ECE4634
Digital Communications
Fall 2007

Instructor: R. Michael Buehrer

Lecture #10: Intersymbol Interference: The impact of bandwidth limits on Pulse Modulation

Analog and Digital Communications

Overview

- Recall that digital communication systems
  - Sample analog signal
  - Quantize sampled signal
  - Map bits to waveforms
- The waveforms determine the bandwidth of the resulting signal
- Let us assume that we employ *polar NRZ* signaling
- The pulse used doesn't have to be a square pulse
  - What pulse shape should be use???
- What to read – Section 6.1-6.2

Lecture Objectives

- The objective of this lecture is to demonstrate and explain the impact of bandwidth limits on pulse modulation.
- Namely we wish to show that by limiting the bandwidth of a train of pulses sent in time, the pulses are elongated in time causing *intersymbol interference.*

Pulse Shaping

Analog and Digital Communications

- The spectrum of the signal is dependent on the spectrum of the pulse used as well as the autocorrelation of the data.

$$P_x(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{j2\pi fkT_s}$$

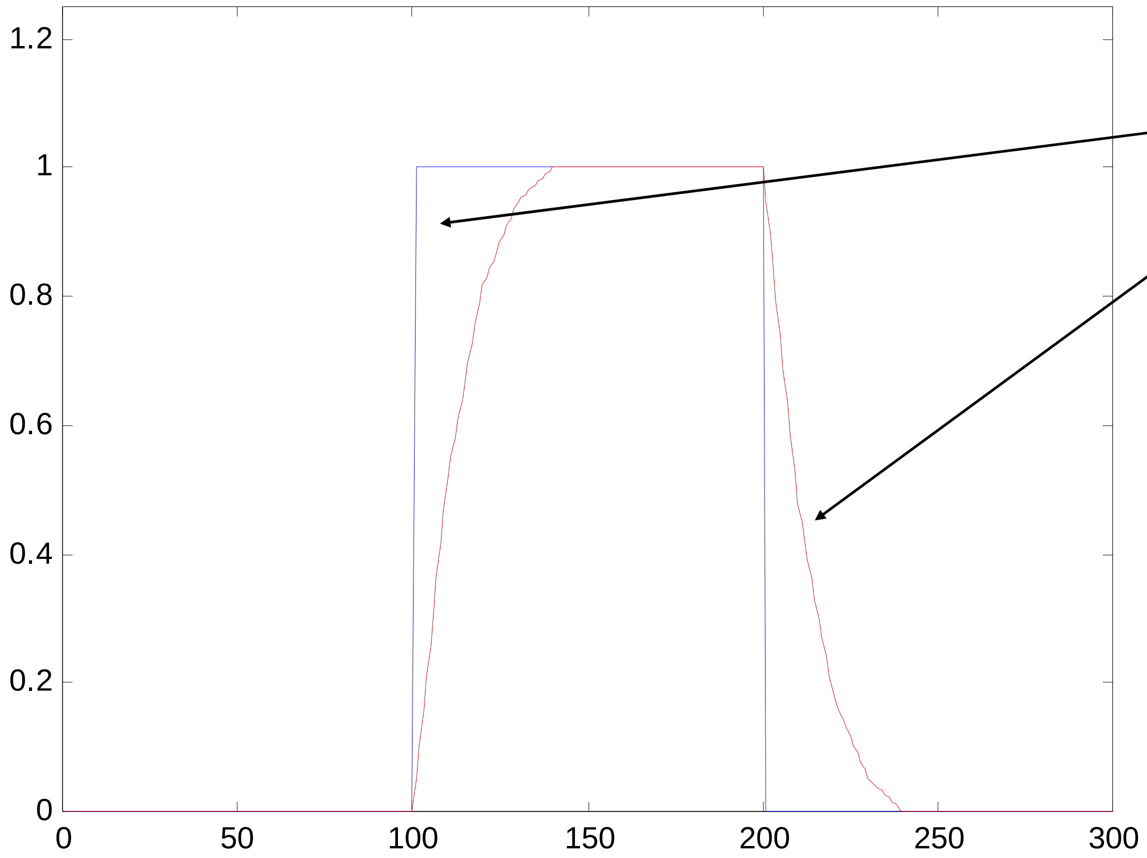
- Often we wish to control aspects of the transmission bandwidth.
- We saw last time how we can control spectral characteristics using different line codes
  - controls the autocorrelation and pulse duration
- Now we turn look at controlling spectrum by controlling the shape of the pulse used.
- This is termed *pulse shaping*.

Inter-symbol Interference (ISI)

Analog and Digital Communications

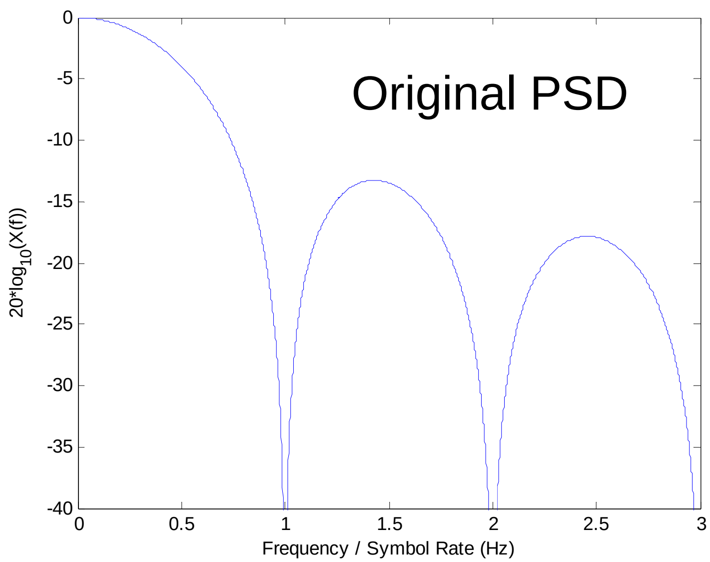
- We know that square pulses in the time domain do not interfere with one another provided that the pulse duration corresponds to the symbol time.
- However, square pulses require infinite absolute bandwidth. Further there is significant energy in the side lobes
- Even if the system requirements didn't lead to bandwidth restrictions, the channel itself will have limited bandwidth
  - This bandwidth limitation can result in ISI

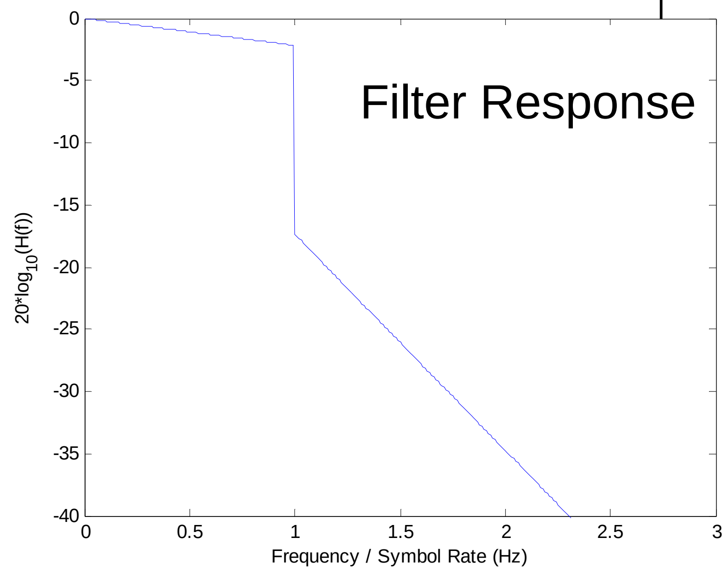
Causes of ISI – Bandwidth restriction

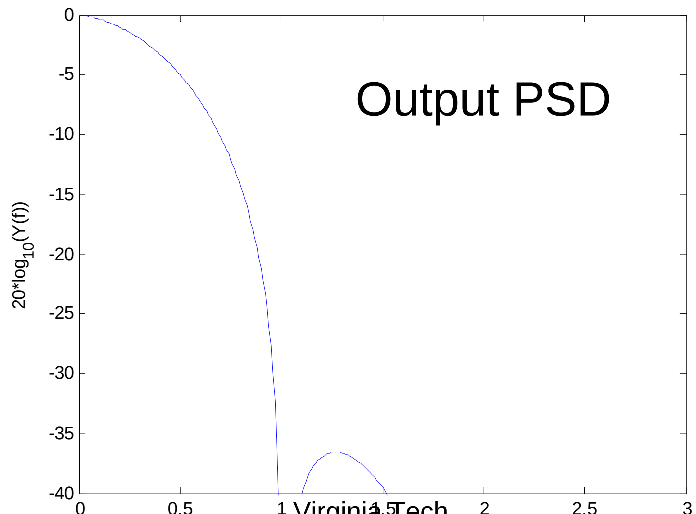


Original Pulse had no ISI
Bandwidth restricted pulse has ISI

Frequency Domain

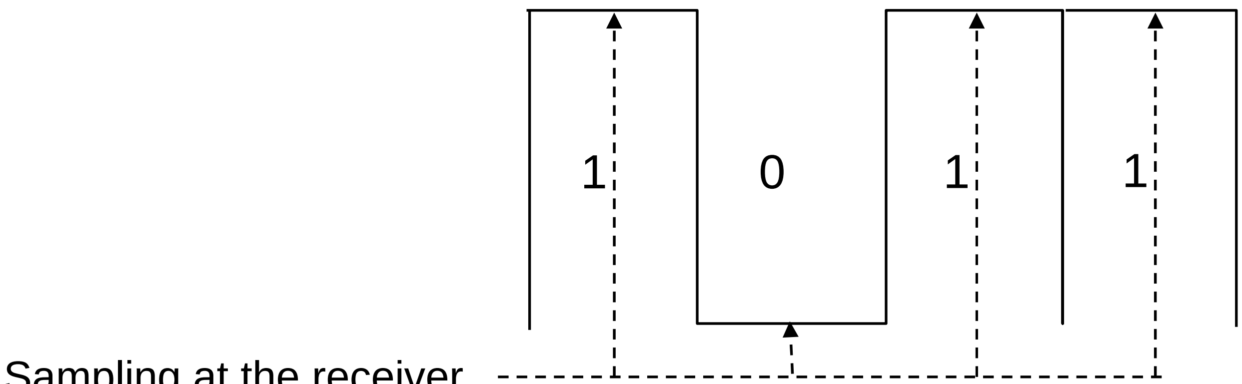




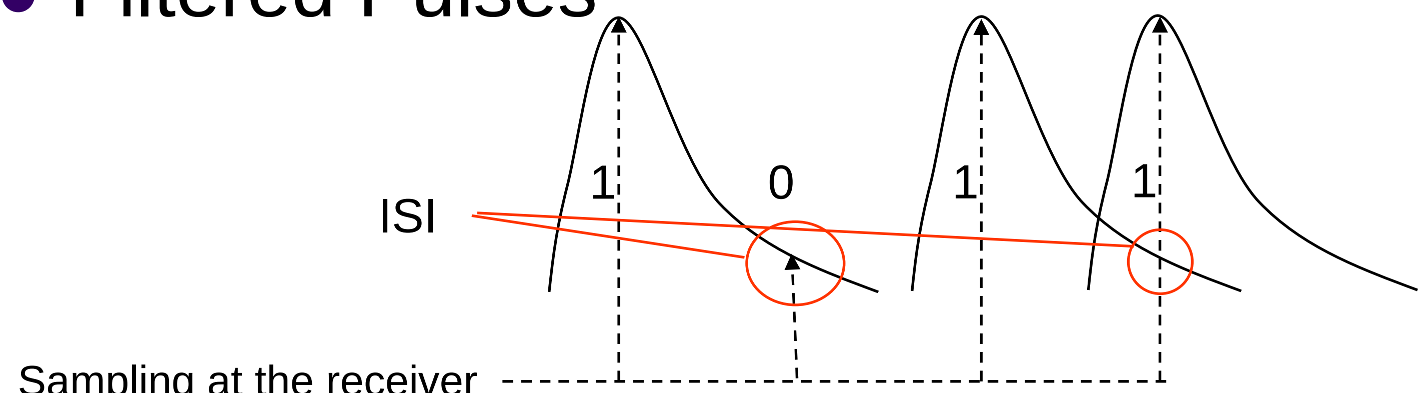


Impact of Bandwidth Limits

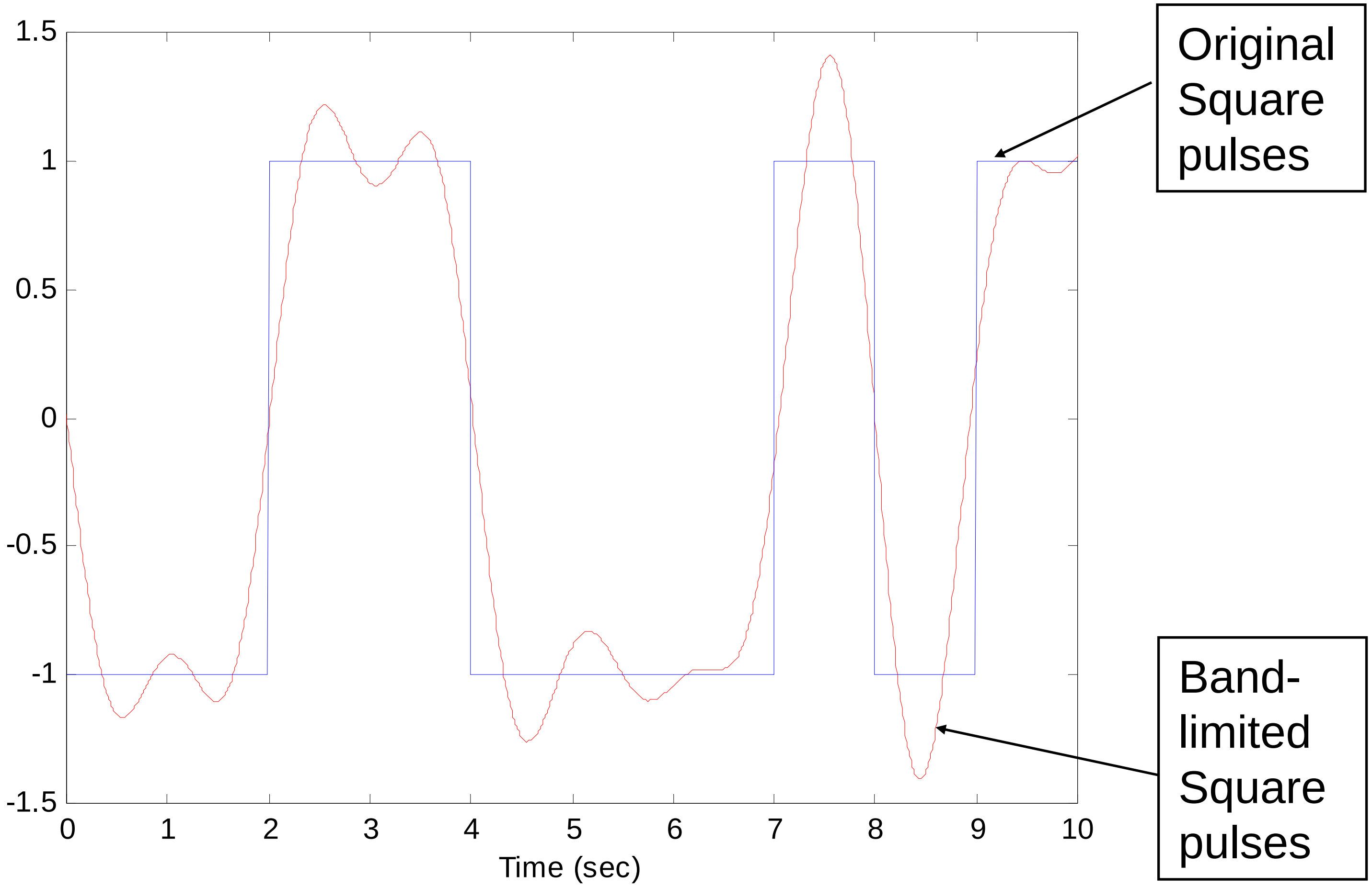
- Unfiltered Pulses



- Filtered Pulses



Band-limiting Square Pulses

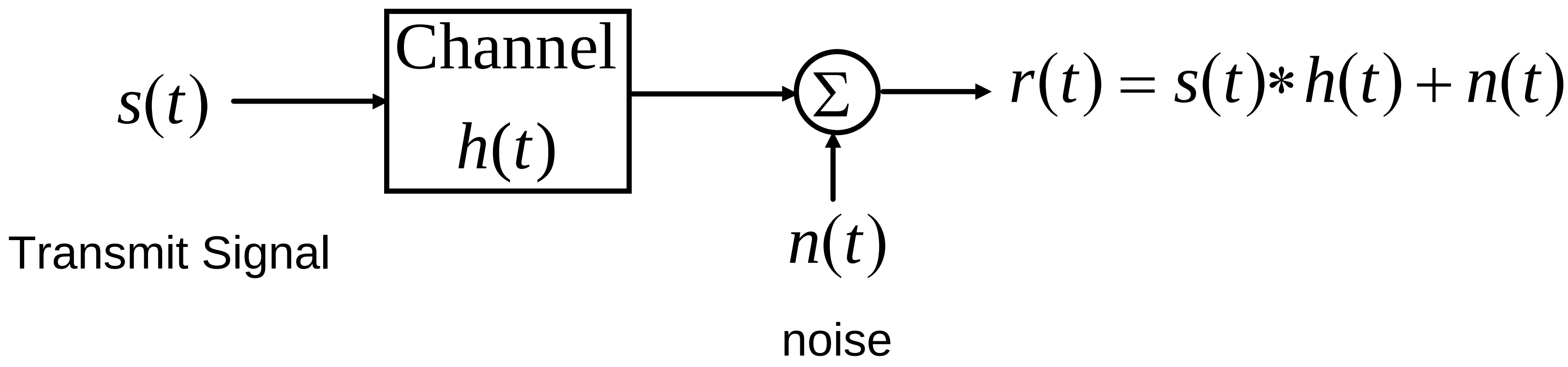


ISI Revisited

- What is Inter-symbol Interference or ISI?
  - ISI is interference caused by one symbol to another at *the sampling instant*
- What causes ISI?
  - Poor pulse design
    - Avoided by using proper pulse design as we will see next class
  - Band-limiting a transmit signal
    - Band-limiting is necessary due to FCC regulations
    - This source of ISI can be avoided by proper symbol design
  - Multipath channel (i.e., channel acts as a filter)
    - Reflections cause ISI between consecutive pulses
    - Can be mitigated by transmitting at lower data rate
    - Can be mitigated by *equalization*

ISI introduced by channel

- Channel has limited bandwidth



- Channel transfer function is just like a filter
- If bandwidth of the channel is less than that of the signal, ISI will result
- At the receiver we also add a filter to minimize noise
  - This filter should be designed along with the transmit filter to avoid ISI

ISI with Pulse Modulation

- The transmit signal for PCM can be written as

$$s(t) = \sum_{k=-\infty}^{\infty} a_k f(t - kT_s)$$

or alternatively as

$$s(t) = f(t) * \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s)$$

- The received signal (ignoring noise) is then

$$r(t) = \underbrace{\left\{ \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s) \right\}}_{\text{data signal}} * \underbrace{f(t)}_{\substack{\text{transmit}\\ \text{pulse shape}}} * \underbrace{h(t)}_{\text{channel}} * \underbrace{q(t)}_{\substack{\text{receive}\\ \text{filter}}}$$

Time View of ISI

$$r(t) = \underbrace{\left\{ \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s) \right\}}_{\text{data signal}} * \underbrace{f(t)}_{\substack{\text{transmit} \\ \text{pulse shape}}} * \underbrace{h(t)}_{\text{channel}} * \underbrace{q(t)}_{\substack{\text{receive} \\ \text{filter}}}$$

$$= \underbrace{\left\{ \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s) \right\}}_{\text{data signal}} * h_e(t)$$

The equivalent system impulse response is

$$h_e(t) = f(t) * h(t) * q(t)$$

The received signal can then be written as

$$r(t) = \sum_{k=-\infty}^{\infty} a_k h_e(t - kT_s)$$

$h_e(t)$ can be viewed as the equivalent pulse shape after the channel and receiver filtering

Time View of ISI (cont.)

- For each symbol we sample the received signal. We can write the sample for the *n*th symbol as

$$r(nT_s) = \sum_{k=-\infty}^{\infty} a_k h_e(nT_s - kT_s)$$

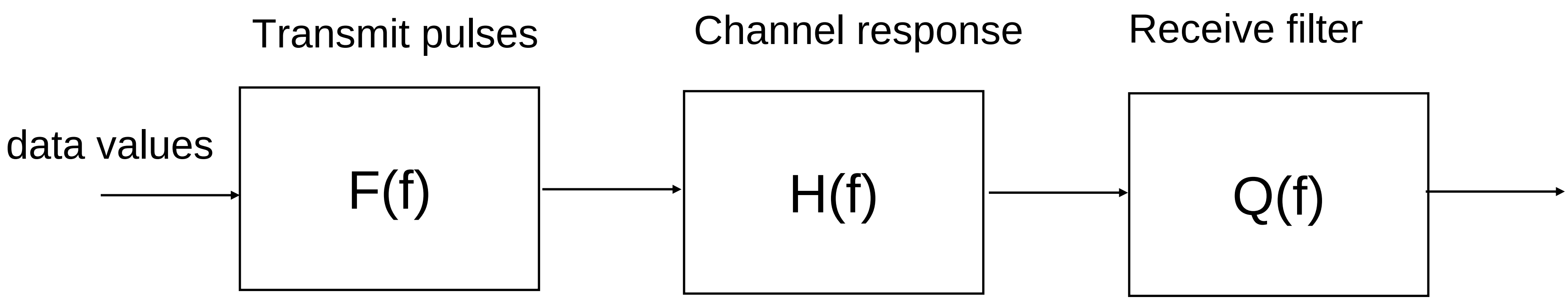
$$= a_n h_e(0) + \sum_{\substack{k=-\infty \\ k\neq n}}^{\infty} a_k h_e([n-k]T_s)$$

$$= a_n h_e(0) + \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} a_m h_e(mT_s)$$

- For zero ISI we require

$$h_e(kT_s) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

Frequency View of ISI



At the output of the receive filter our equivalent system transfer function is

$$H_e(f) = F(f)H(f)Q(f)$$

To avoid ISI, we require $$H_e(f) = \begin{cases} \Pi\left(\dfrac{f}{2B_o}\right) + Y(f) & |f| < 2f_o \\ 0 & else \end{cases}$$

We will show this next time

Intersymbol Interference

- If we transmit pulses that do not have ISI (e.g., they last only one symbol) and the channel has a constant response over the frequency band of interest, than the received filter should also be designed to provide a flat response over the band of interest

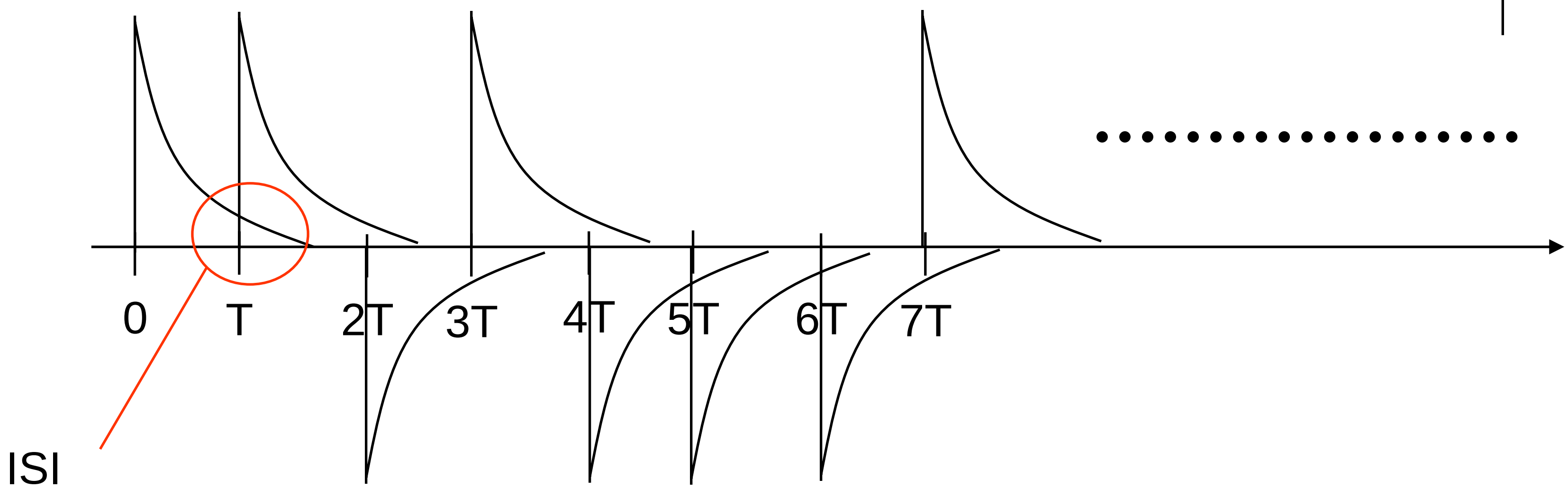
$$|H(f)| \approx 1 \longrightarrow |Q(f)| \approx 1$$

$$H_e(f) \approx F(f)$$

- However, if the channel is not constant we must design the receiver to invert the channel distortion. This is termed an equalizer.

$$|Q(f)| = \left| \frac{1}{H(f)} \right|$$

Example 10.1



$$p(t) = \exp(-a/T)u(t)$$

What is the 40dB bandwidth of the signal if we require the SIR of the signal due to intersymbol interference is 20dB? What if the SIR is required to be 50dB?

Summary

- We have briefly discussed the concept of Intersymbol Interference or ISI
- Bandwidth is a precious commodity
- However, blindly limiting bandwidth can lead to ISI which will degrade performance
- ISI can be viewed in the time-domain or the frequency domain
  - Equalizers, which eliminate ISI, can be designed either in the time or frequency domains
- We next will consider choosing pulse shapes that reduce bandwidth, but avoid ISI