Homework Set #1

Axiomatische Verzamelingentheorie
2013/14: 2nd Semester; block b
Universiteit van Amsterdam

`http://hugonobrega.github.io/teaching/AxVT/`

Homework. There will be seven homework sheets; handed in by each student individually. Homework has to contain the name and student ID of the student. If your homework is handwritten, make sure that it is legible. Also, make sure that you write in complete English sentences.

The *preferred method* for submitting homework solutions is by handing them in before the start of the *werkcollege* on Wednesday morning. Electronic submissions are also possible, by email to `ilin.julia (at) gmail.com` or `hugonobrega (at) gmail.com` before the deadline. These must be in a single, legible PDF file. It is also possible to hand in your homework by putting it into Julia's or Hugo's mailboxes at the ILLC at Science Park 107, but in this case the deadline for submission is at 10:45am on the day of the submission deadline (before Julia and Hugo leave for the *werkcollege*). It is important to respect the strict deadlines stated above; late homework will not be accepted.

Final grade. The final grade will be calculated as the average (according to the guidelines of the OER) with homework counting a third of the grade and the exam counting two thirds.

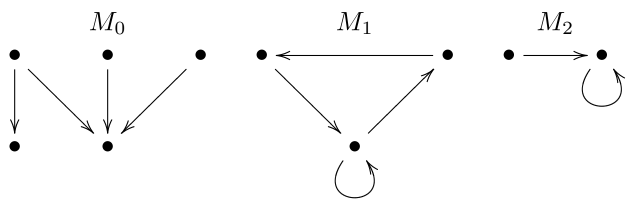
Literature.

1. Herbert B. Enderton, Elements of Set Theory.
2. Yiannis N. Moschovakis, Notes on Set Theory.
3. Heinz D. Ebbinghaus, Einführung in die Mengenlehre.
4. Thomas S. Jech, Set Theory.
5. Kenneth Kunen, Set Theory.

This homework set is due on **Wednesday 9 April 2014, before the morning *werkcollege*.**

1. In the lecture, we talked about the interpretation of the language of set theory $\mathcal{L}_\in$ via directed graphs where the expression $x \in y$ is interpreted as "there is an edge from x to y". Show that there is a model of (Ext), (Sep) together with the statement $\exists x \exists y(\varnothing \in x \wedge x \in y)$ by providing a graph that makes all of these statements true. (Remember that the empty graph is not allowed as a model.)

2. Consider the following three models:



   For each of them, check whether (Ext), (Sep), $\exists x \forall y(y \in x)$, and $\exists x \forall y(y \notin x)$ holds in these models.

3. In class, we proved that (Ext)+(Sep)+(Pair) cannot hold in finite models. Can you show the same for the weaker theory (Ext)+(Pair)?

   If yes, prove it. If no, produce a finite countermodel and show that the theory holds in that model. (Remember once more that the empty graph is not allowed as a model.)