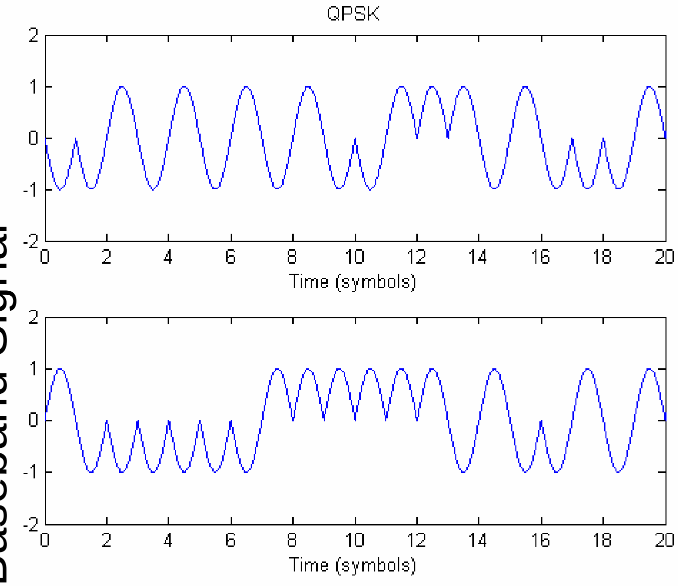
# Cosine Pulses – QPSK



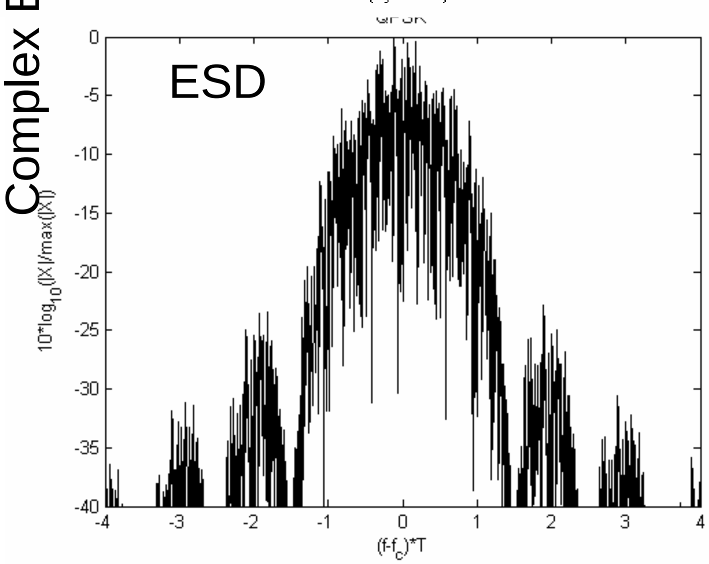

Phase of Carrier

Time

Complex Baseband Signal




ESD

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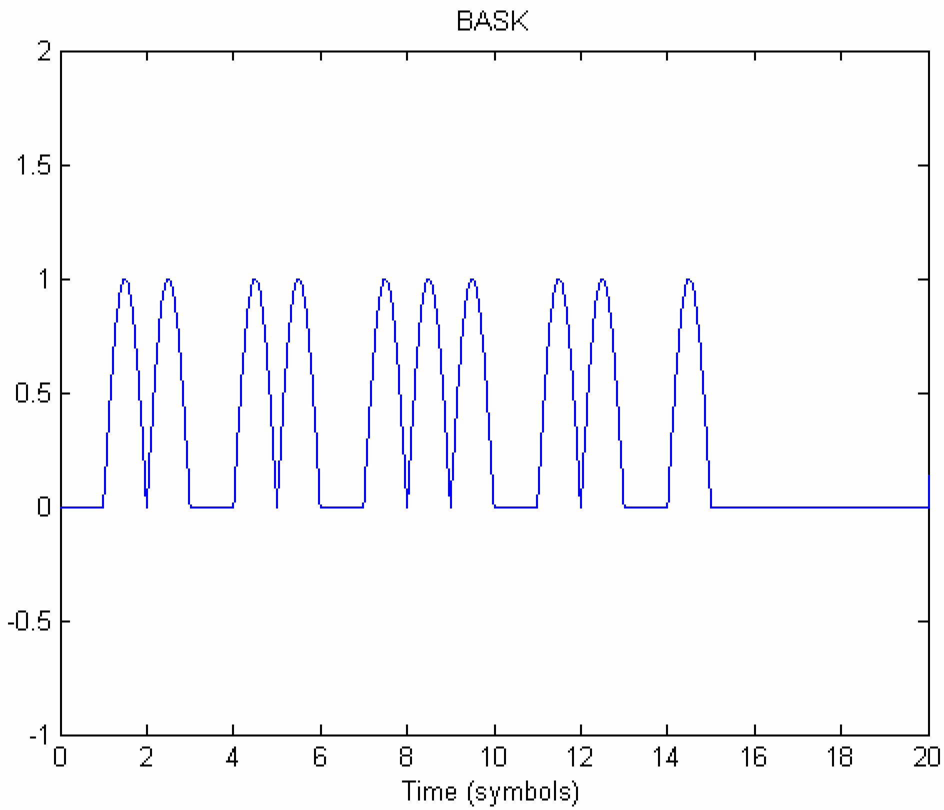
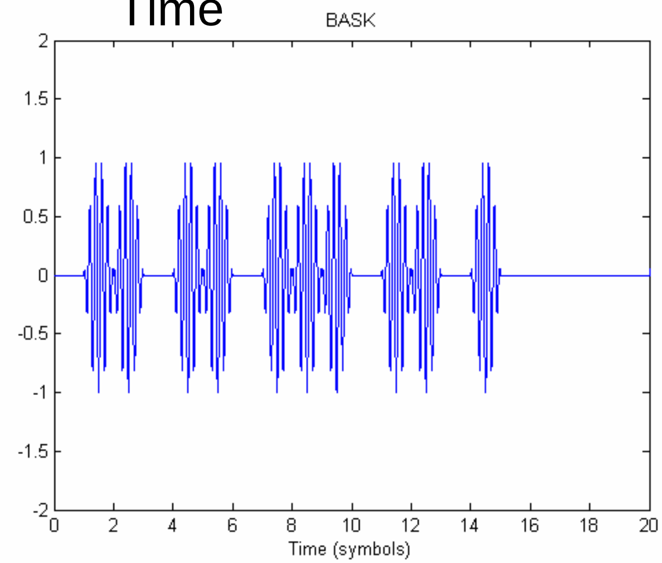
# Cosine Pulses – BASK

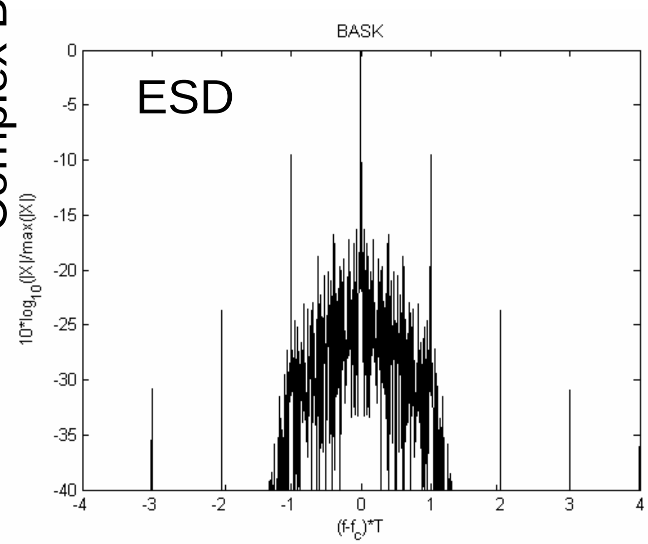



Amplitude of Carrier

Time

Complex Baseband Signal

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# Conclusions

- Pulse shaping does not affect the phase modulation but does affect amplitude modulation
  - Can be ignored with proper sampling time
- Pulse shaping does affect the spectral characteristics
- Using non-square pulses results in non-constant envelope
- We will find that the performance is unaffected by pulse shape provided
  - No ISI introduced
  - Matched filter used