2.1-1 Let us denote the signal in question by $g(t)$ and its energy by E_g. For parts **(a)** and **(b)**

$$E_g = \int_0^{2\pi} \sin^2 t\,dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt = \pi + 0 = \pi$$

$$\textbf{(c)} \qquad E_g = \int_{2\pi}^{4\pi} \sin^2 t\,dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t\,dt = \pi + 0 = \pi$$

$$\textbf{(d)} \qquad E_g = \int_0^{2\pi} (2\sin t)^2\,dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt\right] = 4[\pi + 0] = 4\pi$$

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way we can show that the energy of $kg(t)$ is $k^2 E_g$.

2.1-2 **(a)** $E_x = \int_0^2 (1)^2 dt = 2.\qquad E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2$

$$E_{x+y} = \int_0^1 (2)^2 dt = 4.\ \ E_{x-y} = \int_1^2 (2)^2 dt = 4$$

Therefore $E_{x\pm y} = E_x - E_y$.

$$\textbf{(b)}\, E_x = \int_0^{\pi} (1)^2 dt + \int_{\pi}^{2\pi} (-1)^2 dt = 2\pi, \qquad E_y = \int_0^{\pi/2} (1)^2 dt + \int_{\pi/2}^{\pi} (-1)^2 dt + \int_{\pi}^{3\pi/2} (1)^2 dt + \int_{3\pi/2}^{2\pi} (-1)^2 dt = 2\pi$$

$$E_{x+y} = \int_0^{\pi/2} (2)^2 dt + \int_{\pi/2}^{3\pi/2} (0)^2 dt + \int_{3\pi/2}^{2\pi} (-1)^2 dt = 4\pi$$

Similarly, we can show that $E_{x-y} = 4\pi$ Therefore $E_{x\pm y} = E_x + E_y$. We are tempted to conclude that $E_{x\pm y} = E_x - E_y$ in general. Let us see.

$$\textbf{(c)} \qquad E_x = \int_0^{\pi/4} (1)^2 dt + \int_{\pi/4}^{\pi} (-1)^2 dt = \pi \qquad E_y = \int_0^{\pi} (1)^2 dt = \pi$$

$$E_{x+y} = \int_0^{\pi/4} (2)^2 dt + \int_{\pi/4}^{\pi} (0)^2 dt = \pi \qquad E_{x-y} = \int_0^{\pi/4} (0)^2 dt + \int_{\pi/4}^{\pi} (-2)^2 dt = 3\pi$$

Therefore, in general $E_{x\pm y} \neq E_x + E_y$

2.1-4 This problem is identical to Example 2.2b, except that $\omega_1 \neq \omega_2$. In this case, the third integral in P_g (see p. 19 is not zero. This integral is given by

$$\begin{aligned}
I_3 &= \lim_{T\to\infty} \frac{2C_1C_2}{T}\int_{-T/2}^{T/2} \cos(\omega_1 t + \theta_1)\cos(\omega_1 t + \theta_2)\,dt \\
&= \lim_{T\to\infty} \frac{C_1C_2}{T}\left[\int_{-T/2}^{T/2} \cos(\theta_1 - \theta_2)\,dt + \int_{-T/2}^{T/2} \cos(2\omega_1 t + \theta_1 + \theta_2)\,dt\right] \\
&= \lim_{T\to\infty} \frac{C_1C_2}{T}\,[T\cos(\theta_1 - \theta_2)] + 0 = C_1C_2\cos(\theta_1 - \theta_2)
\end{aligned}$$

Therefore

$$P_g = \frac{C_1^2}{2} + \frac{C_2^2}{2} + C_1C_2\cos(\theta_1 - \theta_2)$$

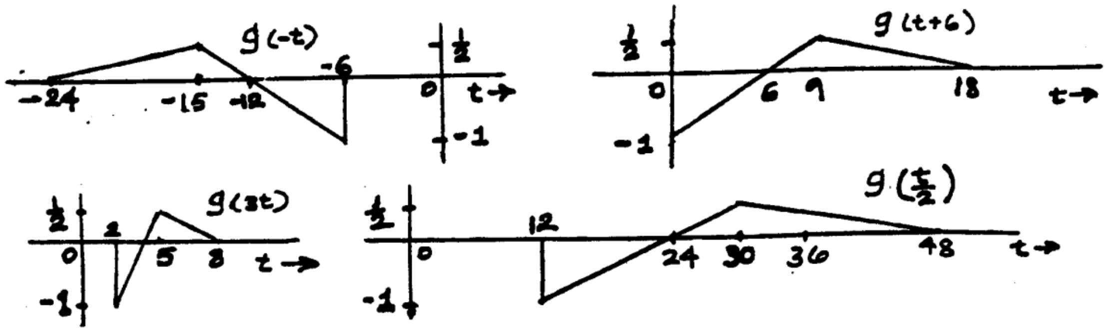

Fig. S2.3-2

Clearly, if a is real, e^{-at} is neither energy not power signal. However, if a is imaginary, it is a power signal with power 1.

2.3-1
$$g_2(t) = g(t-1) + g_1(t-1), \quad g_3(t) = g(t-1) + g_1(t+1), \quad g_4(t) = g(t-0.5) + g_1(t+0.5)$$

The signal $g_5(t)$ can be obtained by (i) delaying $g(t)$ by 1 second (replace t with $t-1$), (ii) then time-expanding by a factor 2 (replace t with $t/2$), (iii) then multiply with 1.5. Thus $g_5(t) = 1.5g(\frac{t}{2} - 1)$.

2.3-2 All the signals are shown in Fig. S2.3-2.

2.8-1 Here $T_0 = 2$, so that $\omega_0 = 2\pi/2 = \pi$, and

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi t + b_n \sin n\pi t \qquad -1 \le t \le 1$$

where

$$a_0 = \frac{1}{2}\int_{-1}^{1} t^2\,dt = \frac{1}{3}, \qquad a_n = \frac{2}{2}\int_{-1}^{1} t^2 \cos n\pi t\,dt = \frac{4(-1)^n}{\pi^2 n^2}, \qquad b_n = \frac{2}{2}\int_{-1}^{1} t^2 \sin n\pi t\,dt = 0$$

Therefore

$$g(t) = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos n\pi t \qquad -1 \le t \le 1$$

Figure S2.8-1 shows $g(t) = t^2$ for all t and the corresponding Fourier series representing $g(t)$ over $(-1, 1)$.

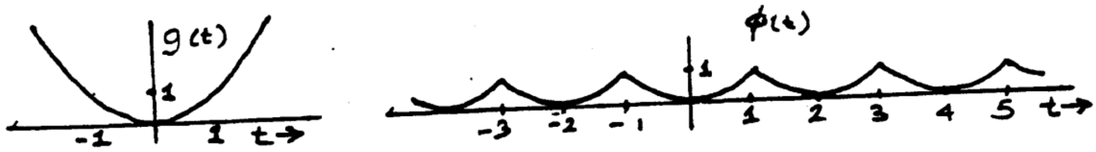

Fig. S2.8-1

The power of $g(t)$ is

$$P_g = \frac{1}{2}\int_{-1}^{1} t^4\,dt = \frac{1}{5}$$

Moreover, from Parseval's theorem [Eq. (2.90)]

$$P_g = C_0^2 + \sum_{1}^{\infty} \frac{C_n^2}{2} = \left(\frac{1}{3}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{\pi^2 n^2}\right)^2 = \frac{1}{9} + \frac{8}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{1}{9} + \frac{8}{90} = \frac{1}{5}$$

(b) If the N-term Fourier series is denoted by $x(t)$, then

$$x(t) = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{N-1} \frac{(-1)^n}{n^2}\cos n\pi t \qquad -1 \le t \le 1$$

The power P_x is required to be $99\% P_g = 0.198$. Therefore

$$P_x = \frac{1}{9} + \frac{8}{\pi^4}\sum_{n=1}^{N-1}\frac{1}{n^4} = 0.198$$

For $N = 1$. $P_x = 0.1111$; for $N = 2$. $P_x = 0.19323$. For $N = 3$. $P_x = 0.19837$, which is greater than 0.198. Thus. $N = 3$.

3.1-1

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} g(t)\cos\omega t\,dt - j\int_{-\infty}^{\infty} g(t)\sin\omega t\,dt$$

If $g(t)$ is an even function of t, $g(t)\sin\omega t$ is an odd function of t, and the second integral vanishes. Moreover, $g(t)\cos\omega t$ is an even function of t, and the first integral is twice the integral over the interval 0 to ∞. Thus when $g(t)$ is even

$$G(\omega) = 2\int_{0}^{\infty} g(t)\cos\omega t\,dt \tag{1}$$

Similar argument shows that when $g(t)$ is odd

$$G(\omega) = -2j\int_{0}^{\infty} g(t)\sin\omega t\,dt \tag{2}$$

If $g(t)$ is also real (in addition to being even), the integral (1) is real. Moreover from (1)

$$G(-\omega) = 2\int_{0}^{\infty} g(t)\cos\omega t\,dt = G(\omega)$$

Hence $G(\omega)$ is real and even function of ω. Similar arguments can be used to prove the rest of the properties.

3.1-2

$$\begin{aligned} g(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|e^{j\theta_g(\omega)}e^{j\omega t}\,d\omega \\ &= \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} |G(\omega)|\cos[\omega t + \theta_g(\omega)]\,d\omega + j\int_{-\infty}^{\infty} |G(\omega)|\sin[\omega t + \theta_g(\omega)]\,d\omega\right] \end{aligned}$$

Since $|G(\omega)|$ is an even function and $\theta_g(\omega)$ is an odd function of ω, the integrand in the second integral is an odd function of ω, and therefore vanishes. Moreover the integrand in the first integral is an even function of ω, and therefore

$$g(t) = \frac{1}{\pi}\int_{0}^{\infty} |G(\omega)|\cos[\omega t + \theta_g(\omega)]\,d\omega$$

For $g(t) = e^{-at}u(t)$, $G(\omega) = \frac{1}{\omega + a}$. Therefore $|G(\omega)| = 1/\sqrt{\omega^2 + a^2}$ and $\theta_g(\omega) = -\tan^{-1}(\frac{\omega}{a})$. Hence

$$e^{-at} = \frac{1}{\pi}\int_{0}^{\infty} \frac{1}{\sqrt{\omega^2 + a^2}}\cos\left[\omega t - \tan^{-1}\left(\frac{\omega}{a}\right)\right]\,d\omega$$

3.1-3

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt$$

Therefore

$$G^*(\omega) = \int_{-\infty}^{\infty} g^*(t)e^{j\omega t}\,dt$$

and

$$G^*(-\omega) = \int_{-\infty}^{\infty} g^*(t)e^{-j\omega t}\,dt$$

3.1-6 (a)

$$g(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} \omega^2 e^{j\omega t}\,d\omega = \frac{1}{2\pi}\,\frac{e^{j\omega t}}{(jt)^3}\left[-\omega^2t^2 - 2j\omega t + 2\right]\Bigg|_{-\omega_0}^{\omega_0} = \frac{(\omega_0^2t^2-2)\sin\omega_0 t + 2\omega_0 t\cos\omega_0 t}{\pi t^3}$$

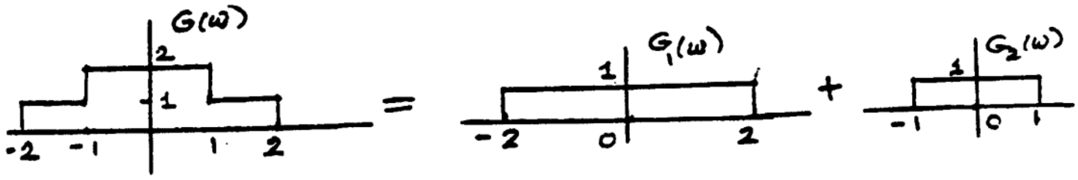

Fig. S3.1-6

(b) The derivation can be simplified by observing that $G(\omega)$ can be expressed as a sum of two gate functions $G_1(\omega)$ and $G_2(\omega)$ as shown in Fig. S3.1-6. Therefore

$$g(t) = \frac{1}{2\pi}\int_{-2}^{2}[G_1(\omega)+G_2(\omega)]e^{j\omega t}\,d\omega = \frac{1}{2\pi}\left\{\int_{-2}^{2} e^{j\omega t}\,d\omega + \int_{-1}^{1} e^{j\omega t}\,d\omega\right\} = \frac{\sin 2t + \sin t}{\pi t}$$

3.1-8 (a)

$$\begin{aligned} g(t) &= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{-j\omega t_0}e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega(t-t_0)}\,d\omega \\ &= \frac{1}{(2\pi)j(t-t_0)}e^{j\omega(t-t_0)}\Bigg|_{-\omega_0}^{\omega_0} = \frac{\sin\omega_0(t-t_0)}{\pi(t-t_0)} = \frac{\omega_0}{\pi}\text{sinc}[\omega_0(t-t_0)] \end{aligned}$$

(b)

$$\begin{aligned} g(t) &= \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} je^{j\omega t}\,d\omega + \int_{0}^{\omega_0} -je^{j\omega t}\,d\omega\right] \\ &= \frac{1}{2\pi t}e^{j\omega t}\Big|_{-\omega_0}^{0} - \frac{1}{2\pi t}e^{j\omega t}\Big|_{0}^{\omega_0} = \frac{1-\cos\omega_0 t}{\pi t} \end{aligned}$$

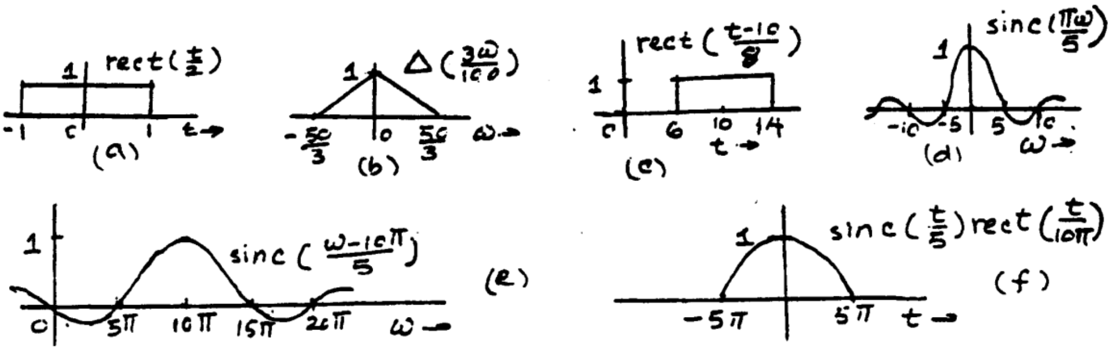

Fig. S3.2-1

3.2-1 Figure S3.2-1 shows the plots of various functions. The function in part **(a)** is a gate function centered at the origin and of width 2. The function in part **(b)** can be expressed as $\Delta\left(\frac{\omega}{100/3}\right)$. This is a triangle pulse centered at the origin and of width 100/3. The function in part **(c)** is a gate function rect$(\frac{t}{8})$ delayed by 10. In other words it is a gate pulse centered at $t = 10$ and of width 8. The function in part **(d)** is a sinc pulse centered at the origin and the first zero occurring at $\frac{\pi\omega}{5} = \pi$, that is at $\omega = 5$. The function in part **(e)** is a sinc pulse sinc$(\frac{\omega}{5})$ delayed by 10π. For the sinc pulse sinc$(\frac{\omega}{5})$, the first zero occurs at $\frac{\omega}{5} = \pi$, that is at $\omega = 5\pi$. Therefore the function is a sinc pulse centered at $\omega = 10\pi$ and its zeros spaced at intervals of 5π as shown in the fig. S3.2-1e. The function in part **(f)** is a product of a gate pulse (centered at the origin) of width 10π and a sinc pulse (also centered at the origin) with zeros spaced at intervals of 5π. This results in the sinc pulse truncated beyond the interval $\pm 5\pi$ ($|t| \geq 5\pi$) as shown in Fig. f.

3.2-2 The function rect $(t-5)$ is centered at $t = 5$, has a width of unity, and its value over this interval is unity. Hence

$$G(\omega) = \int_{4.5}^{5.5} e^{-j\omega t}\,dt = -\frac{1}{j\omega}e^{-j\omega t}\Big|_{4.5}^{5.5} = \frac{1}{j\omega}[e^{-j4.5\omega} - e^{-j5.5\omega}]$$

$$= \frac{e^{-j5\omega}}{j\omega}[e^{j\omega/2} - e^{-j\omega/2}] = \frac{e^{-j5\omega}}{j\omega}\left[2j\sin\frac{\omega}{2}\right]$$

$$= \text{sinc}\left(\frac{\omega}{2}\right)e^{-j5\omega}$$

3.3-1 **(a)**

$$\underbrace{u(t)}_{g(t)} \Longleftrightarrow \underbrace{\pi\delta(\omega) + \frac{1}{j\omega}}_{G(\omega)}$$

Application of duality property yields

$$\underbrace{\pi\delta(t) + \frac{1}{jt}}_{G(t)} \Longleftrightarrow \underbrace{2\pi u(-\omega)}_{2\pi g(-\omega)}$$

or

$$\frac{1}{2}\left[\delta(t) + \frac{1}{j\pi t}\right] \Longleftrightarrow u(-\omega)$$

Application of Eq. (3.28) yields

$$\frac{1}{2}\left[\delta(-t) - \frac{1}{j\pi t}\right] \Longleftrightarrow u(\omega)$$

But $\delta(t)$ is an even function, that is $\delta(-t) = \delta(t)$, and

$$\frac{1}{2}[\delta(t) + \frac{j}{\pi t}] \Longleftrightarrow u(\omega)$$

(b)

$$\underbrace{\cos \omega_0 t}_{g(t)} \Longleftrightarrow \underbrace{\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]}_{G(\omega)}$$

Application of duality property yields

$$\underbrace{\pi[\delta(t + \omega_0) + \delta(t - \omega_0)]}_{G(t)} \Longleftrightarrow \underbrace{2\pi \cos(-\omega_0\omega)}_{2\pi g(-\omega)} = 2\pi \cos(\omega_0\omega)$$

Setting $\omega_0 = T$ yields

$$\delta(t + T) + \delta(t - T) \Longleftrightarrow 2\cos T\omega$$

(c)

$$\underbrace{\sin \omega_0 t}_{g(t)} \Longleftrightarrow \underbrace{j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]}_{G(\omega)}$$

Application of duality property yields

$$\underbrace{j\pi[\delta(t + \omega_0) - \delta(t - \omega_0)]}_{G(t)} \Longleftrightarrow \underbrace{2\pi \sin(-\omega_0\omega)}_{2\pi g(-\omega)} = -2\pi \sin(\omega_0\omega)$$

Setting $\omega_0 = T$ yields

3.3-3 **(a)**

$$g(t) = \text{rect}\left(\frac{t + T/2}{T}\right) - \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$\text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow T\text{sinc}\left(\frac{\omega T}{2}\right)$$

$$\text{rect}\left(\frac{t \pm T/2}{T}\right) \Longleftrightarrow T\text{sinc}\left(\frac{\omega T}{2}\right) e^{\pm j\omega T/2}$$

and

$$G(\omega) = T\text{sinc}\left(\frac{\omega T}{2}\right)[e^{j\omega T/2} - e^{-j\omega T/2}]$$
$$= 2jT\text{sinc}\left(\frac{\omega T}{2}\right)\sin\frac{\omega T}{2}$$
$$= \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

(b) From Fig. S3.3-3b we verify that

$$g(t) = \sin t\, u(t) + \sin(t-\pi)u(t-\pi)$$

Note that $\sin(t-\pi)u(t-\pi)$ is $\sin t\, u(t)$ delayed by π. Now, $\sin t\, u(t) \Longleftrightarrow \frac{\pi}{2j}[\delta(\omega-1)-\delta(\omega+1)] + \frac{1}{1-\omega^2}$ and

$$\sin(t-\pi)u(t-\pi) \Longleftrightarrow \{\frac{\pi}{2j}[\delta(\omega-1)-\delta(\omega+1)] + \frac{1}{1-\omega^2}\}e^{-j\pi\omega}$$

Therefore

$$G(\omega) = \{\frac{\pi}{2j}[\delta(\omega-1)-\delta(\omega+1)] + \frac{1}{1-\omega^2}\}(1+e^{-j\pi\omega})$$

Recall that $g(x)\delta(x-x_0) = g(x_0)\delta(x-x_0)$. Therefore $\delta(\omega\pm 1)(1+e^{-j\pi\omega}) = 0$. and

$$G(\omega) = \frac{1}{1-\omega^2}(1+e^{-j\pi\omega})$$

(c) From Fig. S3.3-3c we verify that

$$g(t) = \cos t\left[u(t) - u\left(t-\frac{\pi}{2}\right)\right] = \cos t\, u(t) - \cos t\, u\left(t-\frac{\pi}{2}\right)$$

But $\sin(t-\frac{\pi}{2}) = -\cos t$. Therefore

$$g(t) = \cos t\, u(t) + \sin\left(t-\frac{\pi}{2}\right)u\left(t-\frac{\pi}{2}\right)$$
$$G(\omega) = \frac{\pi}{2}[\delta(\omega-1)+\delta(\omega+1)] + \frac{j\omega}{1-\omega^2} + \left\{\frac{\pi}{2j}[\delta(\omega-1)-\delta(\omega+1)] + \frac{1}{1-\omega^2}\right\}e^{-j\pi\omega/2}$$

Also because $g(x)\delta(x-x_0) = g(x_0)\delta(x-x_0)$.

$$\delta(\omega\pm 1)e^{-j\pi\omega/2} = \delta(\omega\pm 1)e^{\pm j\pi/2} = \pm j\delta(\omega\pm 1)$$

Therefore

$$G(\omega) = \frac{j\omega}{1-\omega^2} + \frac{e^{-j\pi\omega/2}}{1-\omega^2} = \frac{1}{1-\omega^2}[j\omega + e^{-j\pi\omega/2}]$$

(d)

$$g(t) = e^{-at}[u(t)-u(t-T)] = e^{-at}u(t) - e^{-at}u(t-T)$$
$$= e^{-at}u(t) - e^{-aT}e^{-a(t-T)}u(t-T)$$
$$G(\omega) = \frac{1}{j\omega+a} - \frac{e^{-aT}}{j\omega+a}e^{-j\omega T} = \frac{1}{j\omega+a}[1-e^{-(a+j\omega)T}]$$

3.3-10

A basic demodulator is shown in Fig. S3.3-10a. The product of the modulated signal $g(t)\cos\omega_0 t$ with $2\cos\omega_0 t$ yields

$$g(t)\cos\omega_0 t \times 2\cos\omega_0 t = 2g(t)\cos^2\omega_0 t = g(t)[1+\cos 2\omega_0 t] = g(t) + g(t)\cos 2\omega_0 t$$

The product contains the desired $g(t)$ (whose spectrum is centered at $\omega = 0$) and the unwanted signal $g(t)\cos 2\omega_0 t$ with spectrum $\frac{1}{2}[G(\omega+2\omega_0)+G(\omega-2\omega_0)]$. which is centered at $\pm 2\omega_0$. The two spectra are nonoverlapping because

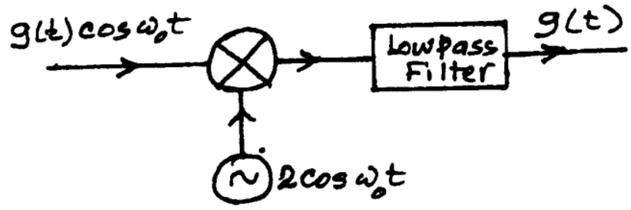

Fig. S3.3-10

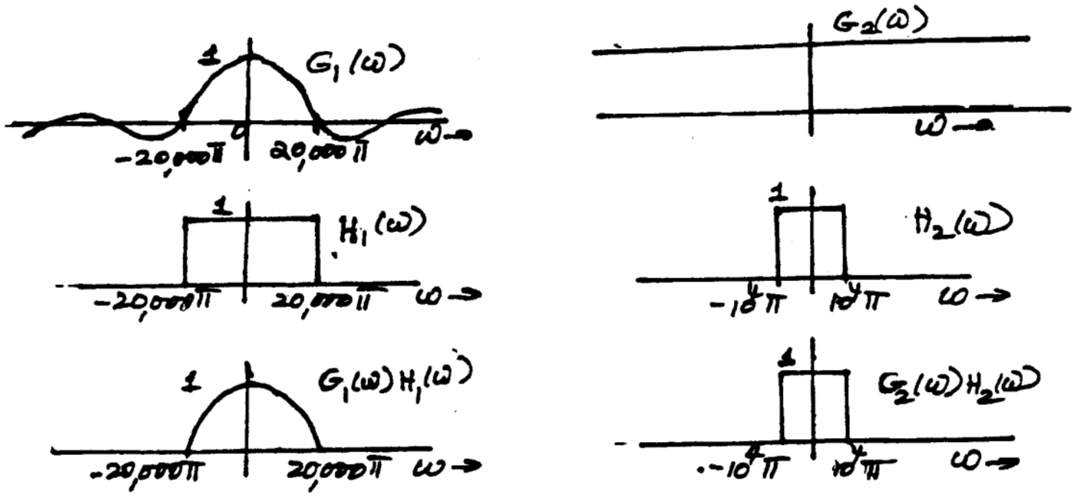

Fig. S3.4-1

$W < \omega_0$ (See Fig. S3.3-10b). We can suppress the unwanted signal by passing the product through a lowpass filter as shown in Fig. S3.3-10a.

3.4-1

$$G_1(\omega) = \text{sinc}\left(\tfrac{\omega}{20000}\right) \qquad \text{and} \quad G_2(\omega) = 1$$

Figure S3.4-1 shows $G_1(\omega)$, $G_2(\omega)$, $H_1(\omega)$ and $H_2(\omega)$. Now

$$Y_1(\omega) = G_1(\omega)H_1(\omega)$$
$$Y_2(\omega) = G_2(\omega)H_2(\omega)$$

The spectra $Y_1(\omega)$ and $Y_2(\omega)$ are also shown in Fig. S3.4-1. Because $y(t) = y_1(t)y_2(t)$, the frequency convolution property yields $Y(\omega) = Y_1(\omega) * Y_2(\omega)$. From the width property of convolution, it follows that the bandwidth of $Y(\omega)$ is the sum of bandwidths of $Y_1(\omega)$ and $Y_2(\omega)$. Because the bandwidths of $Y_1(\omega)$ and $Y_2(\omega)$ are 10 kHz, 5 kHz, respectively, the bandwidth of $Y(\omega)$ is 15 kHz.

3.7-4 In the generalized Parseval's theorem in Prob. 3.7-3, if we identify $g_1(t) = \text{sinc}\,(2\pi Bt - m\pi)$ and $g_2(t) = \text{sinc}\,(2\pi Bt - n\pi)$, then

$$G_1(\omega) = \frac{1}{2B}\,\text{rect}\left(\frac{\omega}{4\pi B}\right) e^{-\frac{jm\omega}{2B}}, \quad \text{and} \quad G_2(\omega) = \frac{1}{2B}\,\text{rect}\left(\frac{\omega}{4\pi B}\right) e^{\frac{jn\omega}{2B}}$$

Therefore

$$\int_{-\infty}^{\infty} g_1(t)g_2(t)\,dt = \frac{1}{2\pi}\,\frac{1}{(2B)^2}\int_{-\infty}^{\infty}\left[\text{rect}\left(\frac{\omega}{4\pi B}\right)\right]^2 e^{\frac{j(n-m)\omega}{2B}}\,d\omega$$

But $\text{rect}\left(\frac{\omega}{4\pi B}\right) = 1$ for $|\omega| \le 2\pi B$, and is 0 otherwise. Hence

$$\int_{-\infty}^{\infty} g_1(t)g_2(t)\,dt = \frac{1}{8\pi B^2}\int_{-2\pi B}^{2\pi B} e^{\frac{j(n-m)\omega}{2B}}\,d\omega = \begin{cases} 0 & n \neq m \\ \frac{1}{2B} & n = m \end{cases}$$

In evaluating the integral, we used the fact that $e^{\pm j2\pi k} = 1$ when k is an integer.

3.8-2 Figure S3.8-2a shows the waveforms $x(t)$ and $x(t-\tau)$ for $\tau < T_b/2$. Let $T = NT_b$. On the average, there are $N/2$ pulses in the waveform of duration T. The area under the product $x(t)x(t-\tau)$ is $N/2$ times $(\frac{T_b}{2} - \tau)$ as shown in Fig. S3.8-2b. Therefore

$$\begin{aligned}
\mathcal{R}_x(\tau) &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)x(t-\tau)\,dt \\
&= \lim_{N\to\infty} \frac{1}{NT_b}\frac{N}{2}\left(\frac{T_b}{2} - \tau\right) = \frac{1}{2}\left(\frac{1}{2} - \frac{\tau}{T_b}\right) \qquad \tau < \frac{T_b}{2} \\
&= \frac{1}{2}\left(\frac{1}{2} - \frac{|\tau|}{T_b}\right) \qquad |\tau| < \frac{T_b}{2}
\end{aligned}$$

For $\frac{T_b}{2} \le |\tau| \le T_b$, there is no overlap between pulses, and $\mathcal{R}_x(\tau) = 0$. For $T_b \le |\tau| \le \frac{3T_b}{2}$, pulses again overlap. But on the average, only half pulses overlap. Hence, $\mathcal{R}_x(\tau)$ repeats every T_b seconds, but only with half the magnitude, as shown in Fig. S3.8-2c. We can express $\mathcal{R}_x(\tau)$ as a sum of two components, as shown in Fig. S3.8-2d. Thus, $\mathcal{R}_x(\tau) = \mathcal{R}_1(\tau) + \mathcal{R}_2(\tau)$. The PSD is the sum of the Fourier transforms of $\mathcal{R}_1(\tau)$ and $\mathcal{R}_2(\tau)$. Hence

$$S_x(\omega) = \frac{T_b}{16}\,\mathrm{sinc}^2\left(\frac{\omega T_b}{4}\right) + S_2(\omega)$$

where $S_2(\omega)$ is the Fourier transform of the periodic triangle function, shown in Fig. S3.8-2d. We find the exponential Fourier series for this periodic signal to be

$$\mathcal{R}_2(\tau) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_b\tau} \qquad \omega_b = \frac{2\pi}{T_b}$$

Using Eq. (2.80), we find $D_n = \frac{1}{16}\mathrm{sinc}^2(\frac{n\pi}{2})$. Hence, according to Eq. (3.41)

$$S_2(\omega) = \frac{\pi}{8}\sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{n\pi}{2}\right)\delta(\omega - n\omega_b) \qquad \omega_b = \frac{2\pi}{T_b}$$

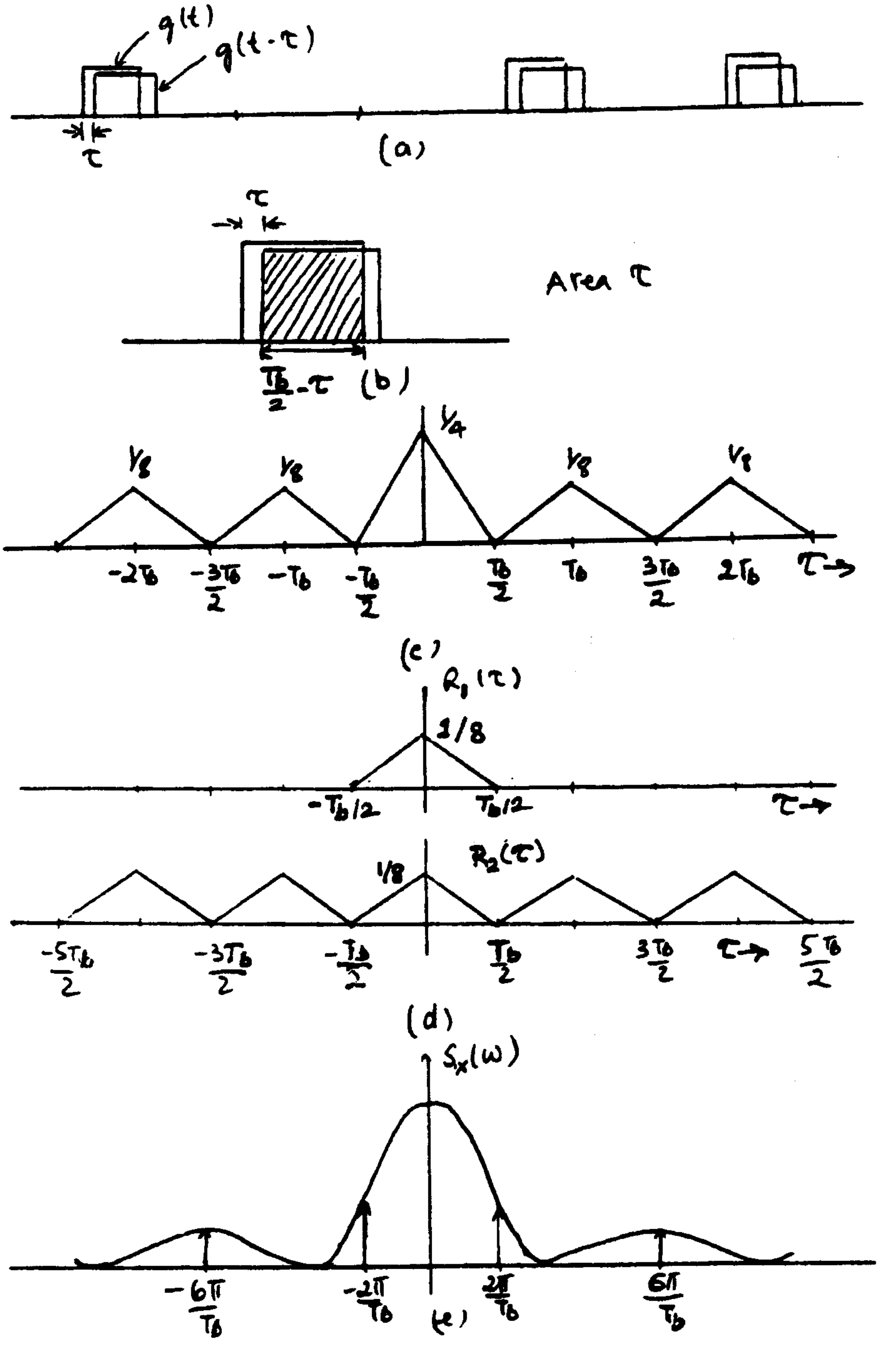

Fig. S3.8-2

Therefore

$$S_x(\omega) = \frac{T_b}{16}\,\mathrm{sinc}^2\left(\frac{\omega T_b}{4}\right) + \frac{\pi}{8}\sum_{n=-\infty}^{\infty}\mathrm{sinc}^2\left(\frac{n\pi}{2}\right)\delta(\omega - n\omega_b) \qquad \omega_b = \frac{2\pi}{T_b}$$

6.1-1 The bandwidths of $g_1(t)$ and $g_2(t)$ are 100 kHz and 150 kHz, respectively. Therefore the Nyquist sampling rates for $g_1(t)$ is 200 kHz and for $g_2(t)$ is 300 kHz.
Also $g_1{}^2(t) \Longleftrightarrow \frac{1}{2\pi} g_1(\omega) * g_1(\omega)$, and from the width property of convolution the bandwidth of $g_1{}^2(t)$ is twice the bandwidth of $g_1(t)$ and that of $g_2{}^3(t)$ is three times the bandwidth of $g_2(t)$ (se also Prob. 4.3-10). Similarly the bandwidth of $g_1(t)g_2(t)$ is the sum of the bandwidth of $g_1(t)$ and $g_2(t)$. Therefore the Nyquist rate for $g_1{}^2(t)$ is 400 kHz. for $g_2{}^3(t)$ is 900 kHz. for $g_1(t)g_2(t)$ is 500 kHz.

6.1-2 (a)

$$\text{sinc} 100\pi t \Longleftrightarrow 0.01 \text{rect}\left(\frac{\omega}{200\pi}\right)$$

The bandwidth of this signal is 100 π rad/s or 50 Hz. The Nyquist rate is 100 Hz (samples/sec).
(b)

$$\text{sinc}^2(100\pi t) \Longleftrightarrow 0.01\Delta\left(\frac{\omega}{400\pi}\right)$$

The bandwidth of this signal is 200 π rad/s or 100 Hz. The Nyquist rate is 200 Hz (samples/sec).
(c)

$$\text{sinc}\,(100\pi t) + \text{sinc}\,(50\pi t) \Longleftrightarrow 0.01 \text{rect}\, 0.01\left(\frac{\omega}{200\pi}\right) + 0.02 \text{rect}\left(\frac{\omega}{100\pi}\right)$$

The bandwidth of the first term on the right-hand side is 50 Hz and the second term is 25 Hz. Clearly the bandwidth of the composite signal is the higher of the two, that is. 100 Hz. The Nyquist rate is 200 Hz (samples/sec).
(d)

$$\text{sinc}(100\pi t) + 3\,\text{sinc}^2(60\pi t) \Longleftrightarrow 0.01\,\text{rect}\left(\frac{\omega}{200\pi}\right) + \frac{1}{20}\,\Delta\left(\frac{\omega}{240\pi}\right)$$

The bandwidth of $\text{rect}\left(\frac{\omega}{200\pi}\right)$ is 50 Hz and that of $\Delta\left(\frac{\omega}{240\pi}\right)$ is 60 Hz. The bandwidth of the sum is the higher of the two, that is. 60 Hz. The Nyquist sampling rate is 120 Hz.
(e)

$$\text{sinc}(50\pi t) \Longleftrightarrow 0.02\,\text{rect}\left(\frac{\omega}{100\pi}\right)$$
$$\text{sinc}(100\pi t) \Longleftrightarrow 0.01\,\text{rect}\left(\frac{\omega}{200\pi}\right)$$

The two signals have bandwidths 25 Hz and 50 Hz respectively. The spectrum of the product of two signals is $1/2\pi$ times the convolution of their spectra. From width property of the convolution, the width of the convoluted signal is the sum of the widths of the signals convolved. Therefore. the bandwidth of sinc$(50\pi t)$sinc$(100\pi t)$ is $25 + 50 = 75$ Hz. The Nyquist rate is 150 Hz.

6.1-3 The pulse train is a periodic signal with fundamental frequency $2B$ Hz. Hence, $\omega_s = 2\pi(2B) = 4\pi B$. The period is $T_0 = 1/2B$. It is an even function of t. Hence, the Fourier series for the pulse train can be expressed as

$$p_{T_s}(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos n\omega_s t$$

Using Eqs. (2.72). we obtain

$$a_0 = C_0 = \frac{1}{T_0}\int_{-1/16B}^{1/16B} dt = \frac{1}{4}, \qquad a_n = C_n = \frac{2}{T_0}\int_{-1/16B}^{1/16B} \cos n\omega_s t\, dt = \frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right). \qquad b_n = 0$$

Hence.

$$\begin{aligned}\overline{g}(t) &= g(t)p_{T_s}(t) \\ &= \frac{1}{4}g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right) g(t)\cos n\omega_s t\end{aligned}$$

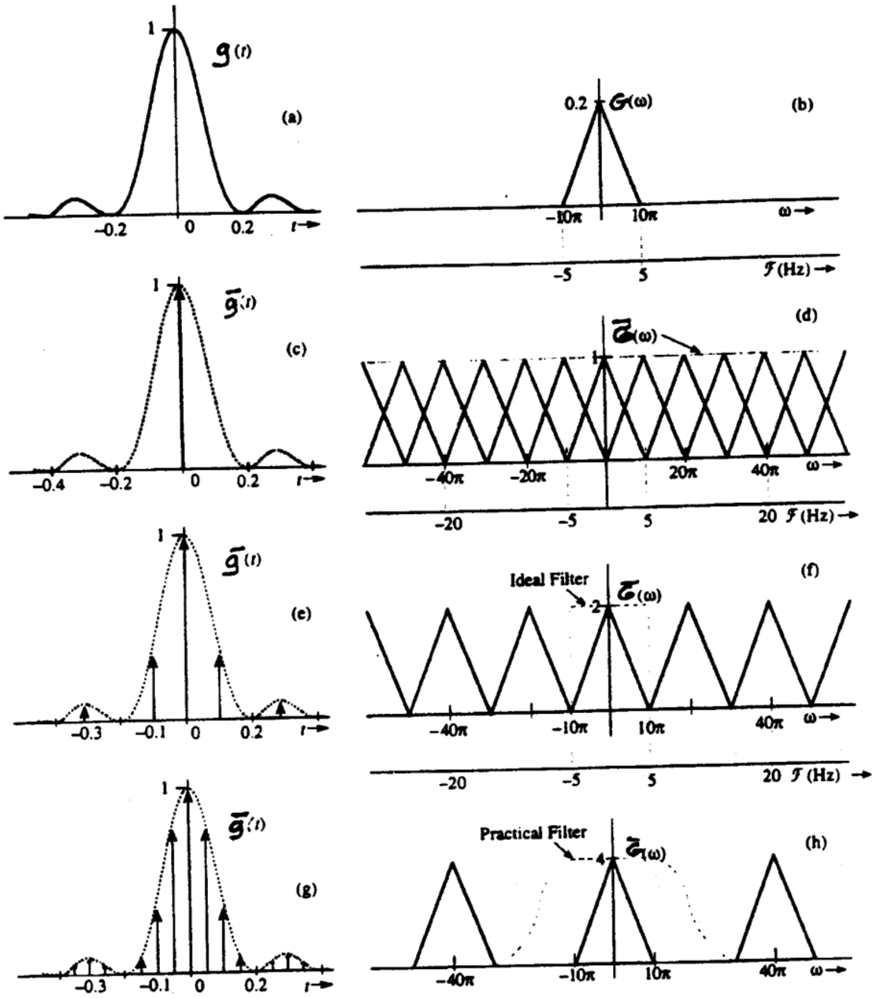

Fig. S6.1-4

6.1-4 For $g(t) = \text{sinc}^2(5\pi t)$ (Fig. S6.1-4a), the spectrum is $G(\omega) = 0.2\,\Delta(\frac{\omega}{20\pi})$ (Fig. S6.1-4b). The bandwidth of this signal is 5 Hz (10π rad/s). Consequently, the Nyquist rate is 10 Hz, that is, we must sample the signal at a rate no less than 10 samples/s. The Nyquist interval is $T = 1/2B = 0.1$ second.
Recall that the sampled signal spectrum consists of $(1/T)G(\omega) = \frac{0.2}{T}\,\Delta(\frac{\omega}{20\pi})$ repeating periodically with a period equal to the sampling frequency f_s Hz. We present this information in the following Table for three sampling rates: $f_s = 5$ Hz (undersampling), 10 Hz (Nyquist rate), and 20 Hz (oversampling).

sampling frequency f_s	sampling interval T	$\frac{1}{T}G(\omega)$	comments
5 Hz	0.2	$\Delta\left(\frac{\omega}{20\pi}\right)$	Undersampling
10 Hz	0.1	$2\Delta\left(\frac{\omega}{20\pi}\right)$	Nyquist Rate
20 Hz	0.05	$4\Delta\left(\frac{\omega}{20\pi}\right)$	Oversampling

In the first case (undersampling), the sampling rate is 5 Hz (5 samples/sec.), and the spectrum $\frac{1}{T}G(\omega)$ repeats every 5 Hz (10π rad/sec.). The successive spectra overlap, as shown in Fig. S6.1-4d, and the spectrum $G(\omega)$ is not recoverable from $\overline{G}(\omega)$, that is, $g(t)$ cannot be reconstructed from its samples $\overline{g}(t)$ in Fig. S6.1-4c. If the sampled signal is passed through an ideal lowpass filter of bandwidth 5 Hz, the output spectrum is $\text{rect}\left(\frac{\omega}{20\pi}\right)$.

and the output signal is $10\,\text{sinc}\,(20\pi t)$, which is not the desired signal $\text{sinc}^2(5\pi t)$. In the second case, we use the Nyquist sampling rate of 10 Hz (Fig. S6.1-4e). The spectrum $\overline{G}(\omega)$ consists of back-to-back, nonoverlapping repetitions of $\frac{1}{T}G(\omega)$ repeating every 10 Hz. Hence, $G(\omega)$ can be recovered from $\overline{G}(\omega)$ using an ideal lowpass filter of bandwidth 5 Hz (Fig. S6.1-4f). The output is $10\,\text{sinc}^2(5\pi t)$. Finally, in the last case of oversampling (sampling rate 20 Hz), the spectrum $\overline{G}(\omega)$ consists of nonoverlapping repetitions of $\frac{1}{T}G(\omega)$ (repeating every 20 Hz) with empty band between successive cycles (Fig. S6.1-4h). Hence, $G(\omega)$ can be recovered from $\overline{G}(\omega)$ using an ideal lowpass filter or even a practical lowpass filter (shown dotted in Fig. S6.1-4h). The output is $20\,\text{sinc}^2(5\pi t)$.

6.1-5 This scheme is analyzed fully in Problem 3.4-1, where we found the bandwidths of $y_1(t)$, $y_2(t)$, and $y(t)$ to be 10 kHz, 5 kHz, and 15 kHz, respectively. Hence, the Nyquist rates for the three signals are 20 kHz, 10 kHz, and 30 kHz, respectively.

6.2-2

(a) The bandwidth is 15 kHz. The Nyquist rate is 30 kHz.

(b) $65536 = 2^{16}$, so that 16 binary digits are needed to encode each sample.

(c) $30000 \times 16 = 480000$ bits/s.

(d) $44100 \times 16 = 705600$ bits/s.

6.2-8 Here $\mu = 100$ and the SNR = 45 dB= 31,622.77. From Eq. (6.18)

$$\frac{S_o}{N_o} = \frac{3L^2}{(\ln 101)^2} = 31,622.77 \quad \Longrightarrow L = 473.83$$

Because L is a power of 2, we select $L = 512 = 2^9$. The SNR for this value of L is

$$\frac{S_o}{N_o} = \frac{3(512)^2}{(\ln 101)^2} = 36922.84 = 45.67 \text{ dB}$$