ECE4634
Introduction to
Digital Communications
Fall 2007

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Lecture #13: Introduction to
Digital Bandpass Modulation

Analog and Digital Communications

Overview

- Modulation is the process for transferring information using EM waves
- Baseband systems – information signal modulates a baseband pulse stream
  - Up until this point we have considered this modulation
- Bandpass systems – information modulates a sinusoid
  - Necessary to allow frequency division multiplexing/multiple access
- We now consider a sinusoidal carrier signal
  - Analog – AM, FM (3614)
  - Digital – PSK, FSK, ASK (in the next 2-3 weeks)
- What to read – Section 7.1

Lecture Objective

- The objective of today's lecture is to introduce digital sinusoidal carrier modulation

Modulation

- Modulation is defined as a process where an *information-bearing signal* causes changes to some characteristic of a carrier signal.
- The carrier signal can be either a pulse stream (a series of pulses in time) as in baseband communications or a sinusoid as in bandpass communications
- We first examined pulse modulation but will now consider sinusoidal carrier modulation

Types of Sinusoidal Modulation

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- In general, time-varying modulation of a sinusoid can be written as

$$s(t) = A(t)\cos\left(2\pi f_c t + \theta(t)\right)$$

  - f_c – the nominal carrier frequency
  - $A(t)$ – time varying amplitude
  - $\theta(t)$ – time varying angle
- The information-bearing signal can be used to modulate (change) the amplitude or the angle of the sinusoid
  - The receiver can then recover the original information by examining the amplitude or angle of the received sinusoid

Types of Sinusoidal Modulation

- Amplitude Modulation
  - The amplitude of the carrier is varied according to the message signal
  - Let $F_{AM}(m(t))$ be the function or mapping of the message to the amplitude:

$$s(t) = F_{AM}(m(t))\cos(2\pi f_c t)$$

- Angle Modulation
  - The angle of the carrier is varied according to the message of the signal
  - Frequency modulation – message directly affects the carrier frequency

$$s(t) = A_c \cos\left(2\pi\left[f_c + F_{FM}(m(t))\right]t\right)$$

  - Phase modulation – message directly affects the carrier phase

$$s(t) = A_c \cos\left(2\pi f_c t + F_{PM}(m(t))\right)$$

Analog vs. Digital Modulation

- Whether the modulation scheme is analog or digital depends on the message signal
  - If the message takes on a continuum of values, we have analog modulation
  - If the message (not the waveform but the information) takes on a discrete number of values, we have digital modulation
- Types of analog modulation
  - Amplitude modulation (AM) – broadcast radio
  - Phase modulation (PM) - not widely used
  - Frequency modulation (FM) – broadcast radio, TV, original cell phones

Amplitude Modulation

- Recall the signal for Large Carrier Amplitude Modulation or simply AM

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

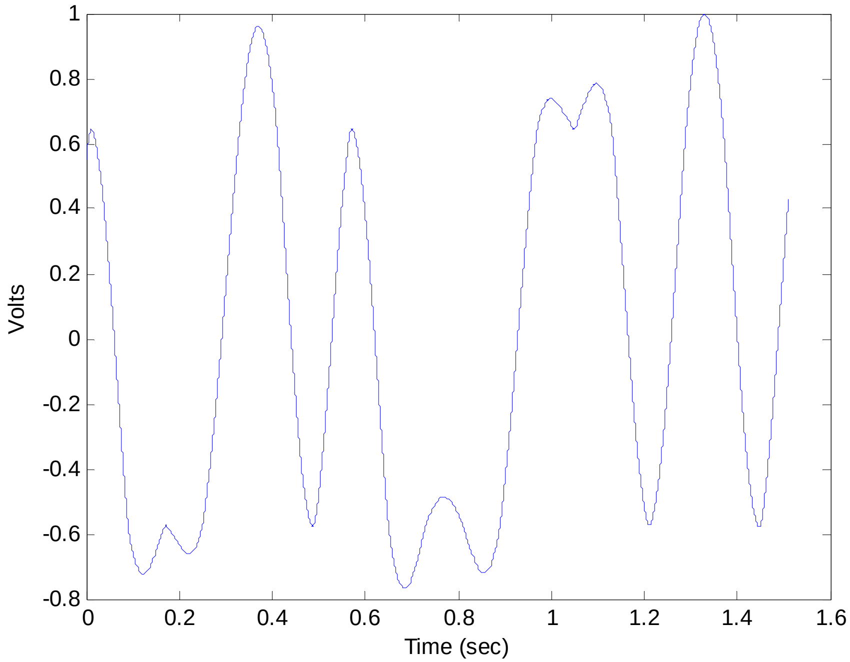
s(t) = transmit signal, *m(t)* is the analog message signal, k_a is a sensitivity constant and f_c is the carrier frequency (assumed to be much greater than the bandwidth of the signal)

- For Double-Sideband Suppressed Carrier (DSBSC) AM

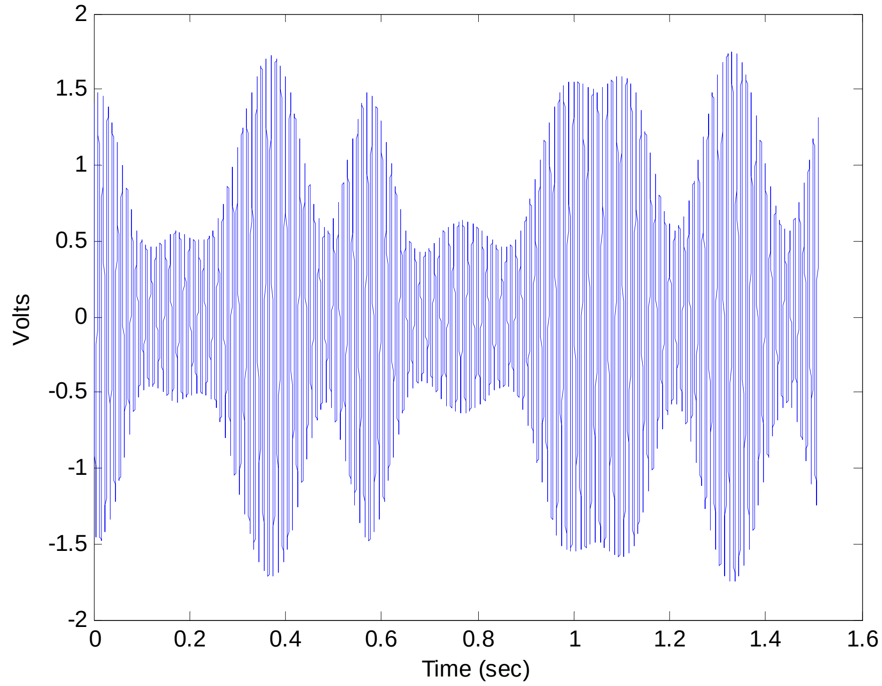
$$s(t) = A_c k_a m(t)\cos(2\pi f_c t)$$

Example – k_a = 0.75, max{m(t)} = 1

Message signal



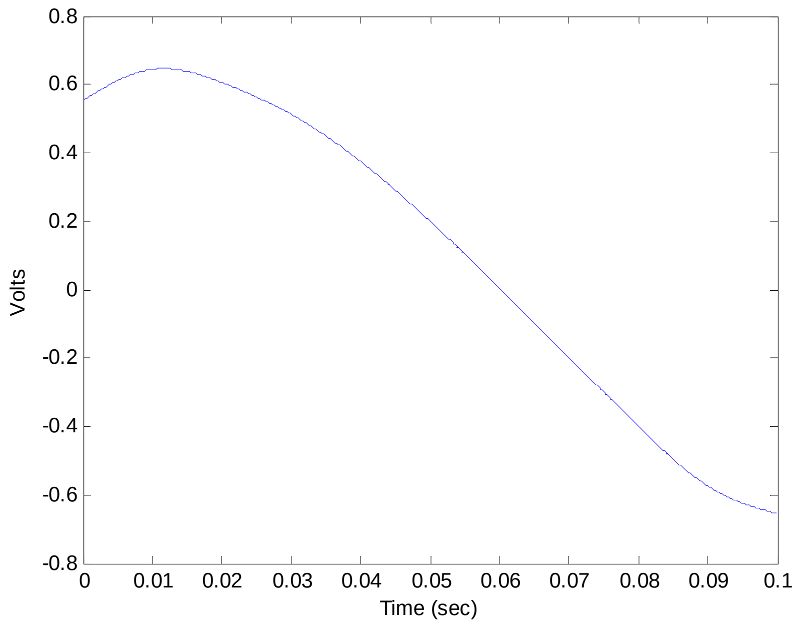
Modulated carrier



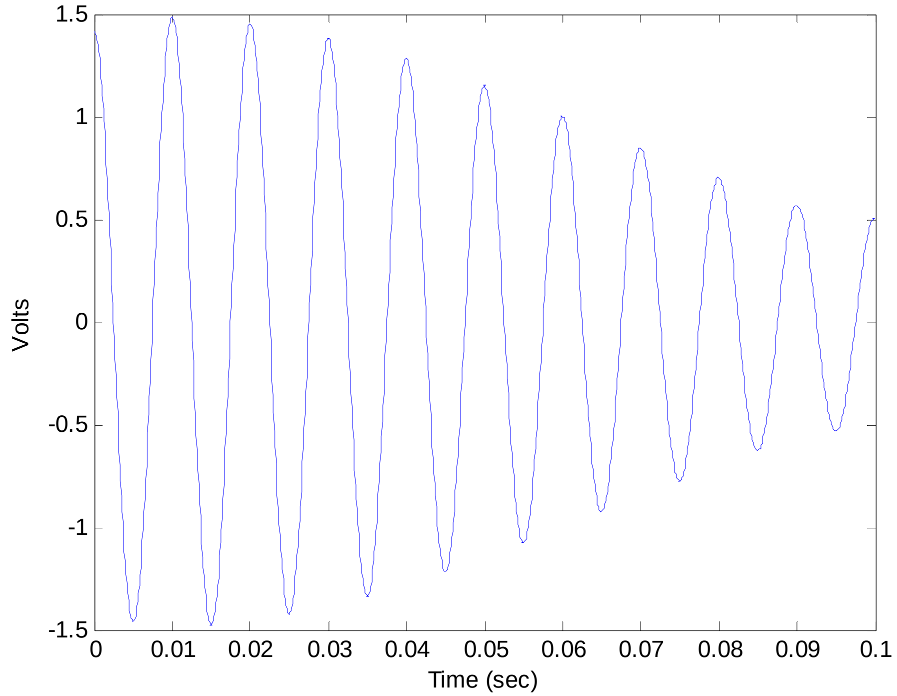
Example – $k_a = 0.75$, max{m(t)} = 1

- Close-up

Message signal

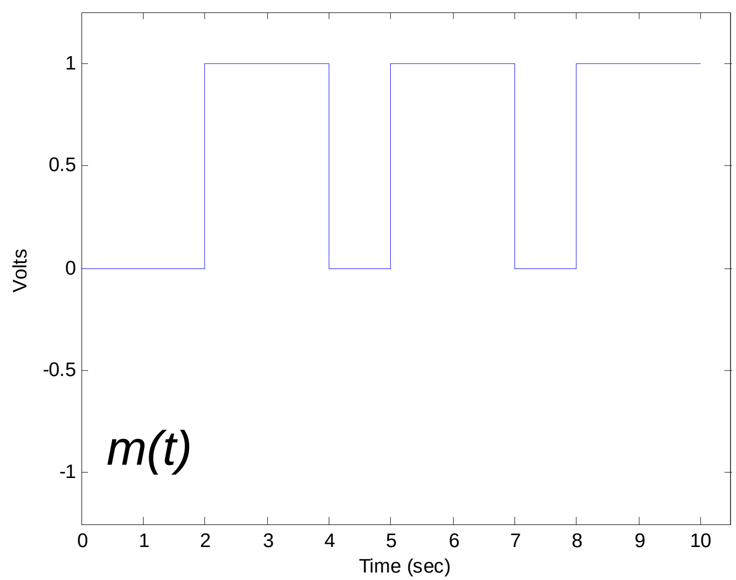


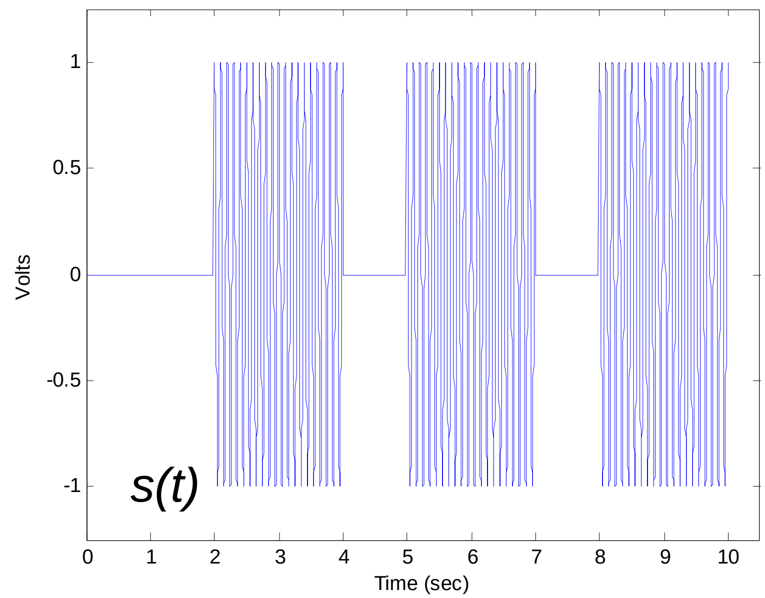
Modulated carrier



Digital Versions of AM

- If the message signal *m(t)* is digital, e.g., a unipolar NRZ line code, we have





- This is simply Binary Amplitude Shift Keying (BASK)

Example – DSB-SC with Digital Message

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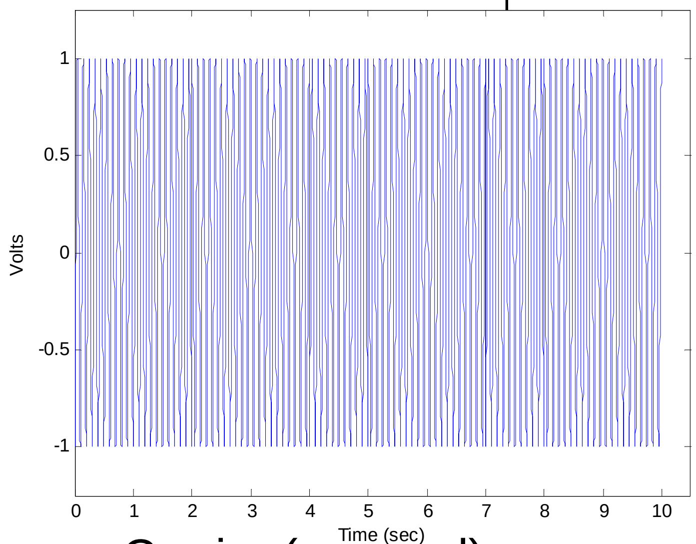
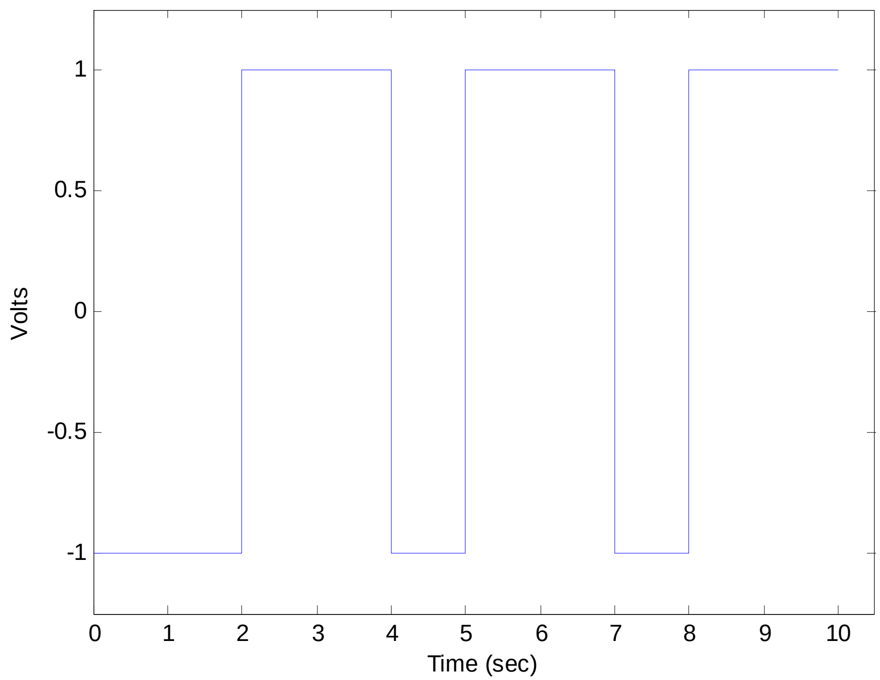
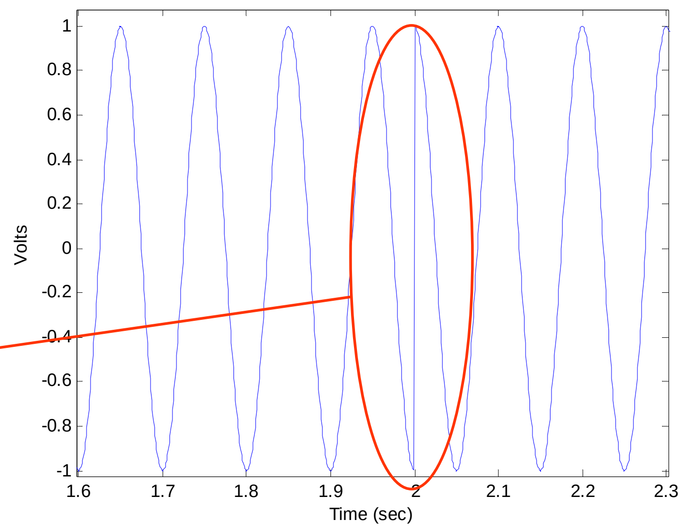
Carrier



Message



Carrier (zoomed)



Note phase change.
This is simply BPSK

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Digital Versions of AM

- If *m(t)* is a unipolar NRZ line code and we use DSB-SC AM we end up with the digital scheme known as Binary Amplitude Shift Keying (BASK)
  - Could also get BASK with Polar NRZ line code and regular large carrier AM with 50% modulation
- If *m(t)* is a polar NRZ line code and we use DSB-SC AM we end up with the digital scheme known as Binary Phase Shift Keying (BPSK)
  - Note that “negative” amplitudes are really phase changes

Bandpass Assumption

- A bandpass signal is one whose frequency content is concentrated about some center frequency f_c.
- Additionally, for bandpass signals, if W is the bandwidth of the signal, we assume that

$$f_c >> W$$

- This means that the message signal changes much more slowly than the carrier
- No overlap between positive and negative frequencies
- Complex baseband notation can be applied (to be discussed next class)

Notation

- Symbol – signal transmitted over one symbol interval (equal to a bit interval in binary modulation)
  - Example: linear modulation (where symbol is line code times the carrier)

$$s(t) = A_c b(t)\cos(2\pi f_c t + \phi_c) \quad 0 \le t \le T_b$$

b(t) represent the data bits

- Often we typically define the amplitude of the carrier A_c, in terms of the bit duration T_b such that it has unit energy during one bit/symbol time

$$A_c = \sqrt{\frac{2}{T_b}}$$

  - For linear modulation this results in

$$\begin{aligned} s(t) &= \sqrt{\frac{2}{T_b}} b(t)\cos(2\pi f_c t + \phi_c) \quad 0 \le t \le T_b \\ &= b(t)c(t) \end{aligned}$$

Notation (cont.)

- Proof:

$$E_c = \int_0^{T_b} c^2(t)\,dt$$

$$= \int_0^{T_b} \left(\sqrt{\frac{2}{T_b}} \cos\left(2\pi f_c t + \phi\right) \right)^2 dt$$

$$= \frac{2}{T_b} \int_0^{T_b} \cos^2\left(2\pi f_c t + \phi\right) dt$$

$$= \frac{2}{T_b}\frac{1}{2} \int_0^{T_b} dt + \frac{2}{T_b}\frac{1}{2} \int_0^{T_b} \cos\left(4\pi f_c t + \phi\right) dt$$

$$\approx 1$$

$f_c >> B \propto 1/T_b$

$$\int_0^{T_b} \cos\left(4\pi f_c t + \phi\right) dt \approx 0$$

"Bandpass Assumption"

Energy per Bit

- The performance of various modulation schemes is compared based on the bit error rate performance for a received energy per bit, E_b
- Since the carrier has unit energy, E_b is determined by the data waveform

$$
\begin{aligned}
E_b &= \int_0^{T_b} s^2(t)\,dt \\
&= \int_0^{T_b} \left(\sqrt{\frac{2}{T_b}}\, b(t) \cos(2\pi f_c t + \phi) \right)^2 dt \\
&= \frac{2}{T_b} \int_0^{T_b} b^2(t) \cos^2(2\pi f_c t + \phi)\,dt \\
&= \frac{2}{T_b}\frac{1}{2} \int_0^{T_b} b^2(t)\,dt + \frac{2}{T_b}\frac{1}{2} \int_0^{T_b} b^2(t) \cos(4\pi f_c t + \phi)\,dt \\
&\approx \frac{1}{T_b} \int_0^{T_b} b^2(t)\,dt
\end{aligned}
$$

Since $f_c >> W$, b(t) remains constant over one cycle of the carrier

$$\int_0^{T_b} b^2(t) \cos(4\pi f_c t + \phi)\,dt \approx 0$$

"Bandpass Assumption"

Types of Binary Digital Modulation

$$\sqrt{\frac{2}{T_b}}\underbrace{A_d(t)}_{\substack{\text{data}\\\text{modulation}}}\cos\left(2\pi f_c t+\underbrace{\theta_d(t)}_{\substack{\text{data}\\\text{modulation}}}+\phi_c\right)$$

- Binary Amplitude Shift Keying (BASK)

$$\sqrt{\frac{2}{T_b}}\underbrace{b(t)}_{\text{data}}\cos\left(2\pi f_c t+\phi_c\right)$$

- Binary Phase Shift Keying (BPSK)

$$\sqrt{\frac{2}{T_b}}\cos\left(2\pi f_c t+\underbrace{b(t)}_{\text{data}}\pi+\phi_c\right)$$

- Binary Frequency Shift Keying (BFSK)

$$\sqrt{\frac{2}{T_b}}\cos\left(2\pi\left(f_c+\Delta f\,\underbrace{b(t)}_{\text{data}}\right)t+\phi_c\right)$$

Coherent Demodulation

- At the receiver we "demodulate" the signal (i.e., retrieve the data) by mixing the received signal to baseband for further processing
- This requires a local replica of the carrier wave
- If the phase of our local carrier wave is made to be equal to the incoming wave, our receiver is *coherent.*
  - Requires phase tracking circuitry
- If the phase is not the same, our receiver is *non-coherent*
  - Less complex

Received signal

$$\sqrt{\frac{2}{T_b}}\underbrace{A_d(t)}_{\text{data modulation}}\cos\left(2\pi f_c t + \underbrace{\theta_d(t)}_{\text{data modulation}} + \phi_c\right)$$

Local carrier

$$\sqrt{\frac{2}{T_b}}\cos\left(2\pi f_c t + \phi_r\right)$$

For a coherent receiver

$$\phi_r = \phi_c$$

In-class drill

Summary

- Today we have introduced digital sinusoidal modulation
- Digital schemes are similar to old analog modulation schemes with the analog message replaced with a digital message signal
- We will assume bandpass signals where the bandwidth is much lower than the carrier frequency
- In the coming weeks we will study various modulation schemes, receiver structures and their bit error rate performance