ECE4634
Digital Communications
Fall 2007

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Lecture #18: Multilevel or *M*-ary Modulation

PSK and ASK

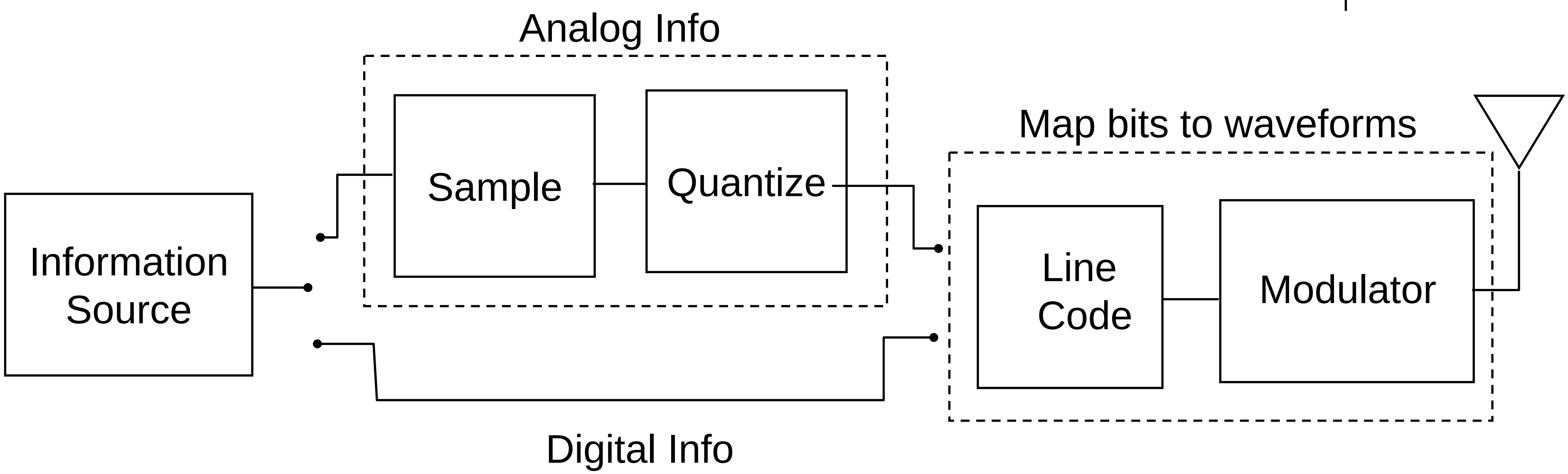
Analog and Digital Communications

Overview

Analog and Digital Communications

- Today we expand our discussion of digital modulation from binary to *M*-ary modulation
- *M*-ary modulation *can (but not necessarily)* reduce the bandwidth requirements of a modulation scheme by reducing the symbol rate
- We will specifically examine *M*-ary PSK and ASK modulation and will examine M-FSK next class
- What to read – Section 7.5, 7.7

Communication System (Transmitter)



- We've talked extensively about converting analog information into digital waveforms (PCM)
- Modulation is using the digital information to *modulate* a sinusoidal carrier.
- Modulation can either be *binary* or *M-ary*.

Multilevel Modulation

In *binary* modulation each bit corresponds to one symbol.

$$b = \{0,1,1,1,0,0,1,0,1,0,0,1,0\ldots,\}$$

$$s = \{s_0, s_1, s_1, s_1, s_0, s_0, s_1, s_0, s_1, \ldots,\}$$

2 Possible symbols $\{s_0, s_1\}$
e.g. $\{\cos(\omega_c t)$, $-\cos(\omega_c t)\}$

In *multi-level* modulation each $log_2(M)$ bits corresponds to one symbol. Example: $M = 8$, $log_2(M) = 3$

$$b = \left\{\underbrace{0,1,1}, \underbrace{1,0,0}, \underbrace{1,0,1}, \underbrace{0,0,1}, \underbrace{0,0,0}, \ldots, \right\}$$

$$s = \{s_3, s_4, s_5, s_1, s_0, \ldots,\}$$

8 Possible symbols
$\{s_0, s_1\ s_2, s_3, s_4, s_5\ s_6, s_7\}$

Multilevel Modulation

- Let $m(t)$ be the information message
- Binary Signaling: $m(t) \in \{0,1\}$
- M-ary Signaling: $m(t) \in \{0,1,\ldots,M-1\}$
  - message signal takes one of M values

$$M = 2^l$$

  - l = # of bits/symbol
- Examples:
  - M different phases (M-ary PSK)
  - M different amplitudes (M-ary ASK)
  - Combinations: Quadrature Amplitude Modulation (QAM)

Advantage of Multilevel Signaling: Saves on Bandwidth *(usually)*

- Let
  - T_b be the duration of one bit
  - T_s be the duration of one symbol
- Then
  - $R_b = 1/T_b$ is the bit rate
  - $R_s = 1/T_s = R_b/l = 1/lT_b$ is the symbol rate
- Information is transmitted at the bit rate
- Bandwidth is proportional to the symbol rate
  - Only one pulse is sent for each symbol
- **Exception**: *M*-FSK – This scheme requires more bandwidth than BFSK. However, *M*-FSK doesn't suffer in energy efficiency as *M*-PSK and *M*-QAM do.

Quadrature Phase Shift Keying

- Recall that with BPSK, the I/Q version of the signal can be written as

$$v(t) = x(t)\cos(\omega_c t)$$

where *x(t)* is a polar NRZ line code.

- Now, since the I and Q channels are orthogonal, in the same bandwidth we could send the signal

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

where y*(t)* is also a polar NRZ line code carrying independent information.

QPSK – cont.

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- Converting this signal to Magnitude/Phase form:

$$v(t) = \sqrt{x^2(t) + y^2(t)}\cos\left(\omega_c t + \tan^{-1}\left\{\frac{y(t)}{x(t)}\right\}\right)$$

- Now, since x(t) and y(t) are both polar NRZ line codes (assumed to use square pulses), there are four separate situations that can arise:

$$x(t) = rect\left(\frac{t}{T}\right), y(t) = rect\left(\frac{t}{T}\right) \quad R(t) = \sqrt{2}rect\left(\frac{t}{T}\right) \quad \theta(t) = \frac{\pi}{4}rect\left(\frac{t}{T}\right)$$

$$x(t) = rect\left(\frac{t}{T}\right), y(t) = -rect\left(\frac{t}{T}\right) \quad R(t) = \sqrt{2}rect\left(\frac{t}{T}\right) \quad \theta(t) = -\frac{\pi}{4}rect\left(\frac{t}{T}\right)$$

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QPSK – cont.

- Thus, the resulting signal is constant amplitude with one of four different phases
- This can be thought of as 4-ary PSK, but is usually termed Quadrature Phase Shift Keying or QPSK due to the use of the quadrature channel
- The signal can then be written as

$$s(t) = A_c \cos\left[\omega_c t + \frac{\pi}{4} + \frac{\pi}{2} m(t)\right]$$

$$m(t) = \sum_{i=-\infty}^{\infty} (m_i)\, rect\left(\frac{t - iT_s}{T_s}\right)$$

$$m_i \in \{0,1,2,3\}$$

M-ary Phase Shift Keying (M-PSK)

Analog and Digital Communications

- Magnitude and Phase Representation:

$$s(t) = A_C \cos\left[\omega_C t + \frac{2\pi}{M} m(t)\right]$$

  - $m(t) = \sum_{i=-\infty}^{\infty} (m_i)\, rect\left(\frac{t - iT_s}{T_s}\right)$

    $m_i \in \{0, 1, \ldots, M-1\}$
  - A_C is a constant representing amplitude
- **Special Case:** M=2 corresponds to BPSK

$$s(t) = \begin{cases} A_c \cos(\omega_c t), & m_i = 0 \\ A_c \cos(\omega_c t + \pi) = -A_c \cos(\omega_c t), & m_i = 1 \end{cases}$$

Other Representations of MPSK

- Complex Envelope:

$$s(t) = \mathrm{Re}\left\{ g(t) e^{j\omega_c t} \right\}$$

$$g(t) = A_c e^{j\theta(t)} = A_c \cos[\theta(t)] + jA_c \sin[\theta(t)]$$

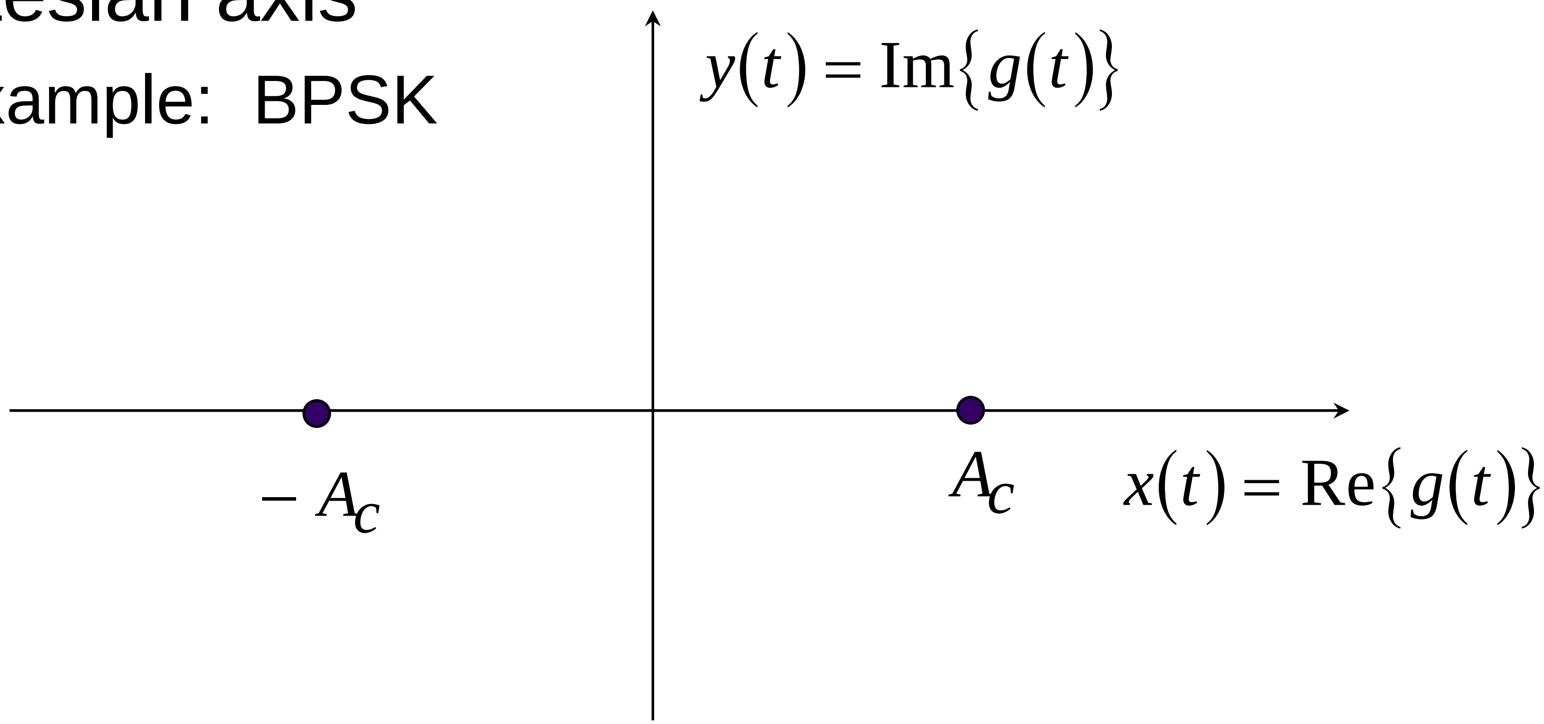
$$\theta(t) = \frac{2\pi}{M} m(t)$$

- In-Phase/Quadrature (I/Q) Representation:

$$s(t) = \underbrace{A_c \cos\left(\frac{2\pi}{M} m(t)\right)}_{x(t)} \cos(\omega_c t) - \underbrace{A_c \sin\left(\frac{2\pi}{M} m(t)\right)}_{y(t)} \sin(\omega_c t)$$

Signal Constellation Representation of Signals

- Plot the real and imaginary parts of *g*(*t*) on Cartesian axis
  - Example: BPSK



Signal Constellation Representation of Signals

- Plot the real and imaginary parts of *g*(*t*) on Cartesian axis
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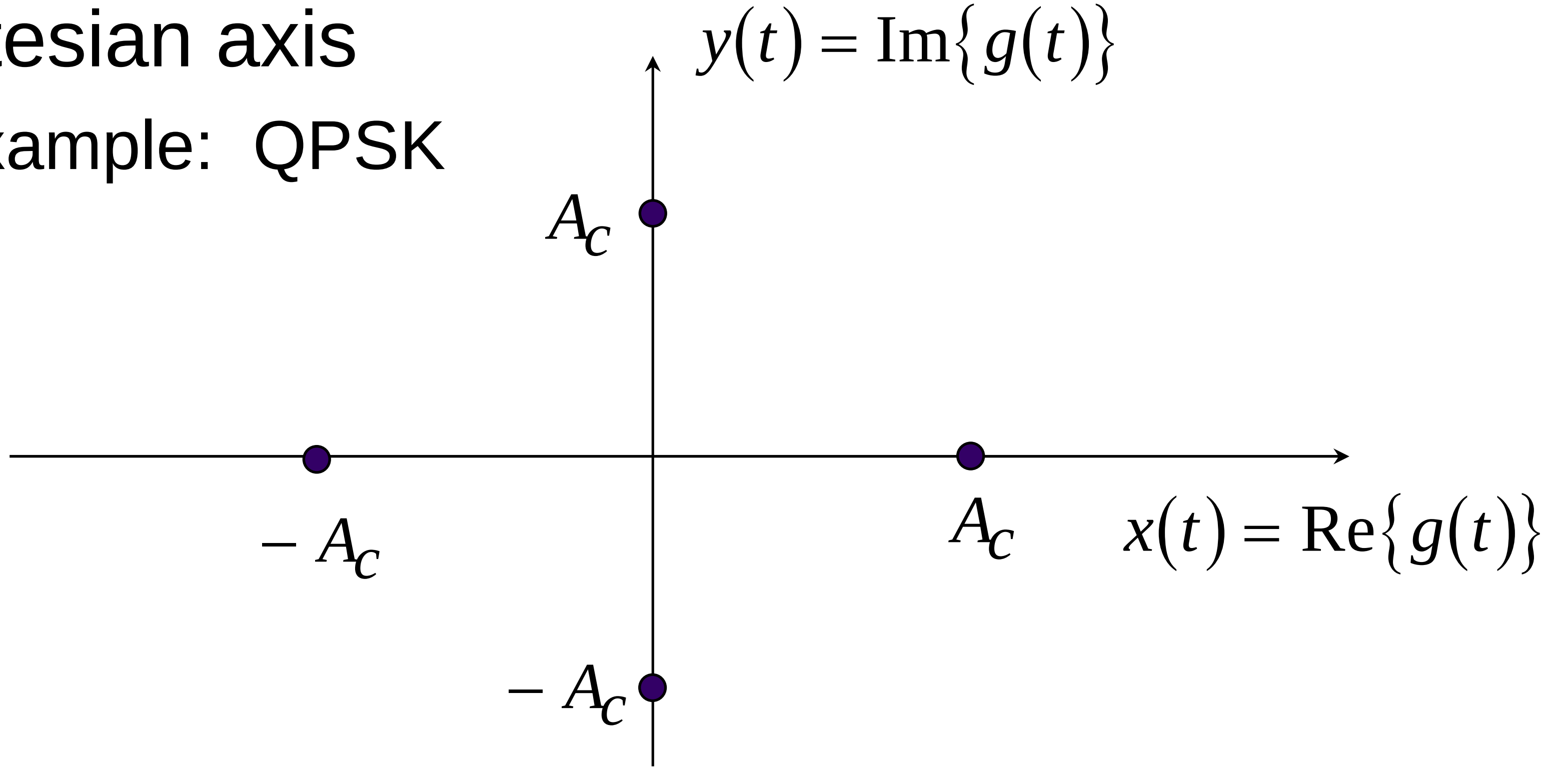
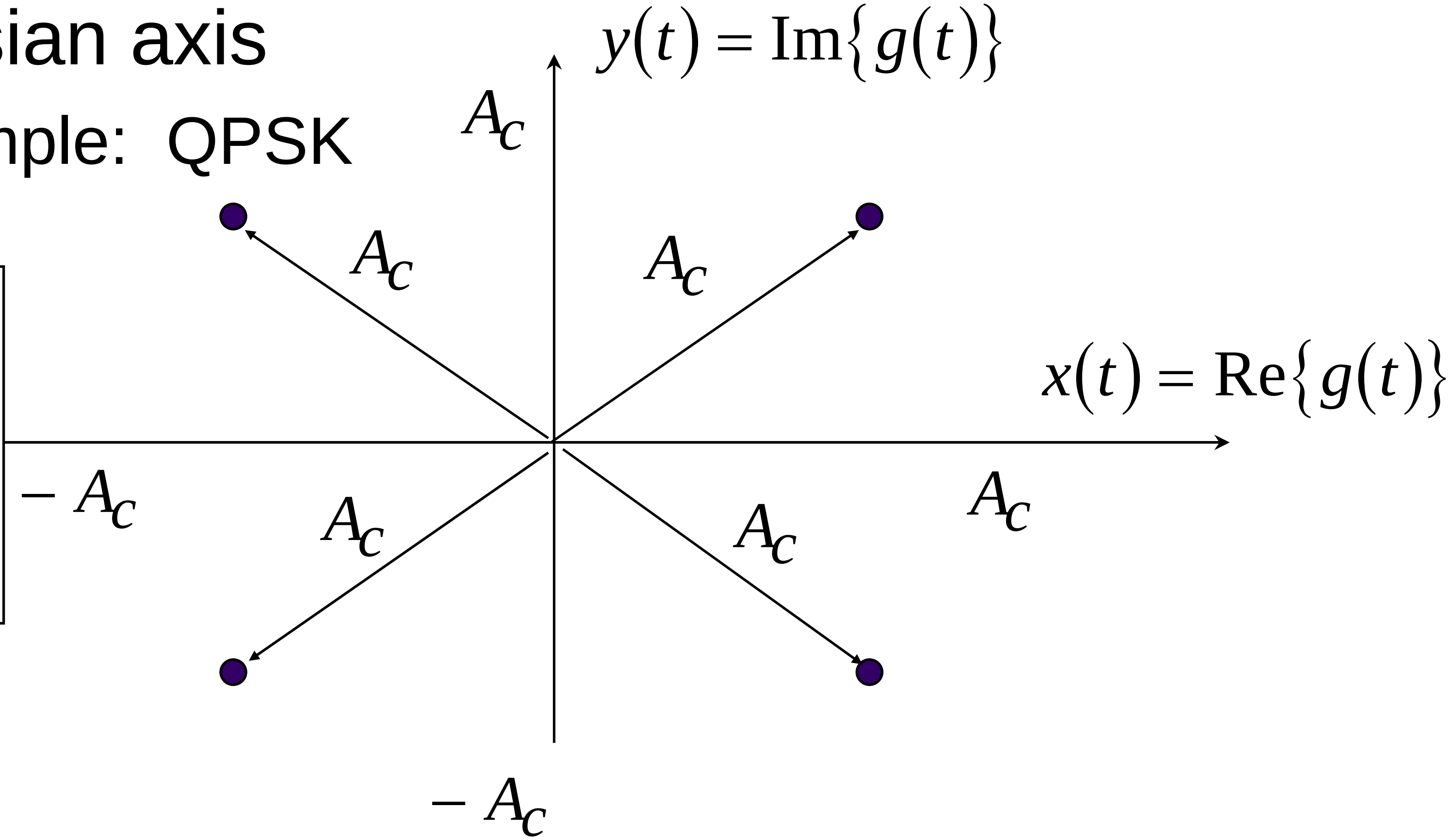


Physical Interpretation of Signal Constellation Diagram

- Real axis shows what modulates cosine wave
- Imaginary axis shows what modulates sine wave
- Distance from origin corresponds to amplitude
  - Distance squared is energy
- Separation of points shows how "far apart" signals are
  - This tells us how likely it is to mistake one signal for another
- Note that different phase offsets can produce the same constellation rotated about the axis
  - example: $s(t) = A_C \cos\left[\omega_C t + \frac{2\pi}{M} m(t) + \frac{\pi}{4}\right]$

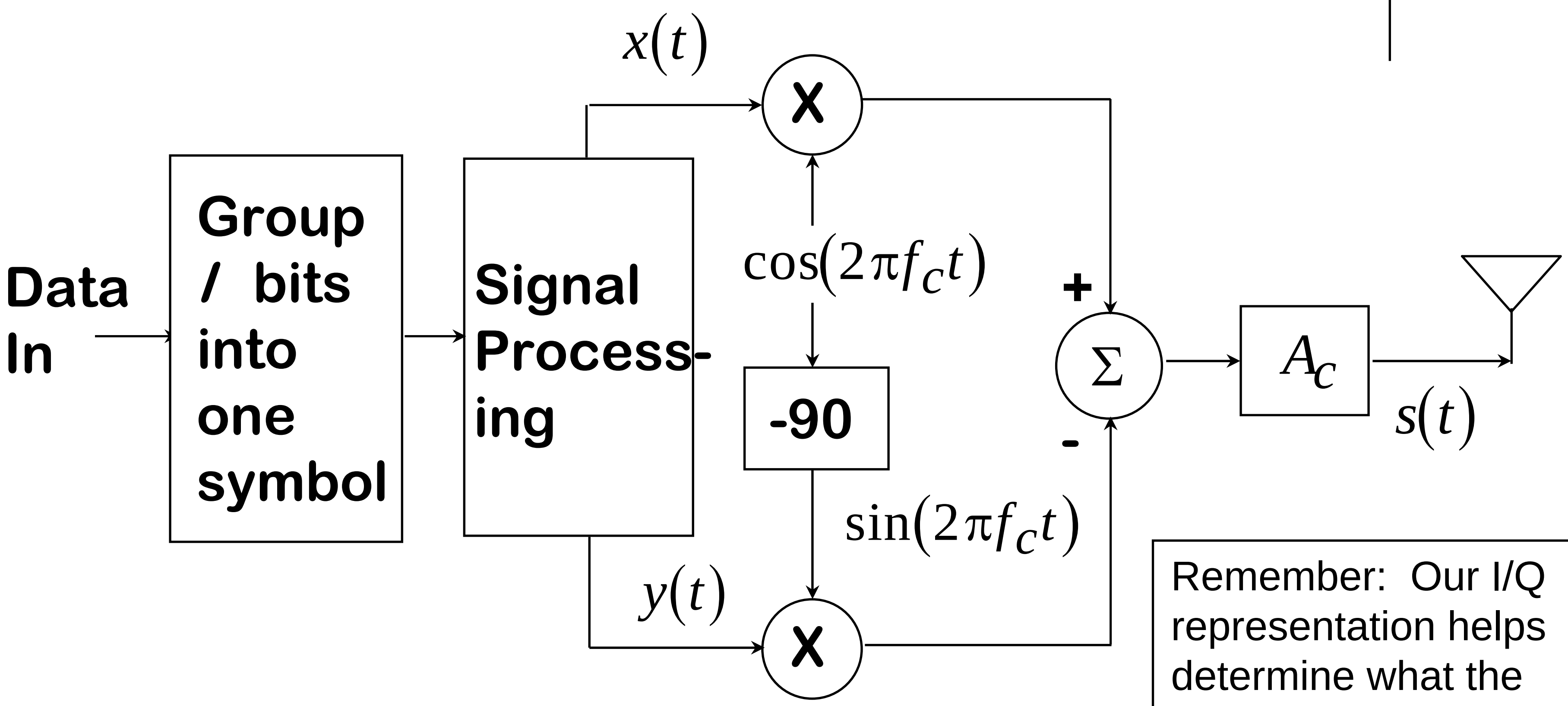
Signal Constellation Representation of Signals

- Plot the real and imaginary parts of *g*(*t*) on Cartesian axis
  - Example: QPSK



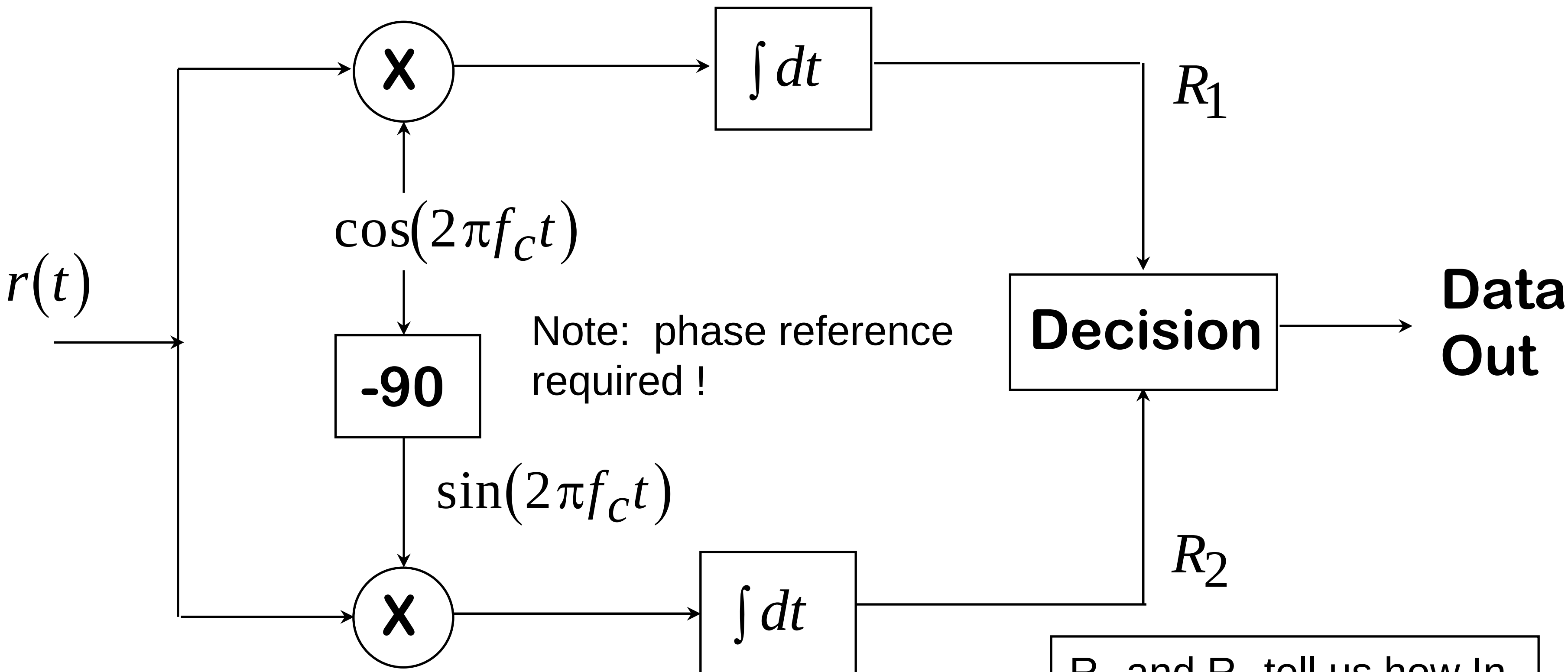
Note: To keep the energy constant, the distance from the origin must be the same

Transmitter for M-ary PSK



Remember: Our I/Q representation helps determine what the transmitter might look like

Receiver for M-ary PSK



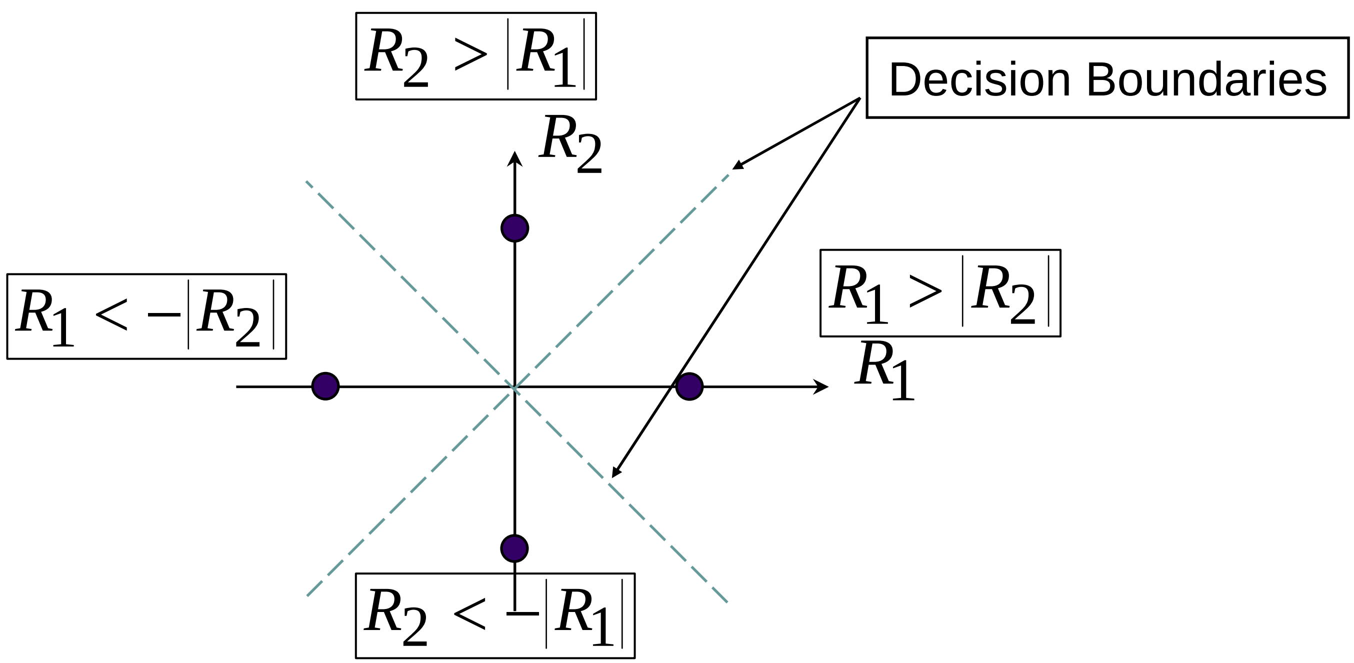
Note: Integrators are low pass filters. Are they optimal?

R_1 and R_2 tell us how In-Phase and Quadrature were modulated respectively

Decision Criteria

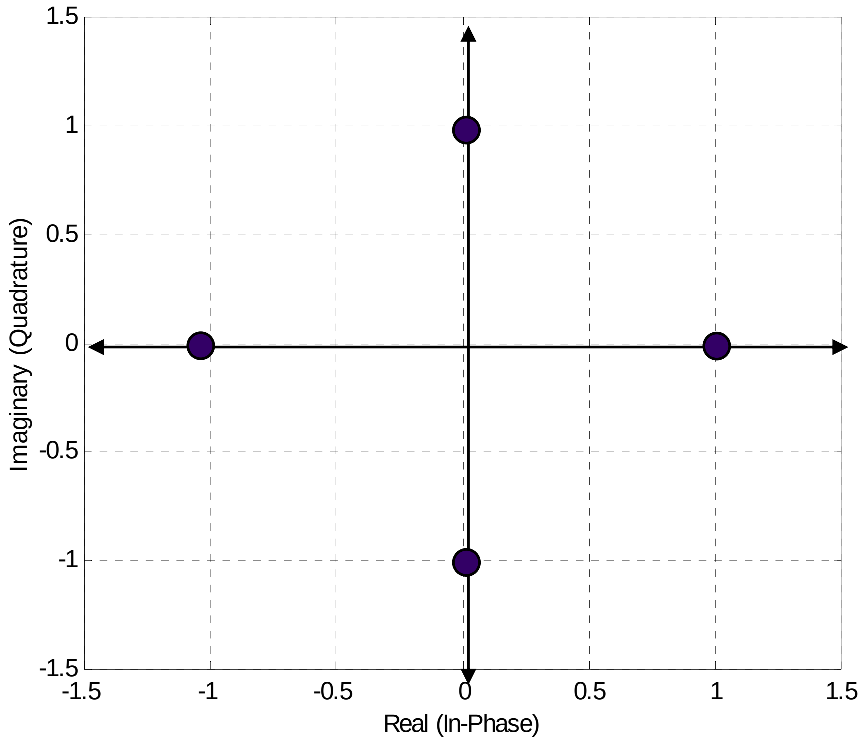
- Generate two variables R_1 and R_2 from correlation
- Decision can be made from constellation diagram
  - Plot the decision variables R_1 and R_2
  - Choose signal which lies closest to decision variables

Four decision regions Corresponding to four possible symbols

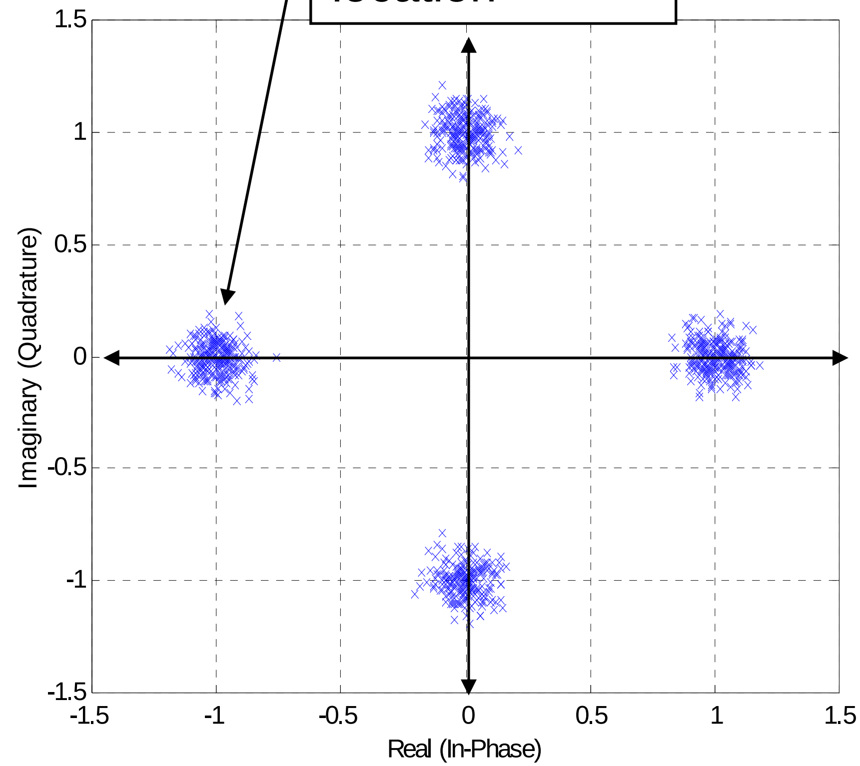


Effect of Noise on Signal Constellation

Noise shifts points away from original location



Original Signal Constellation



Signal Constellation for 1000 received symbols with noise

Quadrature Amplitude Modulation

- Representation
  - Quadrature: $s(t) = A_c x(t)\cos(\omega_c t) - A_c y(t)\sin(\omega_c t)$
  - Complex Envelope: $s(t) = \mathrm{Re}\left\{ g(t) e^{j\omega_c t} \right\},\ g(t) = x(t) + jy(t)$
- *x*(*t*) and *y*(*t*) are both multilevel signals
- Once again:

$$M = \text{ number of possible signals}$$

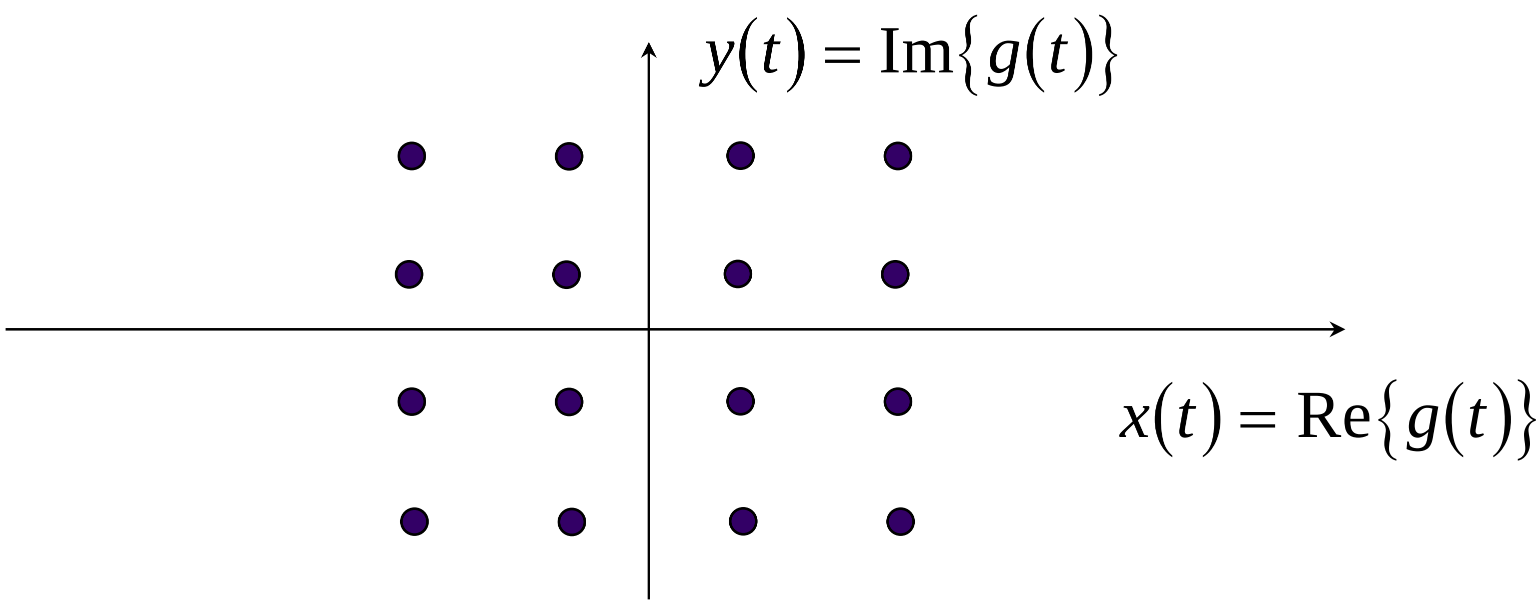
$$l = \log_2(M) = \text{number of bits / symbol}$$

- Bandwidth of QAM:
  - Rectangular Pulses (first null-to-null): $BW_{null} = 2R_s = 2R_b/l = 2n \cdot f_s/l$
  - Raised Cosine (absolute): $BW = (1+r)R_s = (1+r)R_b/l$

$$= (1+r)n \cdot f_s / l$$

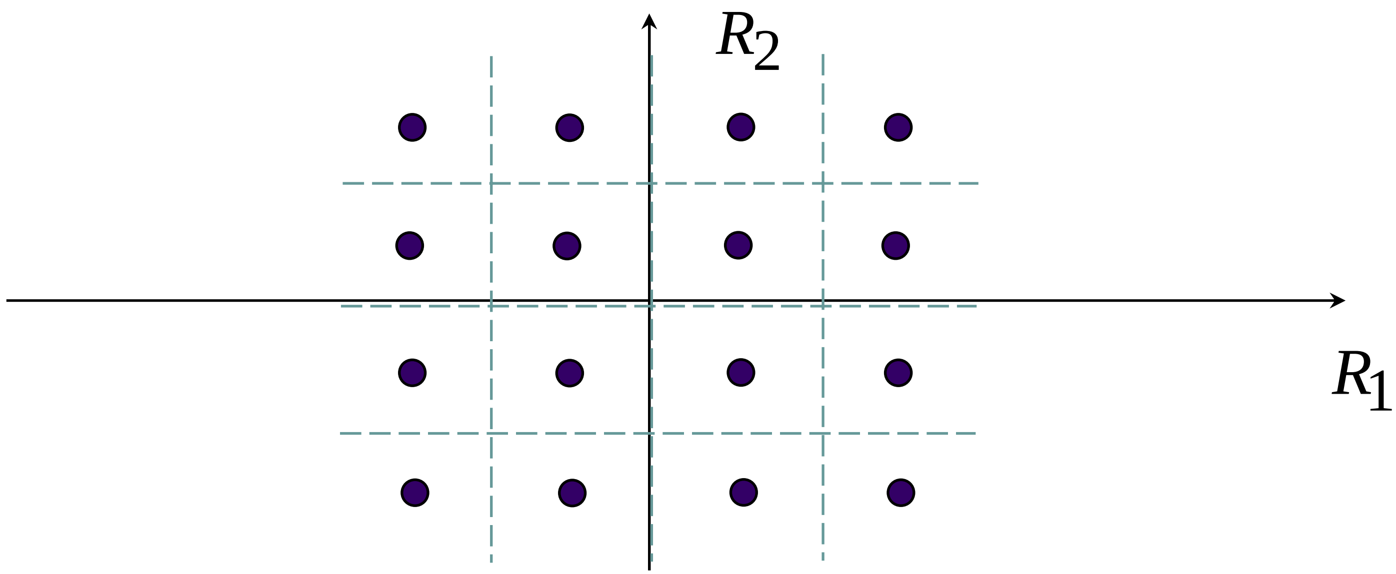
Example: 16 QAM

- M=16 different signal combinations, l = 4 bits/symbol
  - $x(t)$ can take on four different values
  - $y(t)$ can take on four different values
- Signal constellation diagram



Implementation of QAM

- Transmitter and Receiver Representations are identical to MPSK
  - Different signal processing and decision rules are used
- Decision Rule for 16 QAM

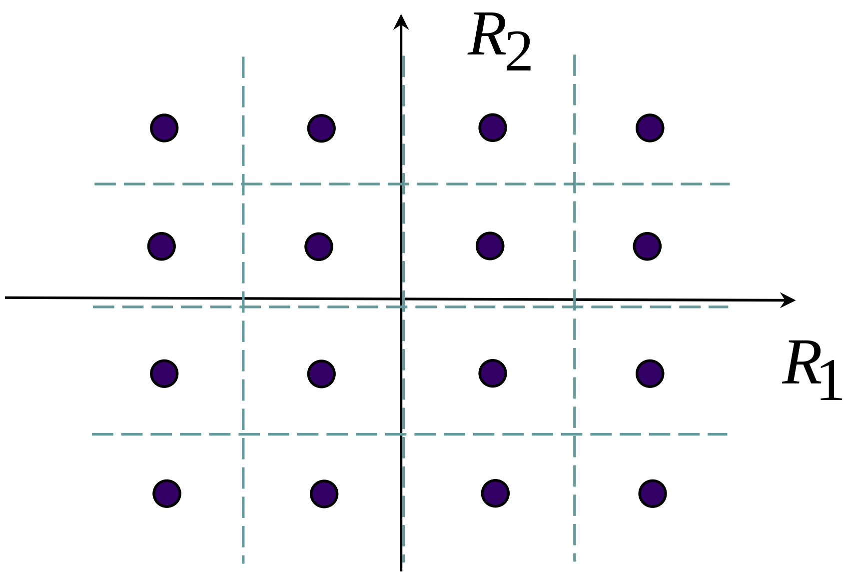


Comparison of QAM and MPSK

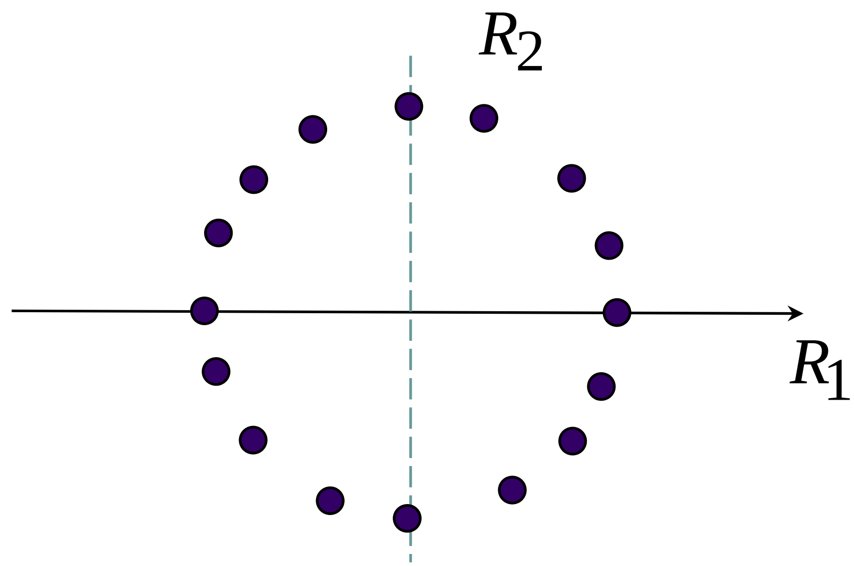
- MPSK has constant amplitude. This has two advantages:
  - More efficient "Class C" power amplifiers may be used
  - MPSK is much less vulnerable to amplitude fading
- QAM signals need not be confined to a circle in the signal constellation diagram
  - More energy efficient since signal constellation more spread out
    - As a result QAM may contain many more levels (as many as 256)
    - For the same transmit power - Potential for higher data rates

Comparison of QAM and MPSK

Note that QAM has more distance between points, thus more energy efficient. (i.e., for the same average energy the probability of making a mistake is less)



16-QAM



16-PSK

Both transmit 4 bits per symbol, thus BW efficiency equal.

Spectral Characteristics of MPSK and QAM

- Power Spectral Densities for MPSK and QAM are the same. For square pulses:

$$P_g(f) = A_c^2 T_s \left(\frac{\sin(\pi f T_s)}{\pi f T_s} \right)^2$$

$$= A_c^2 T_s \operatorname{sinc}^2(f T_s)$$

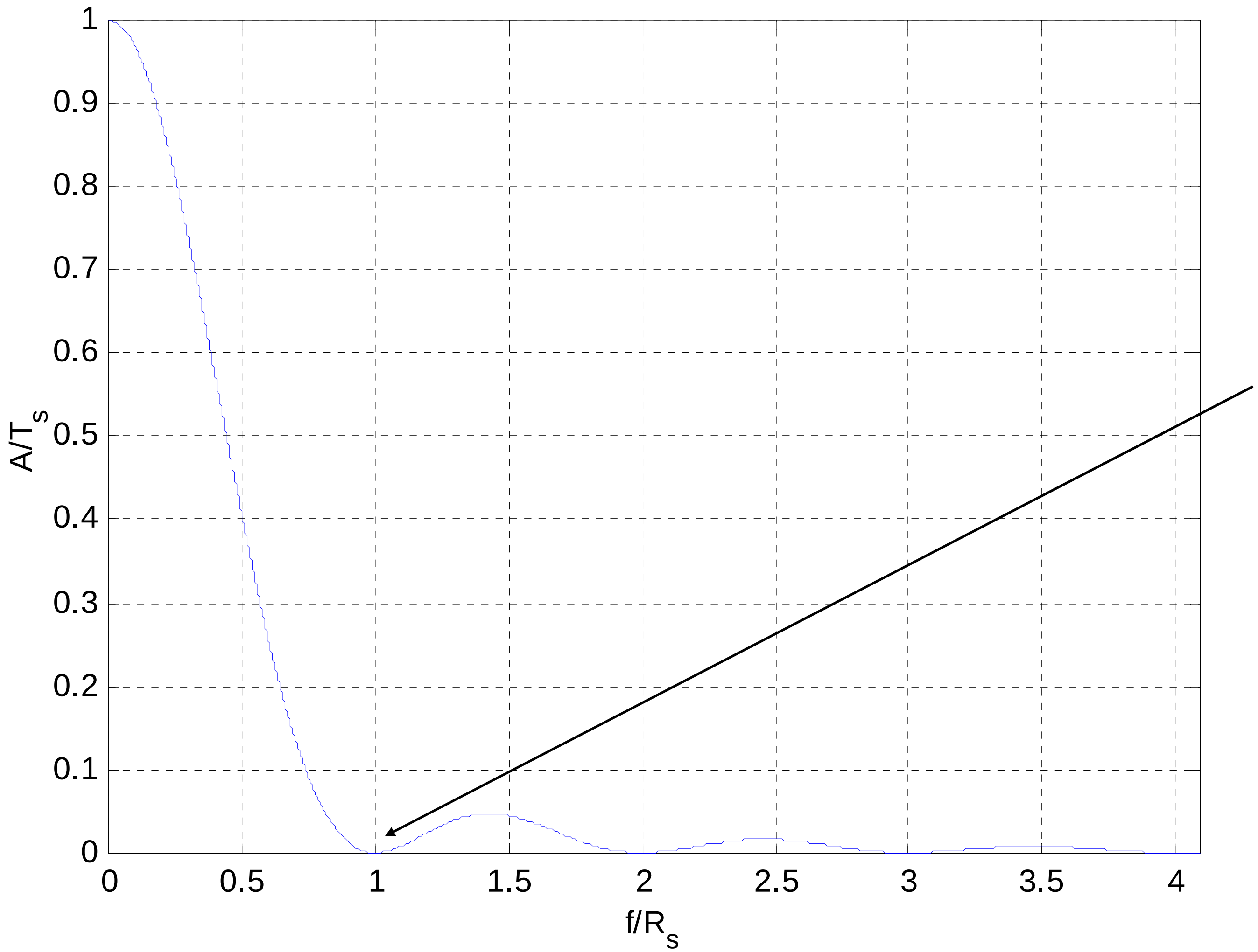
PSD for complex baseband

$$T_s = \frac{1}{R_s} = \frac{l}{R_b}$$

Bits per symbol

First null bandwidth: first null occurs when *P(f)* = 0, i.e, when $f=1/T_s=R_s$

PSD of MPSK and QAM

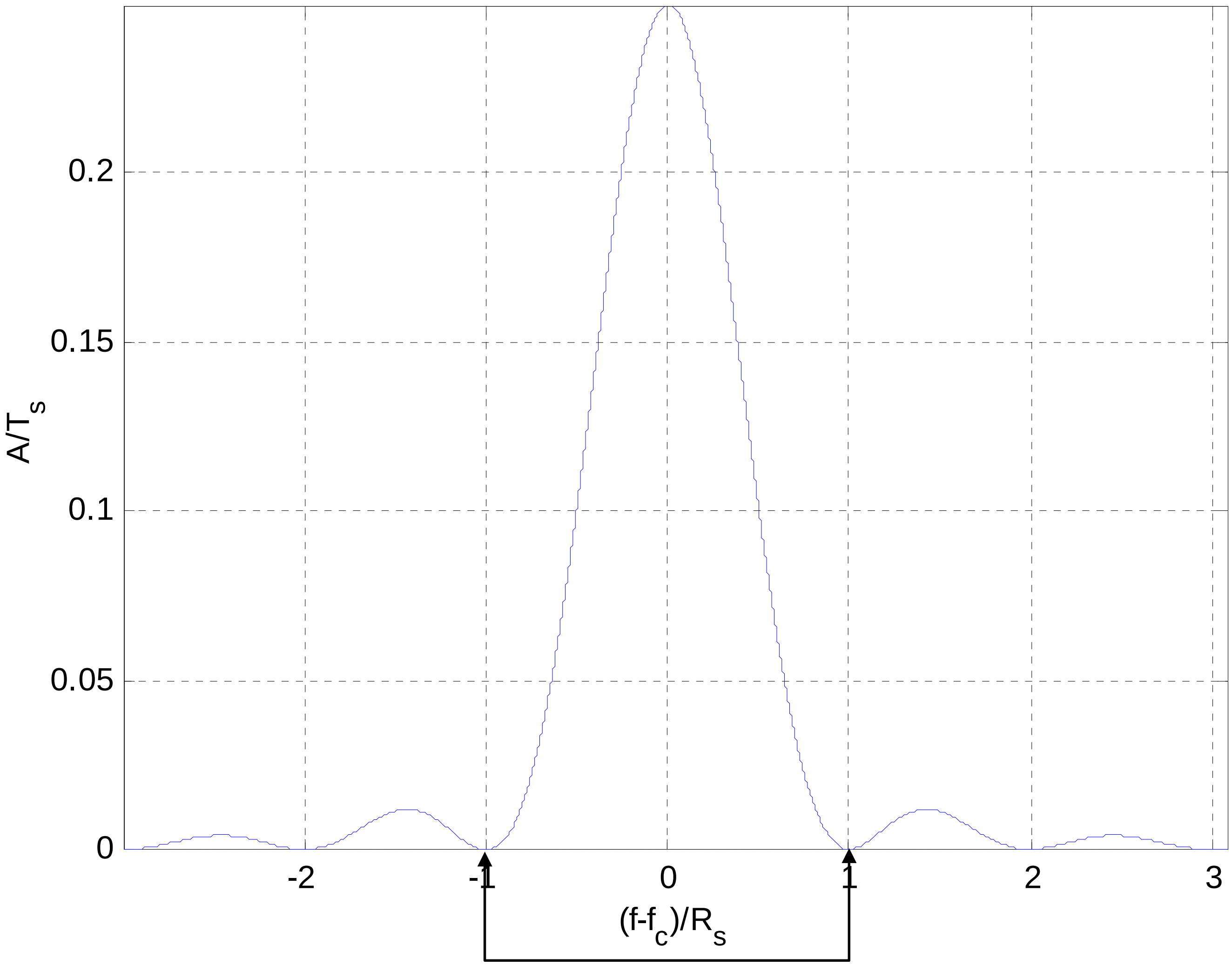


Complex Baseband
$P_g(f)$

First Null = R_s

PSD of MPSK and QAM

$$P_v(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c)\right]$$



Bandpass Spectrum

Null-to-Null = $2R_s$

Bandwidth of MPSK/QAM

- Rectangular Pulses (Null-to-Null)

$$BW = 2R_s$$
$$= 2R_b/l$$
$$= 2n \cdot f_s/l$$

For analog information signals
n = bits from quantizer
f_s = sampling rate

- Raised Cosine Pulses (Absolute)

$$BW = (1+r)R_s$$
$$= (1+r)R_b/l$$
$$= (1+r)n \cdot f_s/l$$

Roll-off factor
$0 \le r \le 1$

Summary

- Today we have investigated two methods of digital bandpass modulation that improve the overall bandwidth efficiency over the previous techniques
- This improvement comes from mapping multiple bits to a single symbol thus reducing the symbol rate (and thus the bandwidth)
- This comes at the price of degraded energy efficiency as we will see
  - This can be guessed from the fact that signal constellation points are closer together
- Note that not all *M*-ary schemes improve bandwidth efficiency (we will see this next class)