Taller 1: Problemas inversos

Y un poco de programación

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Gracias a mis colaboradores,
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Gracias al IGP por recibirme

12 Janvier 2023

Objectivos de este taller

- What is inversion? When do we meet this class of problem (inversion is everywhere in physical, sociological, health, . . .)
- Provide recipes to solve linear inverse problems
- Give simple examples
- Two useful books on the subject, and main credits:
  1. Geophysical Data Analysis: Discrete Inverse Theory (Revised Edition) William Menke (1989), Academic Press
  2. Parameter Estimation and Inverse Problems (Second Edition) Richard C. Aster, Brian Borchers & Clifford H. Thurber (2013), Academic Press

Outline

Mathematical background

Inverse problem (and/or optimization) specific notions

The linear case

Mathematical background

Forward and inverse problems

Forward problem:

Model parameters $\rightarrow$ Forward modeling $\rightarrow$ Predicted data

Inverse problem:

Observed data $\rightarrow$ Inverse modeling $\rightarrow$ Parameter estimation

Principios matemáticos

We are interested in the relationships between physical (or chemical, economic, ...) model parameters m and a set of data d.

- We assume a good knowledge of the laws governing the investigated phenomena (underlying physics), in the form of a function G such that
$$d = G(m) \tag{1}$$
- In the mathematical model $d = G(m)$, the forward modeling operator G can be defined as
  - a linear or nonlinear system of algebraic equations
  - the solution of an ODE or PDE
- Forward problem: find d given m
- Inverse problem: find m given d_c
- Model identification problem: find G knowing some values of d and m

Ejemplos:

m → **Modelo cuantitativo** → d

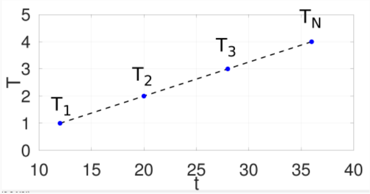
parámetros — datos

$$Gm = d$$

Modelo $\rightarrow$ línea recta

ejemplo:

$$T_1 = a + bt_1$$
$$T_2 = a + bt_2$$
$$\vdots$$
$$T_N = a + bt_N$$



Ejemplos:

$$\underset{\text{parámetros}}{\mathrm{m}} \longrightarrow \boxed{\textbf{Modelo cuantitativo}} \longrightarrow \underset{\text{datos}}{\mathrm{d}}$$

$$\mathrm{Gm} = \mathrm{d}$$

Modelo $\rightarrow$ línea recta

ejemplo:

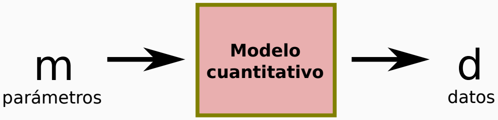
$$\begin{aligned} T_1 &= a + bt_1 \\ T_2 &= a + bt_2 \\ &\vdots \\ T_N &= a + bt_N \end{aligned}$$

en algebra lineal:

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_N \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\underline{d} = \underline{\underline{G}}\,\underline{m}$$

Ejemplos:



$$\mathrm{Gm} = \mathrm{d}$$

Modelo $\rightarrow$ línea recta

ejemplo:

$$\begin{aligned} T_1 &= a + bt_1 \\ T_2 &= a + bt_2 \\ &\vdots \\ T_N &= a + bt_N \end{aligned}$$

en algebra lineal:

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}_{[N\times 1]} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_N \end{bmatrix}_{[N\times 2]} \begin{bmatrix} a \\ b \end{bmatrix}_{[2\times 1]}$$

$$\underline{d} = \underline{\underline{G}}\,\underline{m}$$

Inverse problem (and/or optimization) specific notions

Why inverse problem are difficult?

- We are concerned with far more than just finding an acceptable model m_i that fits the data d_i , assuming a physical model G.
- There may be several acceptable solutions
- Essential issues are i) solution existence, ii) solution uniqueness and iii) instability of the solution process
- There may be no model that exactly fits the data
  - The model is wrong, or approximate
  - Data are noisy
- Rank deficiency in the model space; null space= a set of parameter combination that do not change the data: $d = G(m + lambda * m0) =¿$ model resolution analysis because the solution may be biased with respect to the reality

ii) Several solutions can fit equally the same dataset: in potential field, several density distribution produce the

The linear case