ECE4634
Digital Communications
Fall 2007

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Lecture #15: Bandpass Modulation – BASK

Analog and Digital Communications

Motivation

- Baseband signals *w*(*t*) may be transformed into bandpass signals through multiplication by a sinusoid:

$$w(t)\cos(\omega_c t+\theta) \Leftrightarrow \frac{1}{2}\left[e^{j\theta}W(f-f_c)+e^{-j\theta}W(f+f_c)\right]$$

- Most transmitted signals are modulated onto a carrier because
  - Modulated signals propagate well through the atmosphere
  - Modulation allows many signals with different carrier frequencies to share the spectrum
- We investigate a very simple form of sinusoidal modulation that directly uses the Fourier Modulation property: Binary Amplitude Shift Keying (BASK)
- What to read – Section 7.2

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Lecture Objective

- The objective of today's lecture is to describe the most simple form of digital bandpass modulation, Binary Amplitude Shift Keying (BASK)
- Specifically we will desribe
  - Bandpass representations for BASK
  - Transmitter and Receiver design
  - Spectral characteristics

Three Ways of Representing Bandpass Signals

- We will use the following analytical tools to handle bandpass signals
- Magnitude and Phase

$$v(t) = R(t)\cos\left[\omega_c t + \theta(t)\right]$$

- In Phase and Quadrature

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- Complex Envelope

$$v(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$$

Bandpass Modulation - ASK

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- Amplitude Shift Keying
- Basic Idea:
  - Send a sinusoid for 1, nothing for a 0
  - Let T_s be the duration of one data symbol (also bit)
  - Then we transmit the signal $v(t)$:

$$b_k = 1 \Rightarrow v(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)\Big|_0^{T_s}$$

$$b_k = 0 \Rightarrow v(t) = 0\Big|_0^{T_s}$$

$$\boxed{v(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}}\left(\sum_k b_k rect\left(\frac{t-kT_s}{T_s}\right)\right)}_{x(t)}\cos(\omega_c t)}$$

  - Analogous to Unipolar NRZ signaling
  - Also called On-Off Keying (OOK)

For binary modulation

$$T_b = T_s$$

ASK - Magnitude and Phase Representation

- $v(t) = R(t)\cos\left[\omega_c t + \theta(t)\right]$ where
  - $$1 \Rightarrow R(t) = \sqrt{\frac{2E_b}{T_b}}\Bigg|_0^{T_s}$$
    $$0 \Rightarrow R(t) = 0\Big|_0^{T_s}$$
    $$\theta(t) = 0\Big|_0^{T_s}$$
    $$\Rightarrow \quad R(t) = x(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}}\sum_k b_k rect\left(\frac{t-kT_s}{T_s}\right)}_{\text{unipolar NRZ linecode}}$$
  - All of the information is in the amplitude (i.e., no information in the phase)
  - This modulation is nice since it can be detected non-coherently (i.e., we do not need a phase reference)

ASK - I & Q Representation

- $v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$ where
  - $y(t) = R(t)\sin(\theta(t))$

    $= 0\Big|_0^{T_s}$
    - No Q component
  - $1 \Rightarrow x(t) = R(t)\cos(\theta(t)) = \sqrt{\dfrac{2E_b}{T_b}}\Bigg|_0^{T_s}$ $\Rightarrow$ $x(t) = \underbrace{\sqrt{\dfrac{2E_b}{T_b}} \sum_k b_k \, rect\left(\dfrac{t - kT_s}{T_s}\right)}_{\text{unipolar NRZ linecode}}$

    $0 \Rightarrow x(t) = R(t)\cos(\theta(t)) = 0\Big|_0^{T_s}$
    - I component is just a unipolar NRZ signal
  - If a second ASK signal is transmitted as the Q-component, then we can transmit two bits
  - ASK can also be used with noncoherent reception

ASK - Complex Envelope Representation

- $$v(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$$

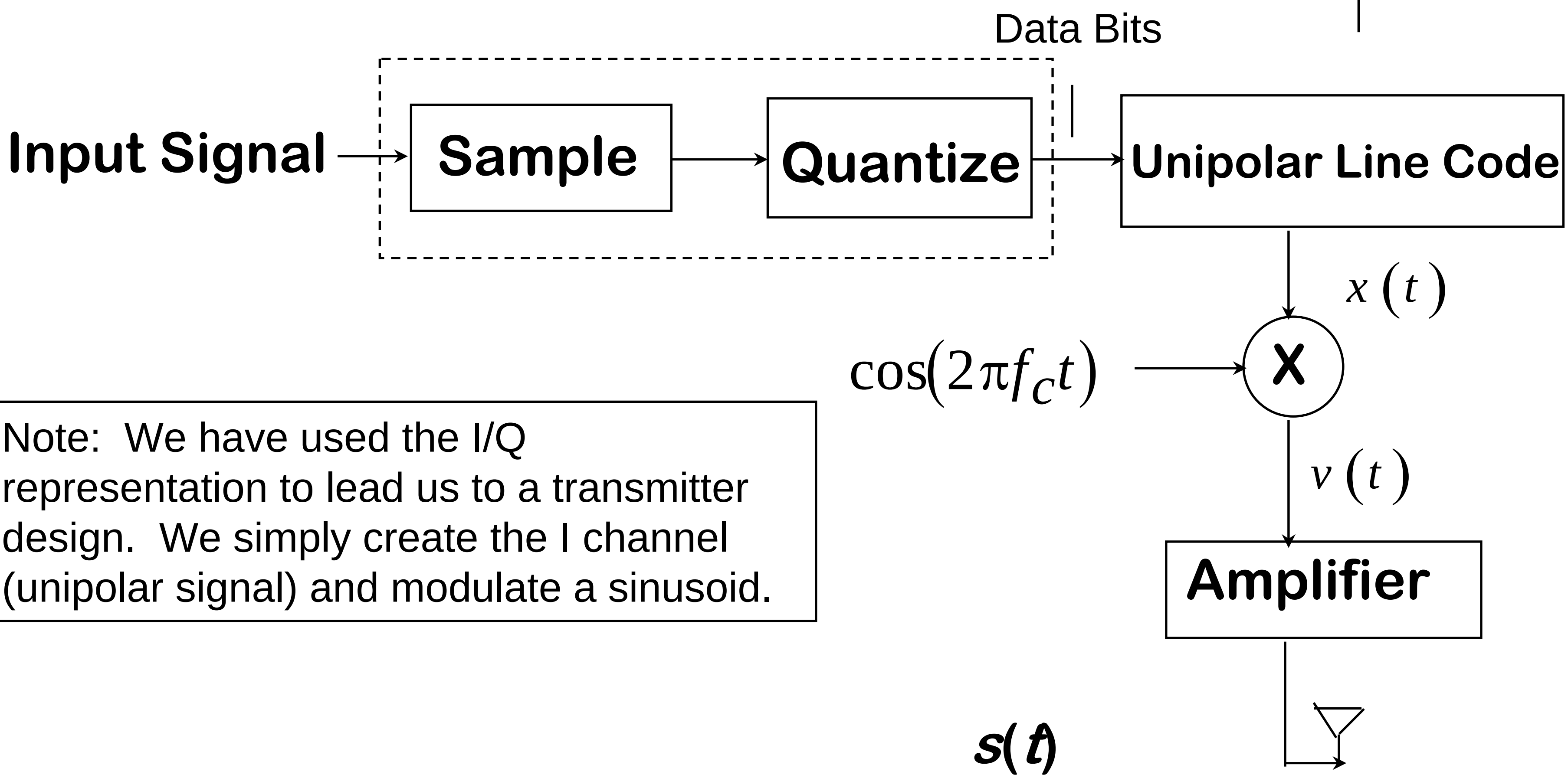
$$1 \Rightarrow g(t) = \sqrt{\frac{2E_b}{T_b}}\Bigg|_0^{T_s}$$

$$0 \Rightarrow g(t) = 0\Big|_0^{T_s}$$

$$g(t) = x(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}}\sum_k b_k rect\left(\frac{t-kT_s}{T_s}\right)}_{\text{unipolar NRZ linecode}}$$

- Complex envelope is entirely real
- Complex envelope is equivalent to unipolar NRZ signaling
- BPSK is more energy efficient
  - Note ASK has energy in impulse at f_c

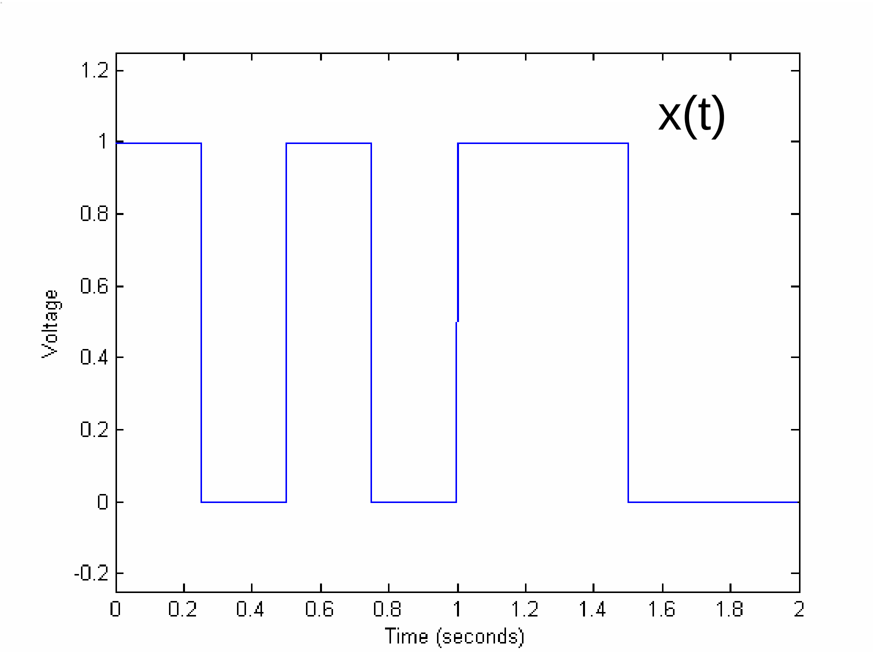
Transmitter for ASK



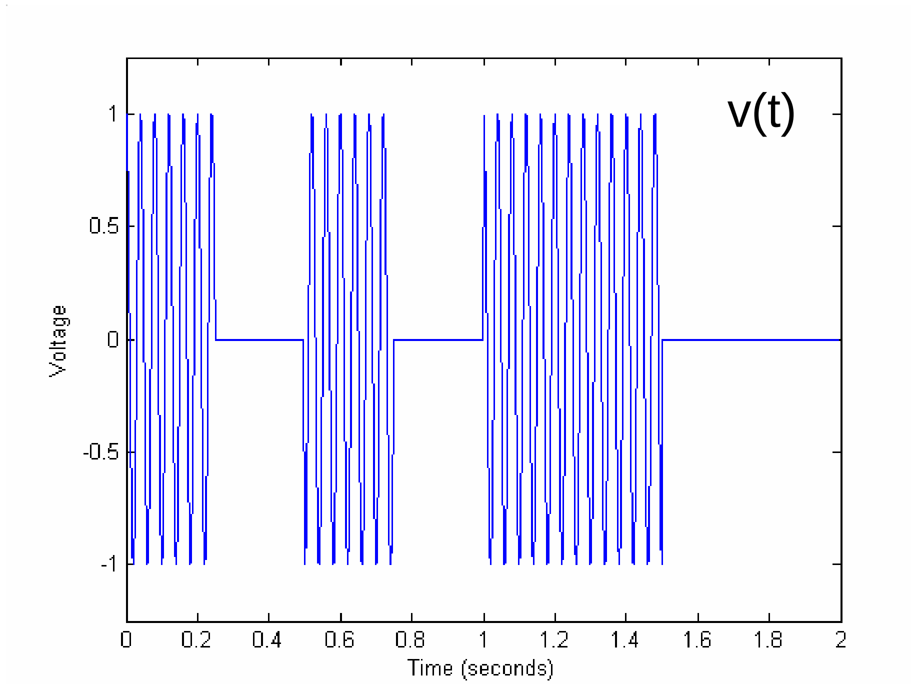
Note: We have used the I/Q representation to lead us to a transmitter design. We simply create the I channel (unipolar signal) and modulate a sinusoid.

Example

- b = [1, 0, 1, 0, 1, 1, 0, 0]



First step is to create a baseband pulse modulated signal



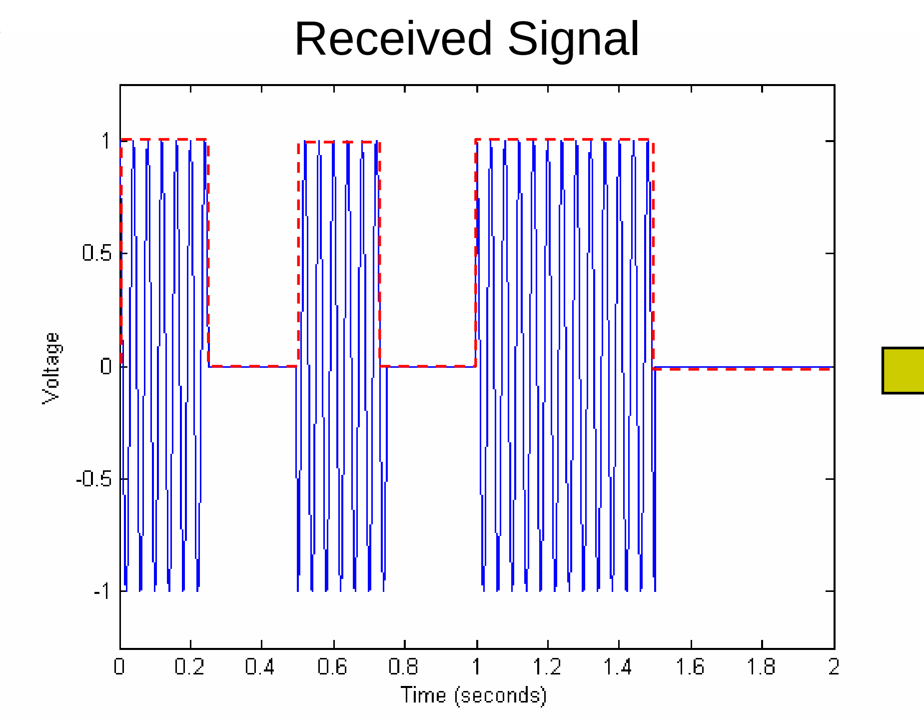
Second step is to create a the bandpass signal by linearly modulating the carrier by the baseband signal

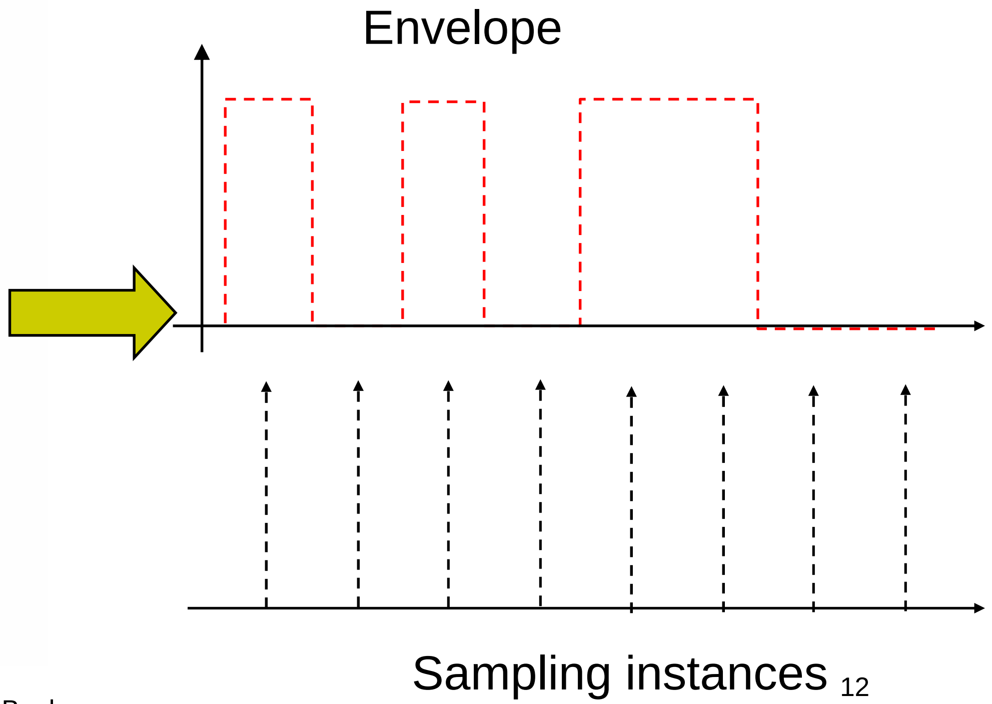
Receivers for BASK

- A digital receiver can be broken down into to two general functions
  - Converting the bandpass signal into a baseband signal (often termed *demodulation*)
  - Determining the data bits from the baseband signal (often termed *detection*)
- Receiver structures for BASK can be found by recognizing the similarity between BASK and AM
  - Envelope detection
  - Product detector
  - Non-coherent product detector
- Sampling vs. Filtering
  - Determining the data bits can be done by simply sampling the bandpass signal and making a decision
  - However in the presence of noise better performance can be obtained by filtering or integrating the signal first and then sampling

Envelope Detector

- The simplest receiver is an envelope detector





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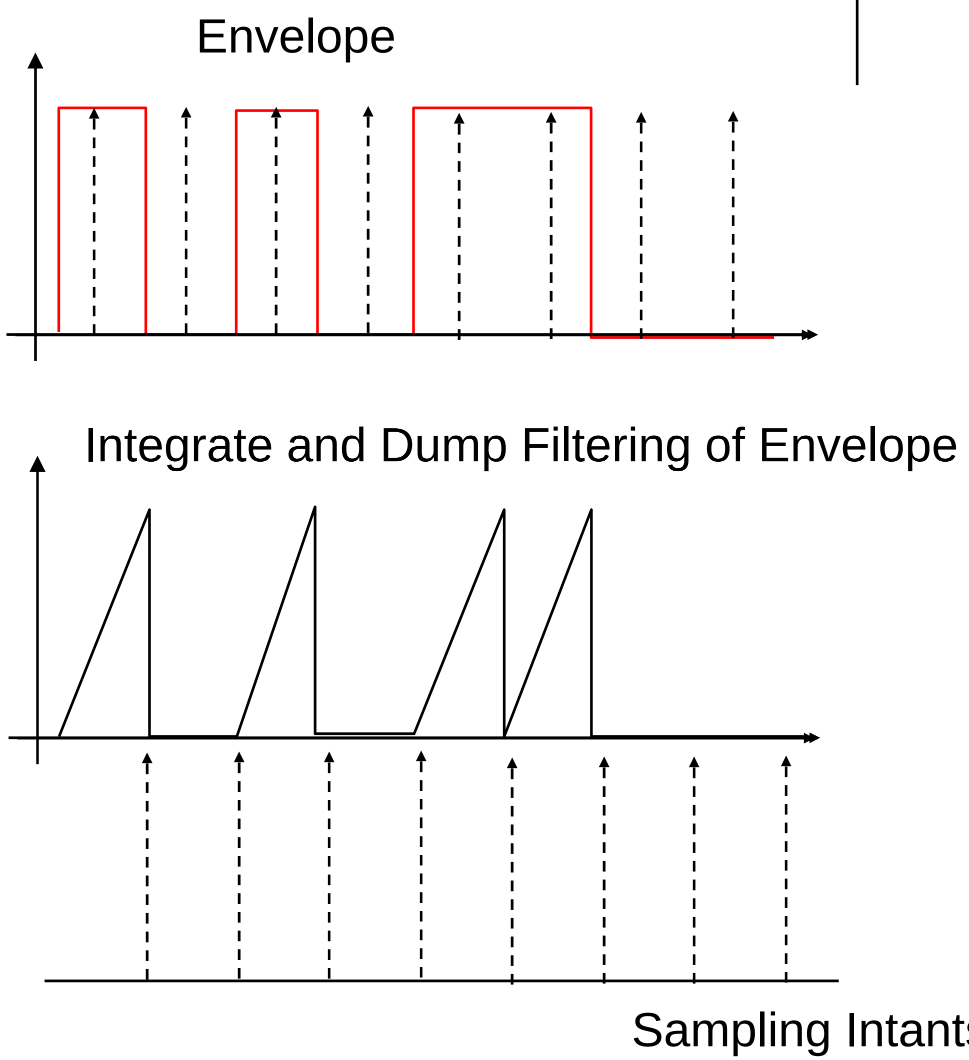
Sampling vs. Filtering

- Integration improves SNR (as we will see later in the course) but may require more accurate timing



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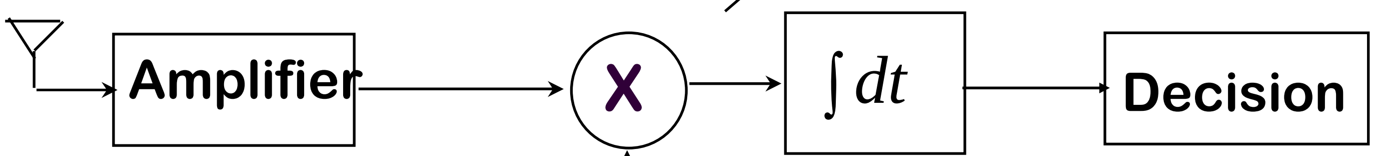
Coherent Product Detector

$$r(t)=\sqrt{\frac{2E_b}{T_b}}\left(\sum_{n=-\infty}^{\infty} d_n p\left(t-nT_s\right)\right)\cos\left(\omega_c t\right)$$

Received Signal *r(t)*

$$d_n=\begin{cases}1 & b=1\\ 0 & b=0\end{cases}$$

Mixer shifts the spectrum down to DC and up to $2f_c$



$$Z \geq \sqrt{\frac{E_b}{8T_b}} \Rightarrow 1$$

$$Z < \sqrt{\frac{E_b}{8T_b}} \Rightarrow 0$$

$$\cos\left(2\pi f_c t\right)$$

The optimal decision threshold depends on the noise which we will examine in a few weeks.

Note that we ASK receiver does not necessarily need coherent phase reference. However, using a coherent phase reference improves performance as we will see.

Integration removes double frequency term and integrates baseband signal providing SNR benefit.

Receiver for ASK

- Decision Variable *Z (assume that b=1)*:

$$
\begin{aligned}
Z &= \frac{1}{T_s}\int_0^{T_s} r(t)\cos(2\pi f_c t)\,dt \\
&= \frac{1}{T_s}\int_0^{T_s} x(t)\cos(2\pi f_c t)\cos(2\pi f_c t)\,dt \\
&= \frac{1}{T_s}\int_0^{T_s} x(t)\frac{1}{2}\left[1+\cancel{\cos(4\pi f_c t)}\right]dt \\
&\approx \sqrt{\frac{E_b}{2T_b}}
\end{aligned}
$$

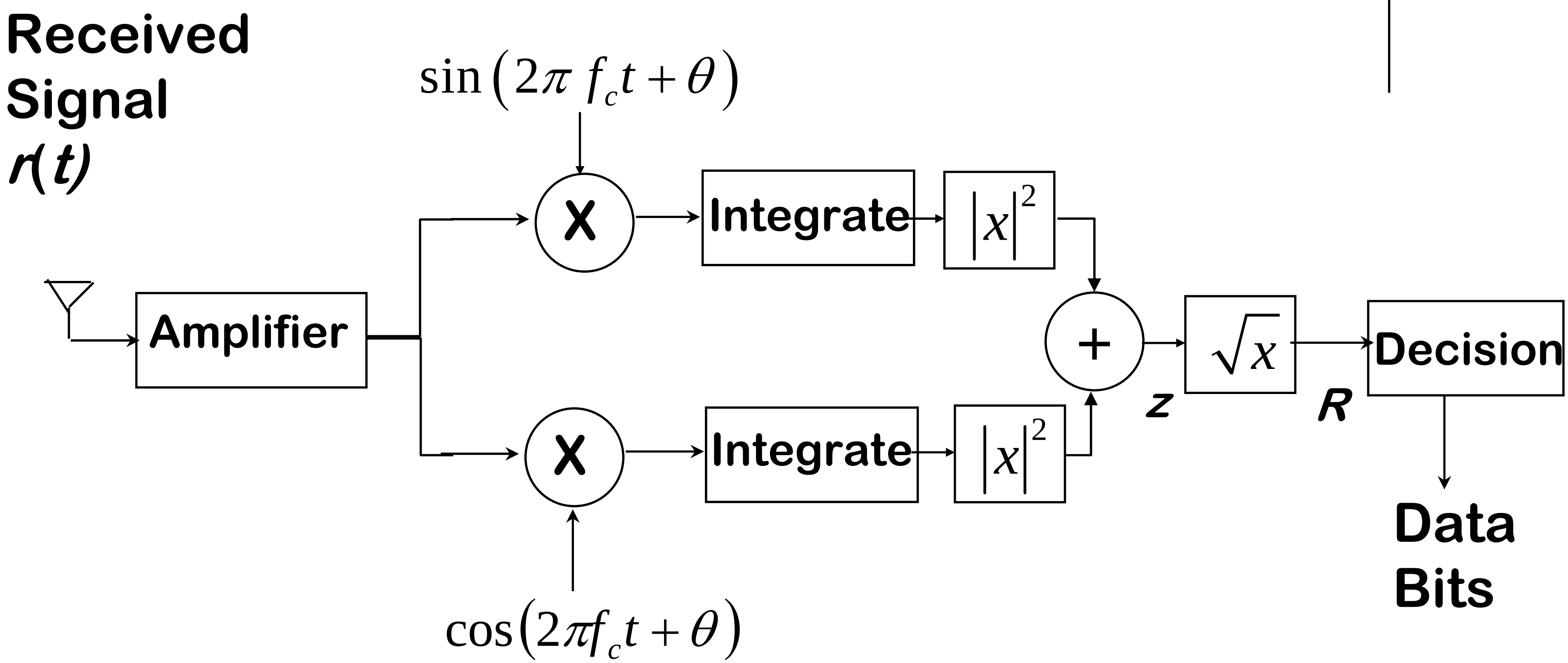
Integrates to ~ 0

$$\boxed{x(t)=\sqrt{\frac{2E_b}{T_b}}\,p(t-nT_s)\cos(\omega_c t)}$$

- Decision Variable *Z (assume that b=0)*:

$$
\begin{aligned}
Z &= \frac{1}{T_s}\int_0^{T_s} r(t)\cos(2\pi f_c t)\,dt \\
&= \frac{1}{T_s}\int_0^{T_s} x(t)\cos(2\pi f_c t)\cos(2\pi f_c t)\,dt \\
&\approx 0
\end{aligned}
$$

Non-coherent Product Detector



Note that this ASK receiver does not need a coherent phase reference. In other words, the value of θ is irrelevant.

$$R \geq \sqrt{\frac{E_b}{8T_b}} \Rightarrow 1$$

$$R < \sqrt{\frac{E_b}{8T_b}} \Rightarrow 0$$

Receiver for ASK

- Decision Variable *R* (ignoring noise for now) :

$$Z = \left(\frac{1}{T_s} \int_0^{T_s} r(t)\cos(2\pi f_c t + \theta)\, dt \right)^2 + \left(\frac{1}{T_s} \int_0^{T_s} r(t)\sin(2\pi f_c t + \theta)\, dt \right)^2$$

$$= \left(\frac{1}{T_s} \int_0^{T_s} x(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta)\, dt \right)^2 + \left(\frac{1}{T_s} \int_0^{T_s} x(t)\cos(2\pi f_c t)\sin(2\pi f_c t + \theta)\, dt \right)^2$$

$$= \left(\frac{1}{T_s} \int_0^{T_s} x(t)\frac{1}{2}\left[\cos(\theta) + \cos(4\pi f_c t)\right] dt \right)^2 + \left(\frac{1}{T_s} \int_0^{T_s} x(t)\frac{1}{2}\left[\sin(\theta) + \sin(4\pi f_c t)\right] dt \right)^2$$

$$\approx \left(\sqrt{\frac{E_b}{2T_b}} \right)^2 \left(\cos^2(\theta) + \sin^2(\theta)\right)$$

$$R = \sqrt{z}$$

$$= \sqrt{\frac{E_b}{2T_b}}$$

$$R \geq \sqrt{\frac{E_b}{8T_b}} \Rightarrow 1,\ R < \sqrt{\frac{E_b}{8T_b}} \Rightarrow 0$$

$$\boxed{x(t) = \sqrt{\frac{2E_b}{T_b}}\, p(t - nT_s)\cos(\omega_c t)}$$

Note: In the absence of noise, we get the same result for coherent or non-coherent reception. However, when noise is added to the equation, we will see a difference.

Power Spectral Density

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- The power spectral density can be determined by examining the complex baseband representation

$$P_v(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c)\right]$$

- However, we know that the complex baseband is simply a unipolar NRZ line code. We derived the PSD for this earlier in the course. Thus we can directly borrow the result.

Power Spectral Density of ASK

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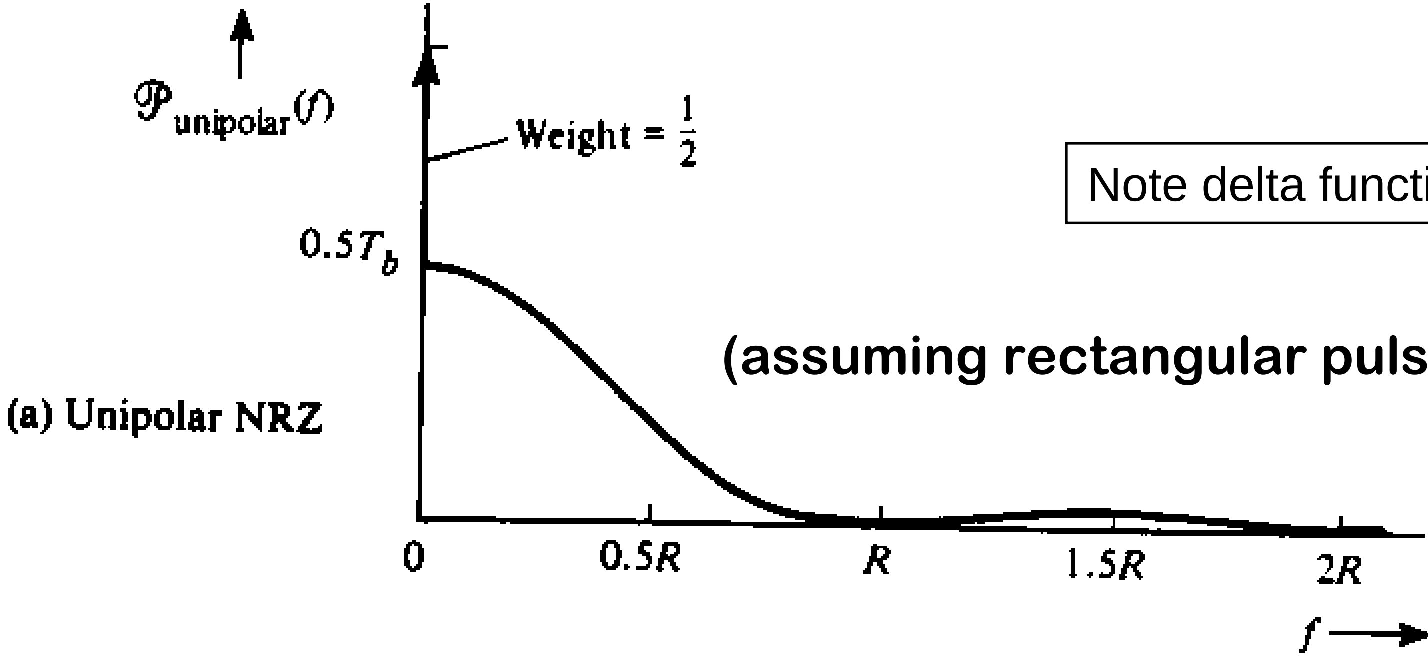
- Complex envelope *g*(*t*) of ASK is unipolar NRZ



Note delta function at f_c

(assuming rectangular pulse shape)

- Can relate power spectral density of ASK:

$$P_v(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c)\right]$$

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Power Spectral Density for ASK

We have shown for unipolar NRZ signaling:

$$P_g(f) = \underbrace{\frac{A^2 T_b}{4}\left(\frac{\sin(\pi f T_b)}{\pi f T_b}\right)^2}_{\text{due to pulse shape}} \left[1 + \underbrace{\frac{1}{T_s}\delta(f)}_{\text{due to DC component}} \right]$$

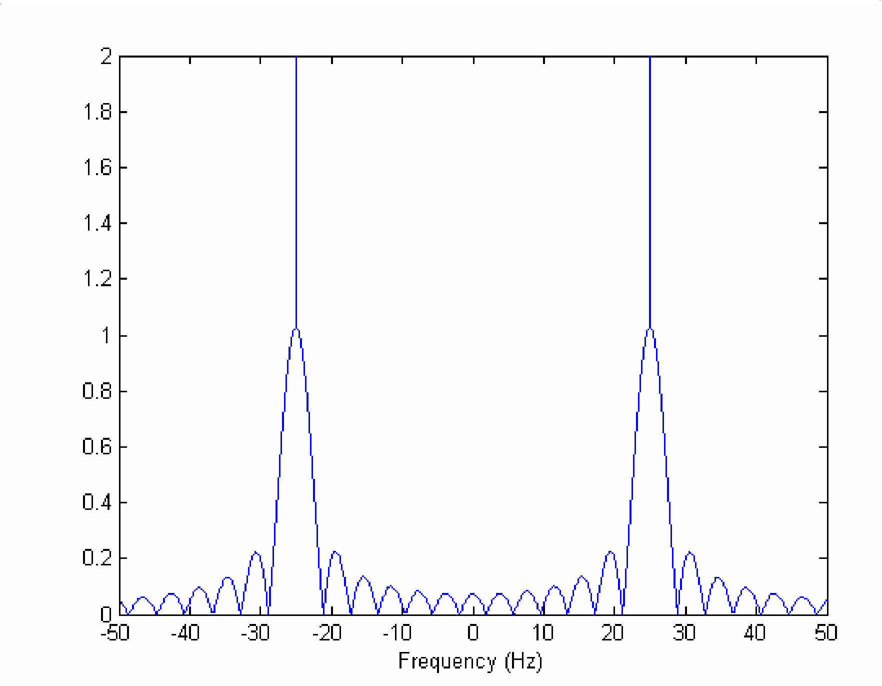
Thus,

$$\begin{aligned}
P_v(f) &= \frac{1}{4}\left[P_g(f-f_c) + P_g(-f-f_c)\right] \\
&= \frac{A^2 T_b}{16}\left[\left(\frac{\sin(\pi(f-f_c)T_b)}{\pi(f-f_c)T_b}\right)^2 + \left(\frac{\sin(\pi(-f-f_c)T_b)}{\pi(-f-f_c)T_b}\right)^2 + \frac{1}{T_b}\delta(f-f_c) + \frac{1}{T_b}\delta(-f-f_c)\right] \\
&= \frac{A^2 T_b}{16}\left[\left(\frac{\sin(\pi(f-f_c)T_b)}{\pi(f-f_c)T_b}\right)^2 + \left(\frac{-\sin(\pi(f+f_c)T_b)}{\pi(-f-f_c)T_b}\right)^2 + \frac{1}{T_b}\delta(f-f_c) + \frac{1}{T_b}\delta(f+f_c)\right] \\
&= \frac{A^2 T_b}{16}\left[\left(\frac{\sin(\pi(f-f_c)T_b)}{\pi(f-f_c)T_b}\right)^2 + \left(\frac{\sin(\pi(f+f_c)T_b)}{\pi(f+f_c)T_b}\right)^2 + \frac{1}{T_b}\delta(f-f_c) + \frac{1}{T_b}\delta(f+f_c)\right]
\end{aligned}$$

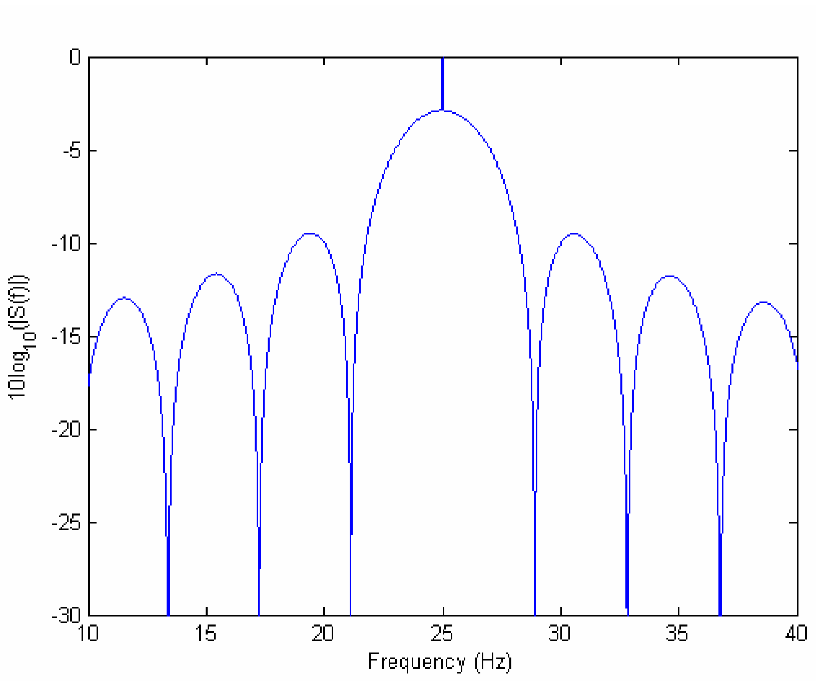
$$\boxed{A = \sqrt{\frac{2E_b}{T_b}}}$$

Example - Theory

- f_c = 25Hz, R_b = 4bps



Linear Plot



Log Plot
(zoomed to 25Hz)

Example – Measured Signal

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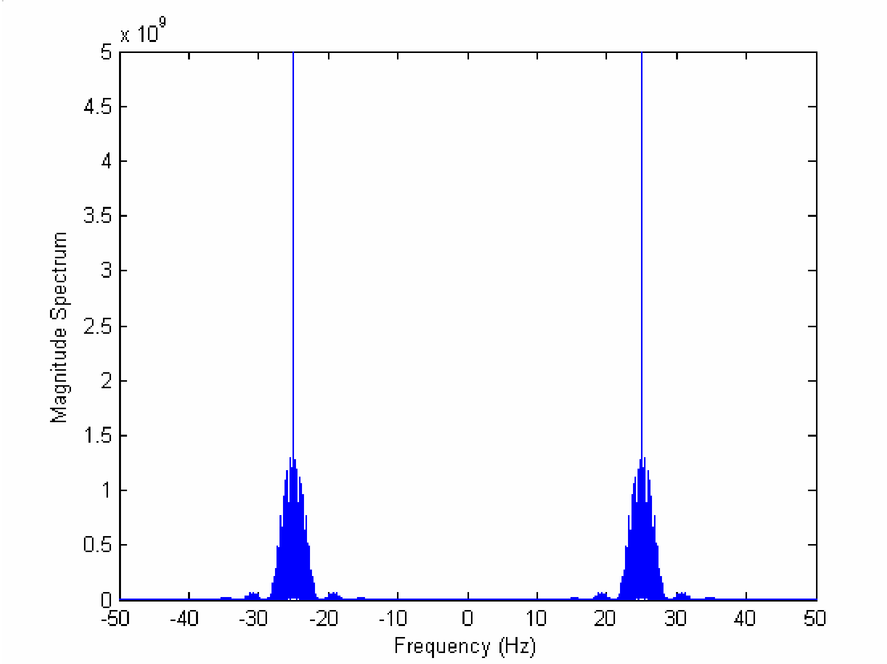
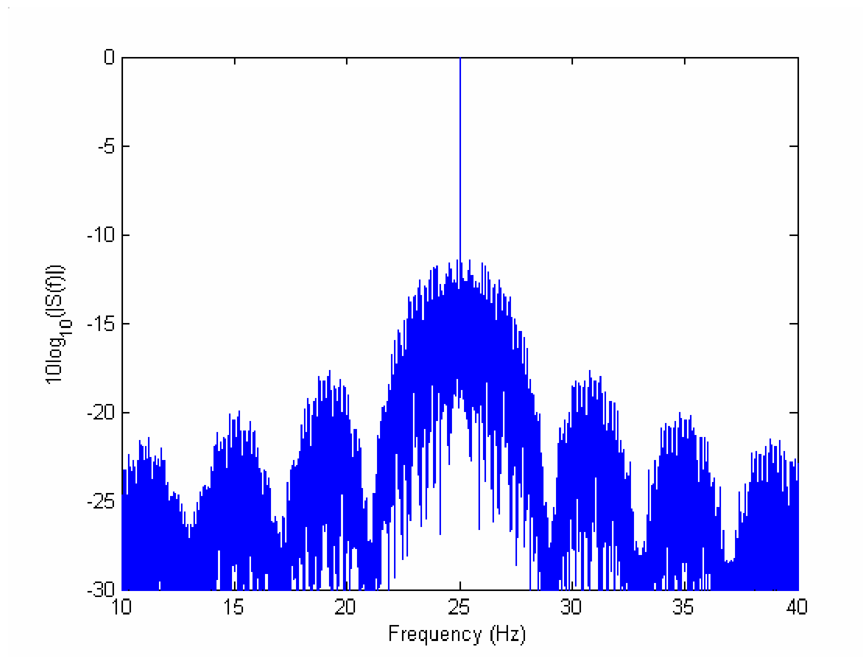
- f_c = 25Hz, R_b = 4bps



Linear Plot



Log Plot
(zoomed to 25Hz)

Pulse Shaping

- Recall that the power spectral density of BASK is dominated by the square pulse from the NRZ line code:

$$P_v(f) = \frac{A^2 T_b}{16}\left[\left(\frac{\sin\left(\pi\left(f-f_c\right)T_b\right)}{\pi\left(f-f_c\right)T_b}\right)^2 + \left(\frac{\sin\left(\pi\left(f+f_c\right)T_b\right)}{\pi\left(f+f_c\right)T_b}\right)^2 + \frac{1}{T_b}\delta(f-f_c) + \frac{1}{T_b}\delta(f+f_c)\right]$$

- Thus, we can improve the spectral characteristics of BASK by changing the pulse shape *p(t)* associated with the NRZ line code:

$$x(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}}\sum_k b_k p\left(\frac{t-kT_s}{T_s}\right)}_{\text{unipolar NRZ linecode}}$$

Pulse Shape

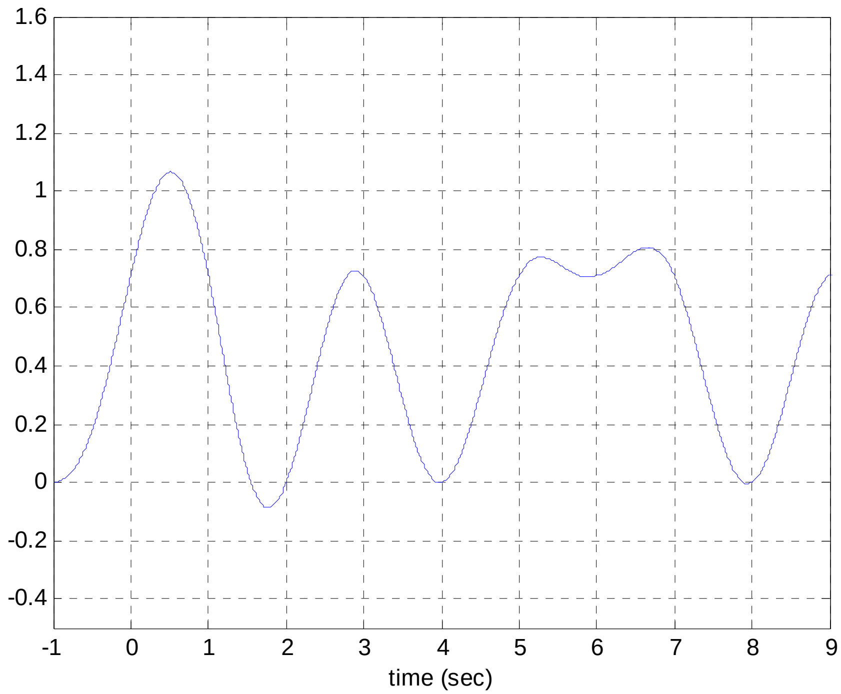
$$p(t) \rightleftharpoons P(f)$$

- And the new PSD is

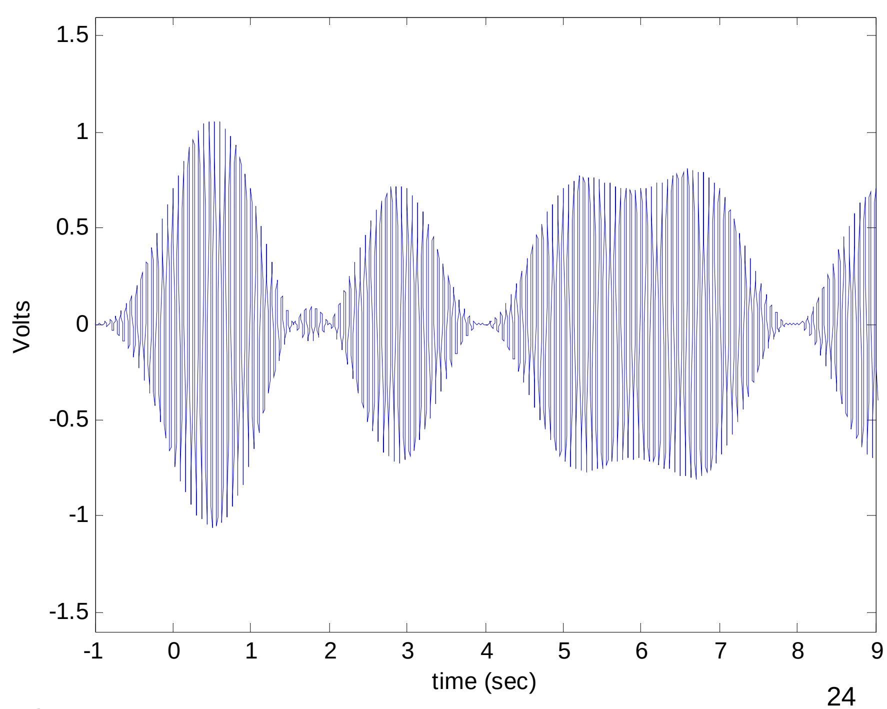
$$P_v(f) = \frac{A^2 T_b}{16}\left[P^2(f) + P^2(f) + \frac{1}{T_b}\delta(f-f_c) + \frac{1}{T_b}\delta(f+f_c)\right]$$

Example – Sinc Functions

- Truncation length – 20 symbols
- Carrier frequency – 15Hz
- Data rate – 1sps



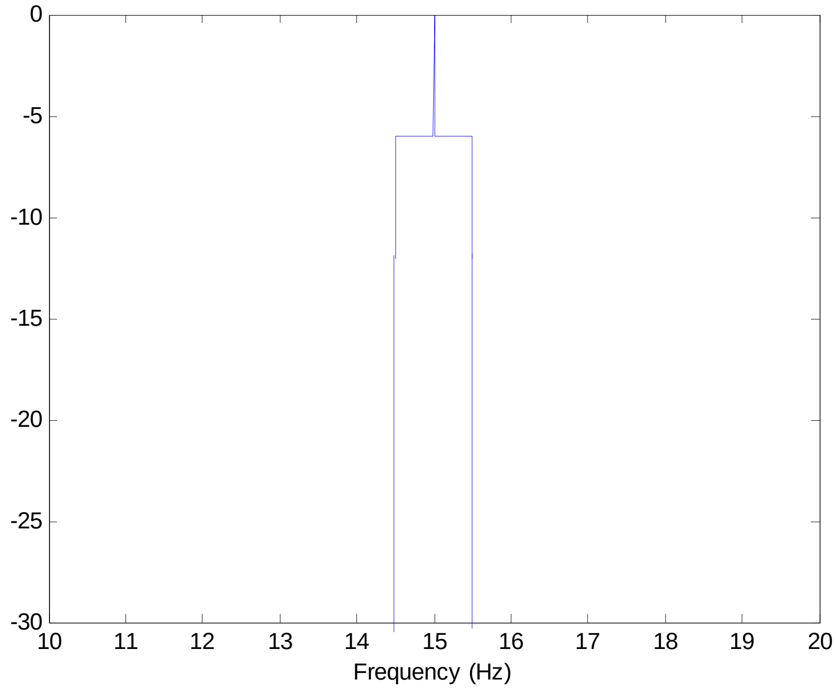
baseband signal



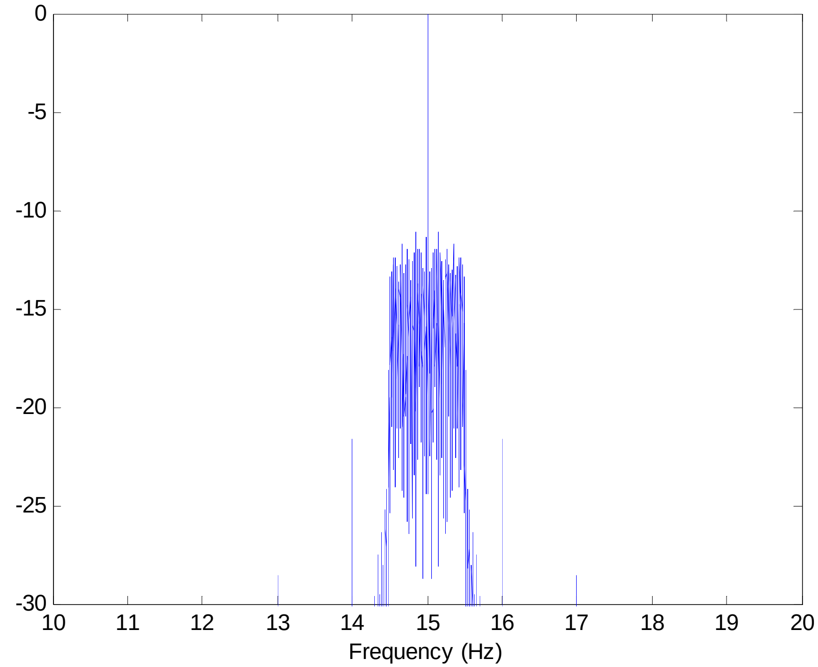
BASK signal

Example – PSD.

Theory



Measured



In class Drill

- If time permits…

Summary

- We have begun our investigation of binary digital bandpass modulation schemes with perhaps the simplest scheme Binary Amplitude Shift Keying (BASK)
- BASK can be viewed as a simple binary NRZ line code linearly modulating a sinusoid
- The transmitter and receiver are similar to those for analog Amplitude Modulation
- The spectral properties of BASK can be easily modified through changing the pulse shape in the NRZ line code