ECE4634
Digital Communications
Fall 2007

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Lecture #16: Bandpass Signals

Bandpass Modulation - BPSK

Analog and Digital Communications

Lecture Objective

- The objective of this lecture is to introduce the concept of Binary Phase Shift Modulation, a form of digital bandpass modulation.

Modulation

- Baseband signals *w*(*t*) may be transformed into bandpass signals through multiplication by a sinusoid:

$$w(t)\cos(\omega_c t + \theta) \Leftrightarrow \frac{1}{2}\left[e^{j\theta}W(f - f_c) + e^{-j\theta}W(f + f_c)\right]$$

- Most transmitted signals are modulated onto a carrier because
  - Modulated signals propagate well through the atmosphere
  - Modulation allows many signals with different carrier frequencies to share the spectrum
  - Efficient antennas can be built at a reasonable size

Three Ways of Representing Bandpass Signals

- Magnitude and Phase

$$v(t) = R(t)\cos\left[\omega_c t + \theta(t)\right]$$

- In Phase and Quadrature

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- Complex Envelope

$$v(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$$

Transmitting Information

- Previously we saw that information (bits) can be transmitted by changing the amplitude of pulses.
  - Baseband communications
- With bandpass communications information is transmitted by modulating (changing) the phase, frequency or amplitude of a carrier (sinusoid)
- Modulation can be envisioned in the time domain as changing or modulating the sinusoid with time
- Modulation in the frequency domain can be envisioned as shifting the frequency components of the signal from baseband to bandpass around some center frequency.

Bandpass Modulation - BPSK

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- Binary Phase Shift Keying
- Basic Idea:
  - Data determines the **phase** of transmit sinusoid over

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t + \theta_i\right)$$

$$1 \Rightarrow \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t + 0\right)$$

$$0 \Rightarrow \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t + \pi\right)$$

$$0 \le t < T_s$$

  - Since cos(π) = -1, this is equivalent to:

$$1 \Rightarrow \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t\right)\Big|_0^{T_s}$$

$$0 \Rightarrow -\sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t\right)\Big|_0^{T_s}$$

Remember: We can transmit information by varying either the (1) amplitude, (2) phase, or (3) frequency. Here we use the phase.

BPSK - Magnitude and Phase Representation

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- $s(t) = R(t)\cos[\omega_C t + \theta(t)]$ where
  - $R(t) = \sqrt{\dfrac{2E_b}{T_b}}$ (constant envelope)
  - $1 \Rightarrow \theta(t) = 0\Big|_0^{T_s}$

    $0 \Rightarrow \theta(t) = \pi\Big|_0^{T_s}$

Note: Constant envelope signals are beneficial in fading channels because fading corrupts the amplitude, but here the amplitude carries no information.

- All of the information is in the phase

BPSK - I & Q Representation

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$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

$$x(t) = R(t)\cos(\theta(t))$$

$$= \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} & \theta = 0 \\ -\sqrt{\dfrac{2E_b}{T_b}} & \theta = \pi \end{cases} \quad 0 \le t \le T_s$$

$$y(t) = R(t)\sin(\theta(t))$$

$$= \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} * 0 & \theta = 0 \\ \sqrt{\dfrac{2E_b}{T_b}} * 0 & \theta = \pi \end{cases} \quad 0 \le t \le T_s$$

$$= 0$$

$$\boxed{v(t) = \underbrace{x(t)}_{\text{polar NRZ signal}} \cos(\omega_c t)}$$

- No Q component
- I component is just a polar NRZ signal

Note: Since there is no signal in the Q component we could send an additional BPSK signal in the Q portion. This is termed Q (Quadrature) PSK signal

BPSK - Complex Envelope Representation

- $v(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$ where
  - $g(t) = x(t) + j \cdot y(t)$

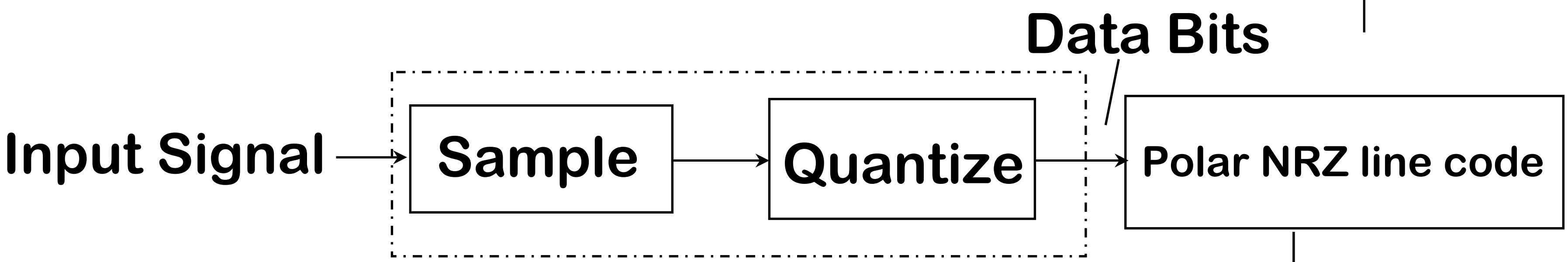
$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} & \theta = 0 \\ -\sqrt{\frac{2E_b}{T_b}} & \theta = \pi \end{cases}$$

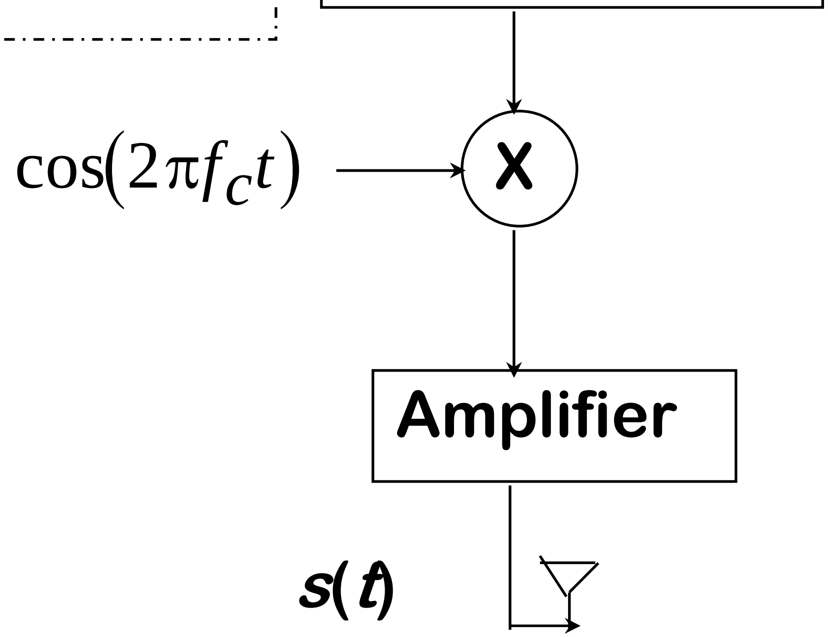
$$g(t) = x(t)$$

$$= \underbrace{\sqrt{\frac{2E_b}{T_b}} \sum_k (2b_k - 1)\, rect\left(\frac{t - kT_s}{T_s}\right)}_{\text{polar NRZ linecode}}$$

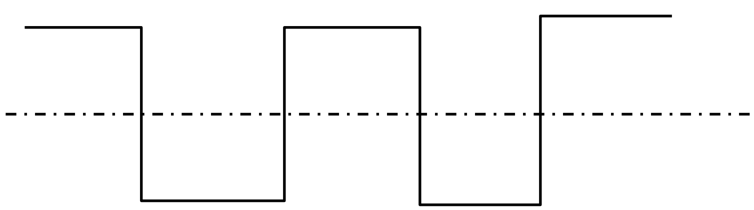
  - Complex envelope is entirely real
  - Complex envelope is equivalent to polar NRZ signaling
  - Imaginary portion of complex envelope corresponds to Q component

Transmitter for BPSK





If we use Polar NRZ line codes, we can simply multiply the sinusoid with data signal as suggested by I/Q representation



Polar NRZ format

Alternative Representation for BPSK

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- We can also think of BPSK as a binary modulated pulse stream multiplied a carrier

$$v(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}}\left(\sum_k d_k p(t - kT_s)\right)}_{x(t)}\cos\left(\omega_c t\right)$$

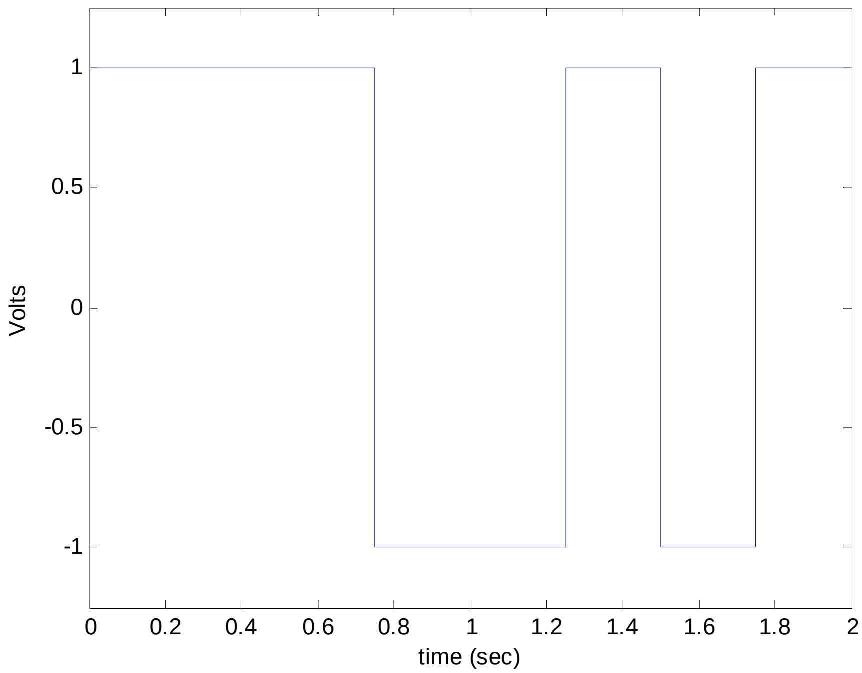
where

$$p(t) = rect\left(\frac{t - T_s/2}{T_s}\right)$$ Rectangular pulses

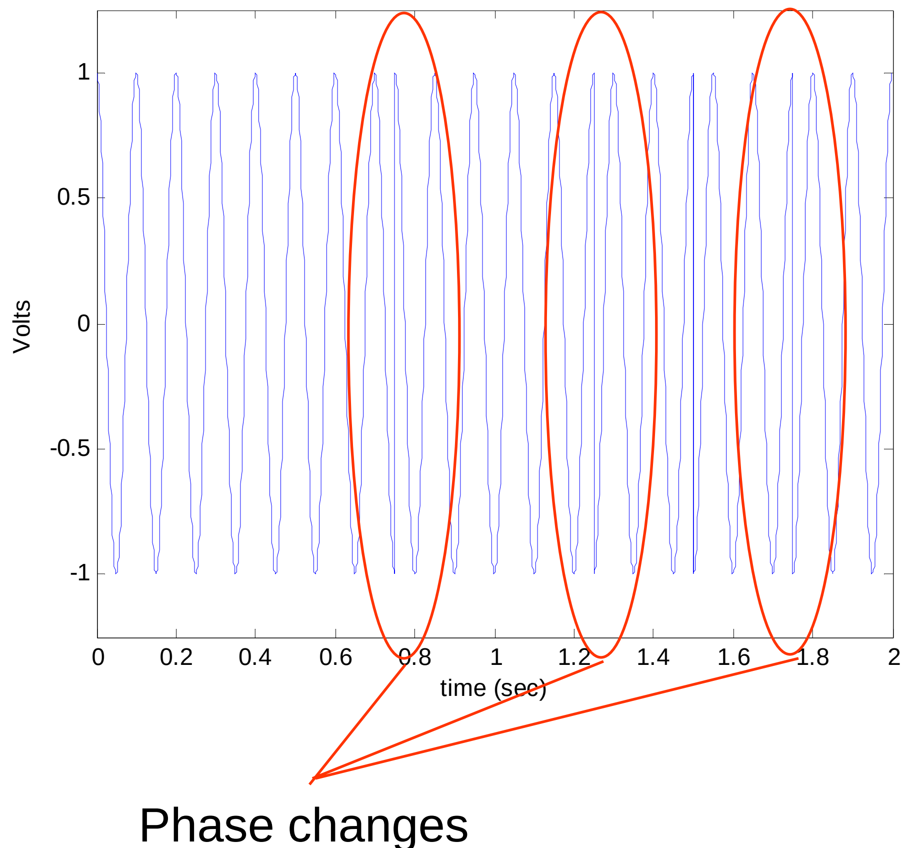
$$d_k = 2b_k - 1$$ Convert bits to {+1,-1}

Example Signal

- Baseband signal



Modulated Signal



Power Spectral Density of BPSK

- Recall that the PSD of a bandpass signal can be related to the PSD of the complex envelope:

$$P_v(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c)\right]$$

- Thus, since
  - we know the complex baseband signal is NRZ signal and
  - we know the PSD of a polar NRZ signal,

  we can determine the PSD of the BPSK signal. The
- The complex envelope representation is useful for determining the PSD of bandpass signals.

Power Spectral Density for BPSK

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We have shown for polar NRZ signaling:

$$P_g(f) = A^2 T_b \left(\frac{\sin(\pi f T_b)}{\pi f T_b} \right)^2$$

Thus,

$$P_v(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c) \right]$$

$$= \frac{A^2 T_b}{4} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{\sin(\pi(-f - f_c)T_b)}{\pi(-f - f_c)T_b} \right)^2 \right]$$

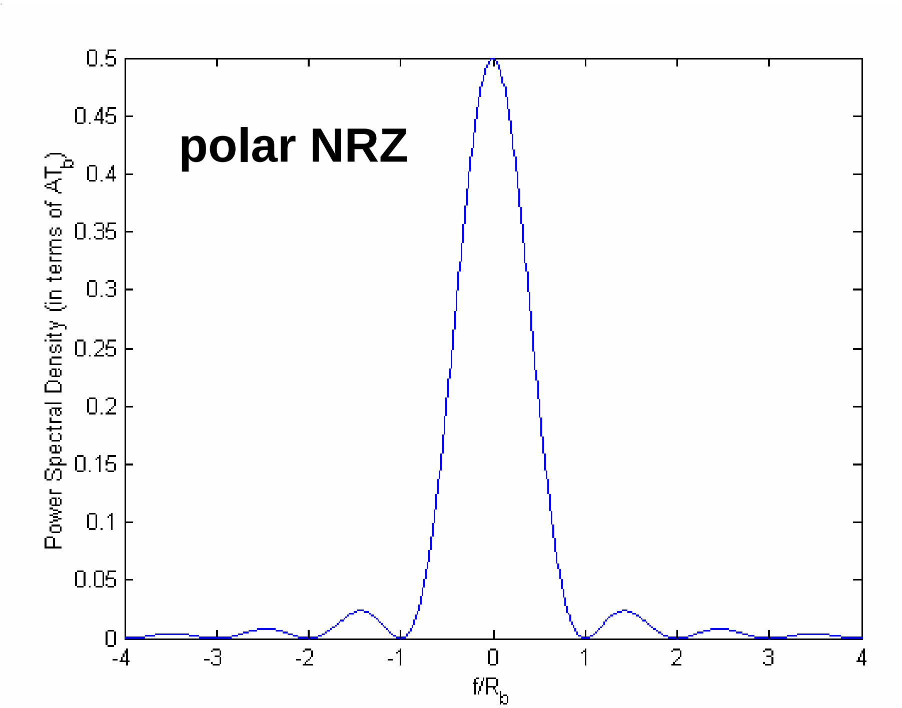
$$= \frac{A^2 T_b}{4} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{-\sin(\pi(f + f_c)T_b)}{\pi(-f - f_c)T_b} \right)^2 \right]$$

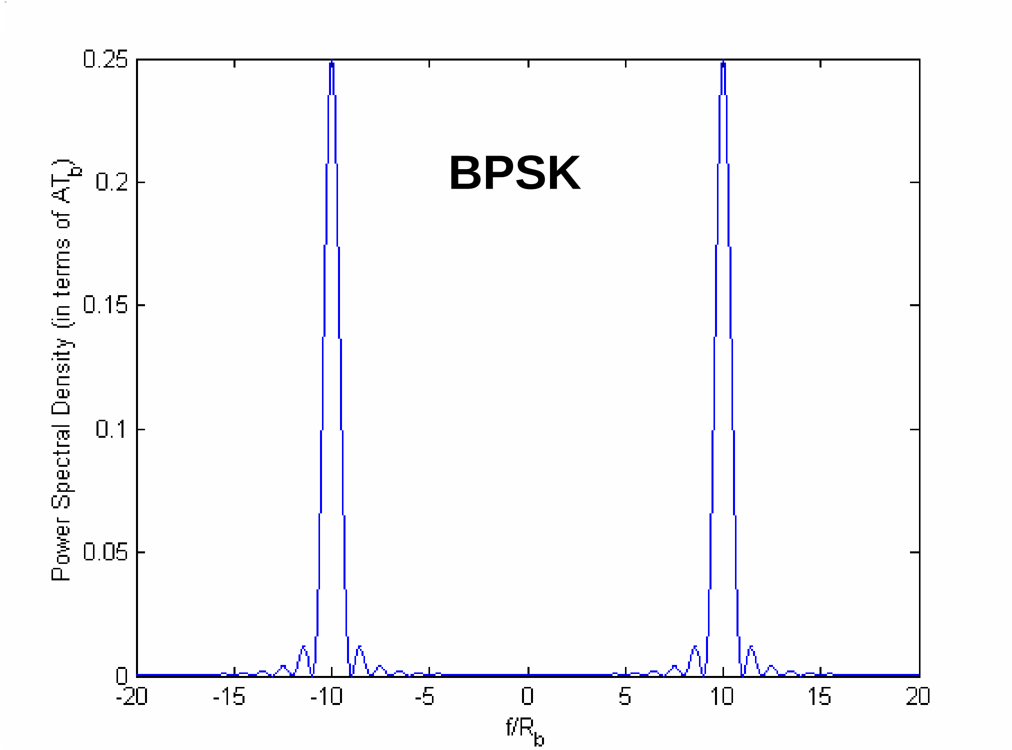
$$\frac{A^2 T_b}{4} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{\sin(\pi(f + f_c)T_b)}{\pi(f + f_c)T_b} \right)^2 \right]$$

$$A = \sqrt{\frac{2E_b}{T_b}}$$

Power Spectral Density of BPSK

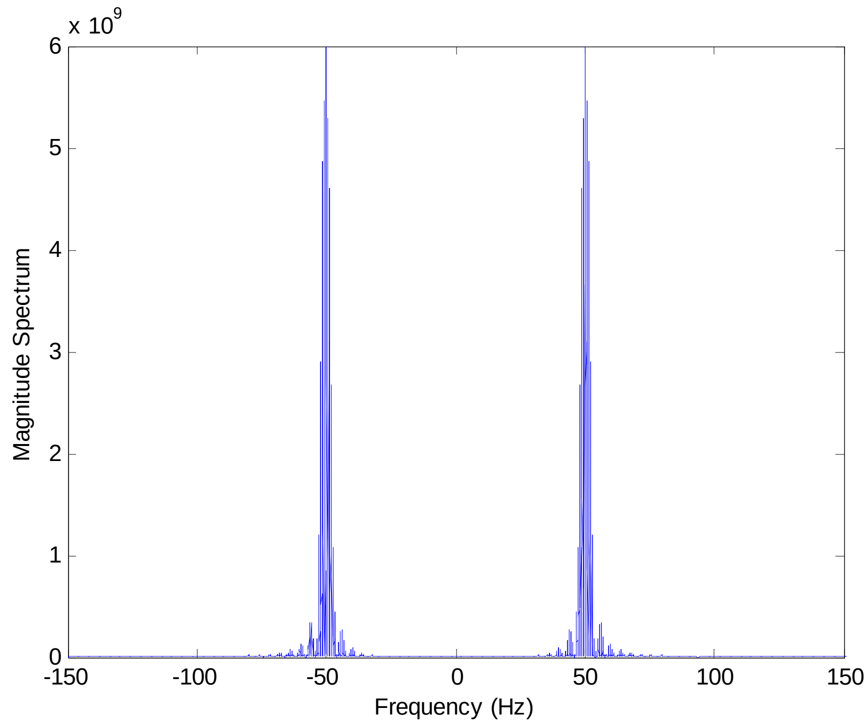
- Complex envelope of BPSK is polar NRZ

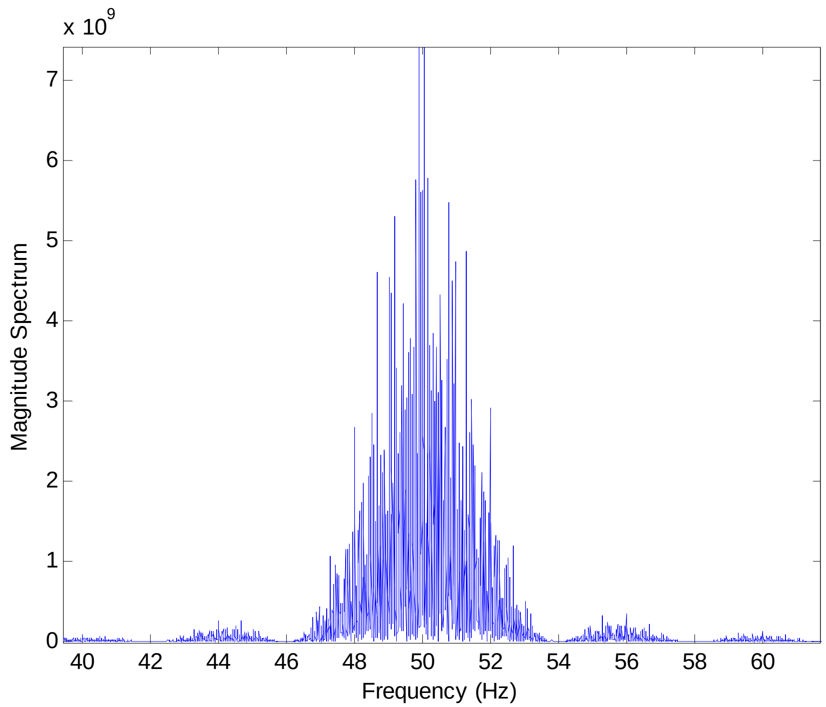




Example Signal (random data)

- f_c = 50Hz, $R_s = R_b$ = 4bps

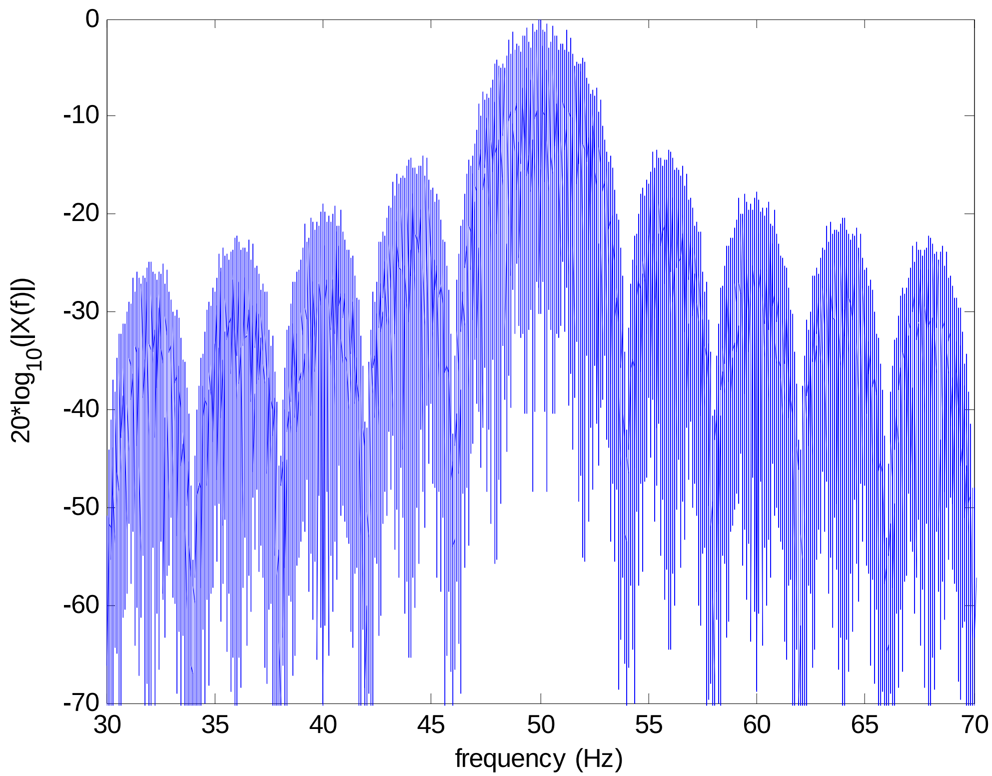




What is the difference between this plot and the PSD on the previous page??

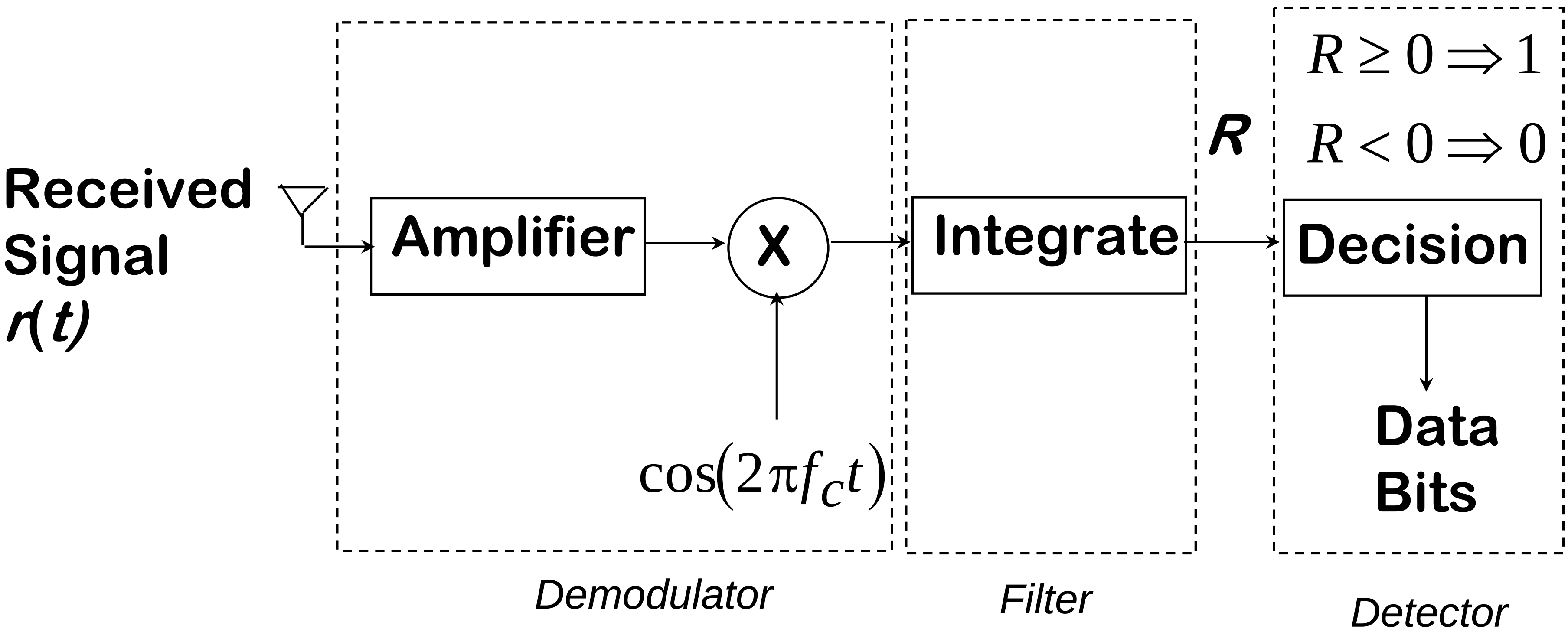
Example – cont.

- Log plot



Spectrum identical to what we expect with a NRZ line code with square pulses. (Notice first sidelobes are ~13dB down)

Receiver for BPSK



Note: We will discuss optimal receiver design shortly. This implementation is optimal only for *square (rectangular) pulses*.

Receiver for BPSK

- Multiply by a sinusoid
  - phase must be aligned with incoming signal (coherent)
- Integrate over duration of one bit
  - Integration is a form of low pass filtering (what is the optimal filter?)
- Compare result with a threshold

- Decision Variable *R* :

$$R = \frac{1}{T_s}\int_0^{T_s} v(t)\cos(2\pi f_c t)\,dt = \frac{1}{T_s}\int_0^{T_s} x(t)\cos(2\pi f_c t)\cos(2\pi f_c t)\,dt$$

$$= \frac{1}{T_s}\int_0^{T_s} x(t)\frac{1}{2}\left[\cos(0)+\cos(4\pi f_c t)\right]dt \approx \sqrt{\frac{E_b}{2T_b}}d_k$$

Decision rule: $R \geq 0 \Rightarrow 1, R < 0 \Rightarrow 0$

Why is this rule optimal?

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Quadrature Phase Shift Keying

- Note:
  - The receiver is coherent – i.e., we must track the phase of the incoming carrier
  - The in-phase and quadrature channels are orthogonal provided that we demodulate the signal coherently
  - In BPSK we haven't transmitted anything in the Q channel
- Thus, we have an opportunity to transmit more data in the Q channel – this is termed Quadrature Phase Shift Keying (QPSK)

Analogy to AM

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- BASK is simply AM transmission with a unipolar NRZ line code as the message and a sensitivity factor of $k_a = 1$
  - Like AM, non-coherent detection is possible
- BPSK is Double Sideband Suppressed Carrier AM with a polar NRZ line code as the message
  - Like DSBSC a coherent receiver is required
- Like AM vs DSBSC, BPSK is more energy efficient than BASK due to the fact that it does not transmit the unmodulated carrier
  - This will become more clear when we examine the BER performance

Pulse Shaping

- Recall that the Power Spectral Density is

$$P_v(f) = \frac{A^2 T_b}{4}\left[\left(\frac{\sin\left(\pi(f - f_c)T_b\right)}{\pi(f - f_c)T_b}\right)^2 + \left(\frac{\sin\left(\pi(f + f_c)T_b\right)}{\pi(f + f_c)T_b}\right)^2\right]$$

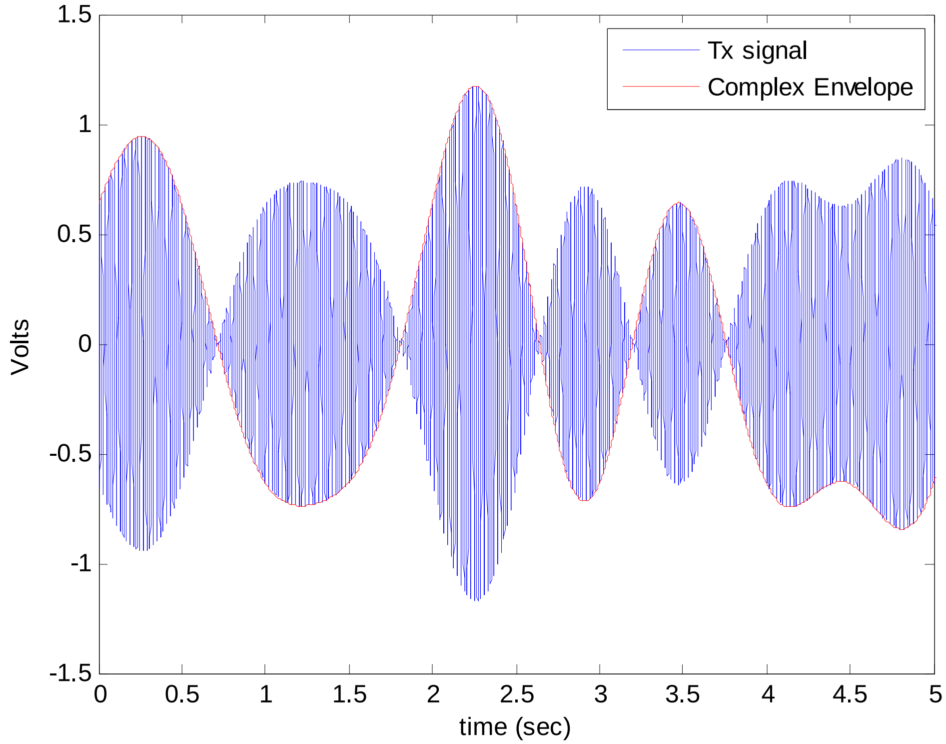
- Which is dominated by the pulse shape

$$P(f) = \left(\frac{\sin\left(\pi f T_b\right)}{\pi f T_b}\right)$$

- Thus, if we replace the pulse shape in the line code, we can control the PSD of BPSK

Pulse Shaping – Example

- Consider sinc pulses with a truncation length of 20 symbols
- Note that the signal is no longer constant amplitude (i.e., the information is still in the phase, but the amplitude variation must be considered in the receiver design)



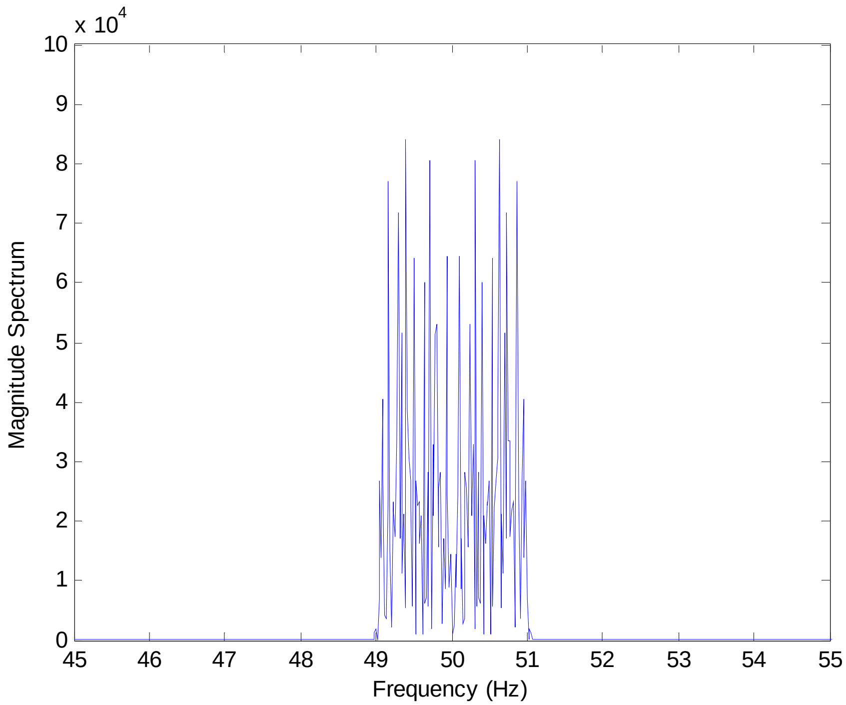
$$v(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}}\left(\sum_k d_k p(t-kT_s)\right)}_{x(t)}\cos\left(\omega_c t\right)$$

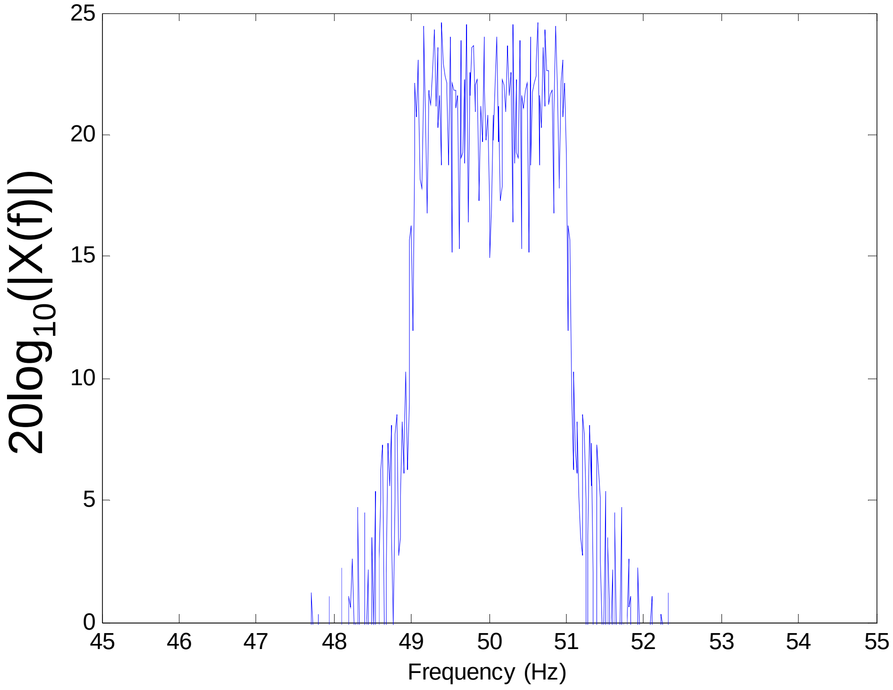
$$p(t) = \mathrm{sinc}\left(\frac{t - T_s/2}{T_s}\right)$$

$$d_k = 2b_k - 1$$

Pulse Shaping – cont.

- Example – f_c = 50Hz, $R_s = R_b$ = 2bps





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In class drill

- If time

Summary

- In this lecture we have expanded our knowledge of sinusoidal modulation techniques by examining binary phase shift keying (BPSK)
- BPSK is analogous to DSBSC AM and is more energy efficient than BASK which we studied last time
- However, the receiver for BPSK must be coherent, unlike BASK
- Since the modulation is *linear* the PSD is directly controlled by the baseband line code used
- Pulse shaping can be employed to modify the spectrum