ECE4634
Digital Communications
Fall 2007

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Lecture #12: Eye Diagrams, Multipath and Equalization

Analog and Digital Communications

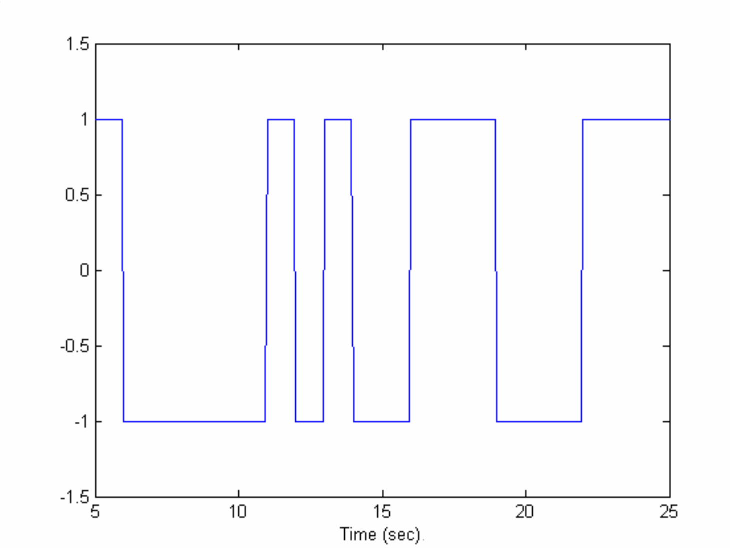
Overview

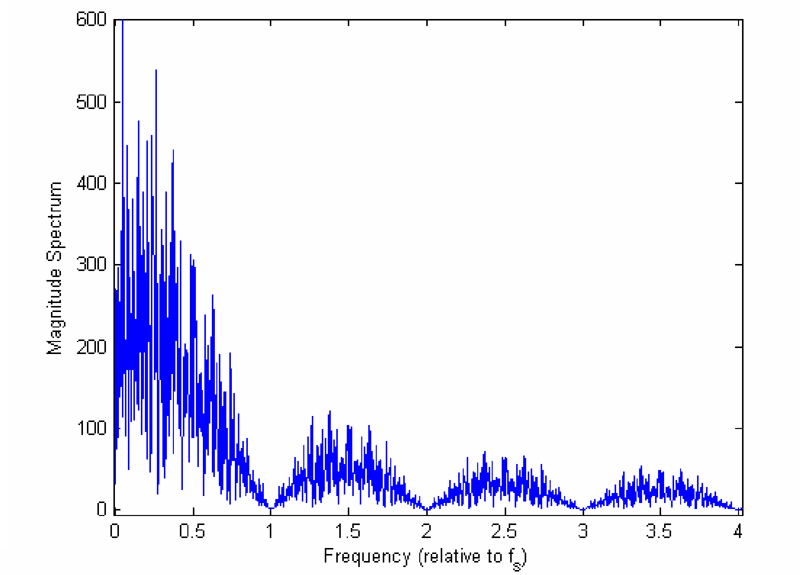
- In this lecture we will discuss another way of viewing intersymbol interference or ISI - eye diagrams
- As we have discussed, ISI due to pulse shape and system bandwidth limitations can be effectively countered through proper pulse design.
- However, the channel can also introduce ISI through *multipath*
- We will discuss a receiver structure that mitigates the effect of multipath termed an *equalizer*
- What to read – Section 6.6-6.8

Eye Diagrams / Patterns

- The impact of intersymbol interference (ISI) can be observed using “eye-patterns” or “eye diagrams”
- Eye patterns are time plots of consecutive symbols typically done on an analog oscilloscope but easily reproduced on a digital computer
- The larger the “eye”, the less susceptible the receiver is to:
  - Noise
  - Sampling Error

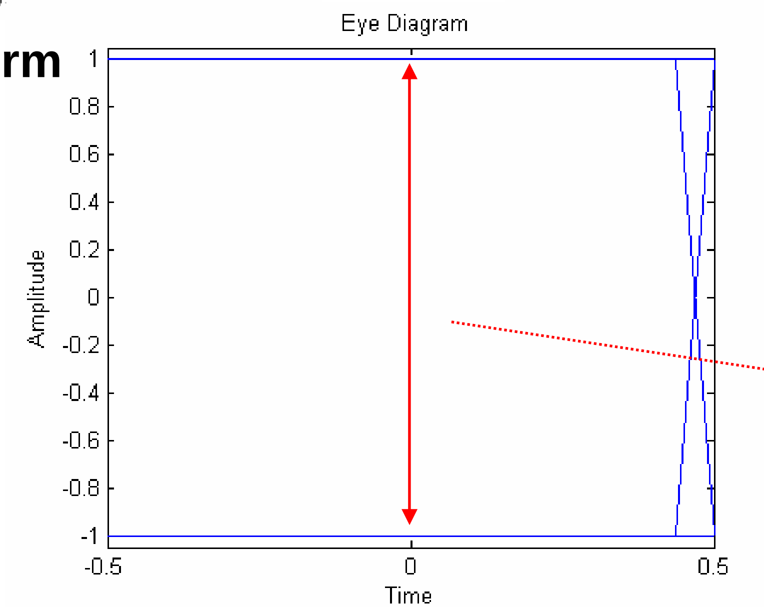
Example: Square Pulses





Example Time Waveform

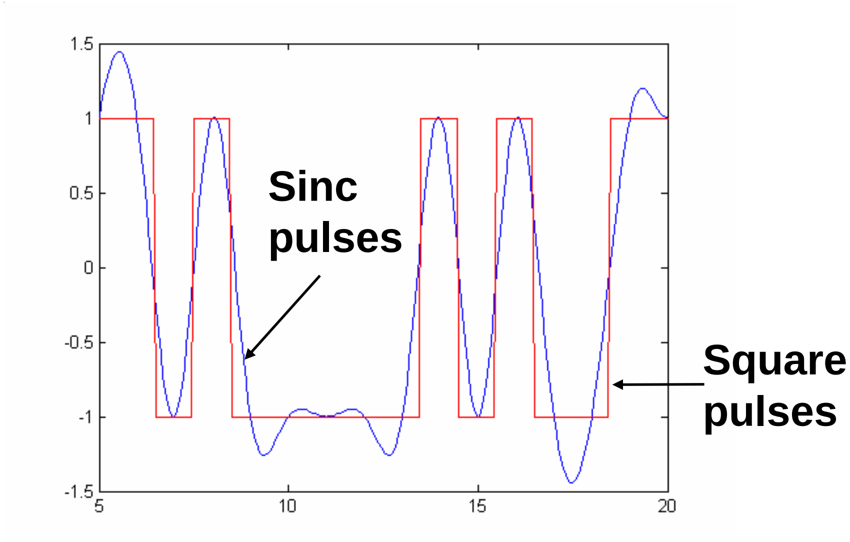
Example Energy Spectral Density



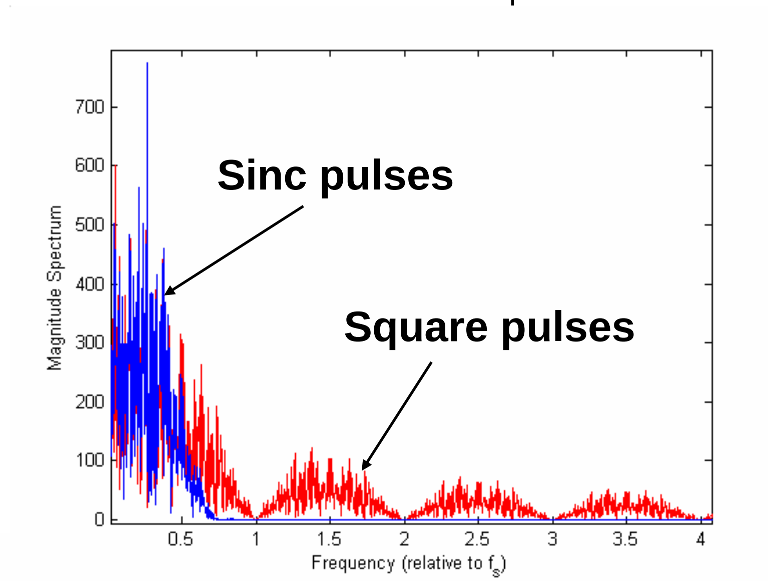
Eye Diagram

Very large “eye”
Low susceptibility to noise and sampling error

Example: Raised Cosine Pulses

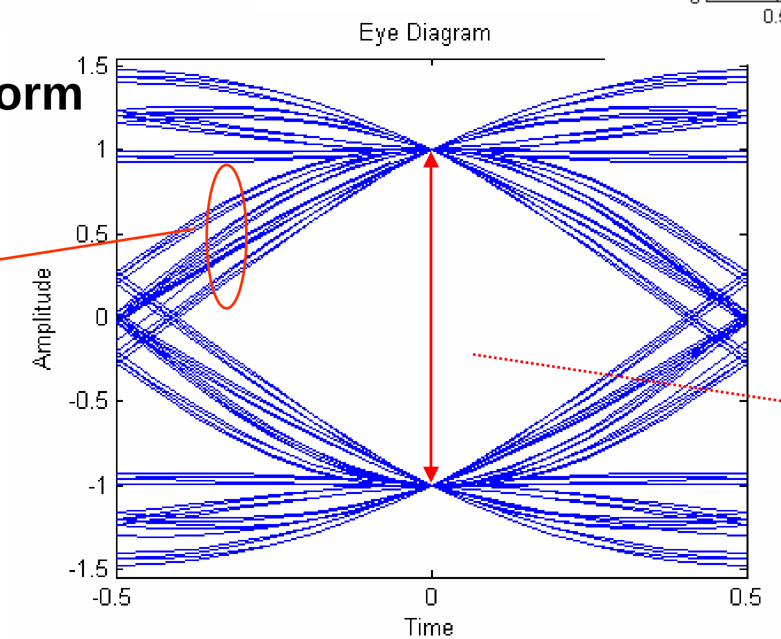


Example Time Waveform



Example Energy Spectral Density

Slope determines sensitivity to timing errors in sampling



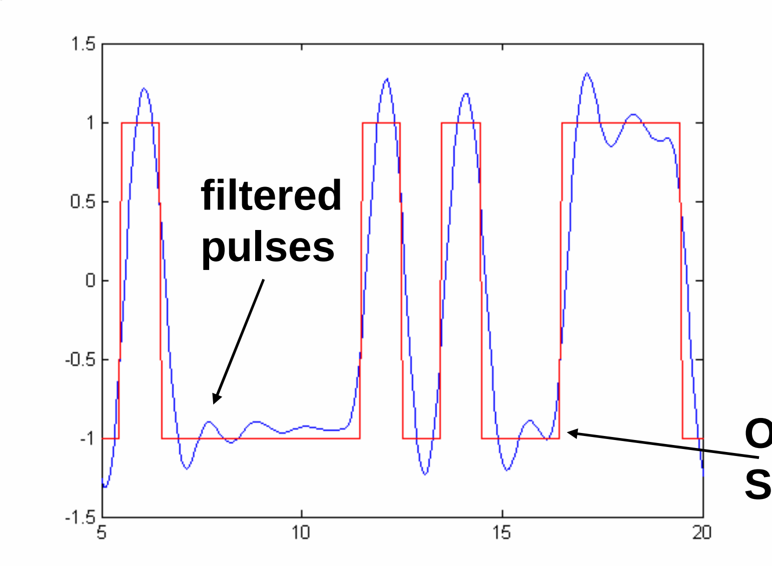
Eye Diagram

Zero ISI provides large "eye", but only at **optimal sampling instant.**

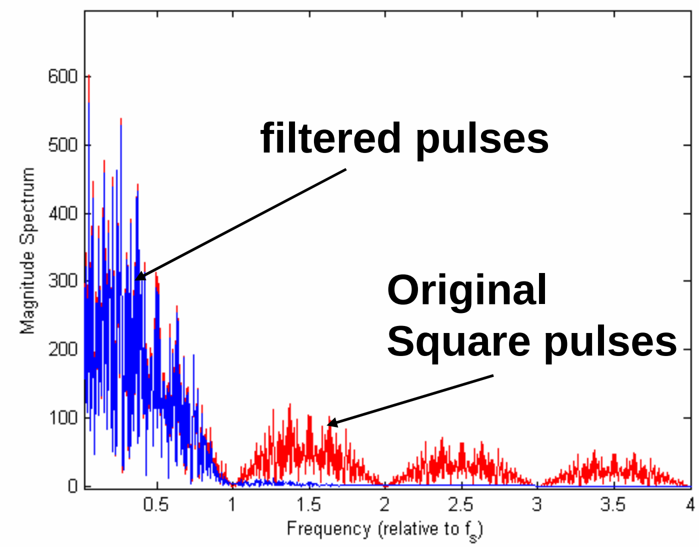
Low susceptibility to noise

Moderate susceptibility to sampling error

Example: Filtered Square Pulses

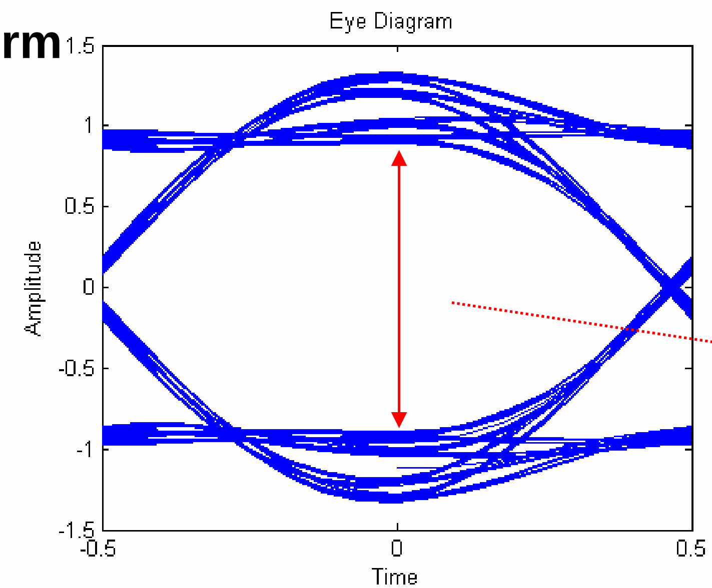


Square pulses sent but channel limits bandwidth



Example Time Waveform

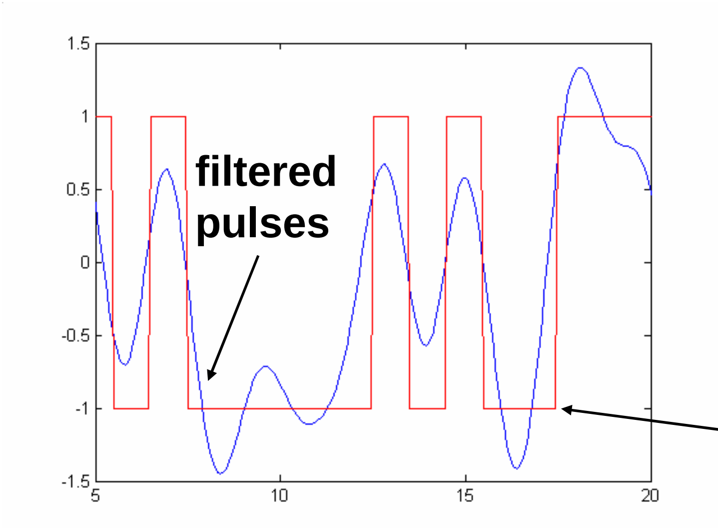
Example Energy Spectral Density



Eye Diagram

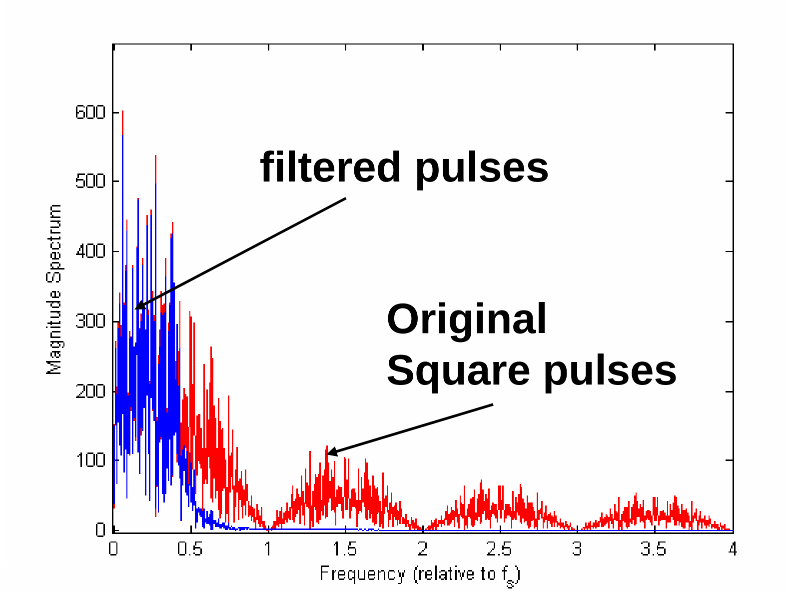
Smaller "eye" due to the presence of ISI
Greater susceptibility to noise
Moderate susceptibility to sampling error

Example: Filtered Square Pulses

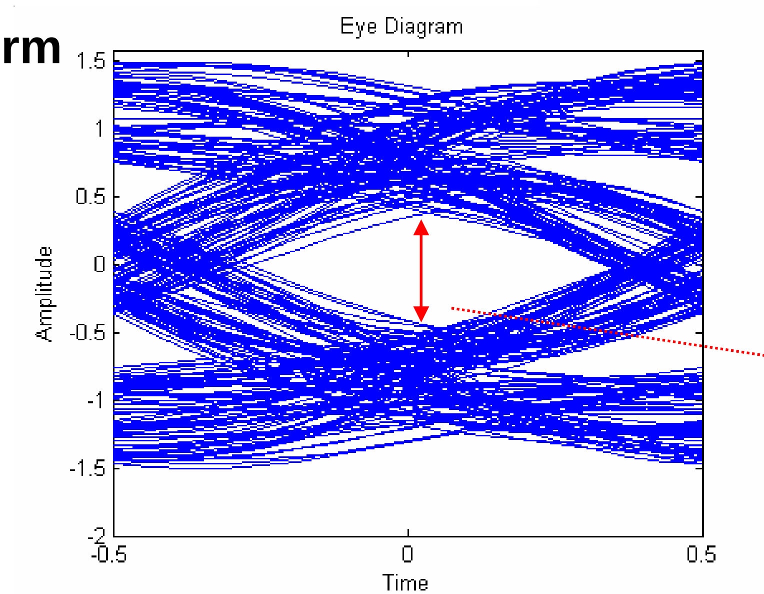


Square pulses sent but channel limits bandwidth

Original Square pulses



Example Time Waveform



Example Energy Spectral Density

Small "eye" due to severe ISI.
Great susceptibility to noise and sampling error

Eye Diagram

Bandwidth Restrictions - Summary

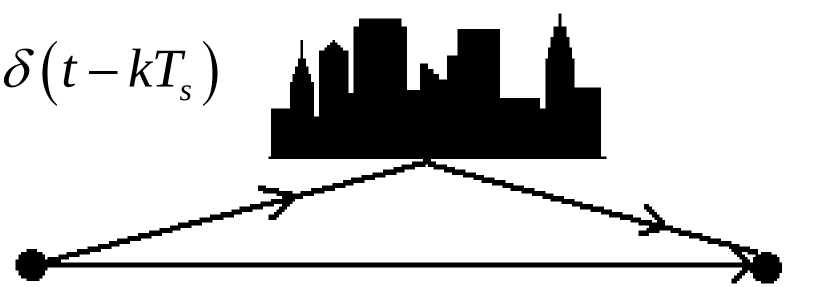
- Reducing the bandwidth of the transmit signal is desirable to improve spectral efficiency (i.e., get the most bits/sec in the smallest bandwidth)
- Reducing bandwidth indiscriminately will smear transmit pulses in time introducing ISI
- Intelligent pulse design can reduce bandwidth without this penalty
  - Nyquist Criterion
- Channel conditions can reduce bandwidth further resulting in channel-induced ISI
  - Requires an equalizer at the receiver to remove
  - Could also be solved by reducing the data rate (i.e., reducing the transmit bandwidth)
  - This channel induced ISI is termed *multipath*

Quiz

Multipath channel

- ISI can be introduced by transmission through a multipath channel that has impulse response *h(τ)*. If the delay spread (a measure of the duration of the impulse response) of the channel is much smaller than the symbol duration, fading can occur but no ISI is observed.
- However, if the delay spread of the channel is larger than the symbol duration, ISI results.

$$s(t) = \underbrace{f(t)}_{\text{transmit pulse}} * \sum_{k=-\infty}^{\infty} \underbrace{a_k}_{\text{info symbol}} \delta(t - kT_s)$$



$$\underbrace{g(t)}_{\text{rx pulse}} = \int_{-\infty}^{\infty} \underbrace{f(\tau)}_{\text{tx pulse}} \underbrace{h(t-\tau)}_{\text{IR of channel}} d\tau$$

$$r(t) = \underbrace{\left\{ \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s) \right\}}_{\text{data signal}} * \underbrace{f(t)}_{\text{transmit pulse shape}} * \underbrace{h(t)}_{\text{channel}} * \underbrace{q(t)}_{\text{receive filter}}$$

Tap Delay Line Model of Multipath Channel

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- A common model for the channel impulse response is

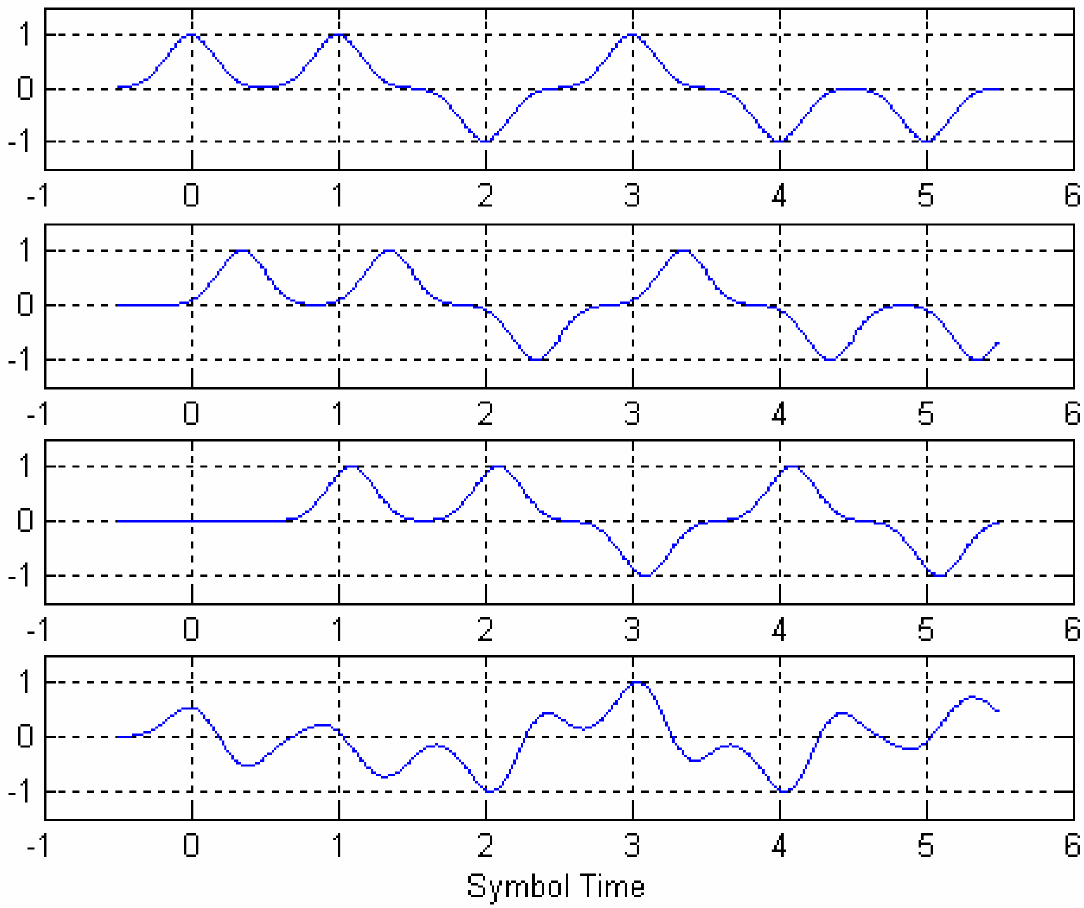
$$h(t) = \sum_{k=0}^{L-1} a_k e^{j2\pi\theta_k} \delta(t - \tau_k)$$

- If the difference between consecutive values of τ_k is small relative to the symbol period T_s, we term this a *narrowband channel* and ISI does not occur

$$(\tau_k - \tau_{k-1}) << T_s \quad \forall k$$

  - In a mobile environment, “fading” can still occur even if there is no ISI. (If no ISI occurs but fading does occur we call this “flat fading”)
- If the difference between consecutive values of τ_k is comparable to the symbol period T_s, we term this a *wideband channel* and ISI does occur
  - This causes frequency distortion and thus requires an equalizer
  - This is termed “frequency selective fading” in a mobile environment

Multipath Example



Path 1

+

Path 2

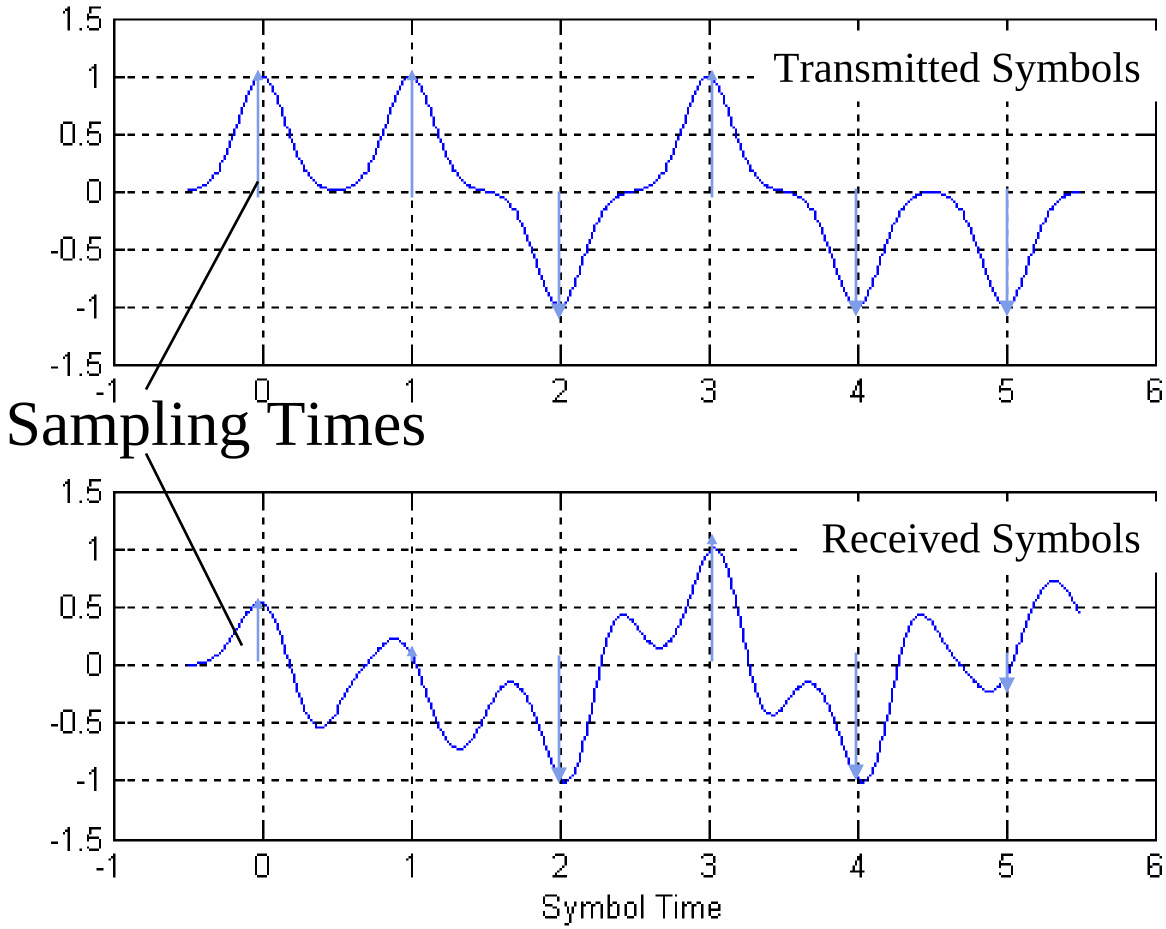
+

Path 3

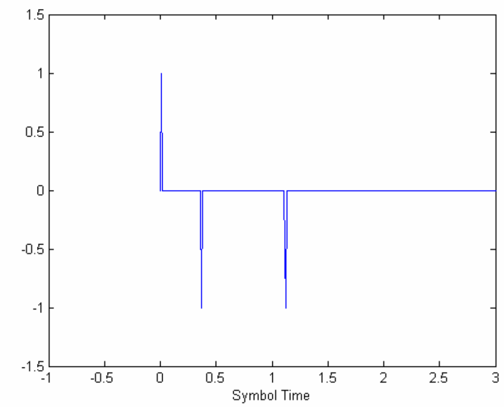
=

Received Signal

Inter-Symbol Interference



Channel Impulse Response



- Multipath Causes Inter-symbol Interference

Multipath Effects

Frequency Selective ⟵ Bandwidth ⟶ Flat

Frequency Selective	Flat	
Static Inter-Symbol Interference	Similar to AWGN Channel	No time variation
Frequency selective fading	Flat fading	Time varying

↕ Time

Flat vs. Frequency Selective

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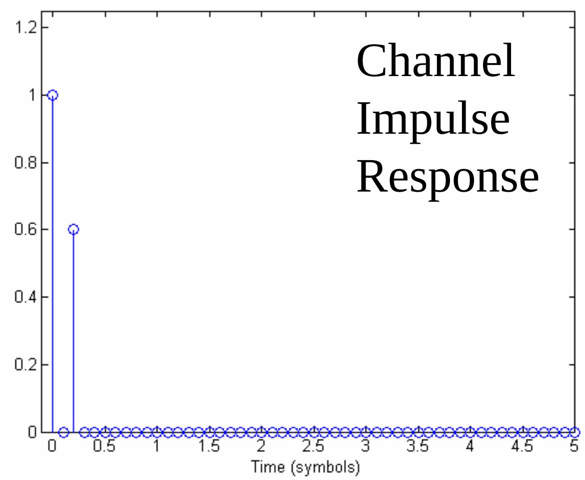
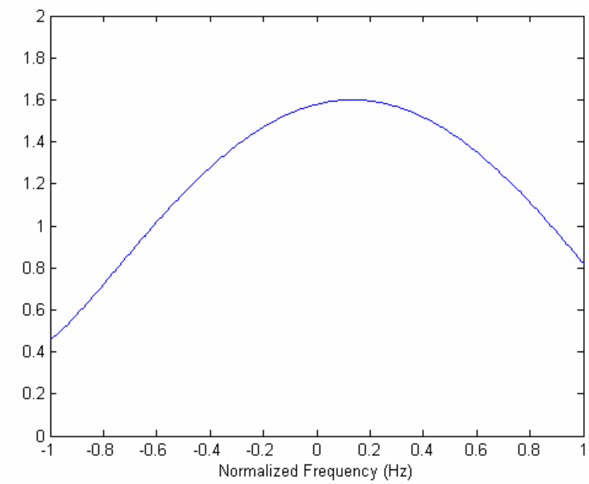


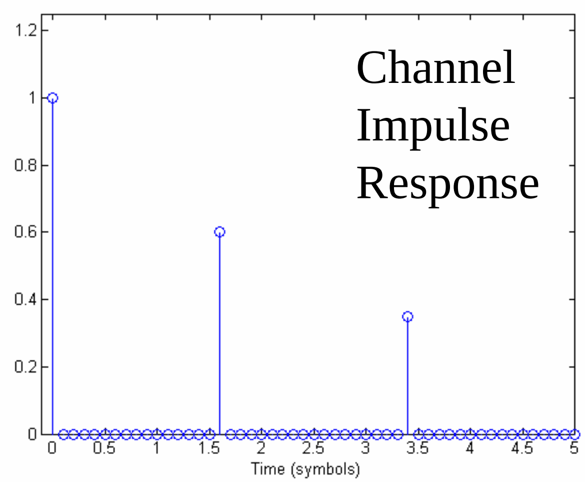
Flat
No ISI

Fourier
Transform

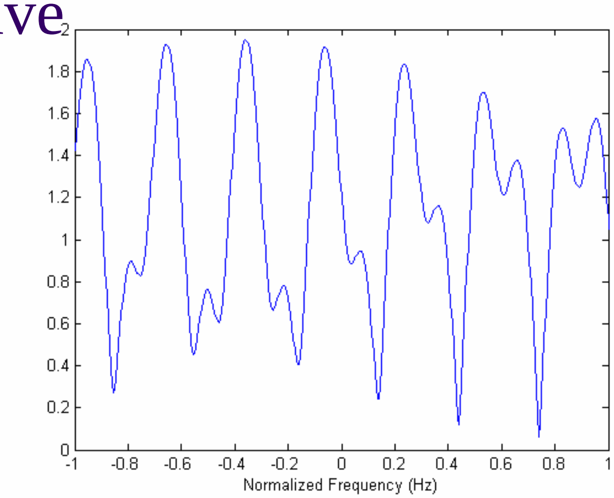


Channel
Transfer
Function

Frequency Selective
ISI



Fourier
Transform



Channel
Transfer
Function

Nyquist's Criteria for Zero ISI

- Recall that for pulse shaping we chose pulses to insure that

$$f\left(kT_s\right)=\begin{cases}C, & k=0\\ 0, & k\neq 0\end{cases}$$

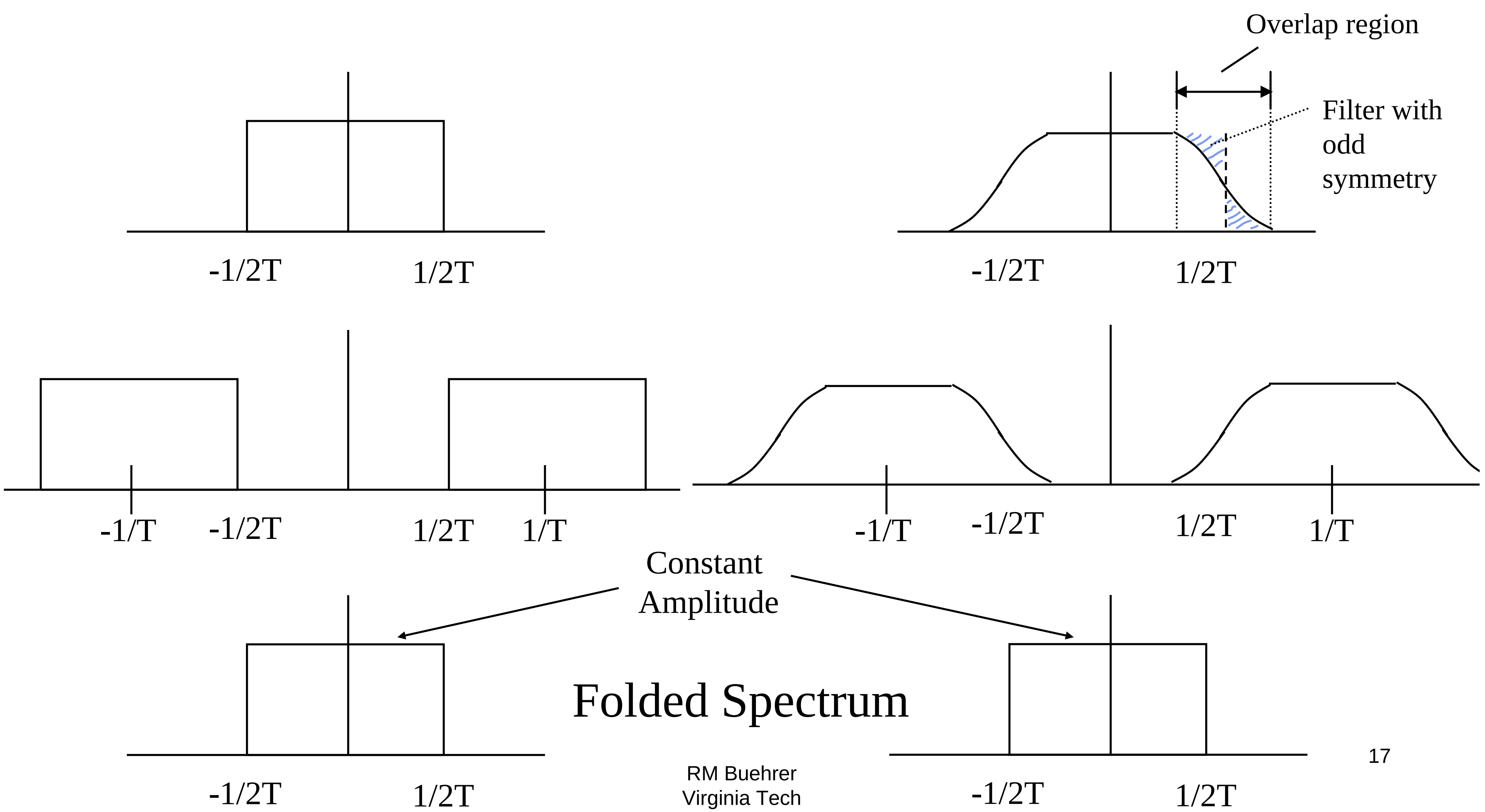
  - where *k* is an integer and T_s is one symbol duration
- This is equivalent to having a transfer function

$$F(f)=\begin{cases}\Pi\left(\dfrac{f}{2B_o}\right)+Y(f) & \left|f\right|<2B_o\\ 0 & else\end{cases}$$

where $B_o = R_s/2$ (i.e., ½ the symbol rate) and *Y(f)* is a real function that is even symmetric about $f=0$ and odd symmetric about $f=B_o$.

$$\boxed{\begin{array}{ll} Y(-f)=Y(f) & |f|<2B_o\\ Y(-f+f_o)=-Y(f+f_o) & |f|<B_o\end{array}}$$

Nyquist Filters



Folded Spectrum

How do we eliminate ISI ?

F(f) = Nyquist Filter – pulse shape with zero ISI

No Channel

$I_n \rightarrow \boxed{F(f)} \rightarrow$

AWGN Channel

Matched filter at rx for noise effects

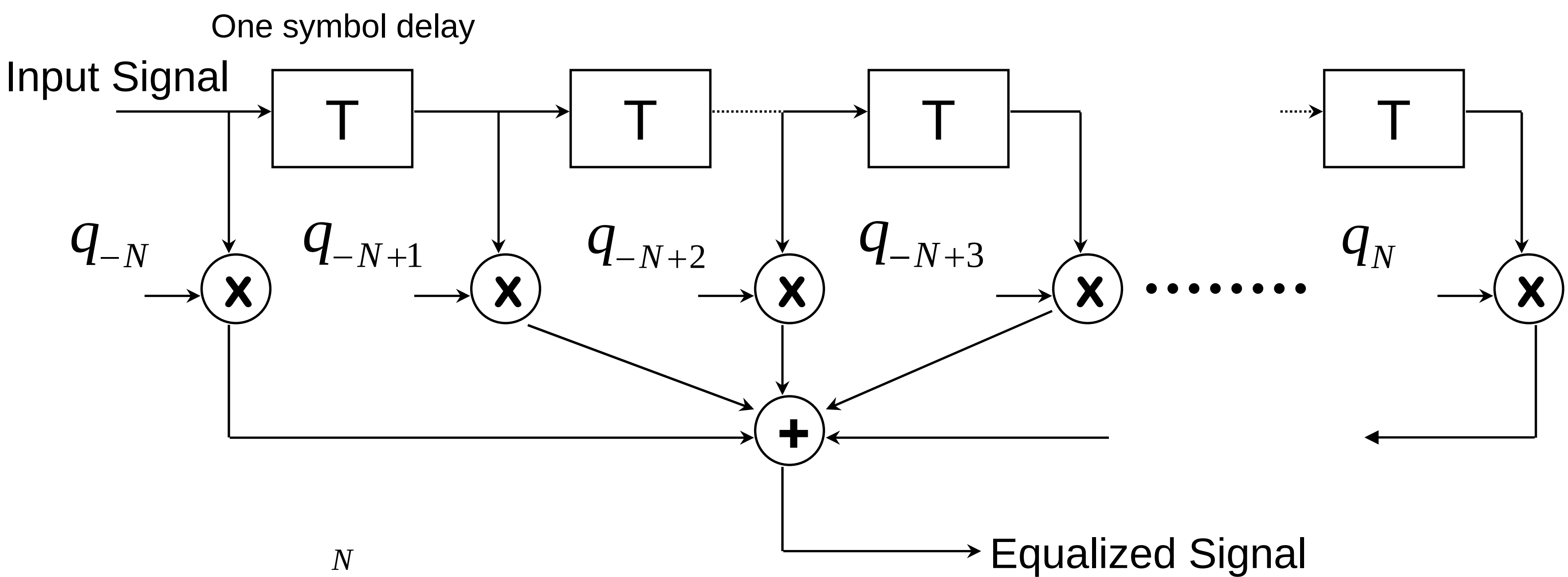
$I_n \rightarrow \underbrace{\boxed{\sqrt{F(f)}}}_{\text{Tx}} \rightarrow \boxed{\text{AWGN}} \rightarrow \underbrace{\boxed{\sqrt{F(f)}}}_{\text{Rx}} \rightarrow$

Multipath Channel

$I_n \rightarrow \underbrace{\boxed{\sqrt{F(f)}}}_{\text{Tx}} \rightarrow \boxed{H(f)} \rightarrow \underbrace{\boxed{\sqrt{F(f)}} \rightarrow \boxed{Q(f)}}_{\text{Rx}} \rightarrow$

$H_e(f) = F(f)H(f)Q(f)$ must satisfy Nyquist criteria

Transversal Filter Equalizer

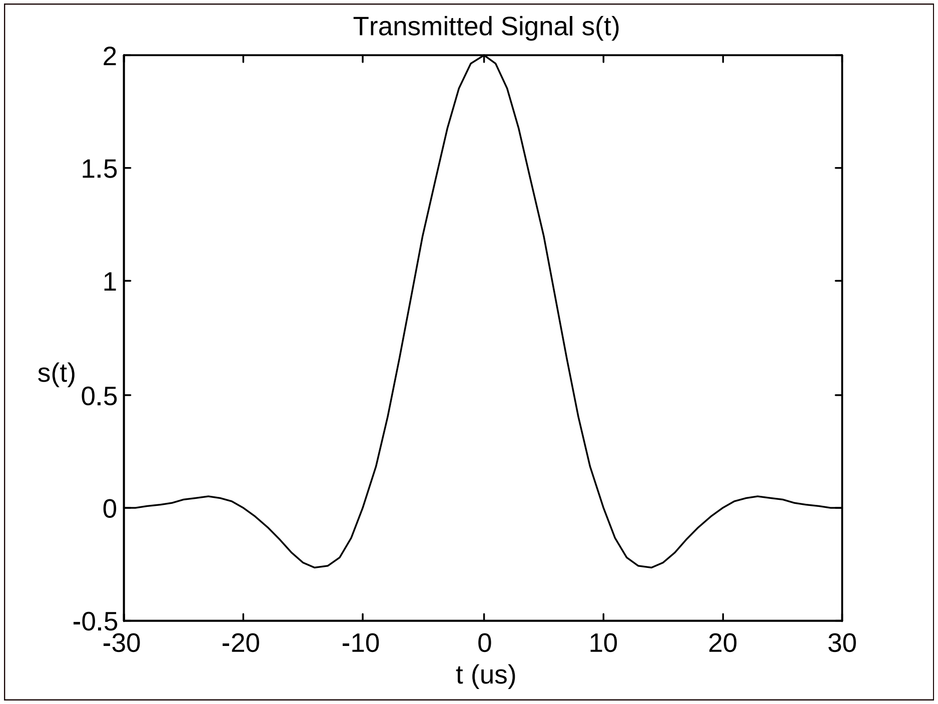


$$q(t) = \sum_{k=-N}^{N} q_k \delta(t - kT_s)$$

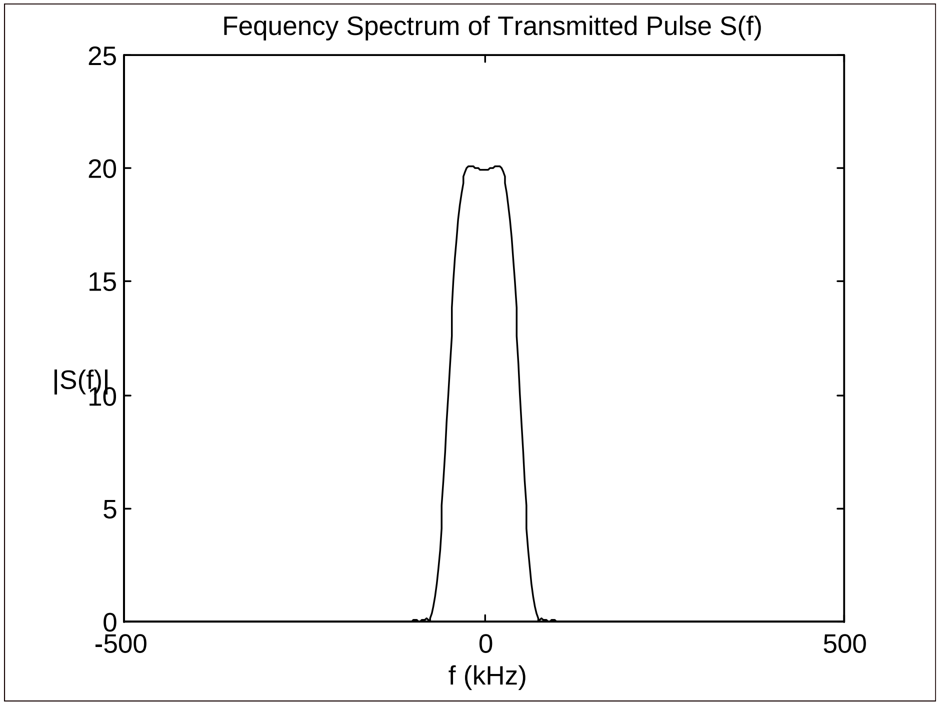
The impulse response of a transversal filter equalizer is simply a series of impulses with the tap coefficients as the weights of the impulses.

Example of Equalization: Transmitted Signal Pulse (Data Rate = 100 kbits/sec)

f(t)

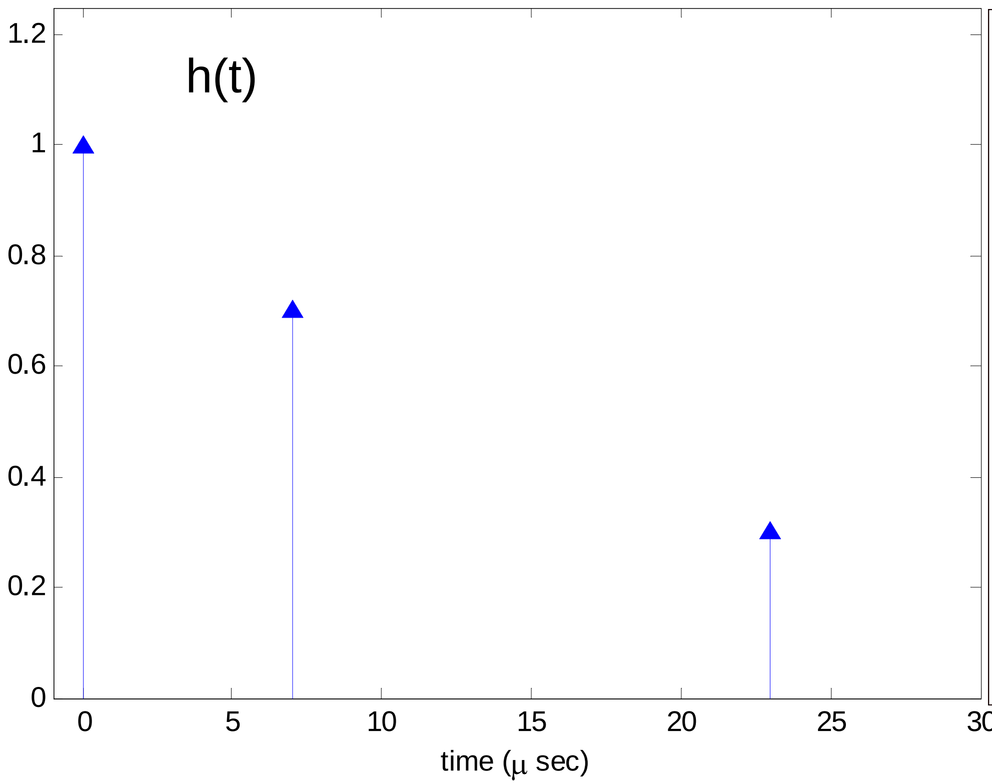


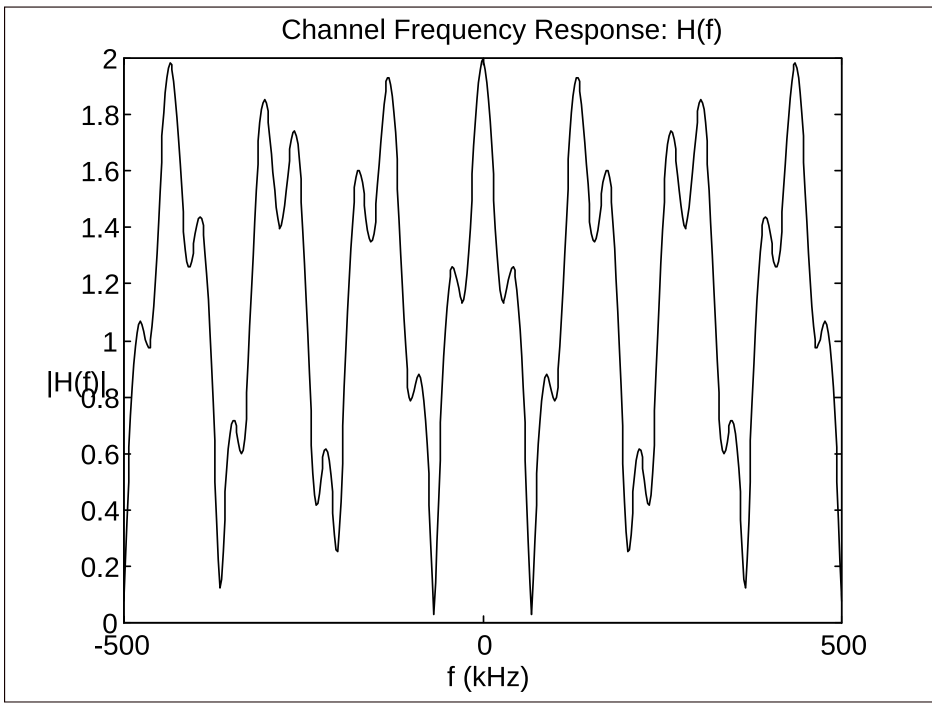
F(f)



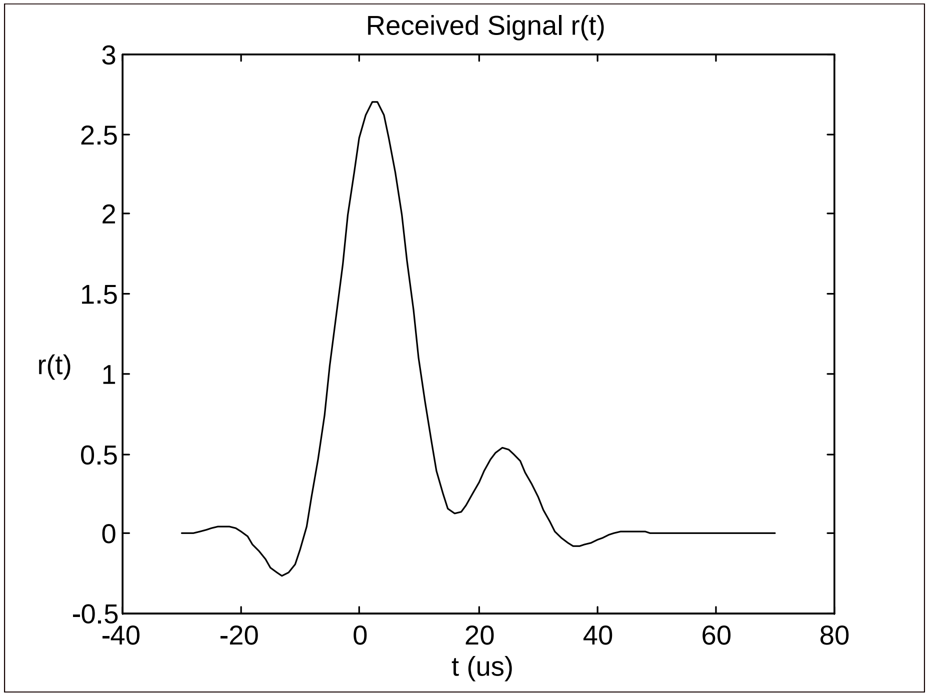
Channel Impulse Response

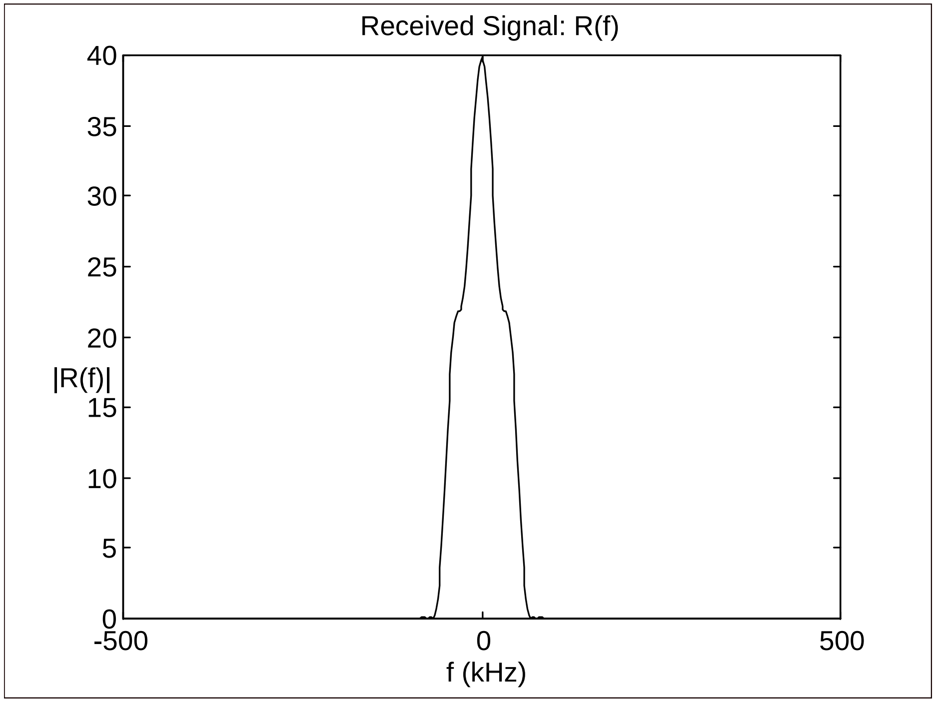
$$h(t) = \delta(t) + 0.7\delta(t - 7\mu s) + 0.3\delta(t - 23\mu s)$$



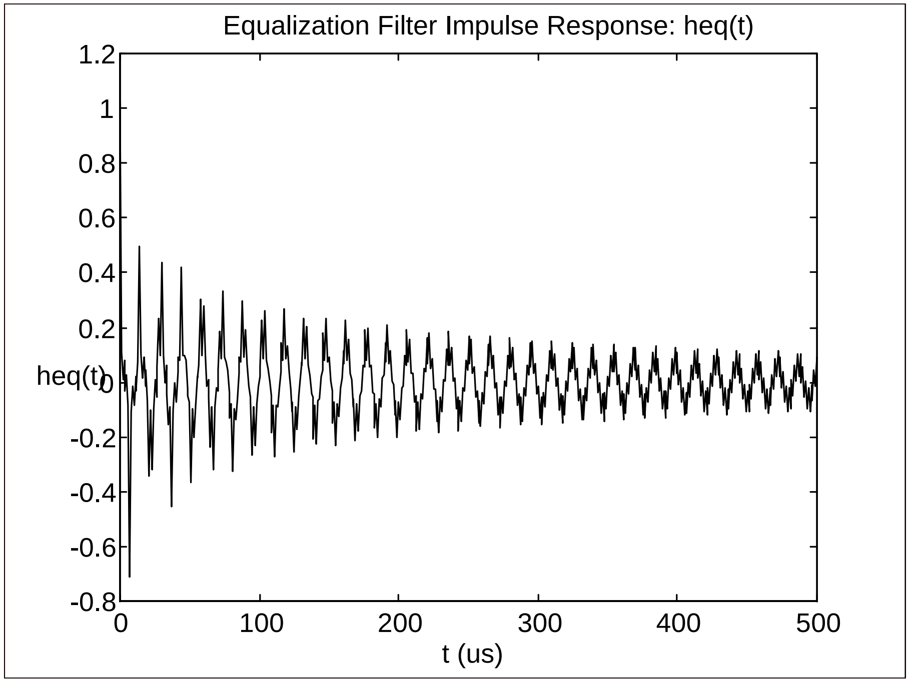


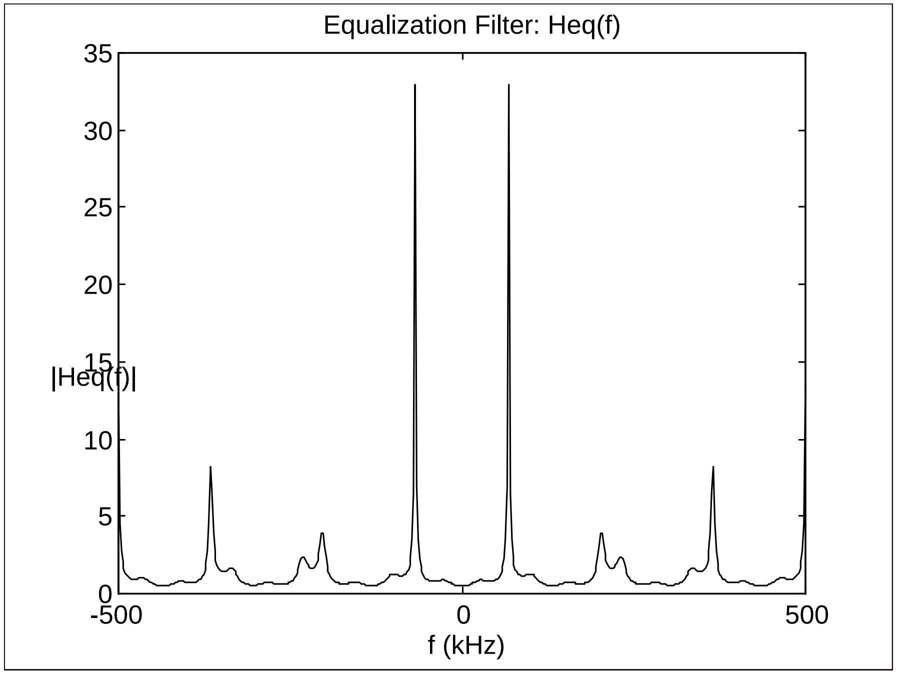
Received Signal: r(t) = s(t)*h(t)



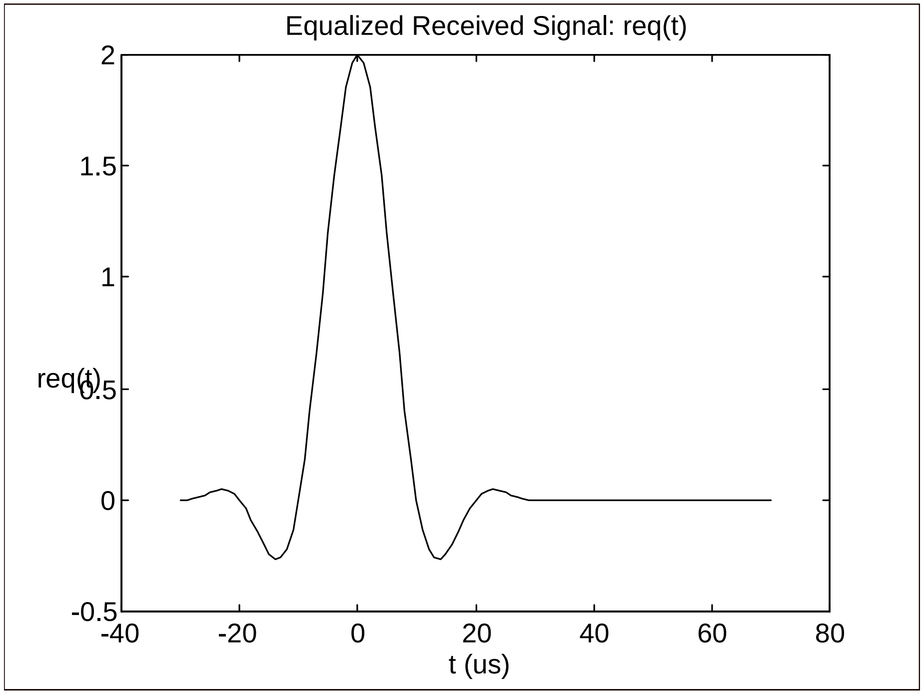


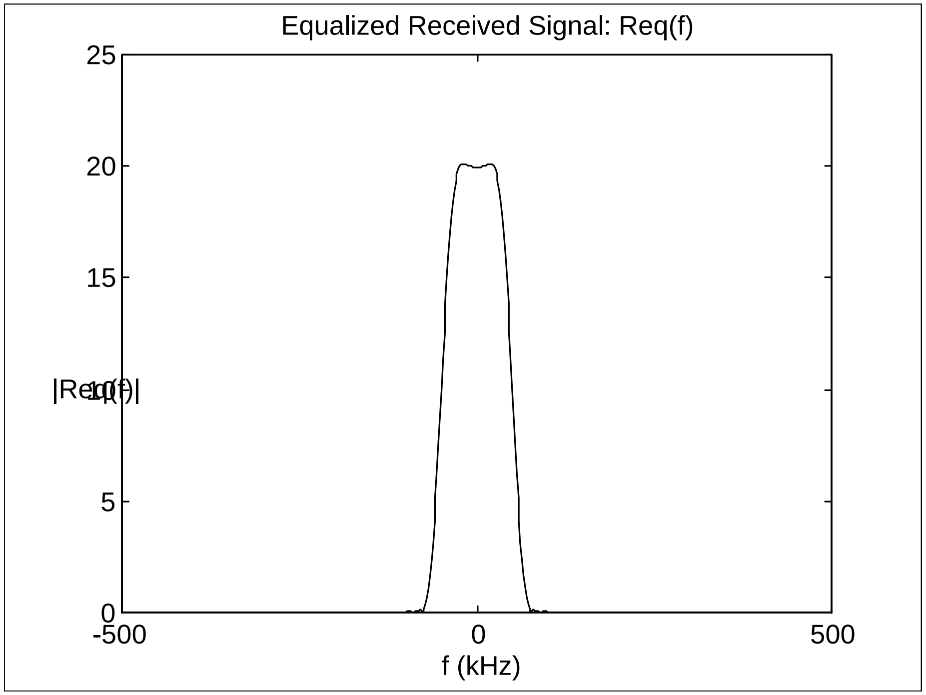
Equalization Filter:





Equalized Received Signal





Complicating factors

- The structure of the inverse filter can become very complicated to implement
- The multipath channel structure is not always known and sometimes must be estimated.
- The channel changes in real time so equalization must be adaptive.
- Further, completely eliminating the ISI may not provide the best bit error rate in the presence of AWGN
  - The minimum BER approach is not necessarily to “invert” the channel
  - Consider the noise gain in bands where the equalizer has peaks

Classes of Equalizer Structures

- Maximum Likelihood Sequence Estimation
  - Optimal equalizer in maximum likelihood sense
  - Viterbi Algorithm
- Linear Equalizers
  - Zero-forcing
  - MMSE
- Decision Feedback
  - Similar to interference cancellation

Conclusions

- Today we have discussed two additional aspects of intersymbol interference.
  - Eye-diagrams
  - Equalizers
- Eye-diagrams help us to visualize the impact of ISI on symbol decisions and timing error.
- Equalizers are receive filters which “equalize” the distortion caused by a multipath channel
- A common equalizer structure is the *transversal filter*