Master Project: Modelling the creep of brittle materials

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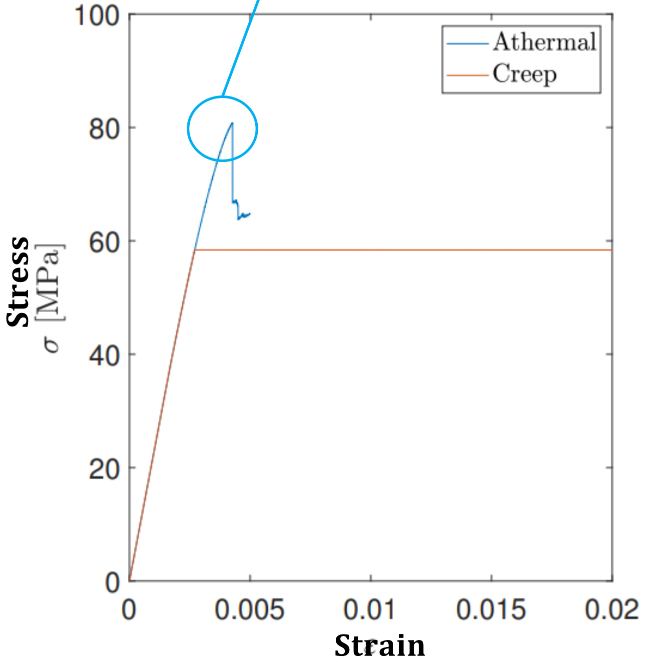
1. Presentation of the problem

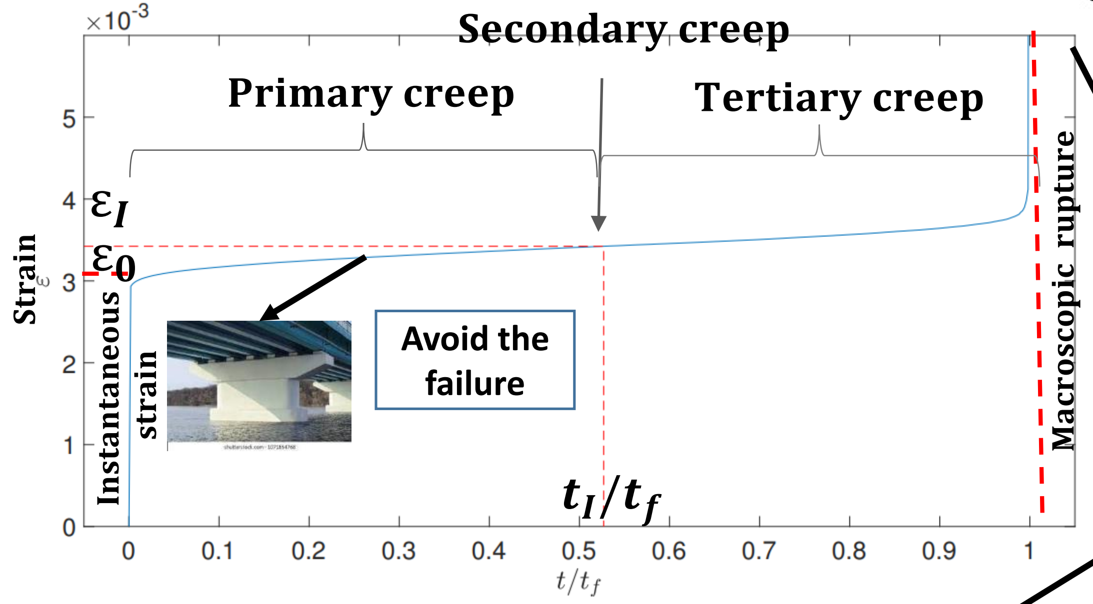
Brittle material

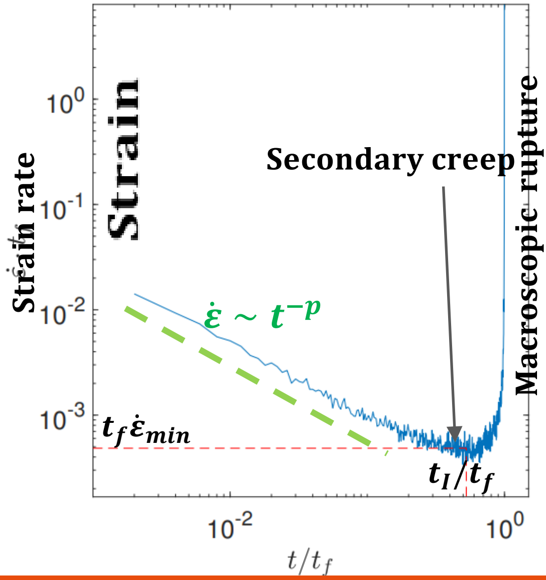
Negligible inelastic strain before failure under athermal conditions

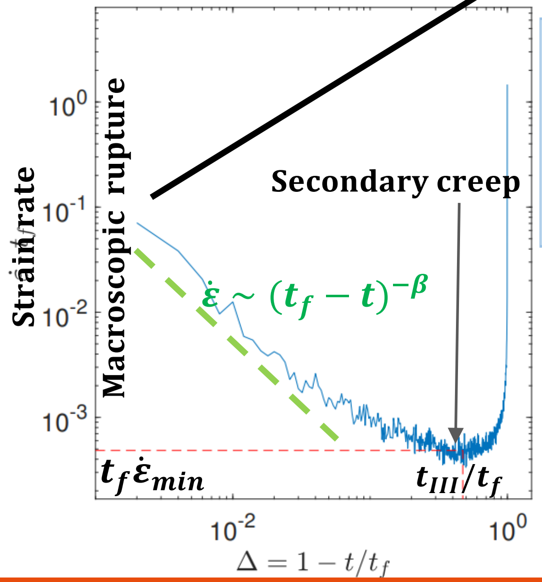


Creep

Time-dependent deformation under constant applied stress







Forecast the failure

Empirical laws

$t_f = A\dot{\varepsilon}_{min}^{-b}$ **Monkman and Grant (1956)**

$\dot{\varepsilon} \sim (t_f - t)^{-\beta}$ **Tertiary creep**

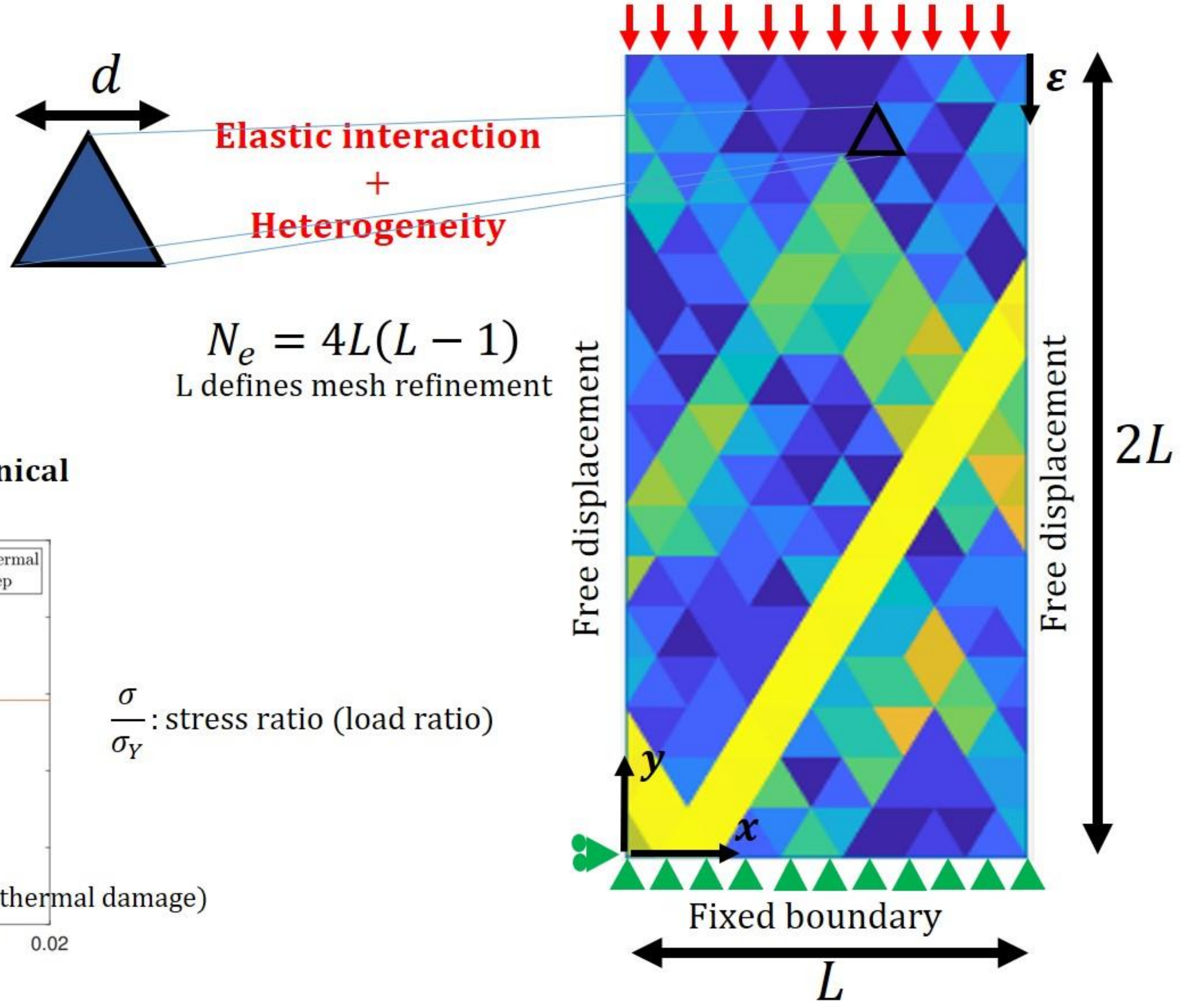
Theoretical explanation??

2. Methods

2. Methods

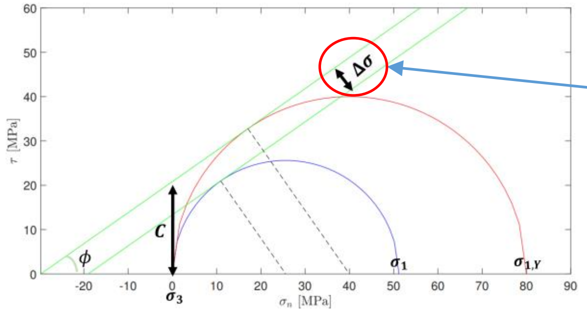
Amitrano, 1999; Amitrano and Helmstetter, 2006; Girard et al. , 2010

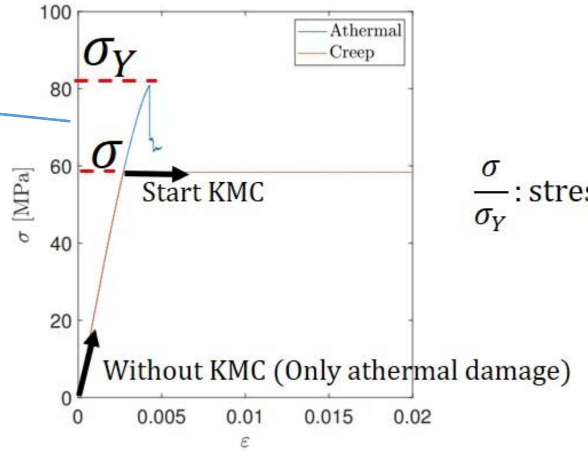
Mesoscale Homogeneous element (REV)



$N_e = 4L(L-1)$
L defines mesh refinement

Macroscopic mechanical behaviour

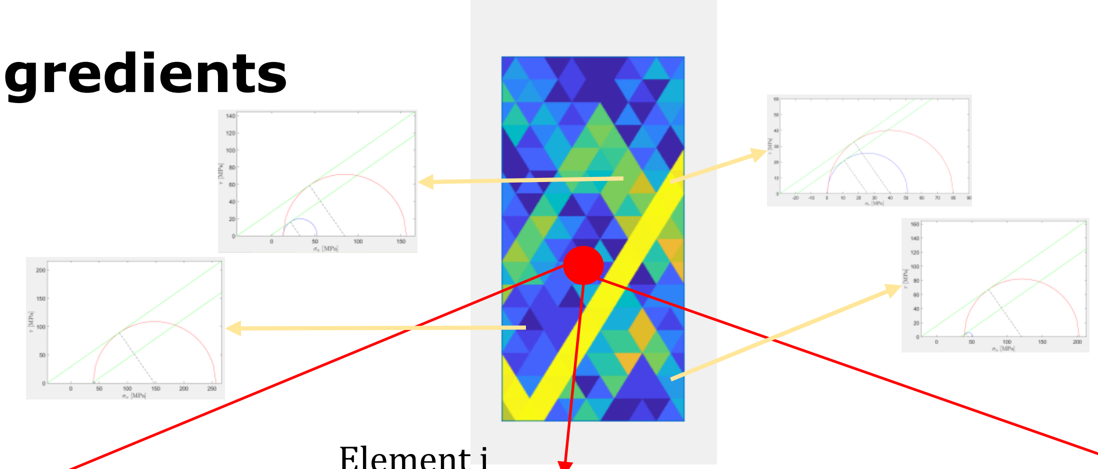




$\frac{\sigma}{\sigma_Y}$: stress ratio (load ratio)

2. Methods

Main ingredients



Element i

All the elements
Σ

1

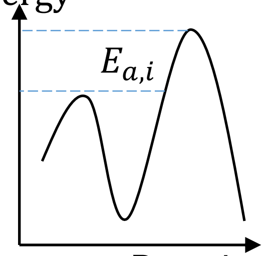
Heterogeneity

Cohesion C_i is set randomly
Uniform distribution
(only once)
$C_i \in [10, 30]$**MPa**

2

Thermal activation

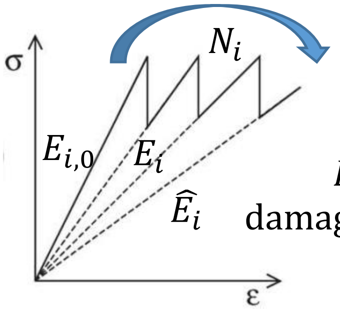
$$\omega_i = \omega_0 \exp\left(-\frac{E_{a,i}}{k_B T}\right)$$



3

Damage

$$\widehat{\boldsymbol{E}_i} = (\boldsymbol{1} - \boldsymbol{D})\boldsymbol{E}_i$$



$D = 0.1$
damage parameter

4

Elastic interaction
(Solved with FEM)

$$\boldsymbol{\sigma}_{ej} = \boldsymbol{E}_{ejkl}\boldsymbol{\varepsilon}_{ej}$$

initial disorder (random variable) + stochastic process

Athermal damage (MC criterium)

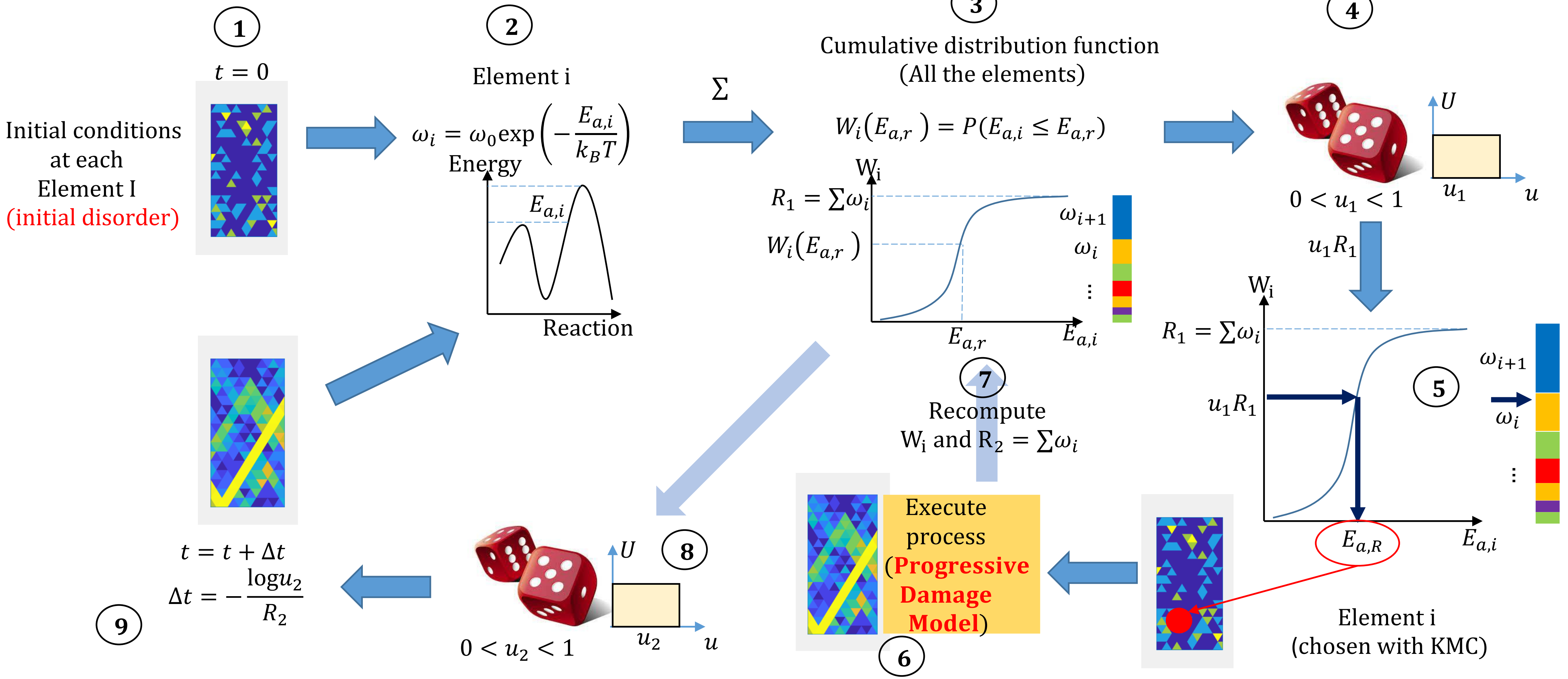
Thermal activated damage (Probability)

2. Methods

Kinetic Monte Carlo algorithm (KMC)

Bortz et al., 1975

Introduction of timescale and thermal effect (Thermally activated damage)

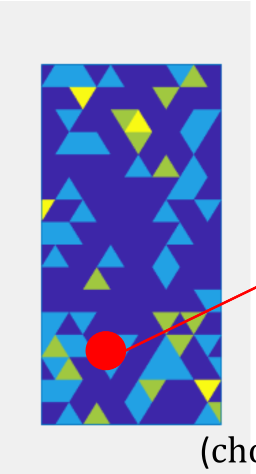


2. Methods

6

Progressive isotropic damage model (PDM)
(athermal damage)

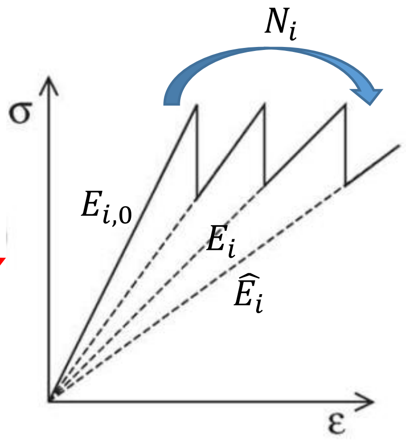
5



Element i
(chosen with KMC)

6, 1

Damage
Element i
$\widehat{E}_i = (1 - D)E_i$

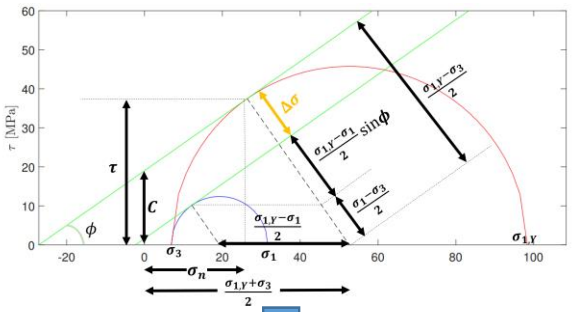


6, 2

Stress redistribution
$\sigma_{ej} = E_{ejkl}\varepsilon_{ej}$

6, 3

MC criterion in all the elements
$\tau_i - \sigma_{n,i}\tan\phi = C_i$
$\Delta\sigma_i = C_i\cos\phi + \sigma_{3,i}\frac{1+\sin\phi}{2} - \sigma_{1,i}\frac{1-\sin\phi}{2}$



$\Delta\sigma_i$>0
not damaged elements

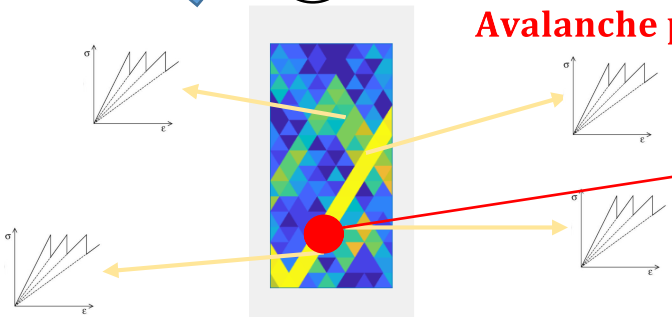
7

Continue KMC

$\Delta\sigma_i \leq 0$
Athermal instantaneous damage

6, 4

Avalanche process



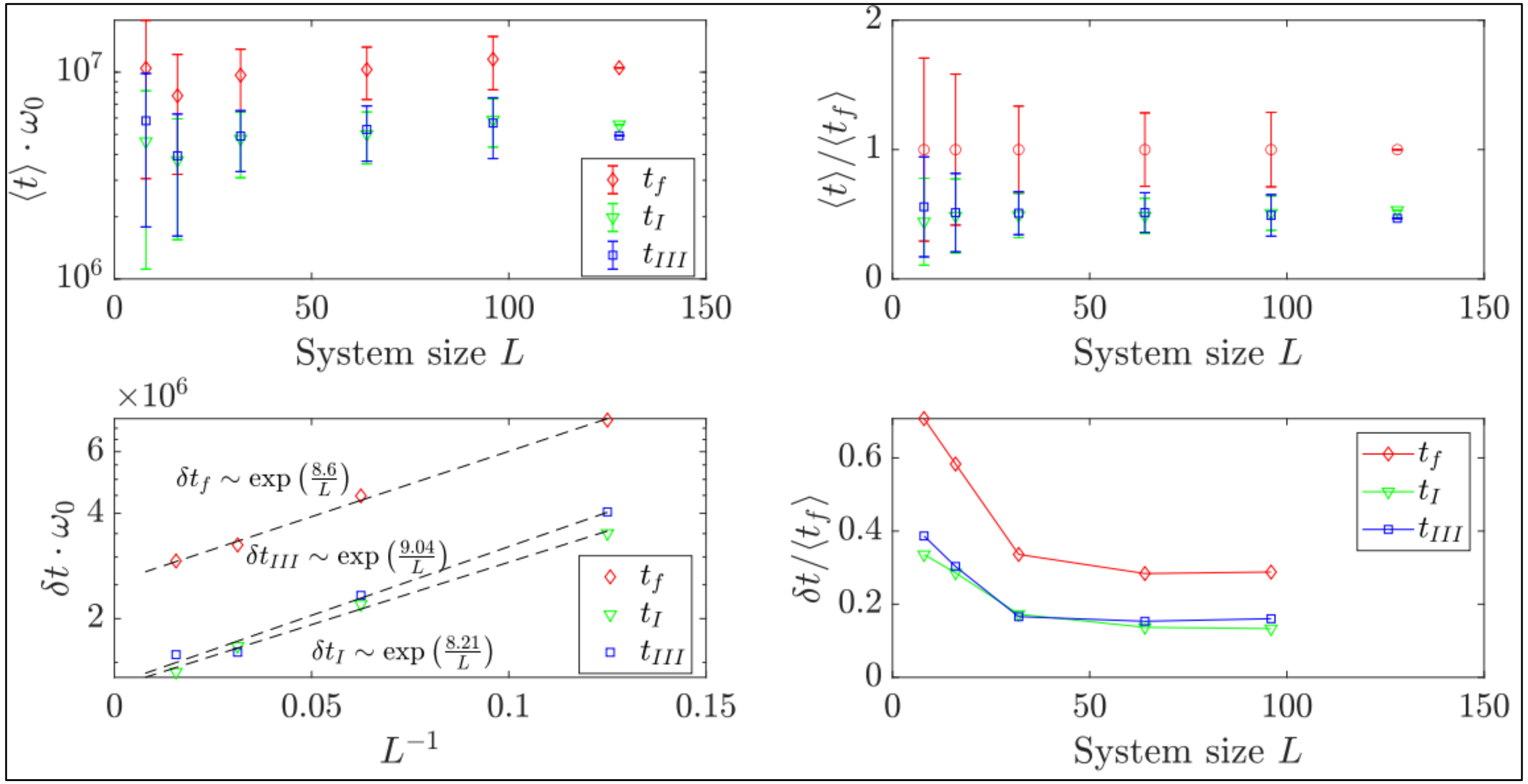
Element i
After N_i damage events
$\widehat{E}_i = (1 - D)^{N_i}E_{i,0}$

4. Results

4.1 Size effects on failure time
4.2 Empirical approximations to predict rupture time
4.3 Response to thermally activated process at macroscale
4.4 Acceleration of deformation during the tertiary creep

4. Results
4.1 Size effects on failure time

Size effects on failure time



$\langle t_f \rangle(L) \sim$ **constant**

$\delta t_f \sim \exp(L^{-1})$

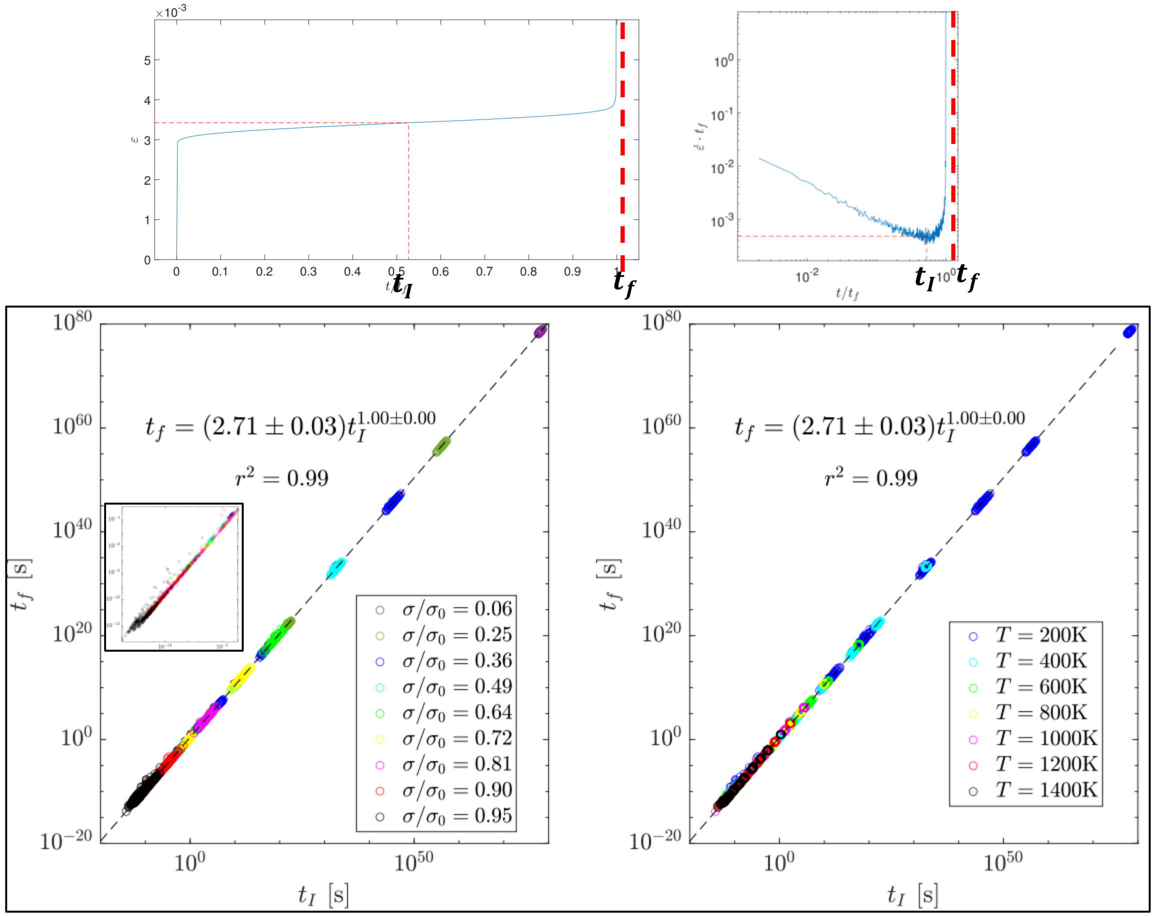
No size effects in mean

Size effects in standard deviation

Standard deviation doesn't vanish with *larger* sizes $\Rightarrow$???

4. Results

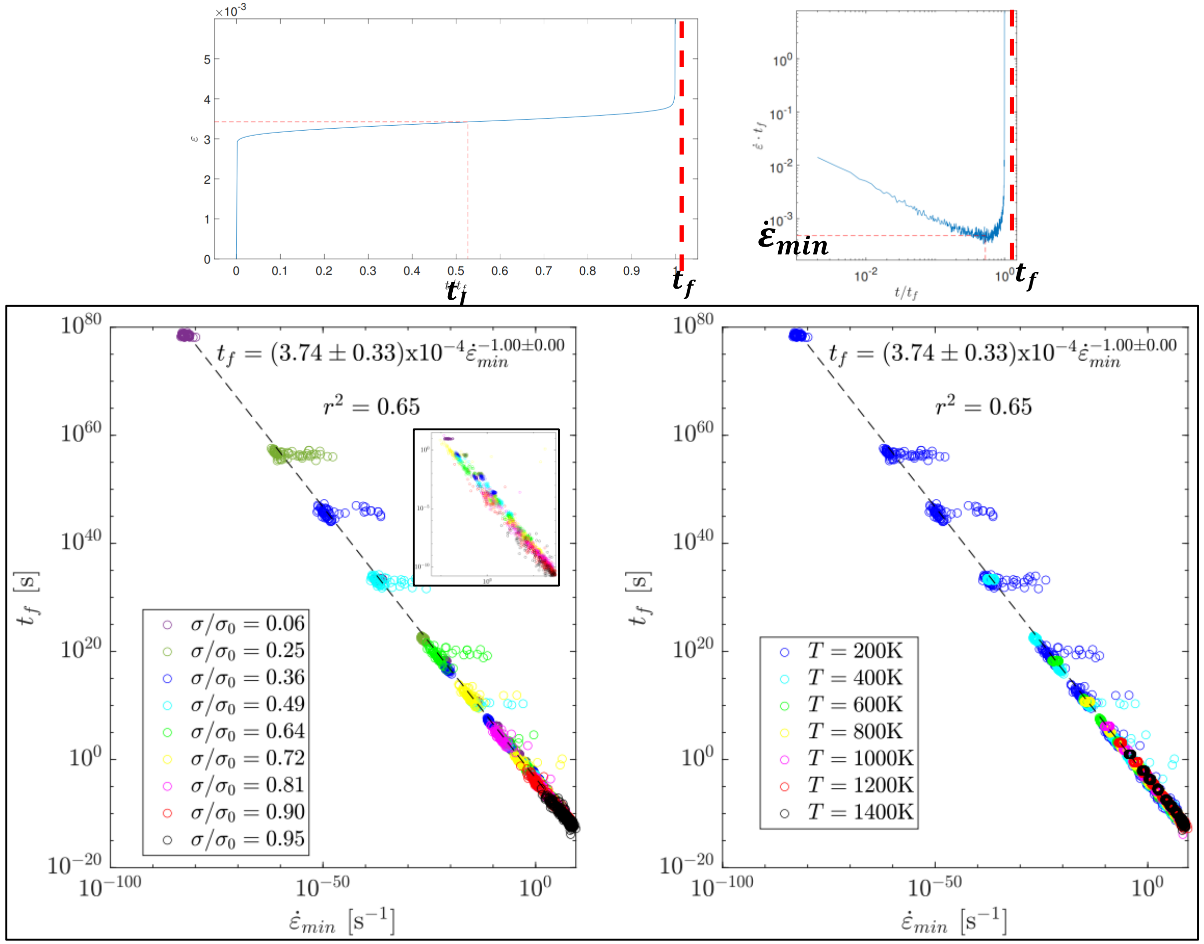
4.2 Empirical approximations to predict rupture time



Nechad et al. (2005)

$$t_f = a t_I^b$$

$b = 1.00 \qquad a = \frac{t_f}{t_I}$



Monkman and Grant (1956)

$$t_f = C_1 \dot{\varepsilon}_{min}^{\;-m}$$

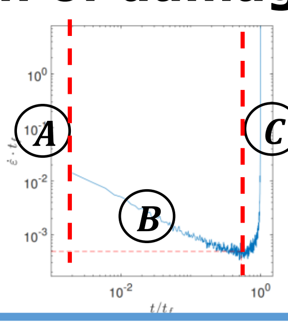
$m = 1.00 \qquad C_1 = t_f \dot{\varepsilon}_{min}$

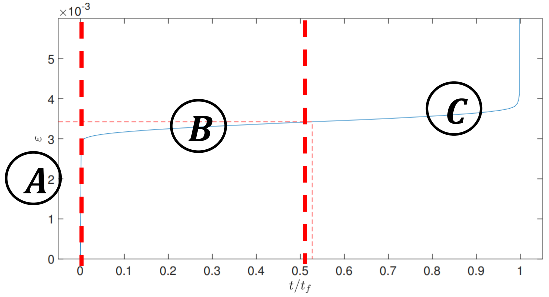
4. Results

4.4 Acceleration of deformation during the tertiary creep

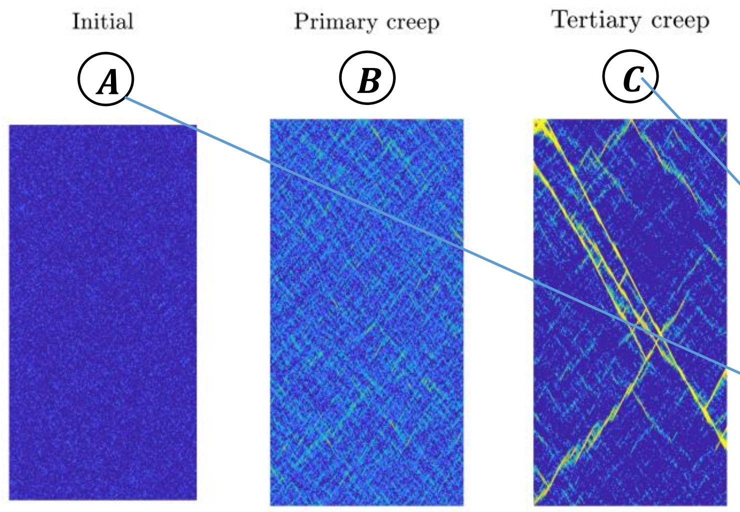
Spatial correlation of damage events

System Sizes	$L = 128$
Number of simulations	$N = 1$
Temperature	$T = 800$ K
Stress ratio	$\sigma/\sigma_Y = 0{,}72 \quad (\sigma = 0{,}72\sigma_Y)$

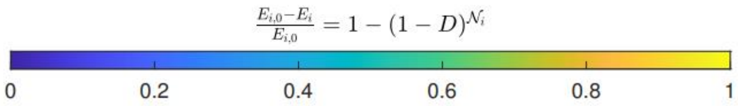




$$dL^{-1} < r < \sqrt{5}\, d$$



Damage events field (Damage localisation)

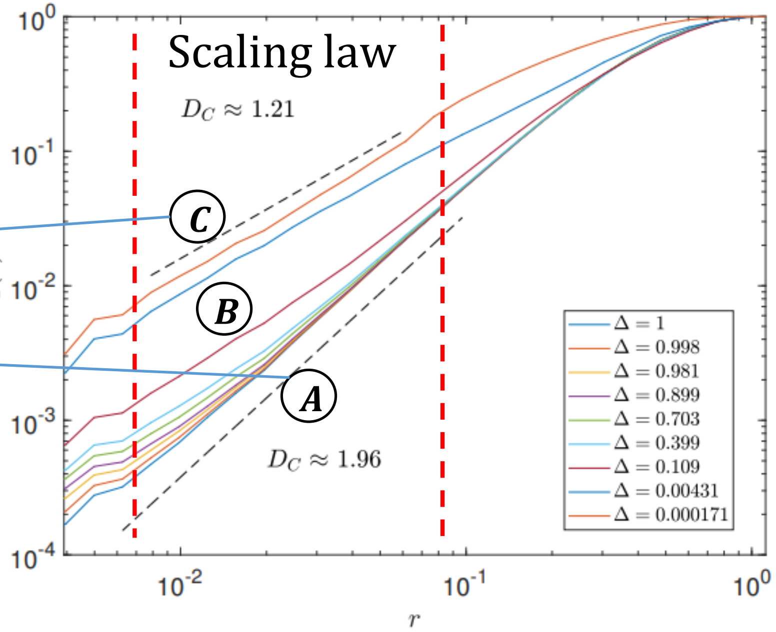


$$C_2(r) = \frac{2\aleph_r(R<r)}{\aleph(\aleph-1)} \sim r^{D_c}$$

Hirata et al., 1987

Fractal behaviour of damage events??

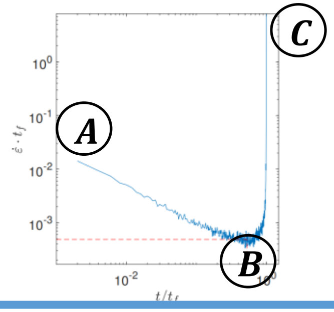
$D_c \approx 1$	*Perfect damage localisation*
$D_c \approx 2$	*Homogeneous damage*

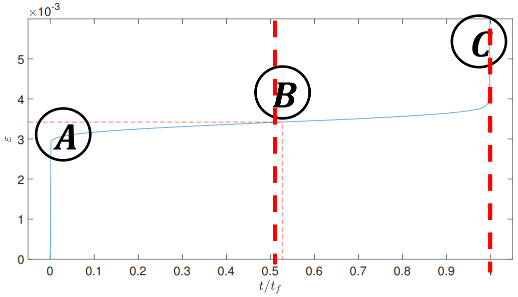


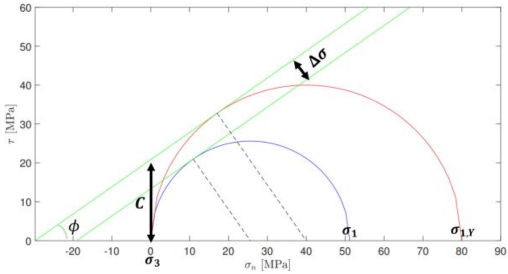
4. Results
4.4 Acceleration of deformation during the tertiary creep

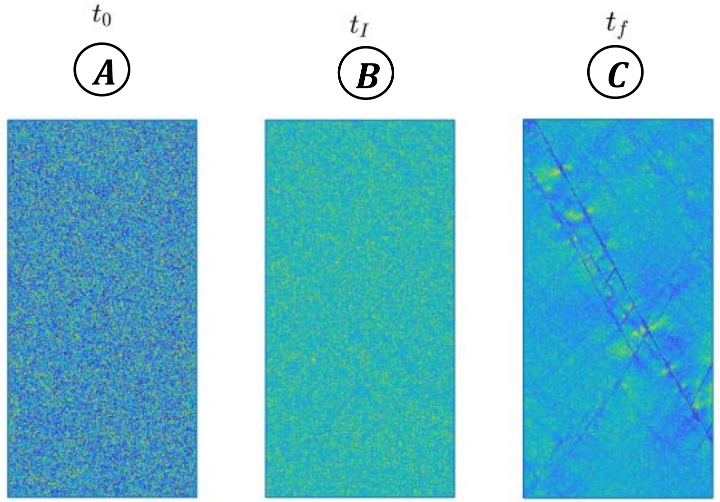
Stress gap distribution

System Sizes	$L = 128$
Number of simulations	$N = 1$
Temperature	$T = 800$ K
Stress ratio	$\sigma/\sigma_Y = 0{,}72 \quad (\sigma = 0{,}72\sigma_Y)$
Macro stress gap	$\Delta\sigma \approx 11$ MPa

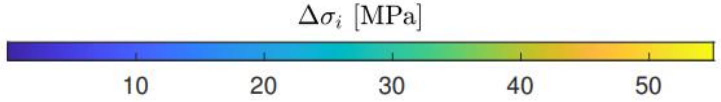








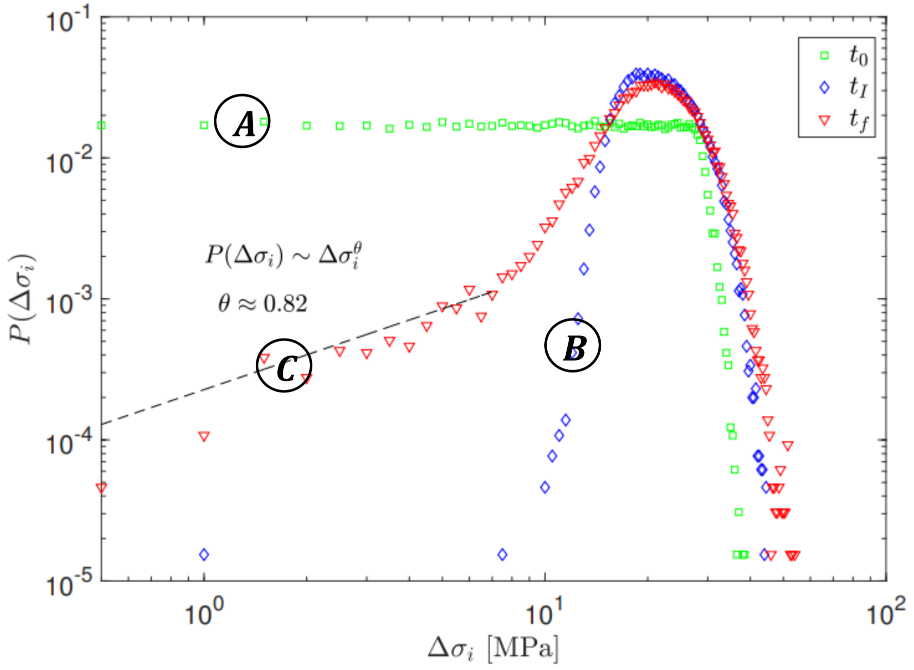
Stress gap field



$P(\Delta\sigma_i) \sim \Delta\sigma_i^{\theta}$

Lin et al., 2014

Power law tail towards small stress gaps



5. Conclusions

- Numerical modelling with small set of physical ingredients (disorder, thermal activation, damage, elastic interaction).
- Numerical modelling based on progressive damage coupled with KMC algorithm.
- Time dependency → through a KMC algorithm
- Numerical modelling allows to obtain information at microscopic scale during different time steps of the creep process.
- For different temperature and stress conditions the model reproduces:
  - the different stages of creep phenomenology and its characteristics (as shown experimentally).
  - damage localisation and fractal structure of the damage field
  - empirical laws of creep (Monkman and Grant, 1956 and Nechad et al., 2005)

- The empirical laws are a consequence of the Arrhenius law with same activation volume, i.e., importance of thermal activation in the phenomenology of creep.
- The empirical laws (MG) could be used for forecasting the failure
- Distribution of the stress gap is the key to understand what is happening during the different stages of creep.

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PhD: Statistical physics of creep rupture of structural materials

Thesis Directors:
Jérôme WEISS, ISTerre
Mikko ALAVA, Aalto University

Creep and relaxation experiments

- **Classic concrete**
- **Light-weight concrete**

(Polystyrene: $1 < D < 5$mm)

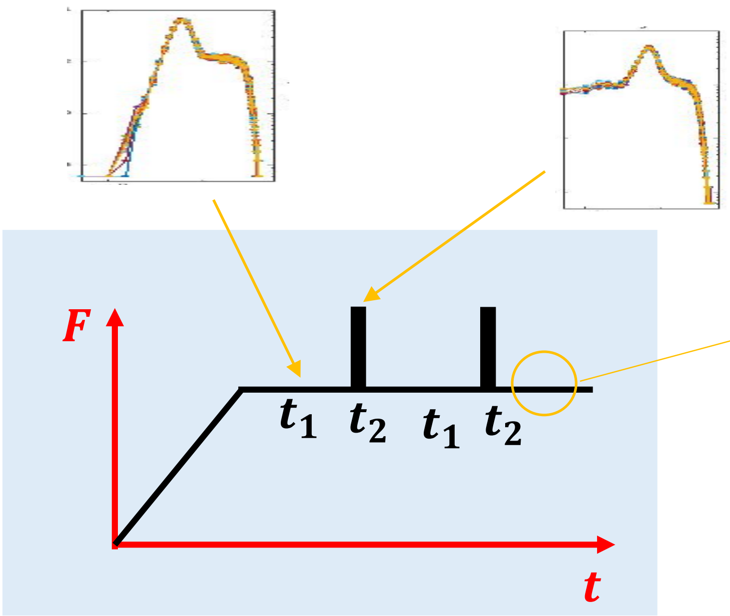
Creep and relaxation experiments

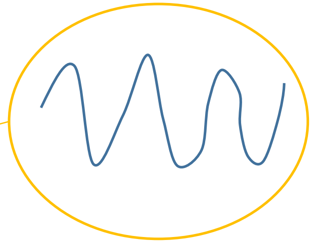
- **Classic concrete**
- **Light-weight concrete**

(Polystyrene: 1 < D < 5mm)

Two kind of experiments ✚ Acoustic emission Tomography

Creep test





Control period and ampltude around a mean

Creep and relaxation experiments

- **Classic concrete**
- **Light-weight concrete**

(Polystyrene: 1 < D < 5mm)

Two kind of experiments ✚ Acoustic emission Tomography

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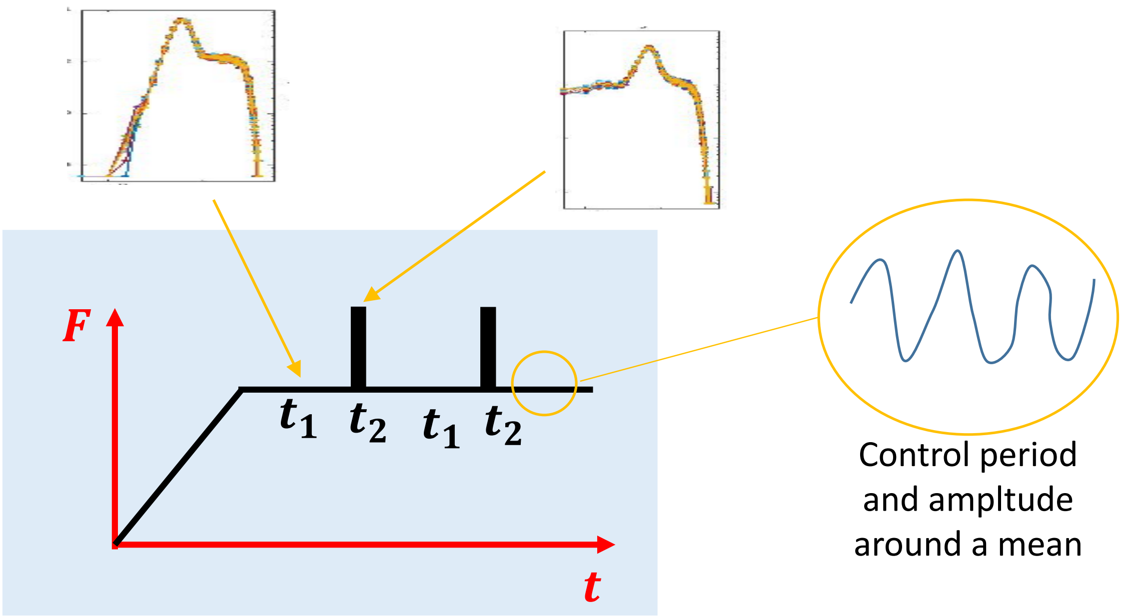
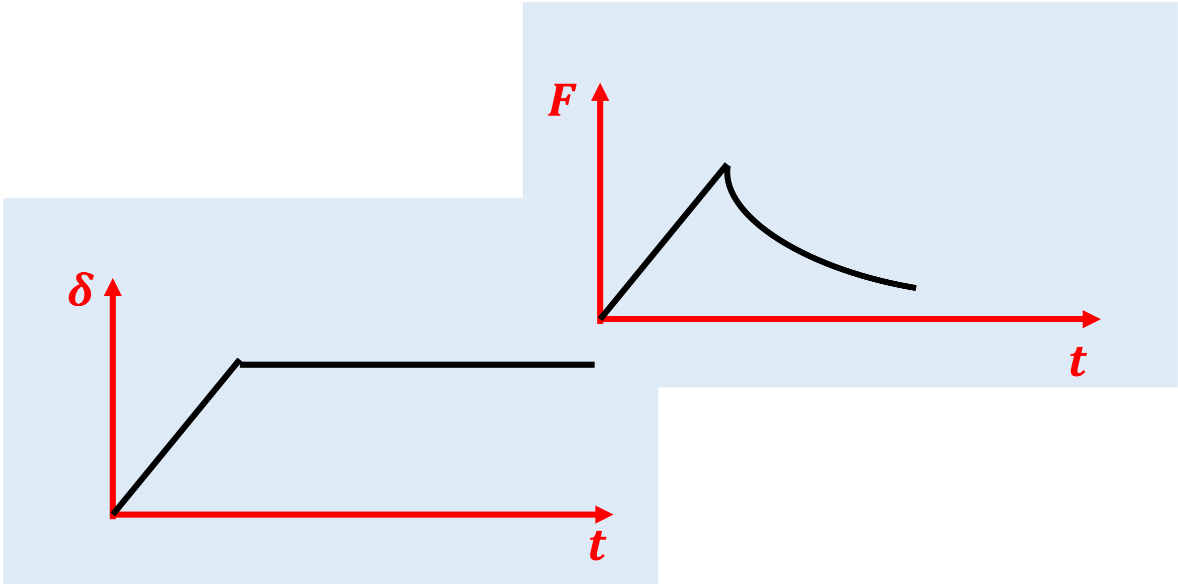


Relaxation test



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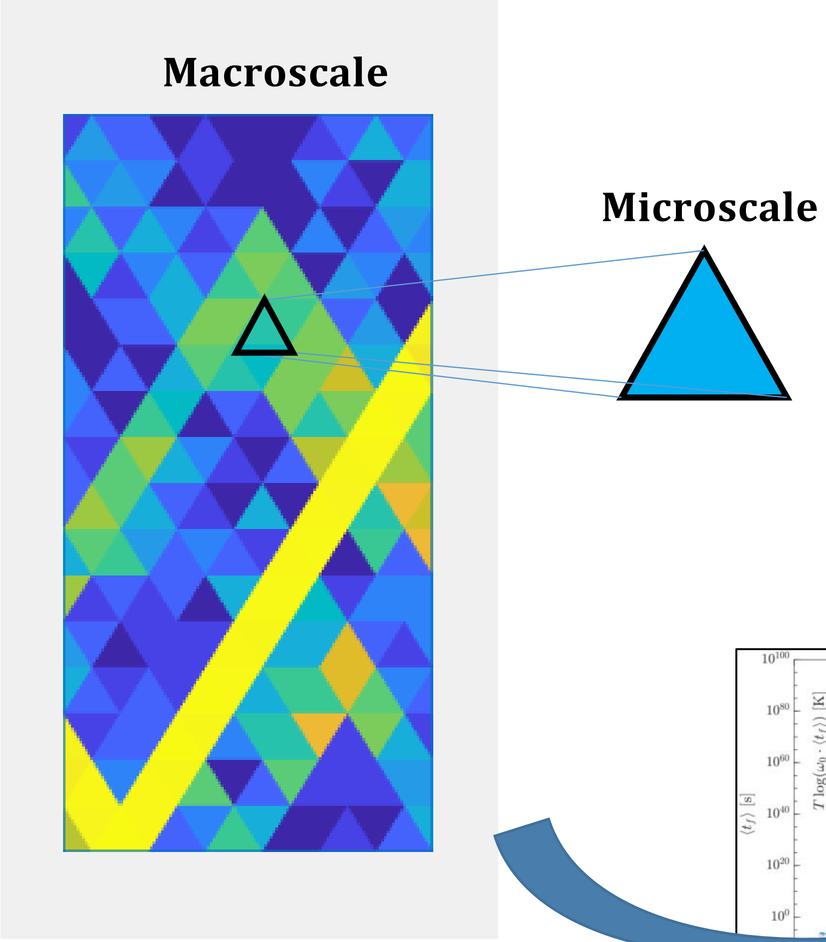
Paper in process:

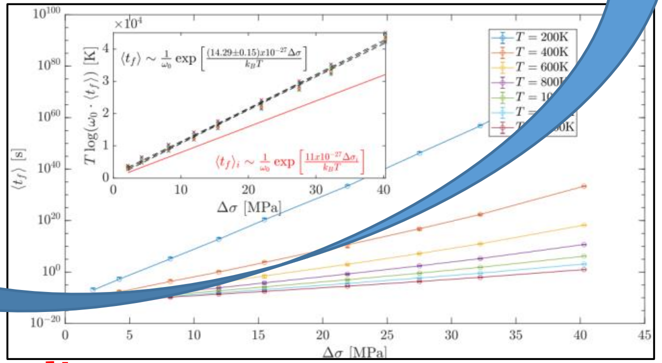
The effect of quenched disorder on creep lifetimes of brittle materials

Problem:



$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T}\right)$$

Arrhenius law



Downscaling

From macro is imposed the temperature and the load, and plotting the failure time as a function of the inverse of the temperature or the stress gap is possible to obtain the activation volume at microscale

This assumption does not take into account the assumption of disorder effect

The effect of disorder on the fracture nucleation process (S. Ciliberto, A. Guarino, R. Scorreti (2001))

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The effect of disorder on the fracture nucleation process

S. Ciliberto*, A. Guarino, R. Scorretti[1]

Ecole Normale Supérieure de Lyon, Laboratoire de Physique, CNRS UMR5672, 46, Allée d'Italie, 69364 Lyon Cedex 07, France

Received 12 January 2001; accepted 28 May 2001
Communicated by S. Fauve

Abstract

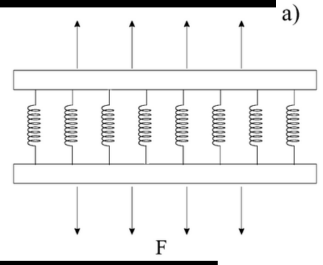
The statistical properties of failure are studied in a fiber bundle model with thermal noise. We show that the macroscopic failure is produced by a thermal activation of microcracks. Most importantly, the effective temperature of the system is amplified by the spatial disorder (heterogeneity) of the fiber bundle. The case of a time-dependent force and the validity of the Kaiser effects are also discussed. These results can give more insight to the recent experimental observations on thermally activated crack and can be useful to study the failure of electrical networks.

The effect of disorder on the fracture nucleation process (S. Ciliberto, A. Guarino, R. Scorreti (2001))

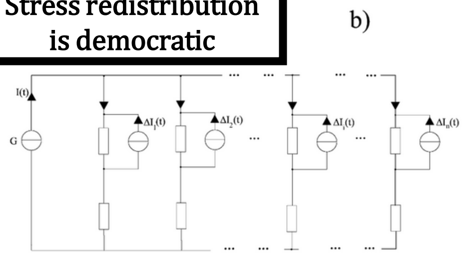
Method: DFBM

1. $F = \sum_{i=1}^{N} f_i .$
2. $f_i = Ye_i ,$
3. $f_c(i) \sim N_d(\langle f_c \rangle, KT_d).$
4. $\Delta f_i(t) \sim N_T(0, KT)$

Element dissapear (Not Memory effect)



Stress redistribution is democratic



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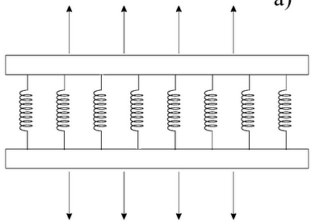
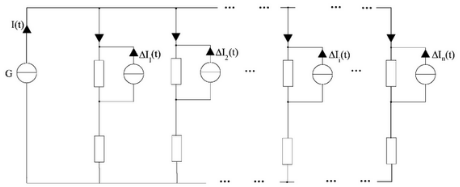
a)



F

Stress redistribution is democratic

b)

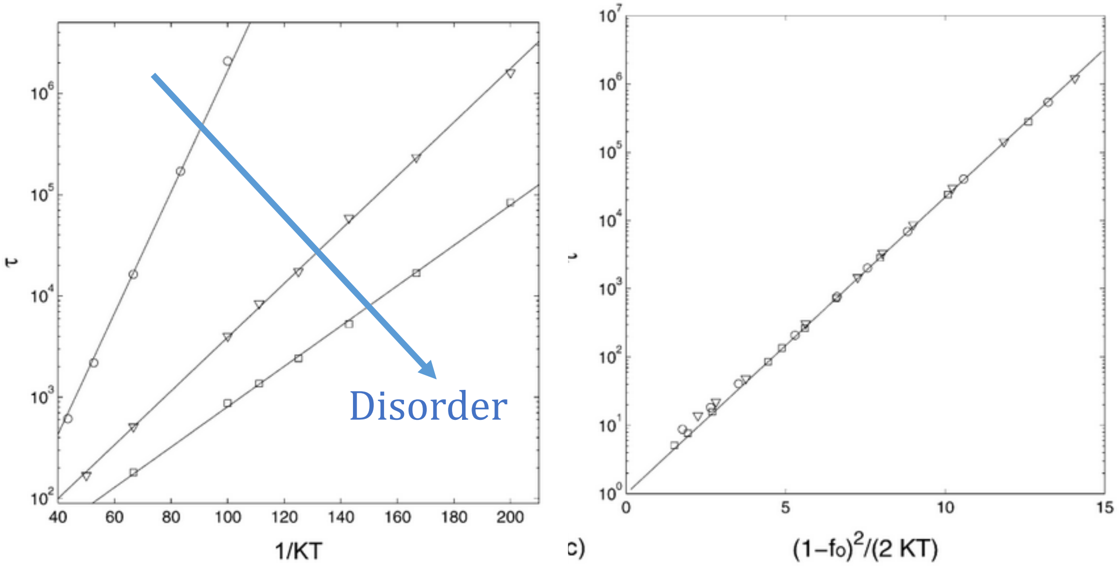


$$\tau \simeq \tau_0 \exp\left(\frac{(1-f_0)^2}{2KT_{\rm eff}}\right)$$

$$KT_{\rm eff} \simeq \frac{KT}{(1-\sqrt{\pi}\sigma_0/2(1-f_0))^2},$$

$$\tau_0 = \frac{2\sqrt{2\pi KT}}{(f_0-\sqrt{\pi}\sigma_0)[1+\exp(-\sqrt{\pi}\sigma_0(1-f_0)/KT)]}.$$

$$\tau \simeq \frac{\sqrt{2\pi KT}}{f_0} \exp\left(\frac{(1-f_0)^2}{2KT}\right)$$



The effect of disorder on the fracture nucleation process (S. Ciliberto, A. Guarino, R. Scorreti (2001))

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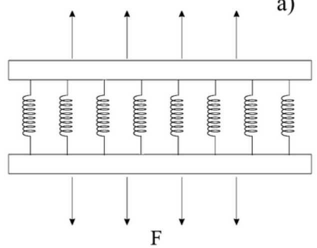
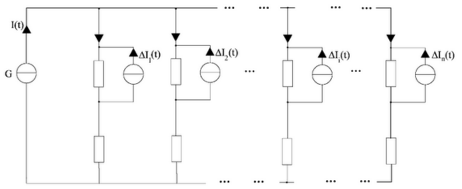


Stress redistribution is democratic

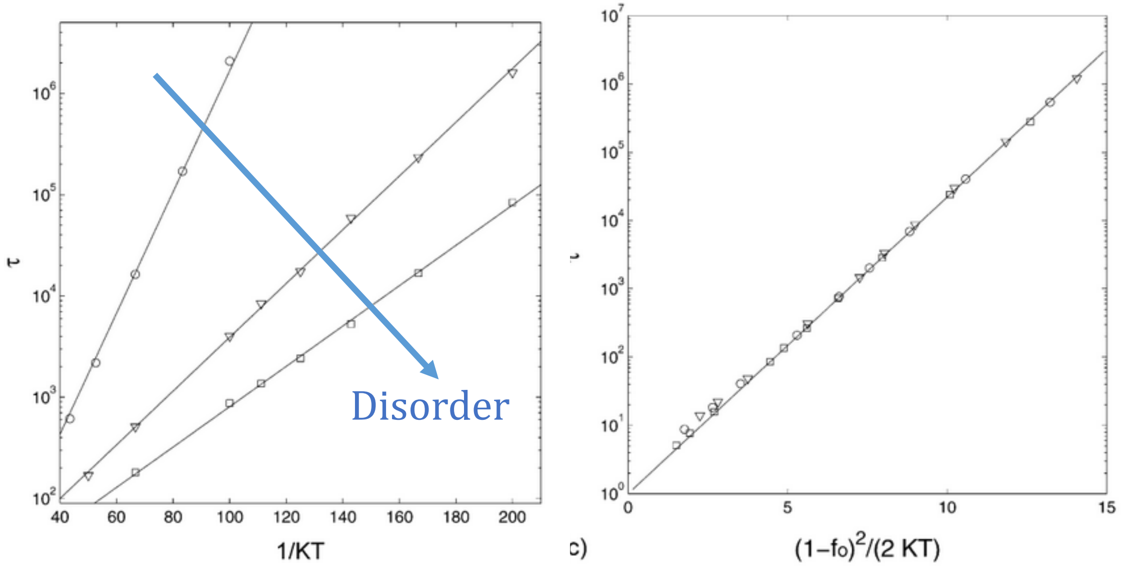


$$\tau \simeq \tau_0 \exp\left(\frac{(1-f_0)^2}{2KT_{\text{eff}}}\right)$$

$$KT_{\text{eff}} \simeq \frac{KT}{(1-\sqrt{\pi}\sigma_0/2(1-f_0))^2},$$

$$\tau_0 = \frac{2\sqrt{2\pi KT}}{(f_0-\sqrt{\pi}\sigma_0)[1+\exp(-\sqrt{\pi}\sigma_0(1-f_0)/KT)]}.$$

$$\tau \simeq \frac{\sqrt{2\pi KT}}{f_0} \exp\left(\frac{(1-f_0)^2}{2KT}\right)$$



- Disorder of the material « amplifies » the thermal noise and produces an effecive temperature
- The failure time decreases as the disorder noise increases (the more the medius heterogeneous, the smaller the failure time is)
- As the disorder noise increases, the derivative of failure time with respect to KT decreases (failure time becomes less sensitive to the effective value of thermal noise.

$$kT_{eff} \geq kT$$

2. Methods: PDM + KMC

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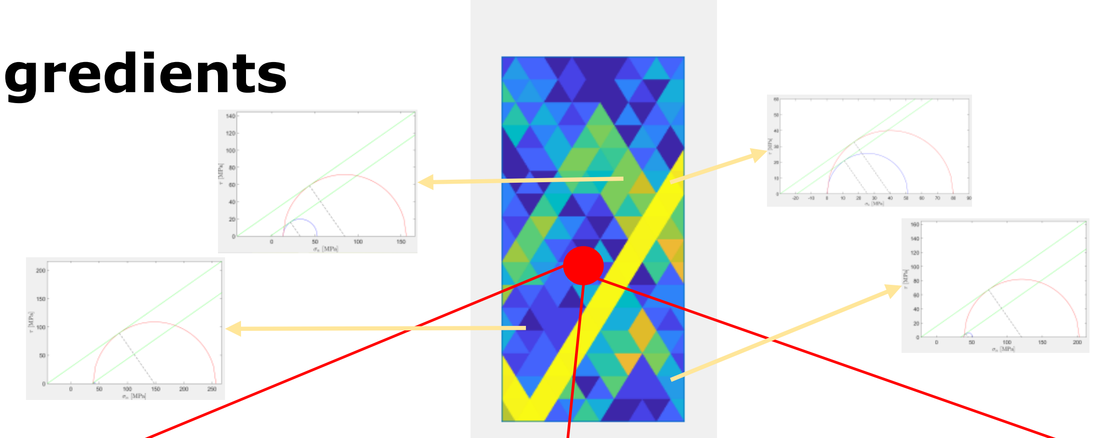
Main ingredients



Element i

All the elements

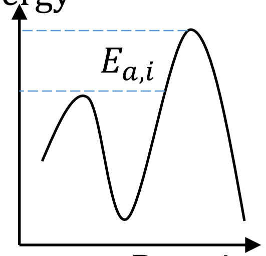
$\sum$

1 **Heterogeneity**

Cohesion C_i is set randomly
Gaussian distribution
(quenched)
$C_i \in [10, 30]$**MPa**

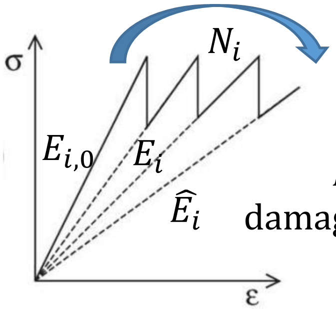
2 **Thermal activation**

$$\omega_i = \omega_0 \exp\left(-\frac{E_{a,i}}{k_B T}\right)$$



3 **Damage**

$$\widehat{\boldsymbol{E}_i} = (\boldsymbol{1} - \boldsymbol{D})\boldsymbol{E}_i$$



$D = 0.1$
damage parameter

4 **Elastic interaction**
(Solved with FEM)

$$\boldsymbol{\sigma_{ej}} = \boldsymbol{E_{ejkl}\varepsilon_{ej}}$$

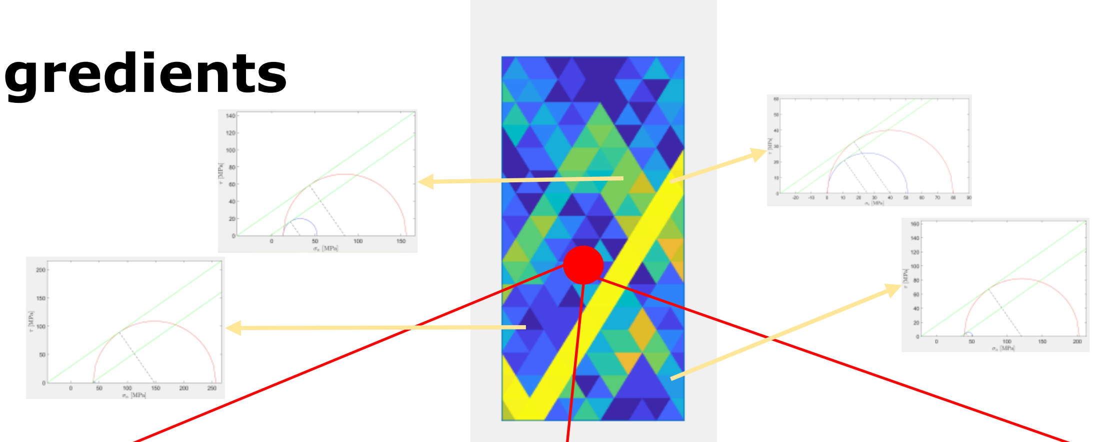
initial disorder (random variable) + stochastic process

Athermal damage (MC criterium)

Thermal activated damage (Probability)

2. Methods: PDM + KMC

Main ingredients



Element i

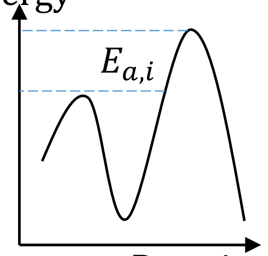
All the elements
$\sum$

1 **Heterogeneity**

Cohesion C_i is set randomly
Gaussian distribution
(quenched)
$C_i \in [10, 30]$**MPa**

2 **Thermal activation**

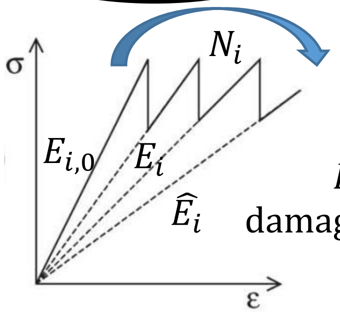
$$\omega_i = \omega_0 \exp\left(-\frac{E_{a,i}}{k_B T}\right)$$



3 **Damage**

Element does not dissapear (Memory effect)

$$\widehat{E}_i = (1 - D) E_i$$



$D = 0.1$
damage parameter

4 **Elastic interaction (Solved with FEM)**

Stress redistribution is not democratic

$$\boldsymbol{\sigma_{ej} = E_{ejkl} \varepsilon_{ej}}$$

initial disorder (random variable) + stochastic process

Athermal damage (MC criterium)

Thermal activated damage (Probability)

Theoretical solution: Expression for failure time (dependence of disorder)

$$\omega_i = \omega_0 \exp\left(-\frac{E_{a,i}}{k_B T}\right)$$
$$R_2 = \sum \omega_i$$
$$\Delta t = -\frac{\log u_2}{R_2}$$
$$t = t + \Delta t$$

$$t_f = -\frac{U}{N\omega_0}\int_U \log(u)p(u)\left[\int_{\Delta\sigma(u)} \exp\left(-\frac{V_{a,0}\Delta\sigma}{k_B T}\right)p(\Delta\sigma)d(\Delta\sigma)\right]^{-1} du,$$

We impose the activation volume V_a

Theoretical solution: Expression for failure time (dependence of disorder)

$$\omega_i = \omega_0 \exp\left(-\frac{E_{a,i}}{k_B T}\right)$$
$$\mathrm{R}_2 = \sum \omega_i$$
$$\Delta t = -\frac{\log u_2}{R_2}$$
$$t = t + \Delta t$$

$$t_f = -\frac{U}{N\omega_0} \int_U \log(u) p(u) \left[\int_{\Delta\sigma(u)} \exp\left(-\frac{V_{a,0}\Delta\sigma}{k_B T}\right) p(\Delta\sigma) d(\Delta\sigma) \right]^{-1} du,$$

$$t_f \sim \frac{1}{\omega_0} \exp\left(\frac{V_a \langle \Delta\sigma \rangle}{k_B T}\right) f(\delta)$$

Disorder effect

We impose the activation volume V_a

Theoretical solution: Expression for failure time (dependence of disorder)

$$\omega_i = \omega_0 \exp\left(-\frac{E_{a,i}}{k_B T}\right)$$
$$R_2 = \sum \omega_i$$
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We impose the activation volume V_a

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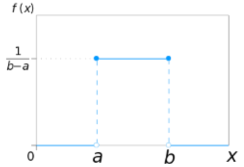
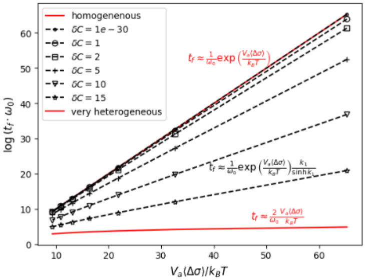
Disorder effect



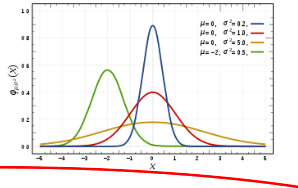
Uniform distribution along the creep behaviour



$$t_f = \frac{1}{\omega_0}\exp\left(\frac{V_{a,0}\langle\Delta\sigma\rangle}{k_B T}\right)\frac{U}{N}\frac{k_1}{\sinh(k_1)}.$$

$$k_1 = Va\delta/k_B T$$

Gaussian distribution along the creep behaviour



$$t_f = \frac{U}{N\omega_0}\exp\left(\frac{V_{a,0}\langle\Delta\sigma\rangle}{k_B T}\right)\exp\left[-\frac{1}{2}\left(\frac{V_{a,0}\delta_{\Delta\sigma}}{k_B T}\right)^2\right] 2\left[\operatorname{erfc}\left[\frac{1}{\sqrt{2}}\left(\frac{V_{a,0}\delta_{\Delta\sigma}}{k_B T} - \frac{\langle\Delta\sigma\rangle}{\delta_{\Delta\sigma}}\right)\right]\right]^{-1}$$

Theoretical solution: Expression for failure time (dependence of disorder)

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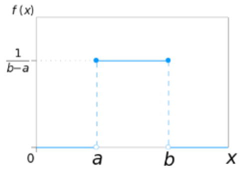
We impose the activation volume V_a

$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T}\right) f(\delta)$$

Disorder effect



Uniform distribution along the creep behaviour

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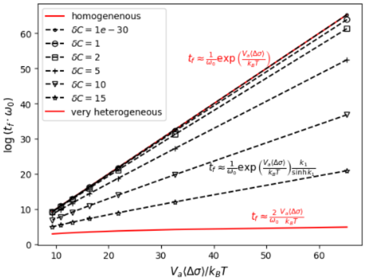
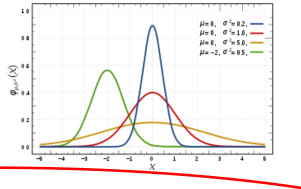


Gaussian distribution along the creep behaviour



$$t_f = \frac{U}{N\omega_0}\exp\left(\frac{V_{a,0}\langle\Delta\sigma\rangle}{k_B T}\right)\exp\left[-\frac{1}{2}\left(\frac{V_{a,0}\delta_{\Delta\sigma}}{k_B T}\right)^2\right] 2\left[\mathrm{erfc}\left[\frac{1}{\sqrt{2}}\left(\frac{V_{a,0}\delta_{\Delta\sigma}}{k_B T} - \frac{\langle\Delta\sigma\rangle}{\delta_{\Delta\sigma}}\right)\right]\right]^{-1}$$

$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T}\right) f(\delta) \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T_{eff}}\right)$$

Theoretical solution: Expression for failure time (dependence of disorder)

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$$\omega_i = \omega_0 \exp\left(-\frac{E_{a,i}}{k_B T}\right)$$
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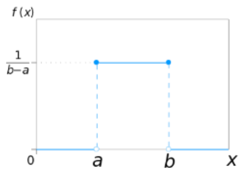
We impose the activation volume V_a

$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T}\right) f(\delta)$$

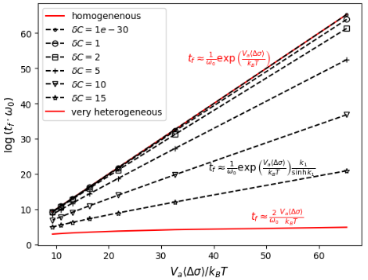
Disorder effect



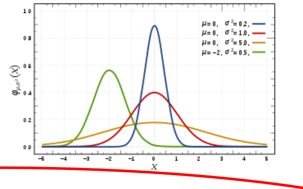
Uniform distribution along the creep behaviour

$$t_f = \frac{1}{\omega_0}\exp\left(\frac{V_{a,0}\langle\Delta\sigma\rangle}{k_B T}\right)\frac{U}{N}\frac{k_1}{\sinh(k_1)}.$$

$$k_1 = V a\delta / k_B T$$

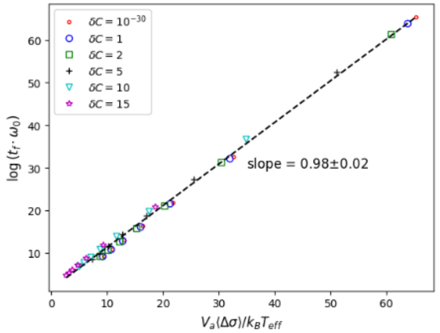


Gaussian distribution along the creep behaviour



$$t_f = \frac{U}{N\omega_0}\exp\left(\frac{V_{a,0}\langle\Delta\sigma\rangle}{k_B T}\right)\exp\left[-\frac{1}{2}\left(\frac{V_{a,0}\delta_{\Delta\sigma}}{k_B T}\right)^2\right] 2\left[\operatorname{erfc}\left[\frac{1}{\sqrt{2}}\left(\frac{V_{a,0}\delta_{\Delta\sigma}}{k_B T} - \frac{\langle\Delta\sigma\rangle}{\delta_{\Delta\sigma}}\right)\right]\right]^{-1}$$

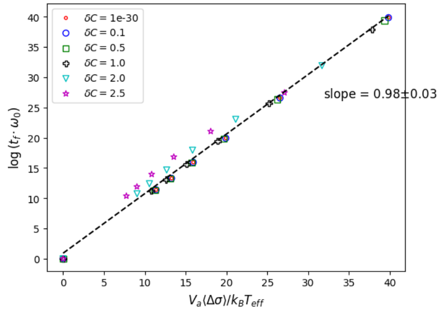
$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T}\right) f(\delta) \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T_{eff}}\right)$$



$$k_B T_{eff} \approx \frac{k_B T}{1 - \frac{\delta}{\langle\Delta\sigma\rangle}}$$

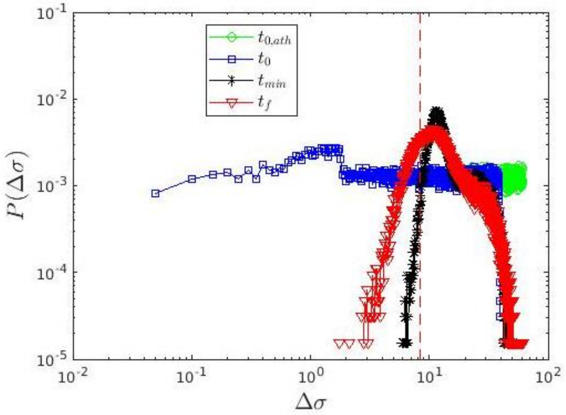
Disorder effect

$$k_B T_{eff} \approx \frac{k_B T}{1 - \alpha\frac{\delta^2}{\langle\Delta\sigma\rangle^2}}$$



Expression for failure time ? – reality: complexity to track the evolution of the stress gap along the deformation

$p(\Delta\sigma)$ is variable



Expression for failure time ? – reality: complexity to track the evolution of the stress gap along the deformation

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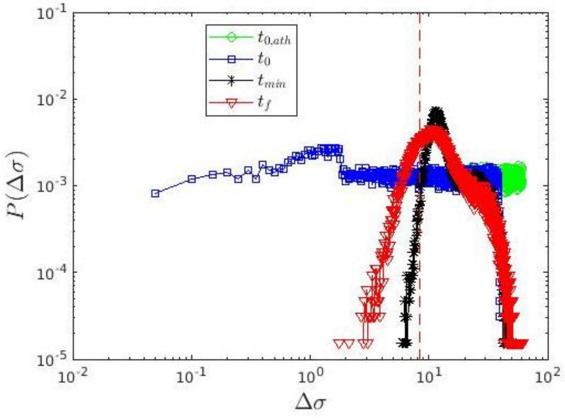
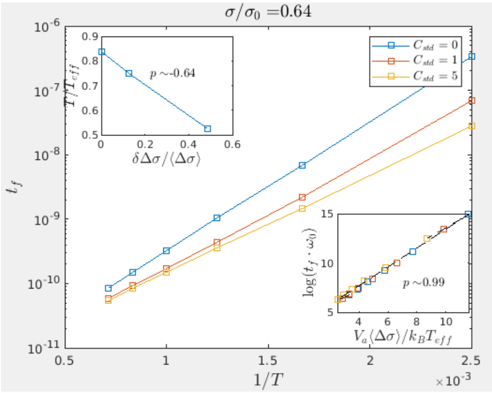
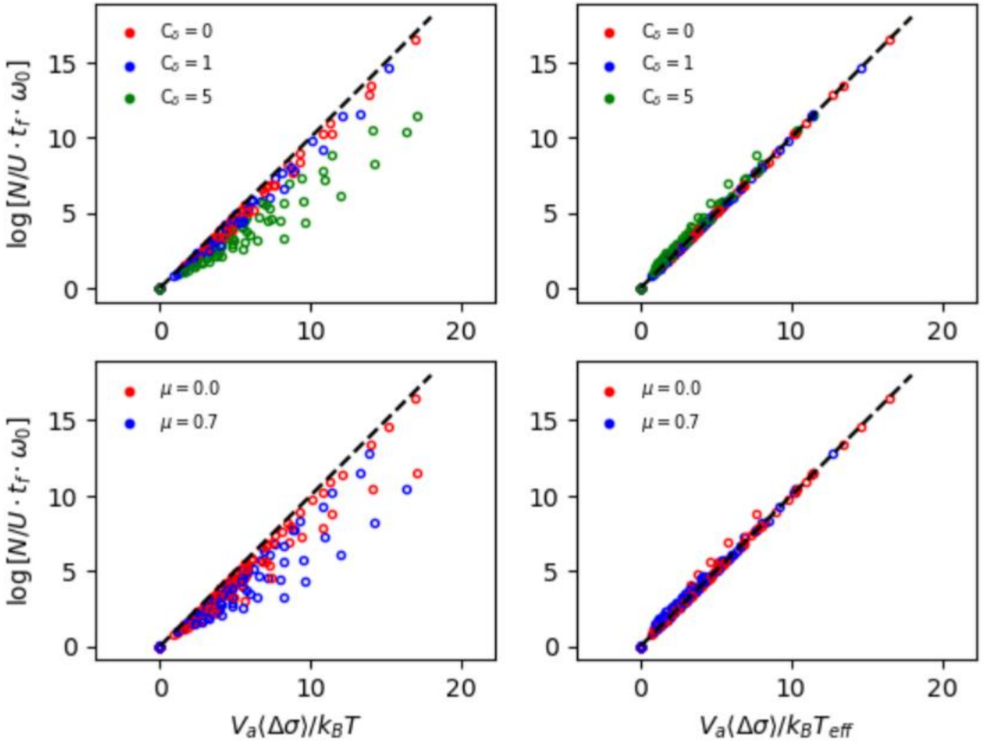


However





Expression for failure time ? – reality: complexity to track the evolution of the stress gap along the deformation

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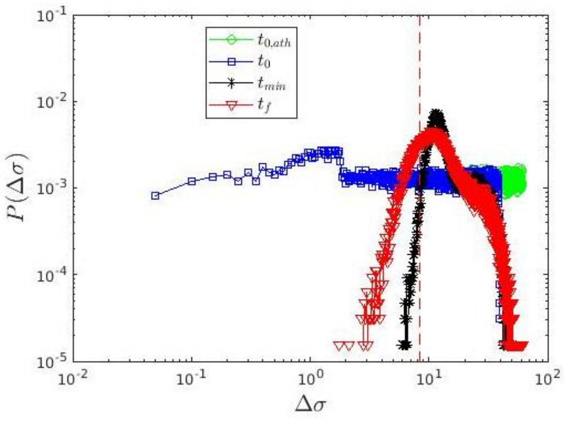
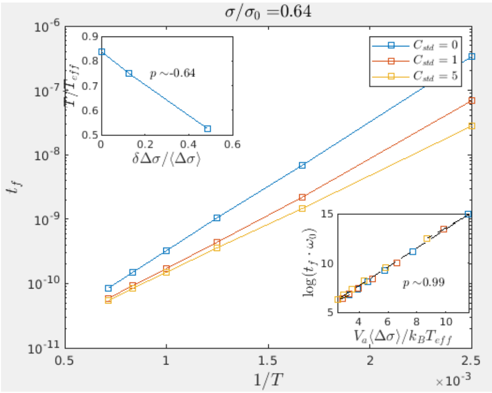
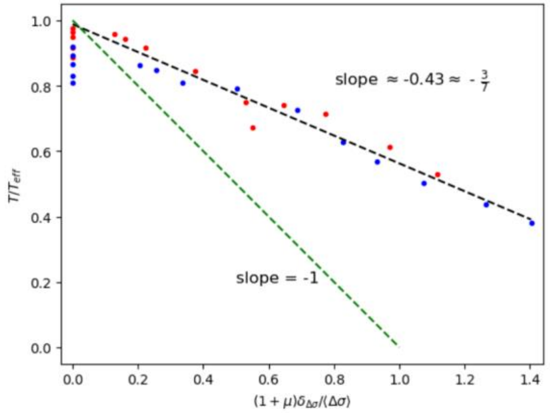
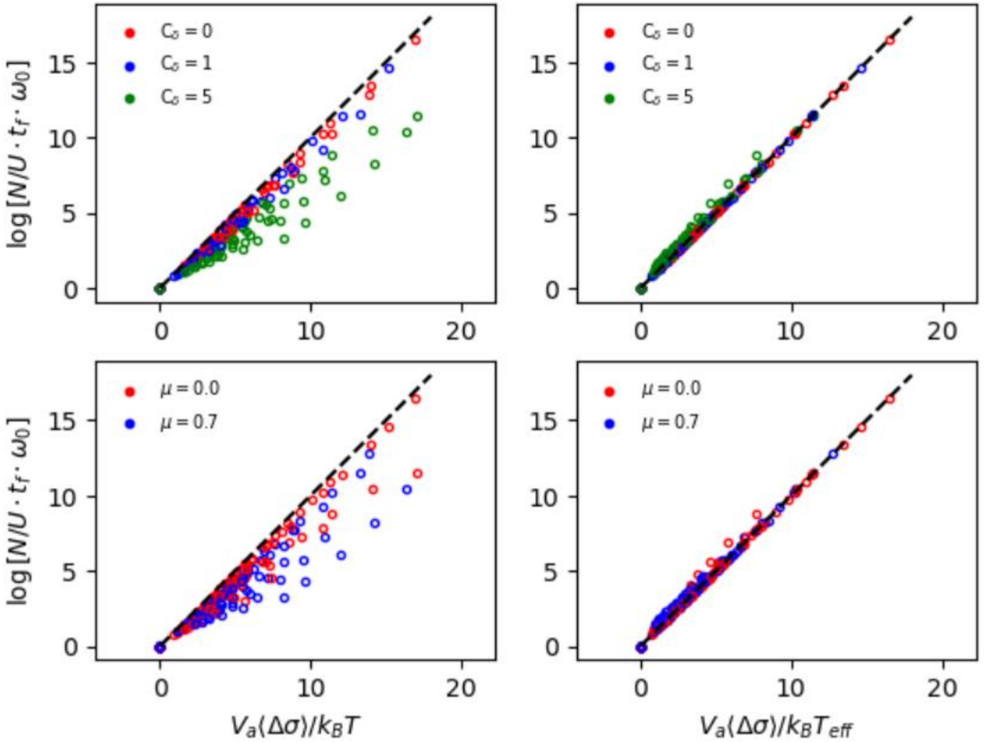


However







$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_BT}\right)f(\delta)$$
$$\sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_BT_{eff}}\right)$$

$$k_BT_{eff} \approx \frac{k_BT}{1-\alpha\dfrac{\delta}{\langle\Delta\sigma\rangle}}$$

$$\alpha \approx \frac{3}{7}(1+\mu)$$
$$\frac{\delta}{\langle\Delta\sigma\rangle} > 0$$

$$k_BT_{eff} \geq k_BT$$
$$\langle\Delta\sigma\rangle \sim \Delta\sigma_{MACRO}$$

Expression for failure time ? – reality: complexity to track the evolution of the stress gap along the deformation

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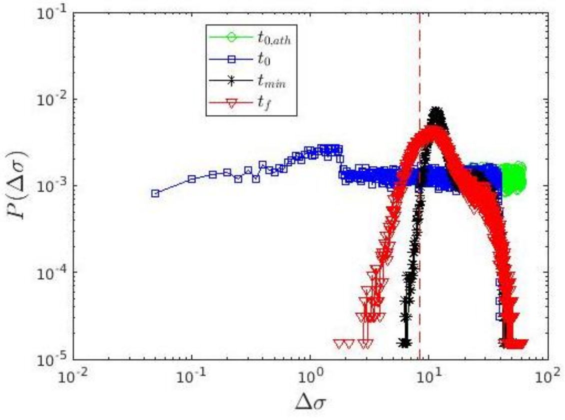
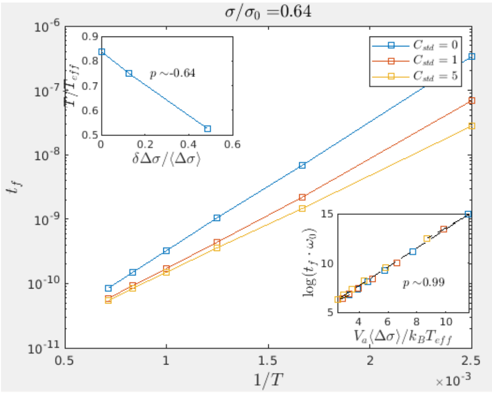
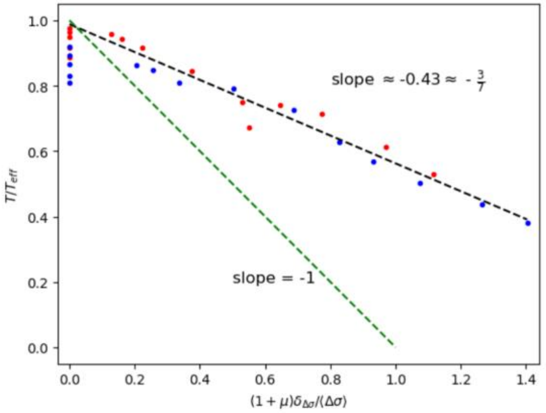
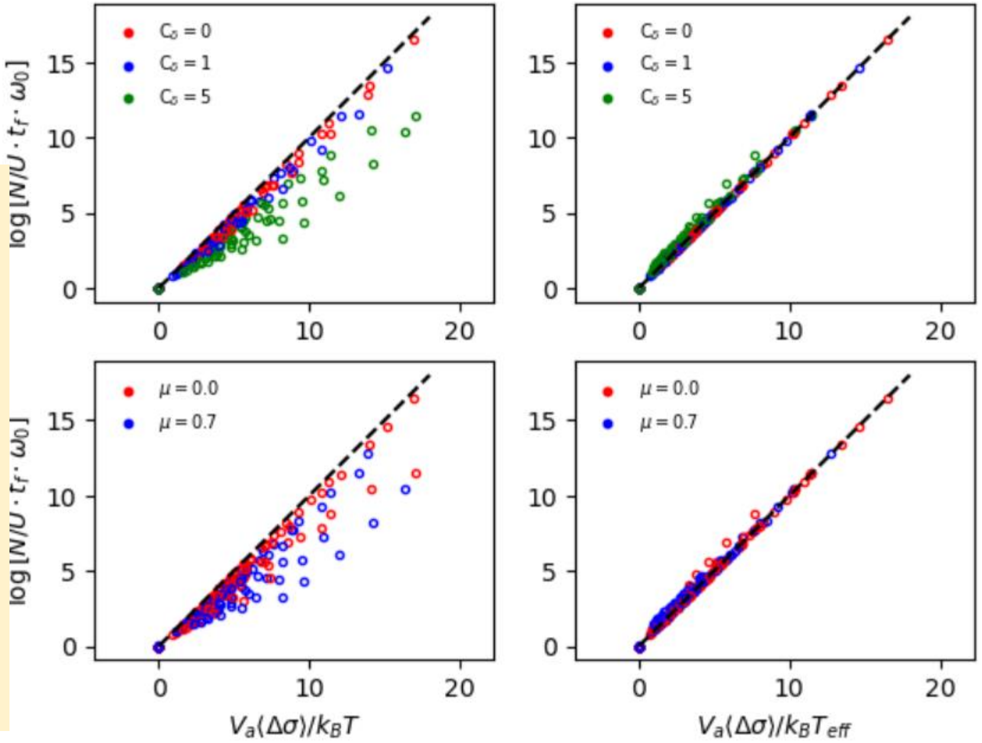
$p(\Delta\sigma)$ is variable



However







- Initial quenched disorder of the material « amplifies » the temperature and produces an effective one
- The failure time decreases as the disorder increases (the more the medius heterogeneous, the smaller the failure time is)
- As the disorder increases, the derivative of failure time with respect to KT decreases (failure time becomes less sensitive to the effective value of thermal noise.

$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T}\right) f(\delta) \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T_{eff}}\right)$$

$$k_B T_{eff} \approx \frac{k_B T}{1 - \alpha \frac{\delta}{\langle\Delta\sigma\rangle}}$$

$$\alpha \approx \frac{3}{7}(1+\mu)$$

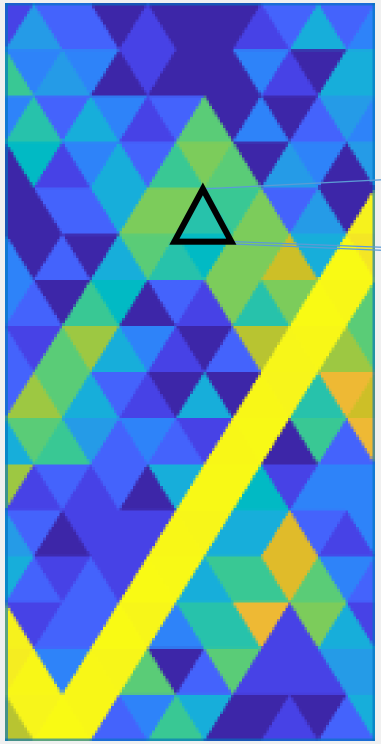
$$\frac{\delta}{\langle\Delta\sigma\rangle} > 0$$

$$k_B T_{eff} \geq k_B T$$

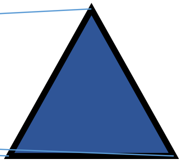
$$\langle\Delta\sigma\rangle \sim \Delta\sigma_{MACRO}$$

Problem:

Macroscale



Microscale

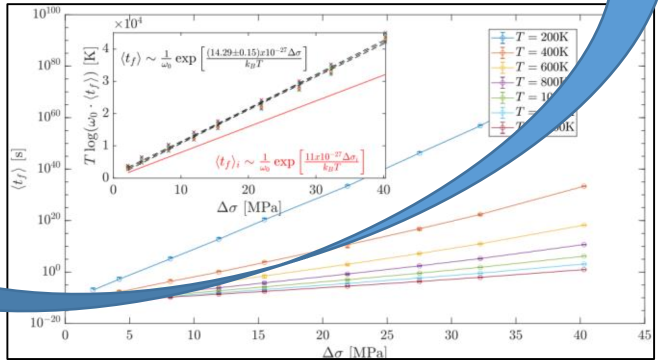


$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T}\right) f(\delta)$$

$$t_f \sim \frac{1}{\omega_0}\exp\left(\frac{V_a\langle\Delta\sigma\rangle}{k_B T_{eff}}\right)$$

Arrhenius law

The obtention of the activation volume at macroscale is more complex than only use the Arrhenius expression



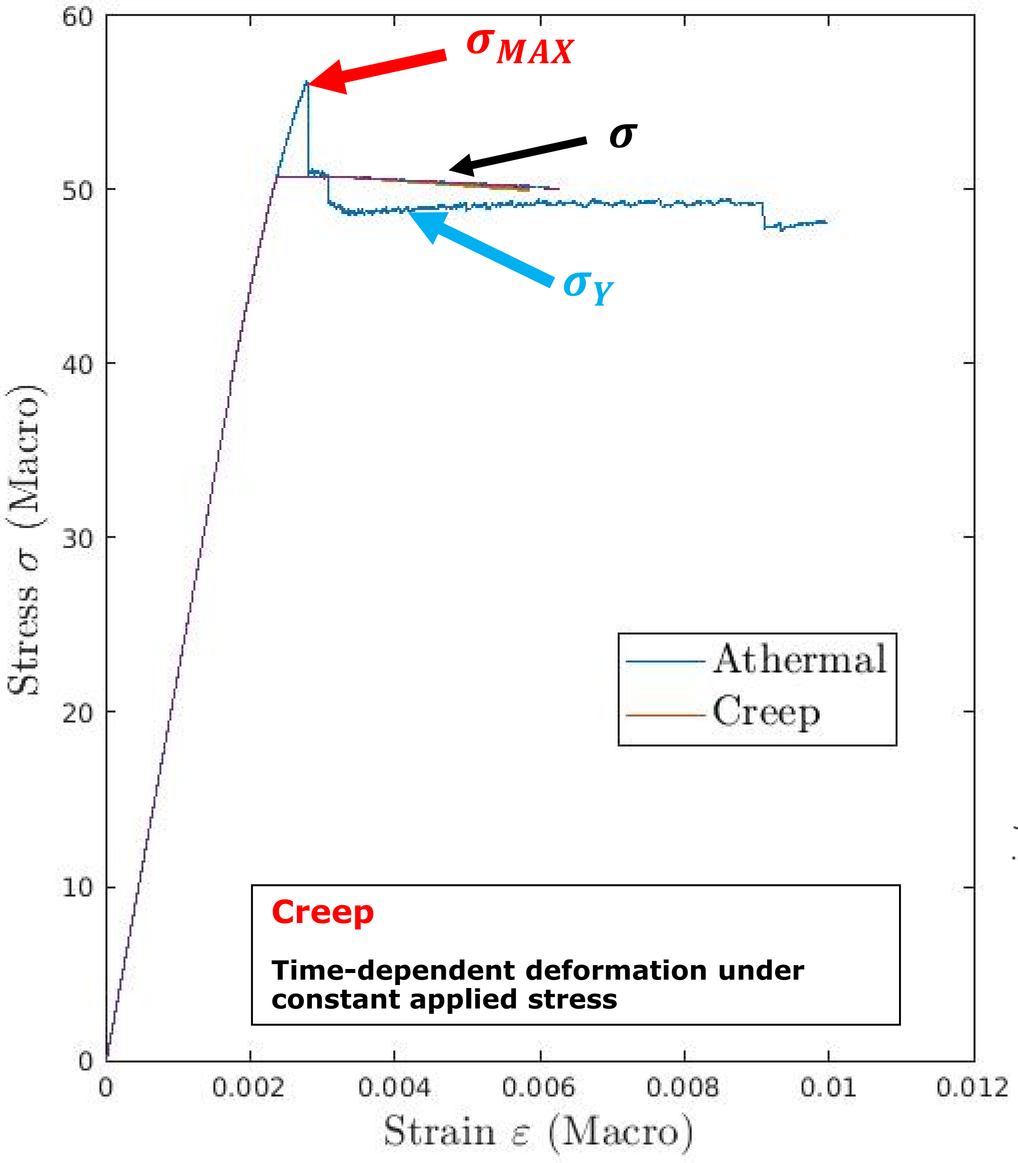
Downscaling

It is important to include a disorder effect to compute the real activation volume

16’

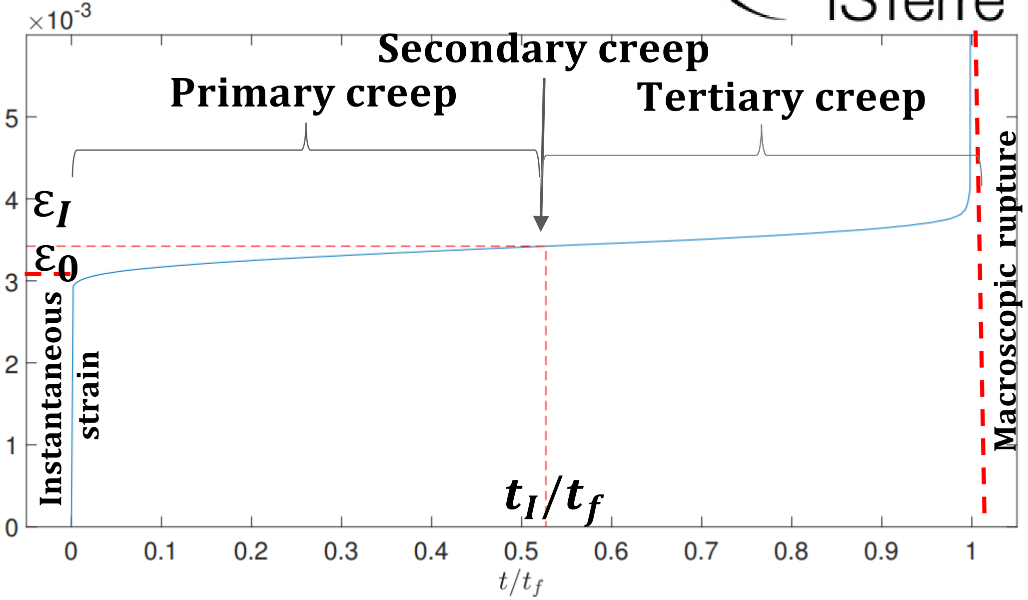
Quenched initial uniform disorder

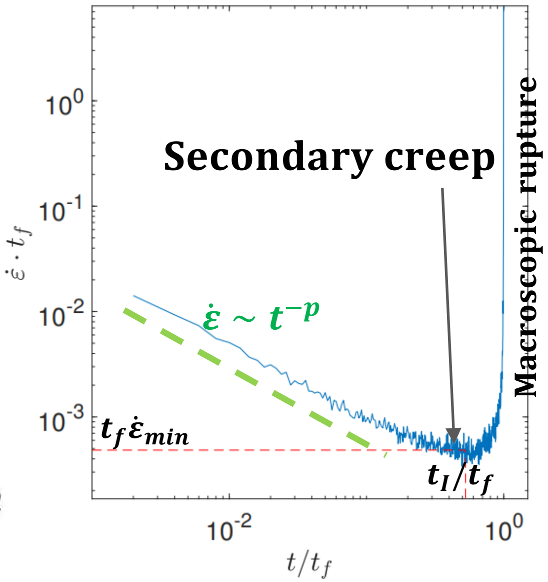
1. Presentation of the problem

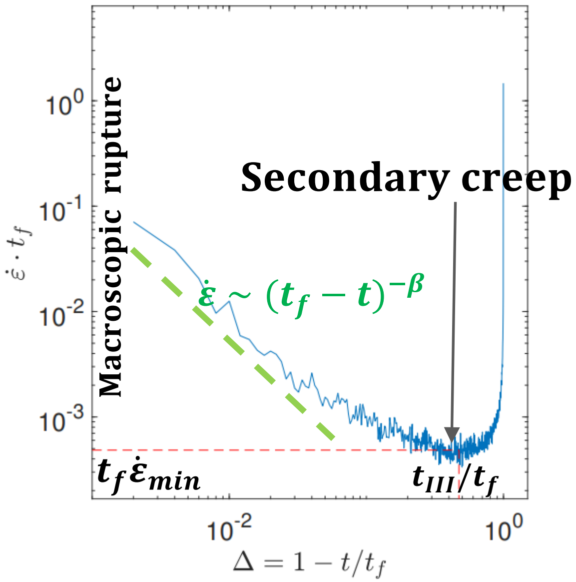


Creep

Time-dependent deformation under constant applied stress





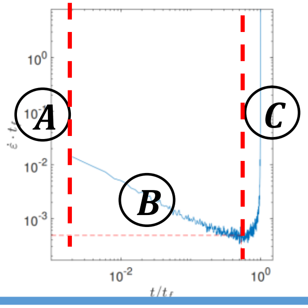


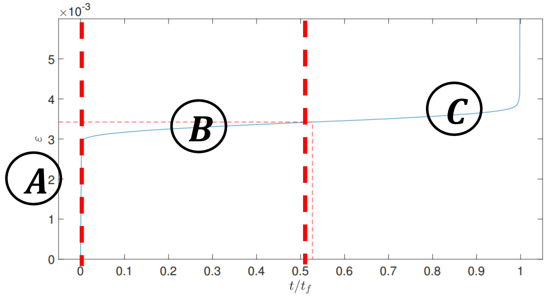
4. Results

4.4 Acceleration of deformation during the tertiary creep

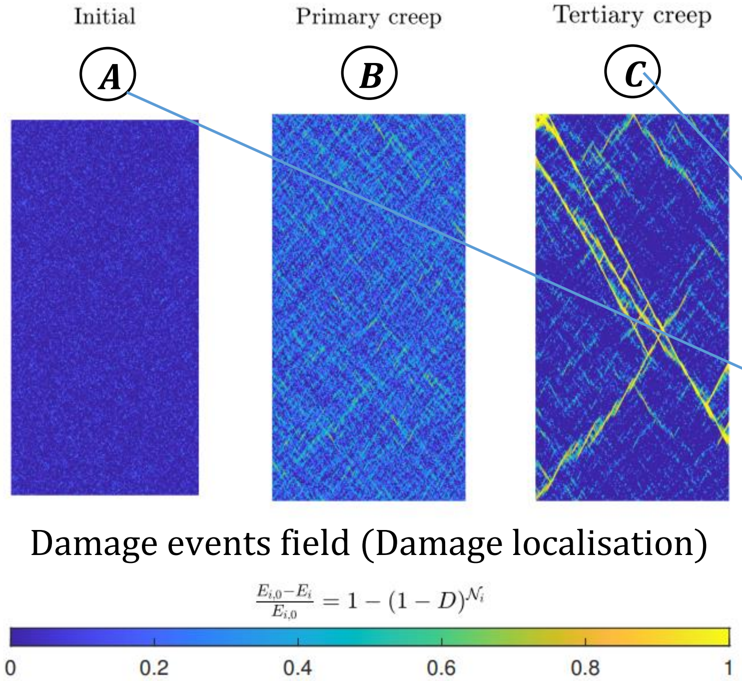
Spatial correlation of damage events

System Sizes	$L = 128$
Number of simulations	$N = 1$
Temperature	$T = 800$ K
Stress ratio	$\sigma/\sigma_Y = 0{,}72 \quad (\sigma = 0{,}72\sigma_Y)$





$dL^{-1} < r < \sqrt{5}\, d$



Damage events field (Damage localisation)

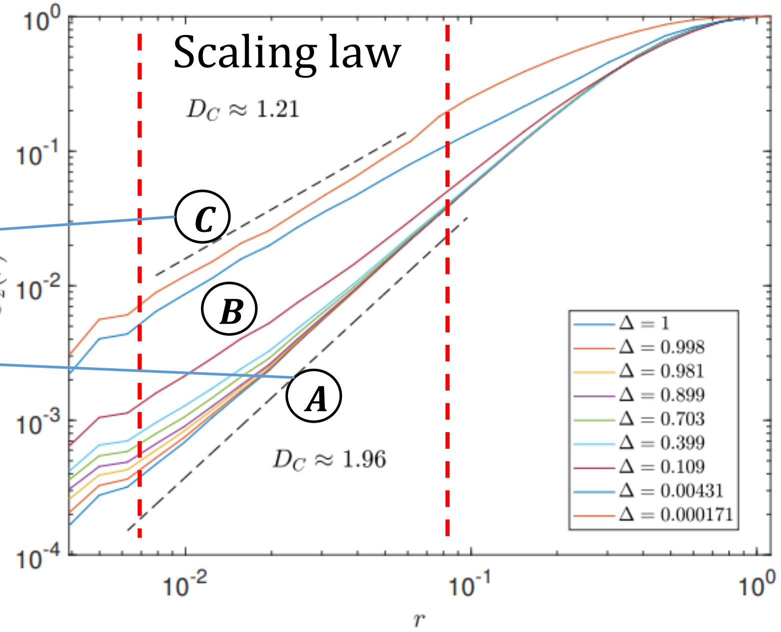
$$\frac{E_{i,0}-E_i}{E_{i,0}} = 1-(1-D)^{N_i}$$

$$C_2(r) = \frac{2\aleph_r(R<r)}{\aleph(\aleph-1)} \sim r^{D_c}$$

Hirata et al., 1987

Fractal behaviour of damage events??

$D_c \approx 1$	*Perfect damage localisation*
$D_c \approx 2$	*Homogeneous damage*

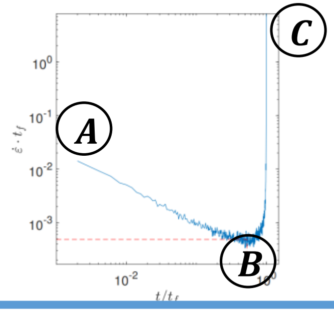


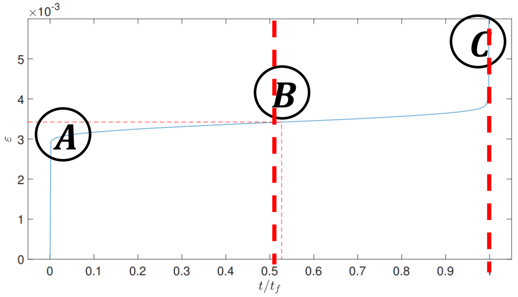
4. Results

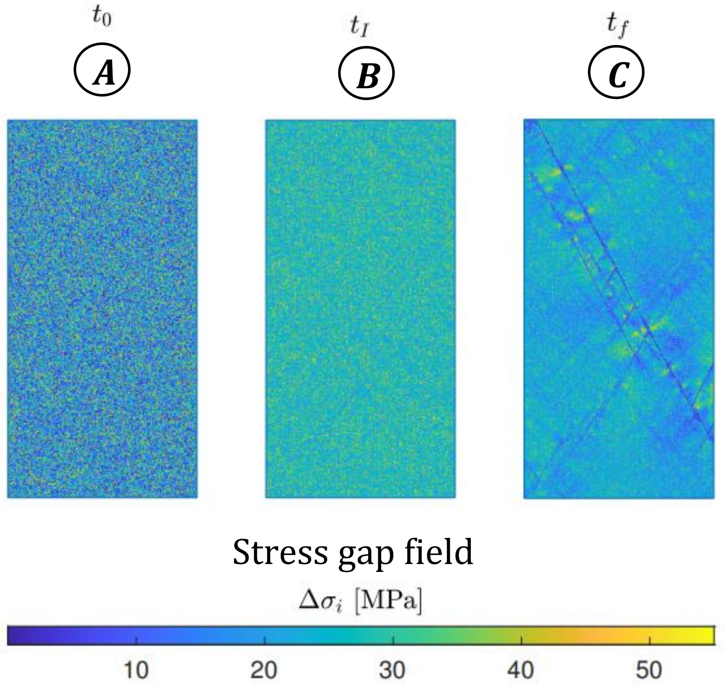
4.4 Acceleration of deformation during the tertiary creep

Stress gap distribution

System Sizes	$L = 128$
Number of simulations	$N = 1$
Temperature	$T = 800$ K
Stress ratio	$\sigma/\sigma_Y = 0{,}72 \quad (\sigma = 0{,}72\sigma_Y)$
Macro stress gap	$\Delta\sigma \approx 11$ MPa





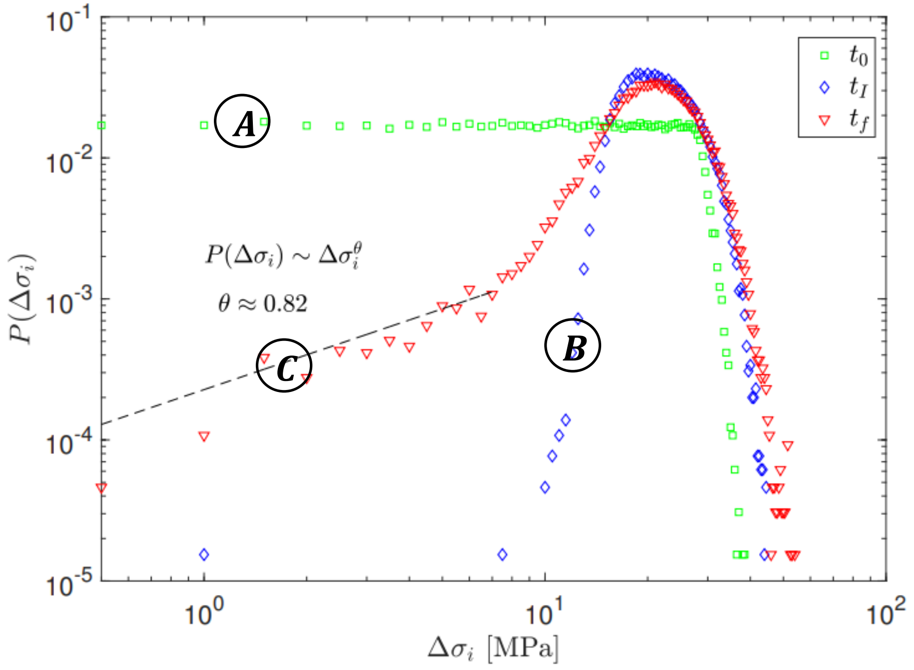


Stress gap field

$P(\Delta\sigma_i) \sim \Delta\sigma_i^{\theta}$

Lin et al., 2014

Power law tail towards small stress gaps

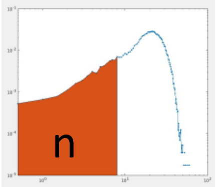


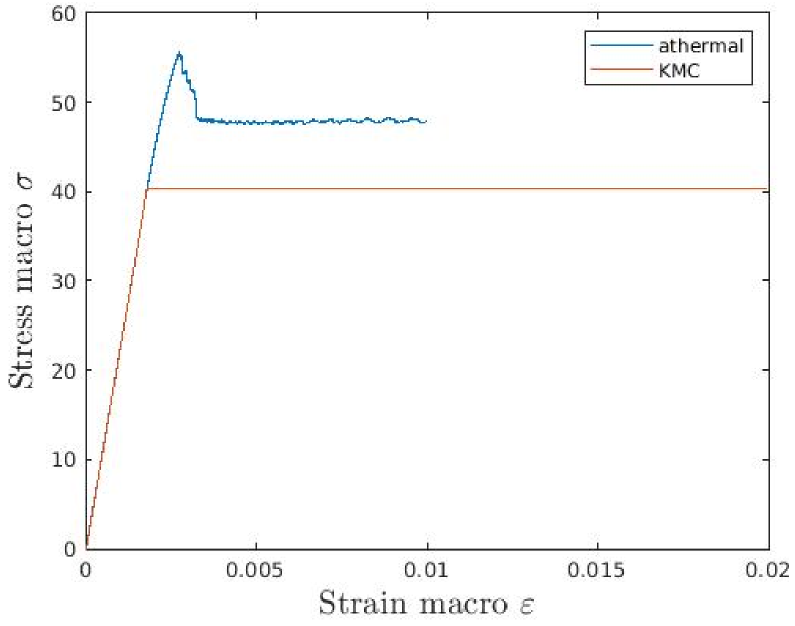
4. Results

4.1 Size effect of the stress gap

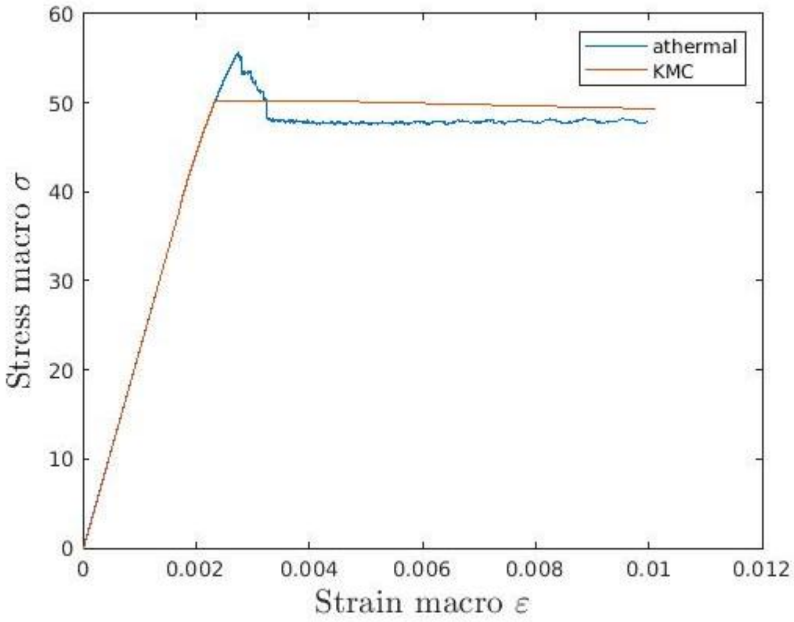
System Sizes	$L = \{8,16,32,64\}$
Number of simulations	$N = \{291, 68,17,4\}$
Temperature	$T = 800$ K
Stress ratio	σ/σ_{MAX} σ/σ_Y
Macro stress gap	

Size effect of the stress gap

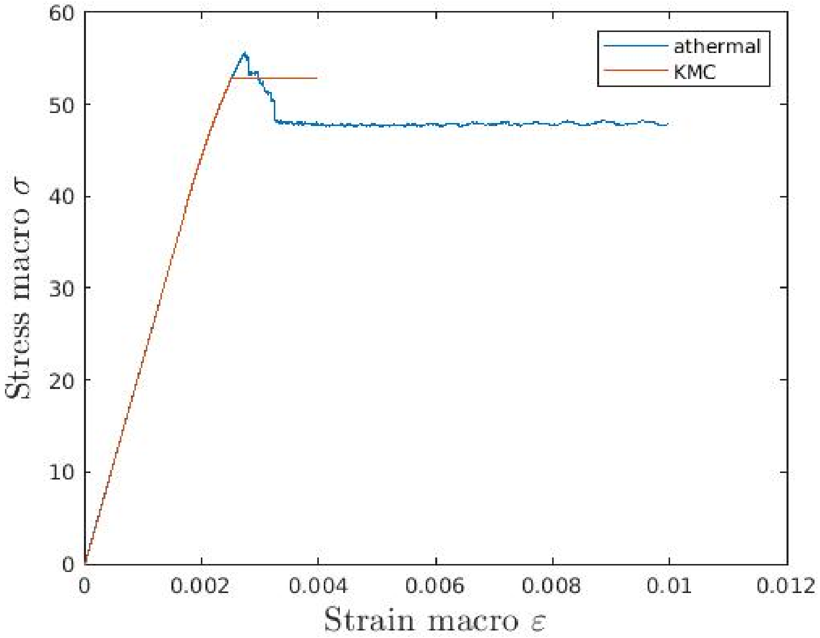




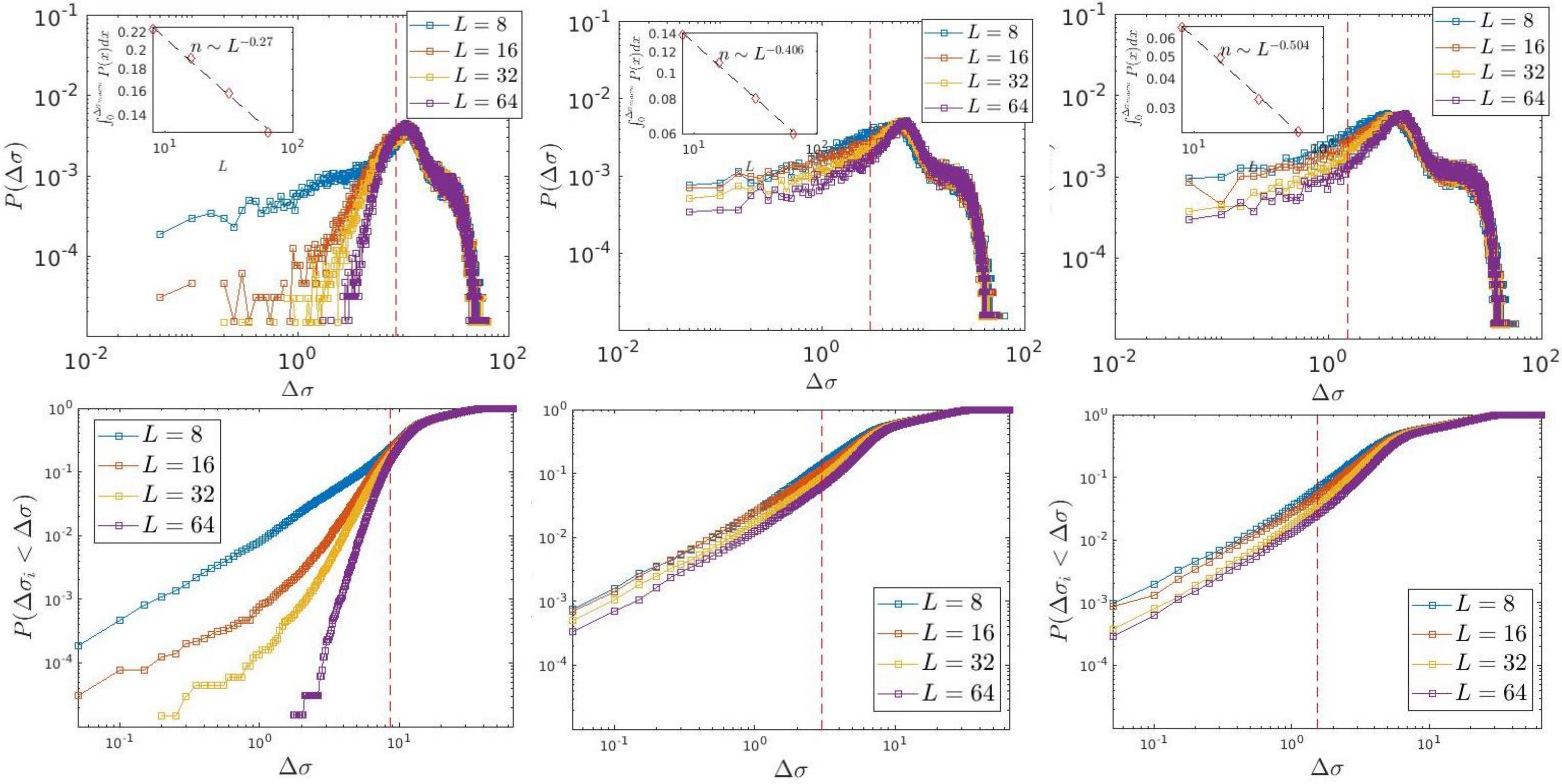
$\Delta\sigma \approx$ 8,57MPa
$\sigma/\sigma_{MAX} = 0{,}72$ $\sigma/\sigma_Y \approx 0{,}80$



$\Delta\sigma \approx$ 3,01MPa
$\sigma/\sigma_{MAX} = 0{,}90$ $\sigma/\sigma_Y \approx 0{,}99$



$\Delta\sigma \approx$ 1,54MPa
$\sigma/\sigma_{MAX} = 0{,}95$ $\sigma/\sigma_Y \approx 1{,}04$



$\Delta\sigma \approx 8{,}57\text{MPa}$
$\sigma/\sigma_{MAX} = 0{,}72 \quad \sigma/\sigma_Y \approx 0{,}80$

$\Delta\sigma \approx 3{,}01\text{MPa}$
$\sigma/\sigma_{MAX} = 0{,}90 \quad \sigma/\sigma_Y \approx 0{,}99$

$\Delta\sigma \approx 1{,}54\text{MPa}$
$\sigma/\sigma_{MAX} = 0{,}95 \quad \sigma/\sigma_Y \approx 1{,}04$

4. Results
4.4 Stress ratio effect

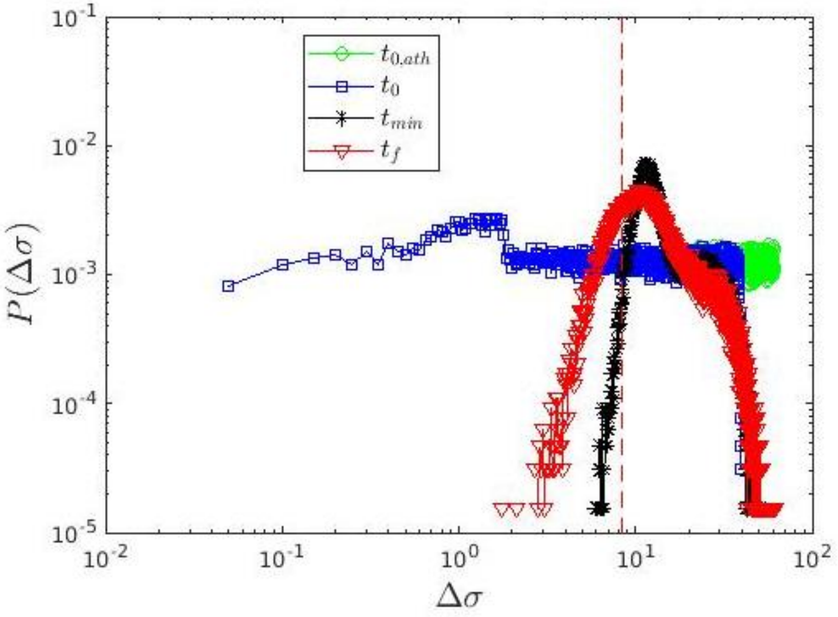
System Sizes	$L = 64$
Number of simulations	$N = 1$
Temperature	$T = 800$ K
Stress ratio	σ/σ_{MAX} σ/σ_Y
Macro stress gap	

Stress ratio dependence

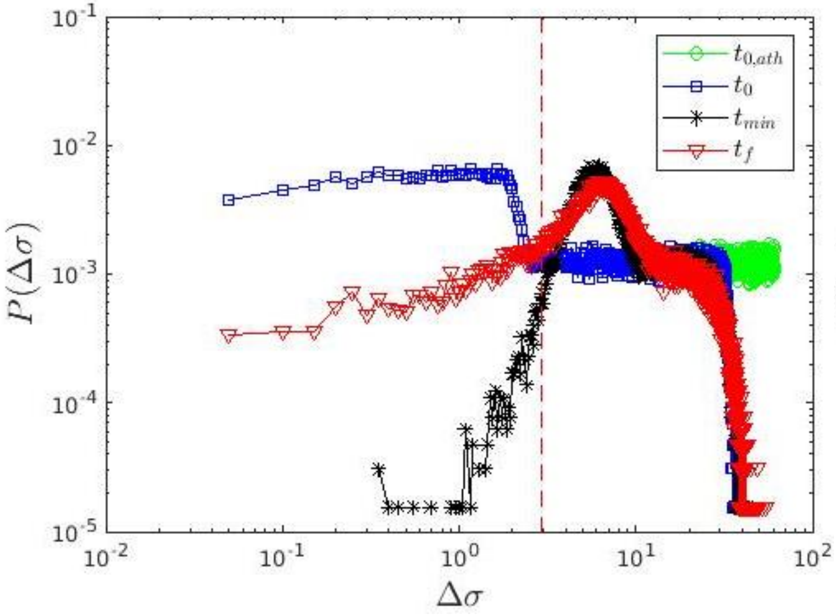
$\boldsymbol{\sigma/\sigma_{MAX}}$ →

$\boldsymbol{\Delta\sigma}$ ←

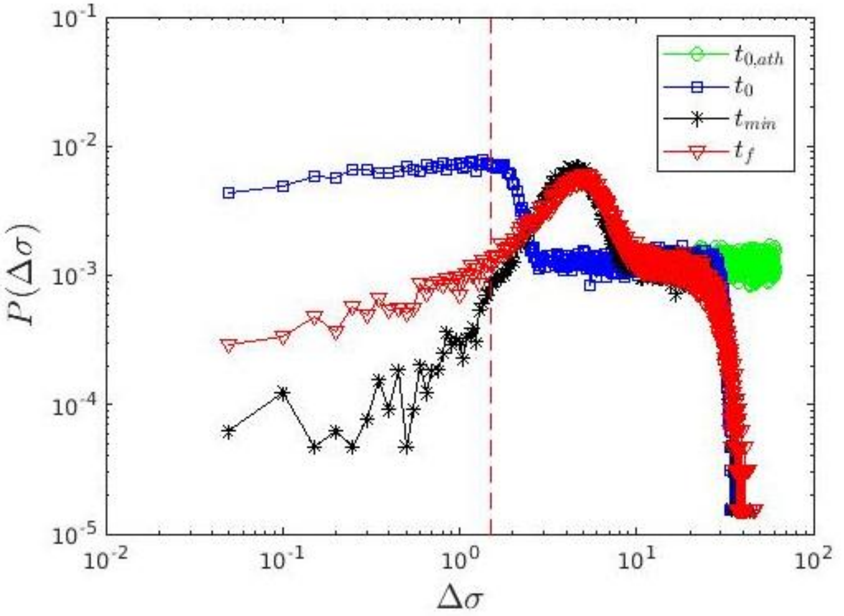
$t_f = 6{,}6x10^{-12}$



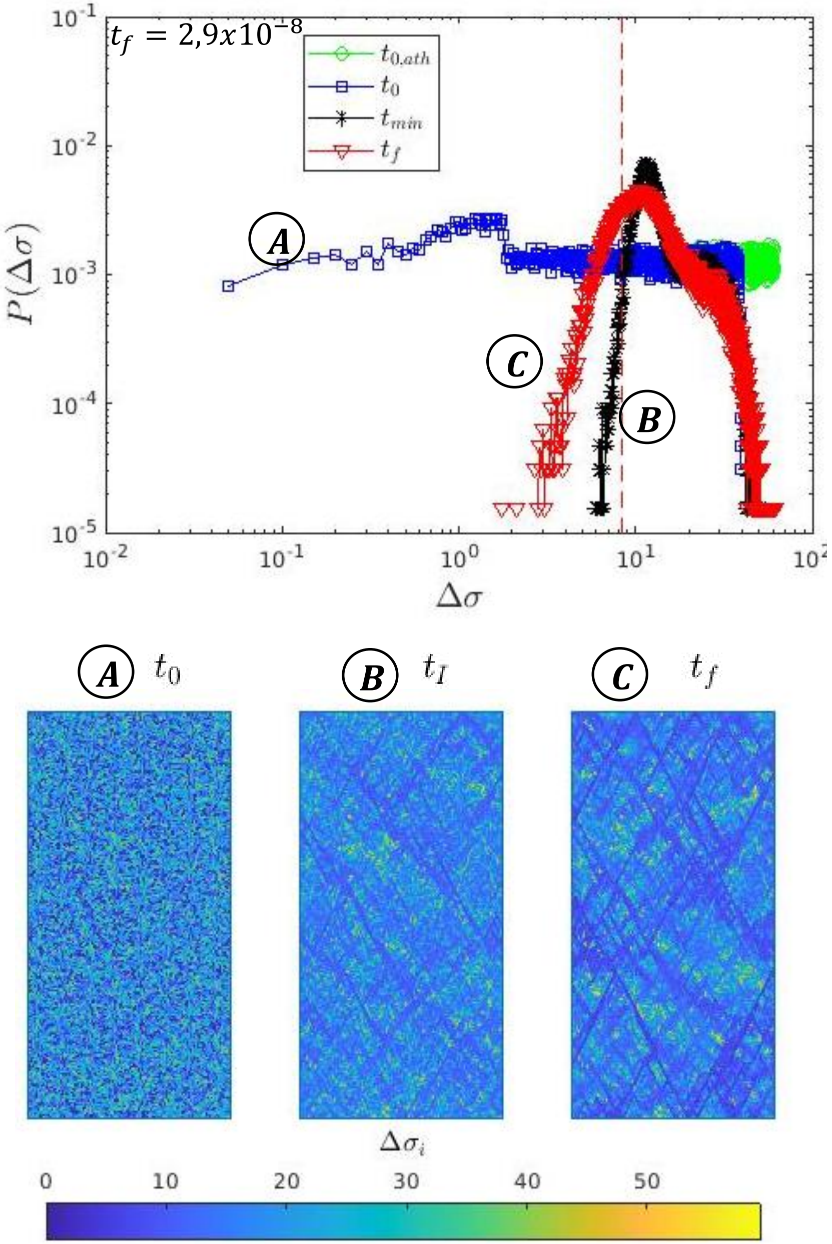
$\Delta\sigma \approx 8{,}57$MPa
$\sigma/\sigma_{MAX} = 0{,}72$ $\sigma/\sigma_Y \approx 0{,}80$



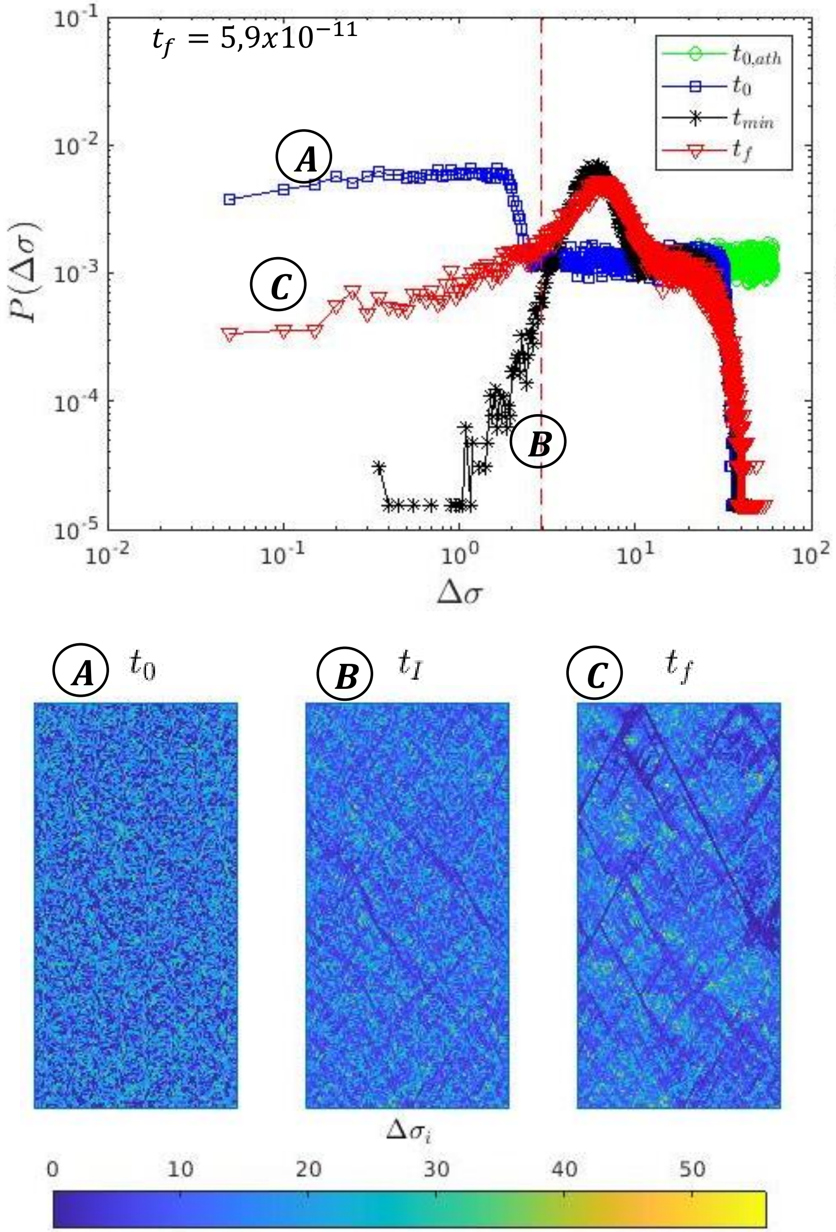
$\Delta\sigma \approx 3{,}01$MPa
$\sigma/\sigma_{MAX} = 0{,}90$ $\sigma/\sigma_Y \approx 0{,}99$



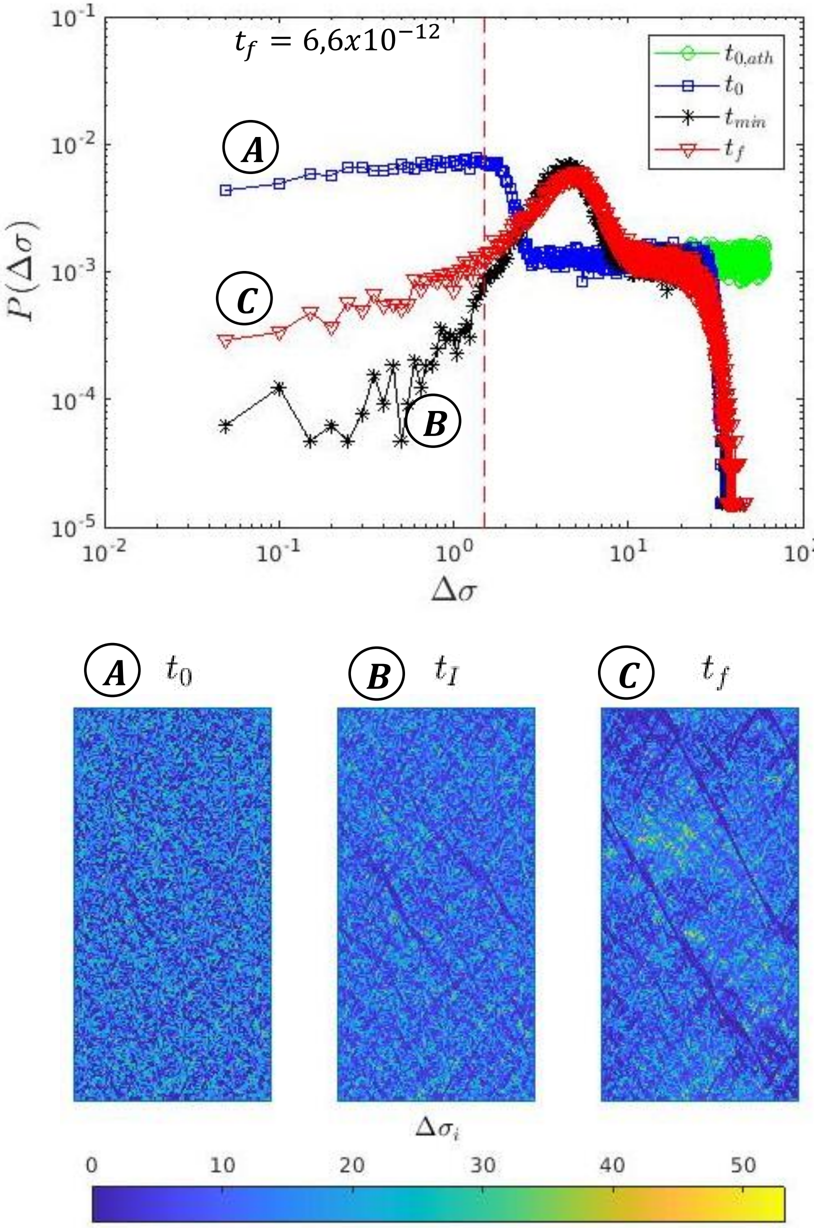
$\Delta\sigma \approx 1{,}54$MPa
$\sigma/\sigma_{MAX} = 0{,}95$ $\sigma/\sigma_Y \approx 1{,}04$



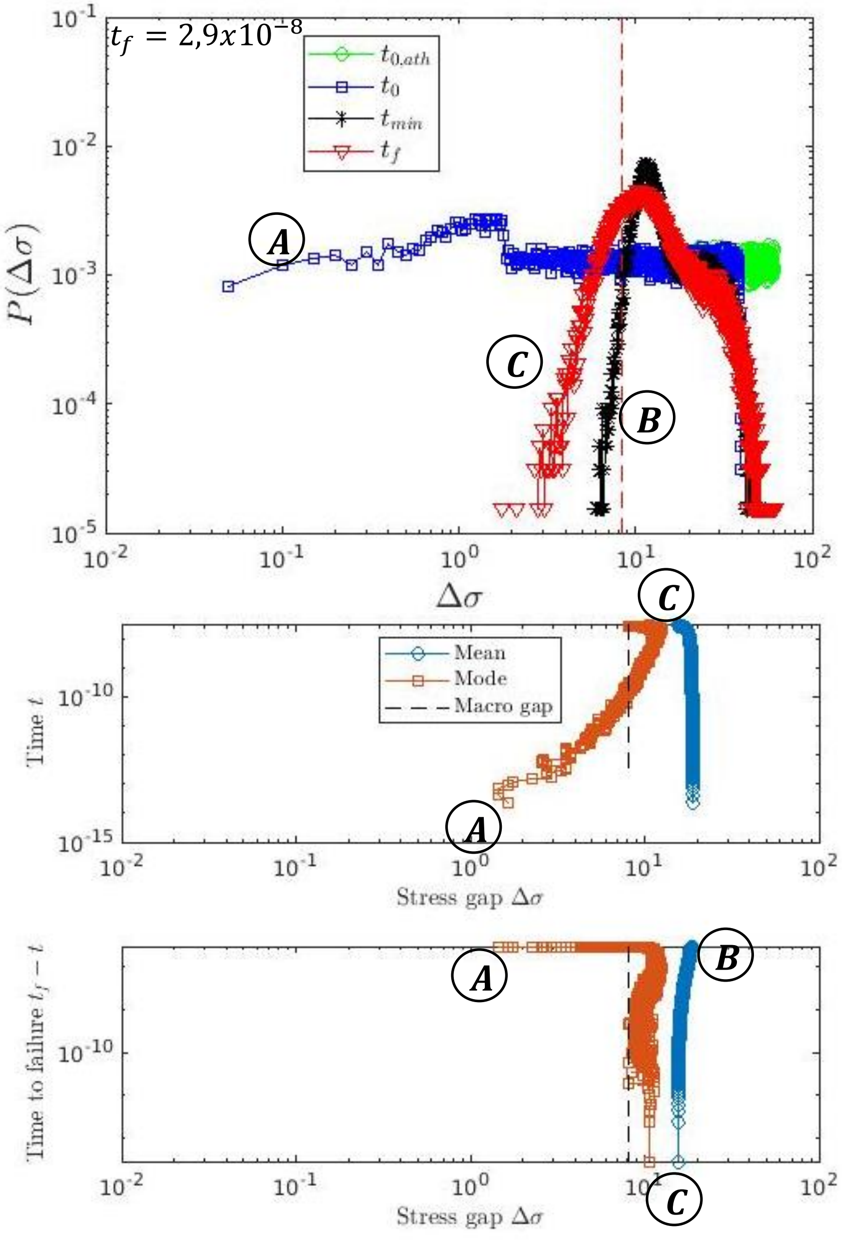
$\Delta\sigma \approx 8,57$MPa
$\sigma/\sigma_{MAX} = 0,72 \quad \sigma/\sigma_Y \approx 0,80$



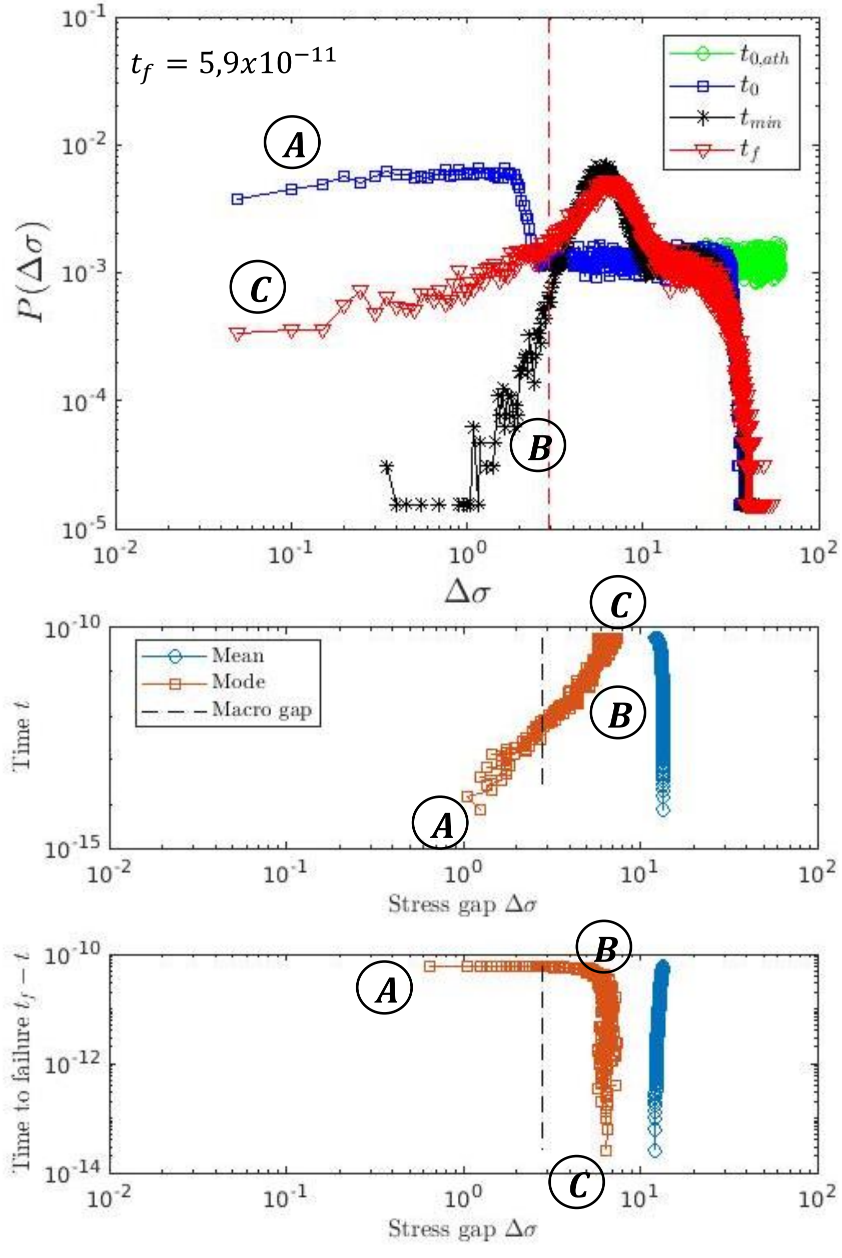
$\Delta\sigma \approx 3,01$MPa
$\sigma/\sigma_{MAX} = 0,90 \quad \sigma/\sigma_Y \approx 0,99$



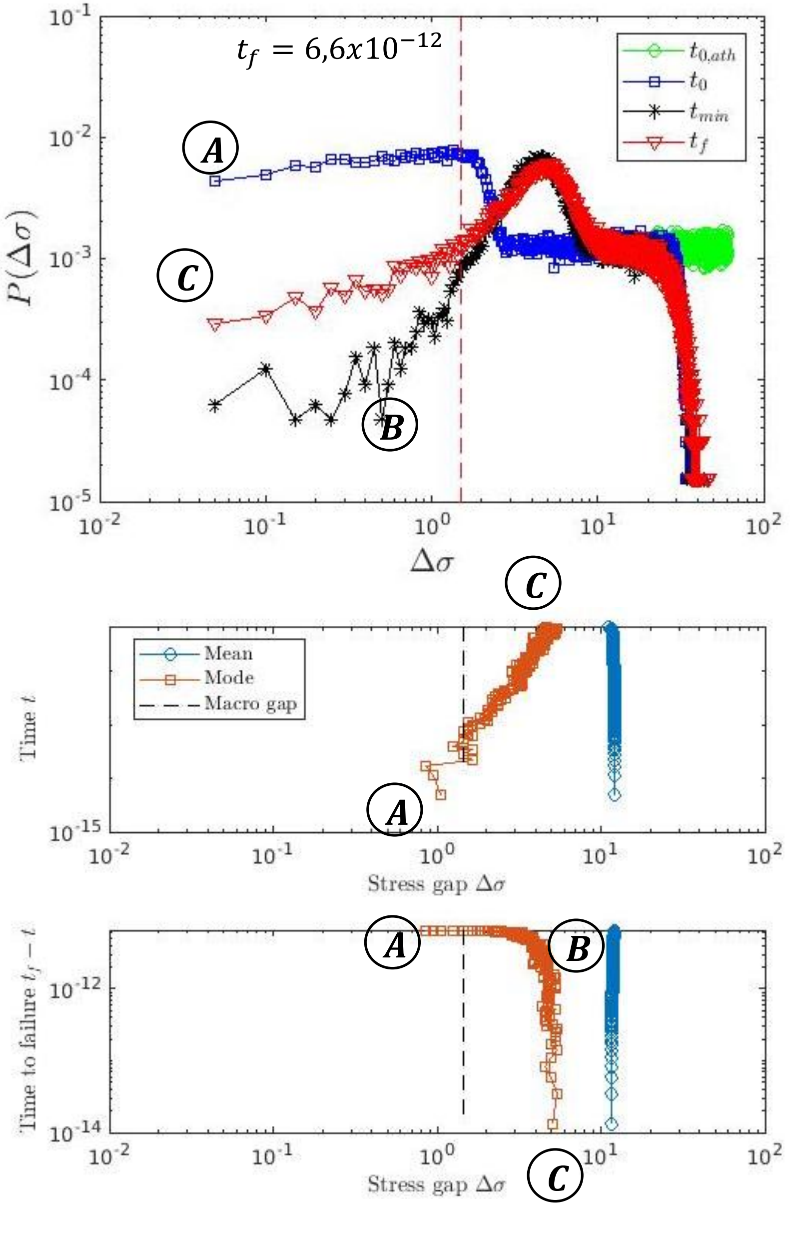
$\Delta\sigma \approx 1,54$MPa
$\sigma/\sigma_{MAX} = 0,95 \quad \sigma/\sigma_Y \approx 1,04$



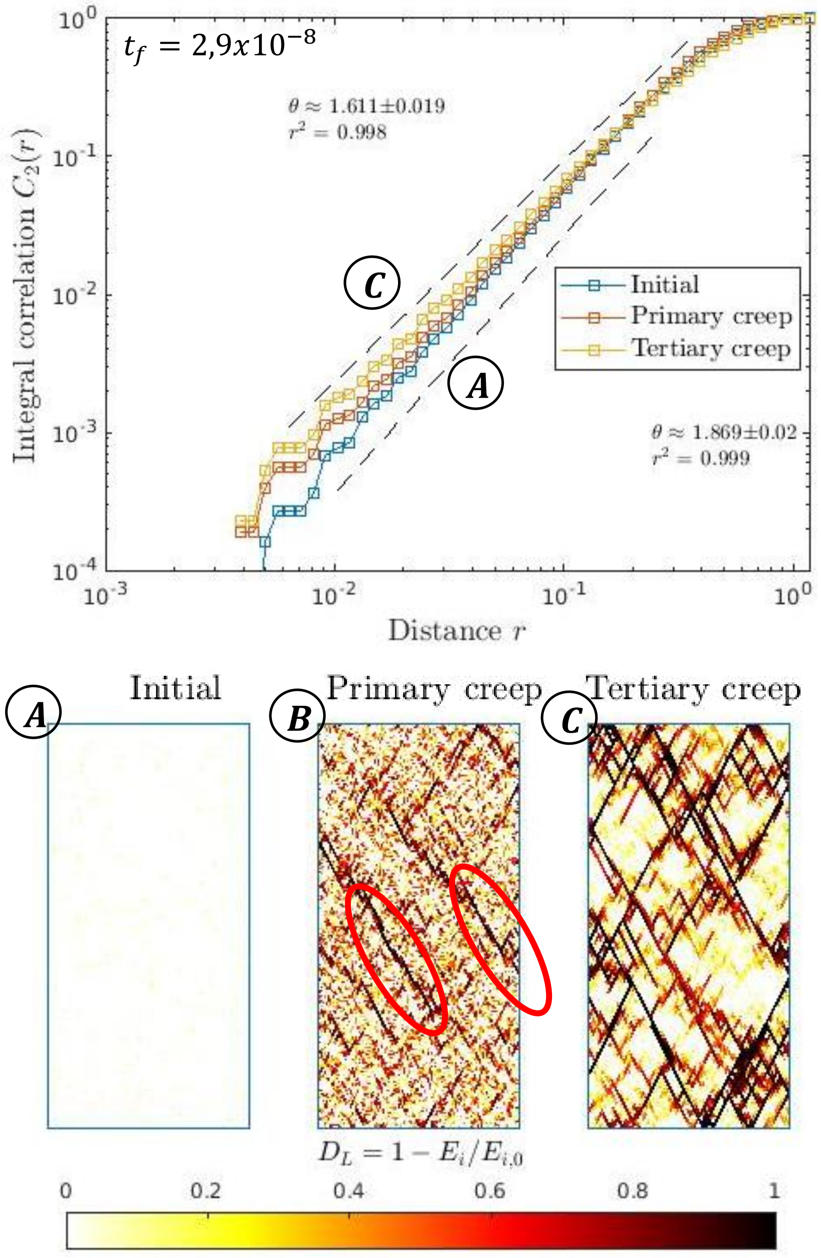
$\Delta\sigma \approx 8{,}57$MPa
$\sigma/\sigma_{MAX} = 0{,}72 \quad \sigma/\sigma_Y \approx 0{,}80$



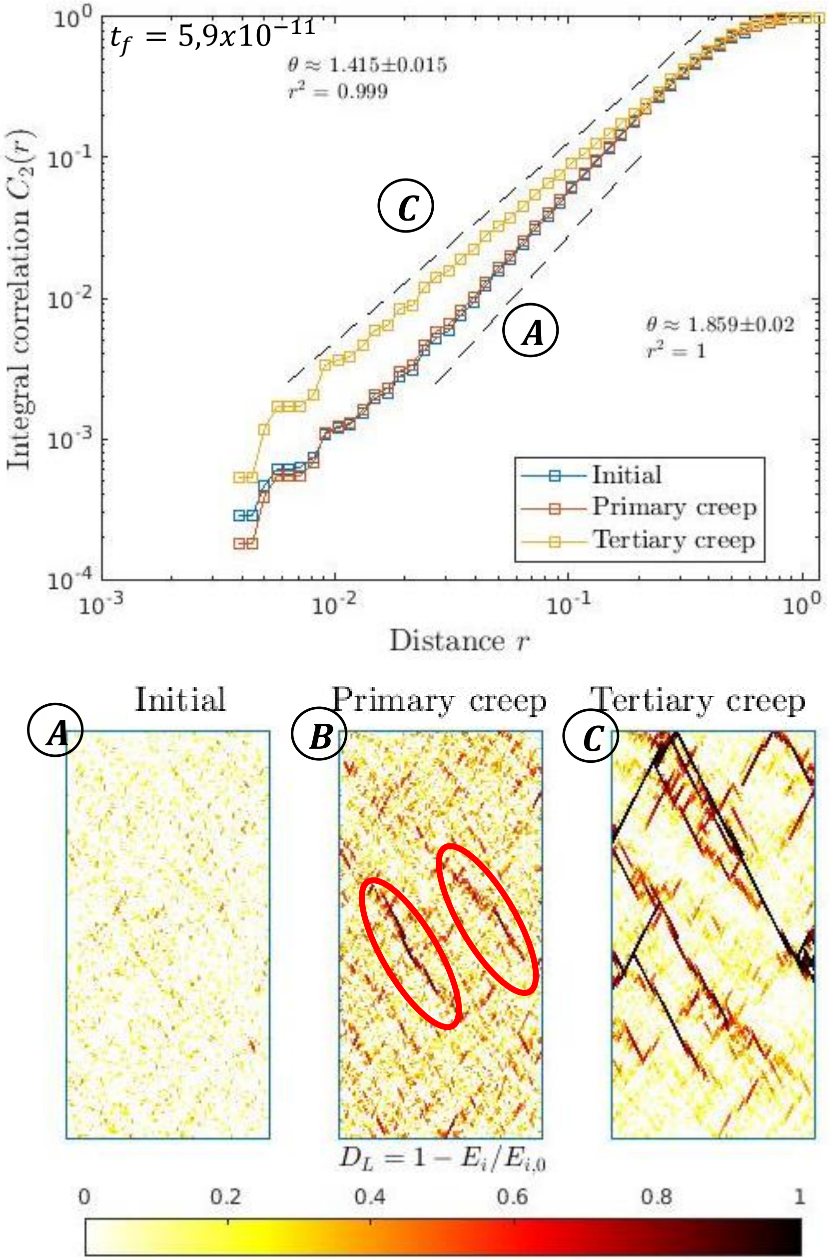
$\Delta\sigma \approx 3{,}01$MPa
$\sigma/\sigma_{MAX} = 0{,}90 \quad \sigma/\sigma_Y \approx 0{,}99$



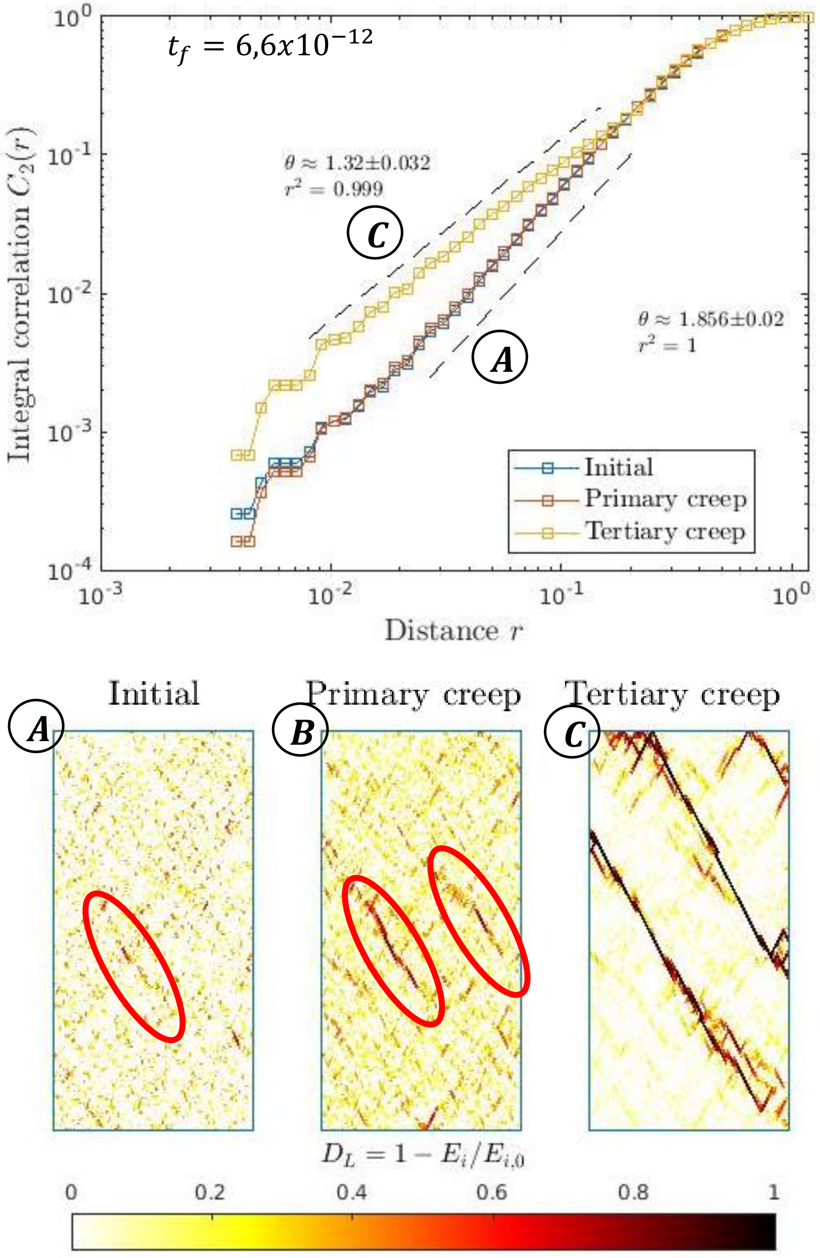
$\Delta\sigma \approx 1{,}54$MPa
$\sigma/\sigma_{MAX} = 0{,}95 \quad \sigma/\sigma_Y \approx 1{,}04$



$\Delta\sigma \approx 8{,}57$MPa
$\sigma/\sigma_{MAX} = 0{,}72 \quad \sigma/\sigma_Y \approx 0{,}80$



$\Delta\sigma \approx 3{,}01$MPa
$\sigma/\sigma_{MAX} = 0{,}90 \quad \sigma/\sigma_Y \approx 0{,}99$



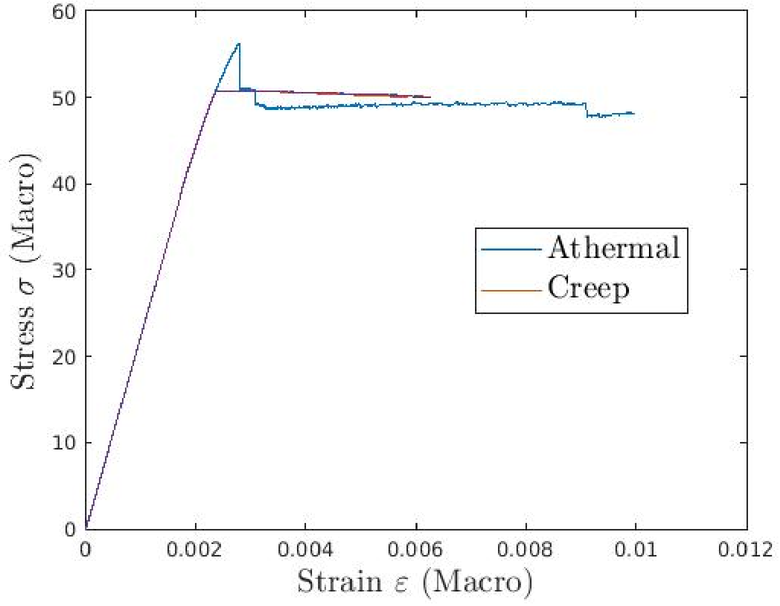
$\Delta\sigma \approx 1{,}54$MPa
$\sigma/\sigma_{MAX} = 0{,}95 \quad \sigma/\sigma_Y \approx 1{,}04$

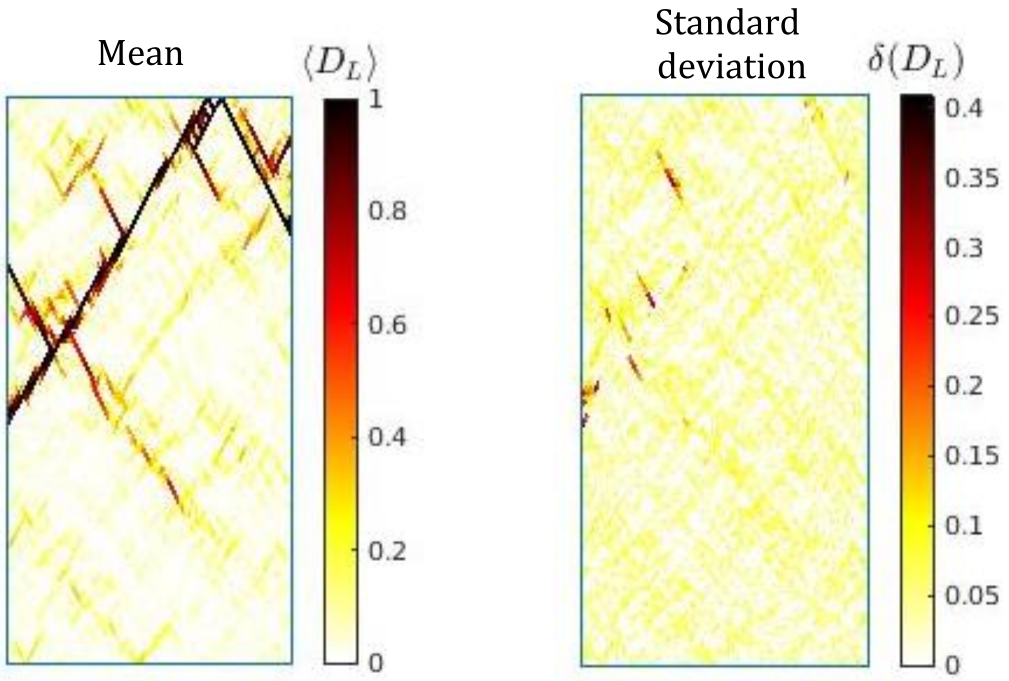
4. Results

4.4 Effect of thermal activation (same initial disorder)

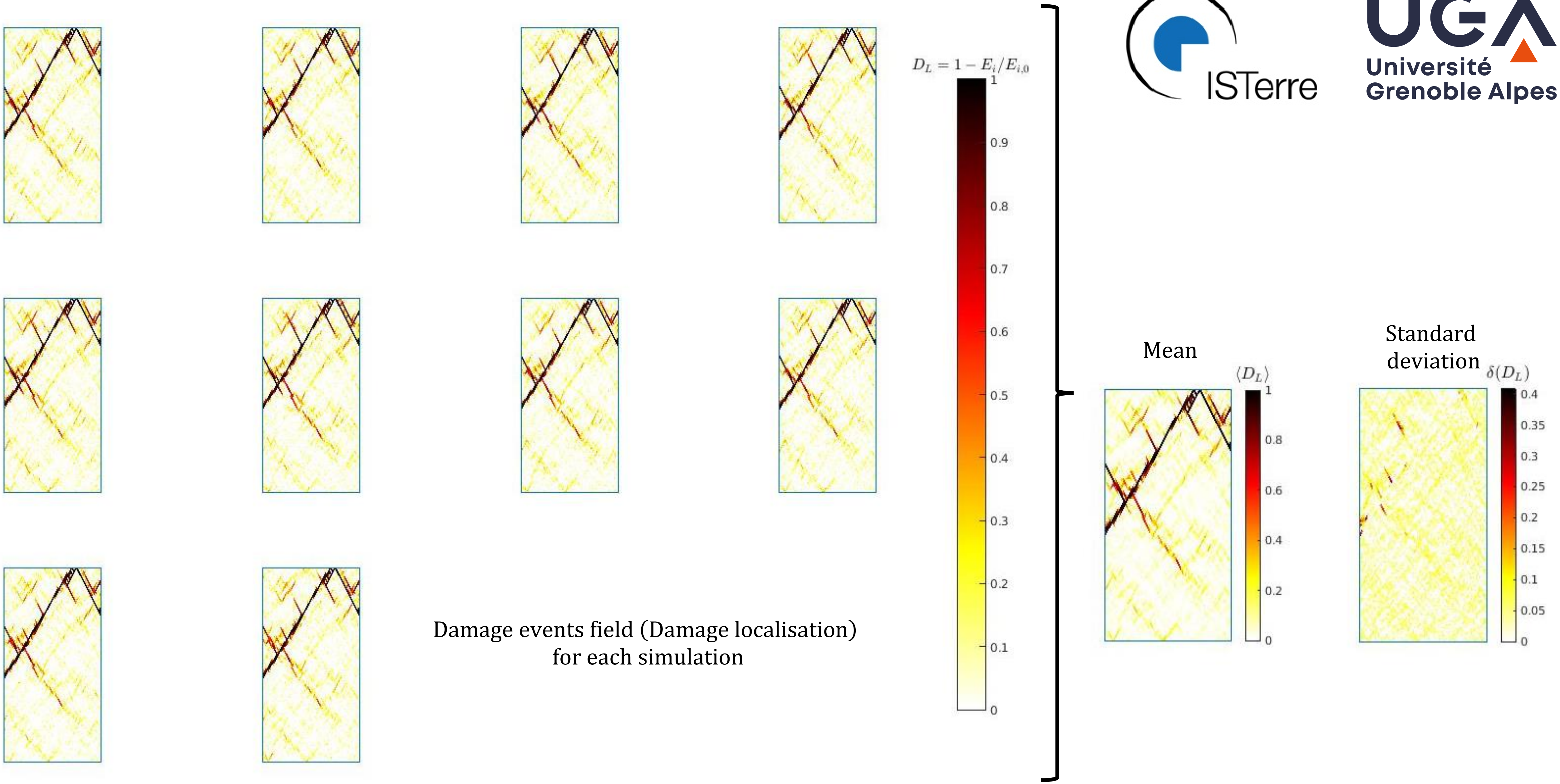
System Sizes	$L = 64$
Number of simulations	$N = 10$
Temperature	$T = 800$ K
Stress ratio	$\sigma/\sigma_{MAX} = 0{,}90 \quad \sigma/\sigma_Y \approx 0{,}99$
Macro stress gap	$\Delta\sigma \approx 3{,}01$MPa

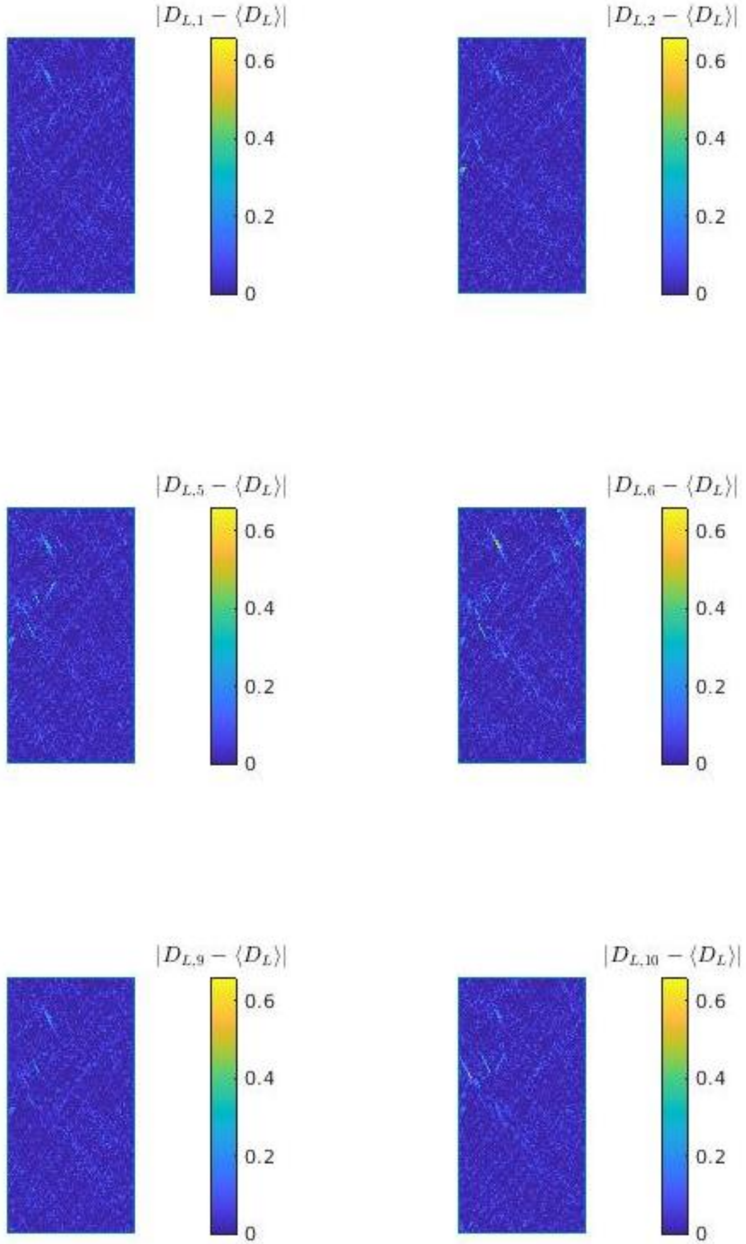
KMC effect: same initial disorder





Damage events field (Damage localisation)

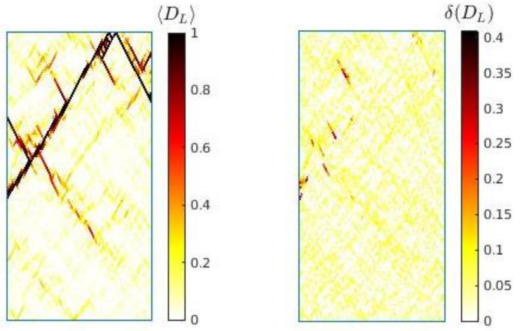

$D_L = 1 - E_i/E_{i,0}$
Damage events field (Damage localisation)
for each simulation
Mean
$\langle D_L \rangle$
Standard
deviation
$\delta(D_L)$

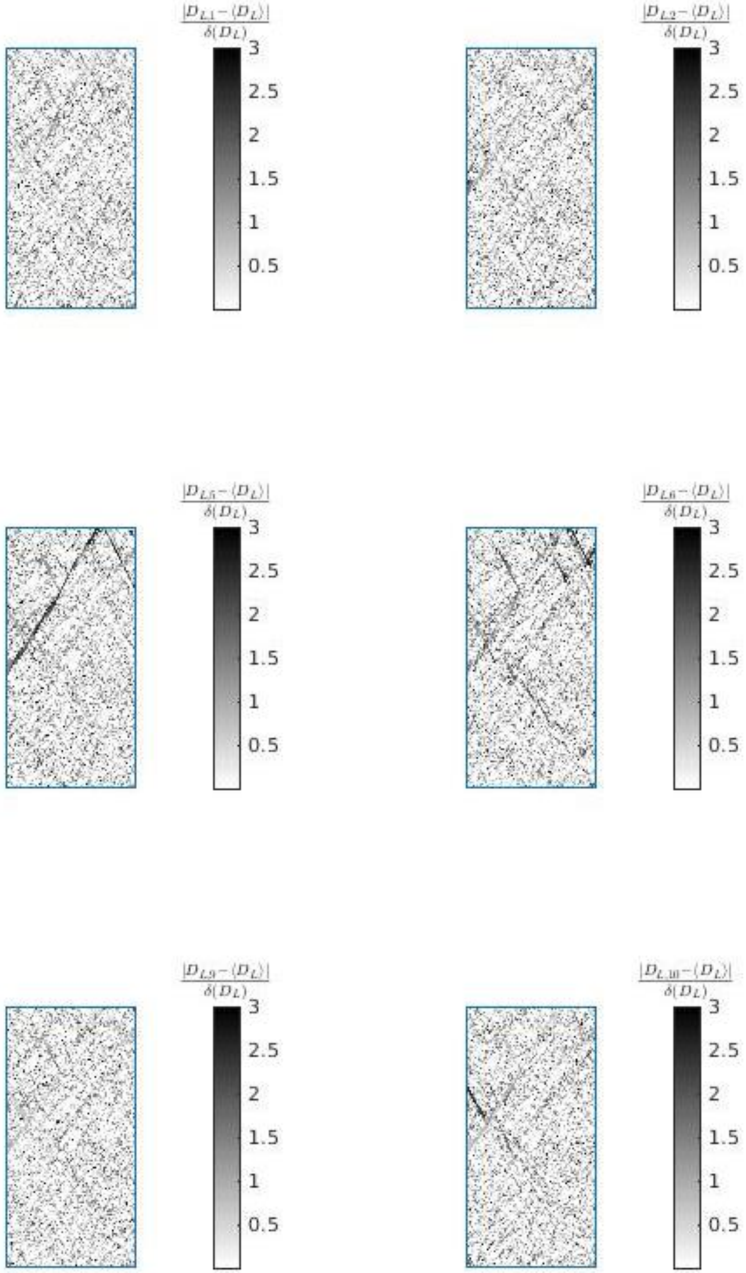


Damage events field (Damage localisation)
for each simulation
Damaged field – Mean of Damaged field
$D_L - < D_L >$

Mean

Standard deviation

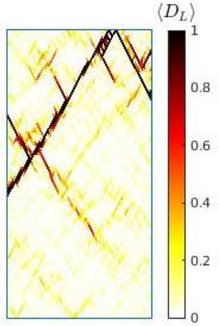




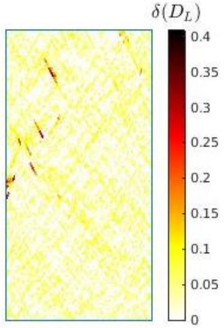
Damage events field (Damage localisation)
for each simulation

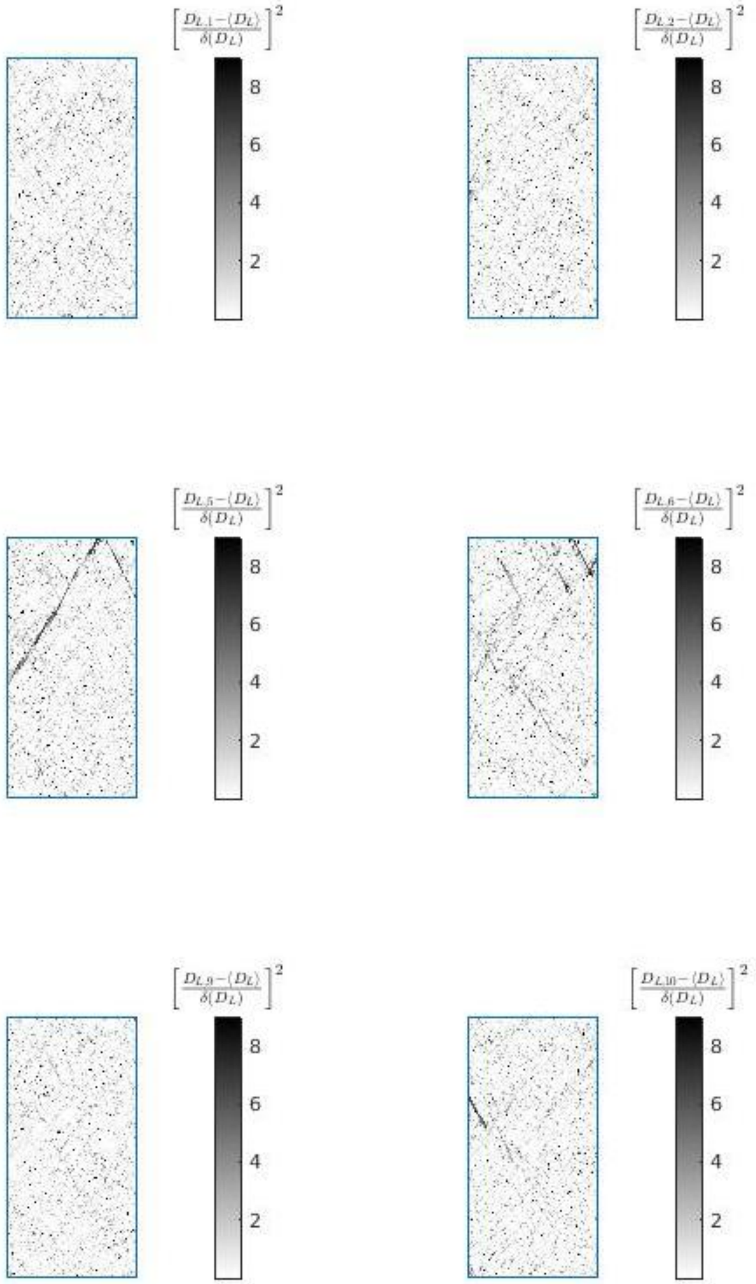
$$z = \frac{D_L - < D_L >}{\delta D_L}$$

Mean



Standard deviation



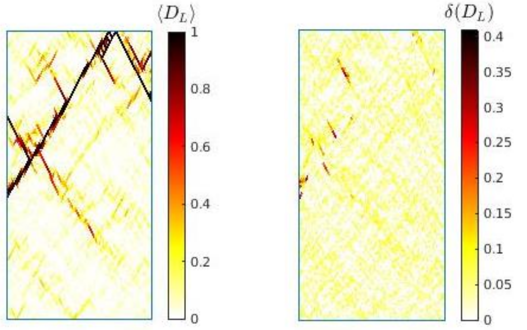


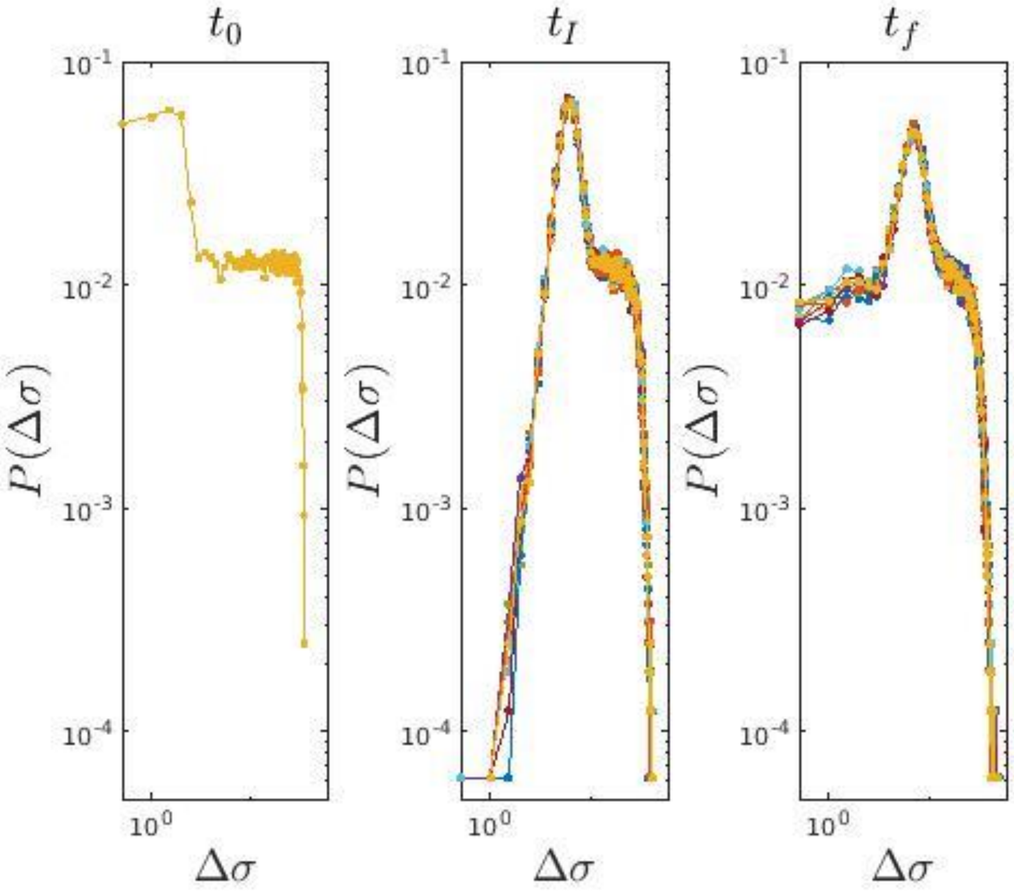
Damage events field (Damage localisation)
for each simulation

$$z^2 = \left(\frac{D_L - < D_L >}{\delta D_L}\right)^2$$

Mean

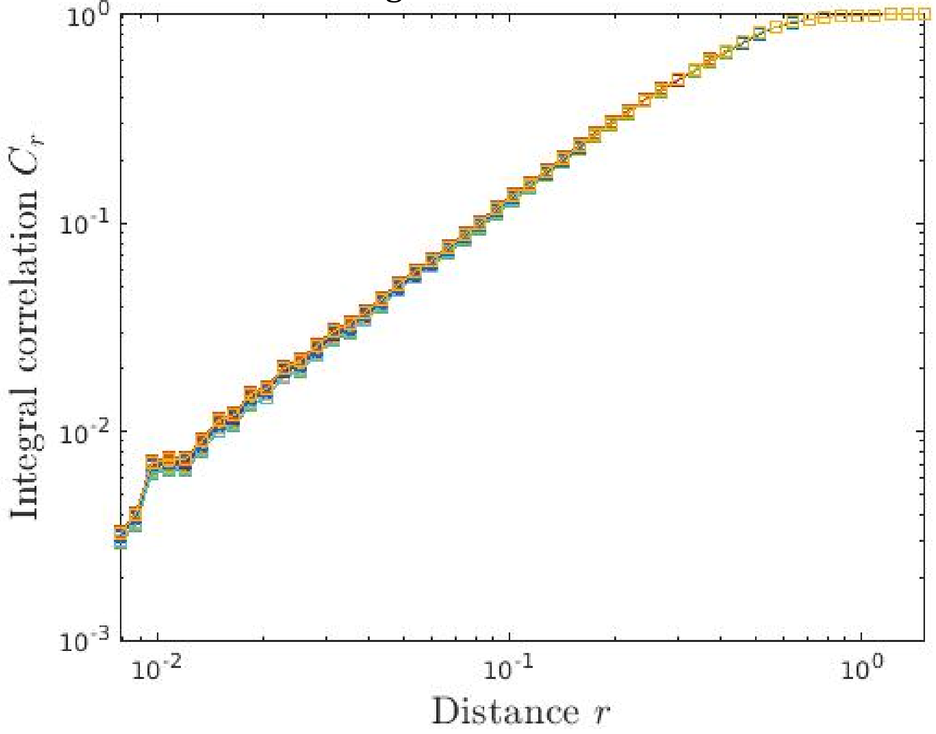
Standard
deviation





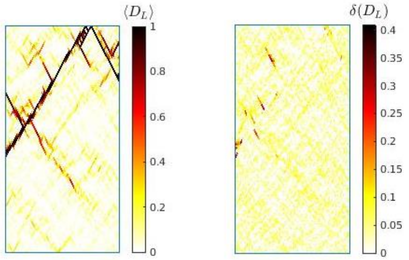
Stress gap probability distributions

Integral correlations



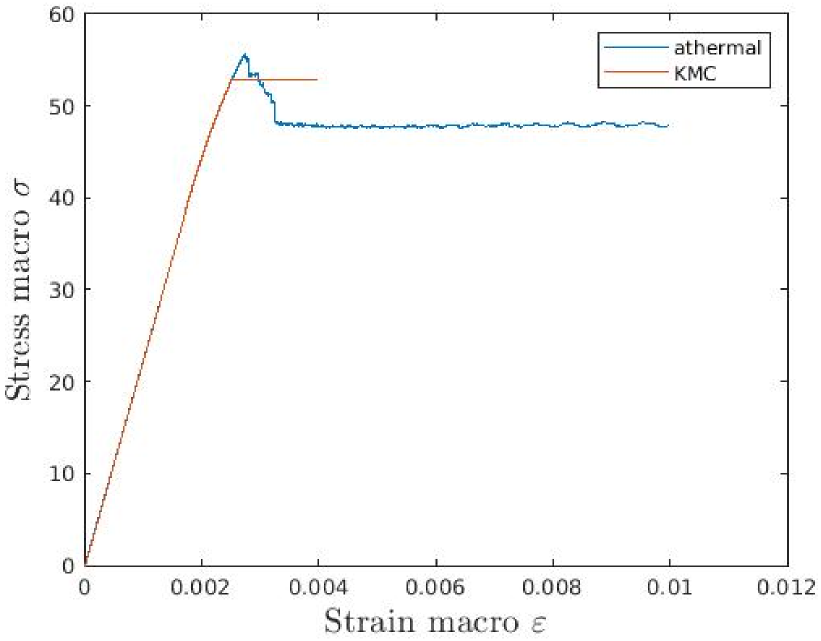
Mean

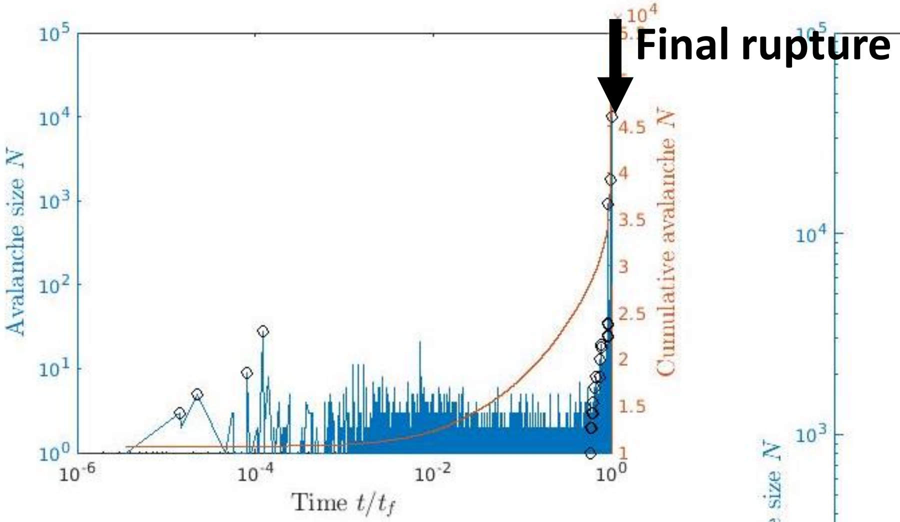
Standard deviation

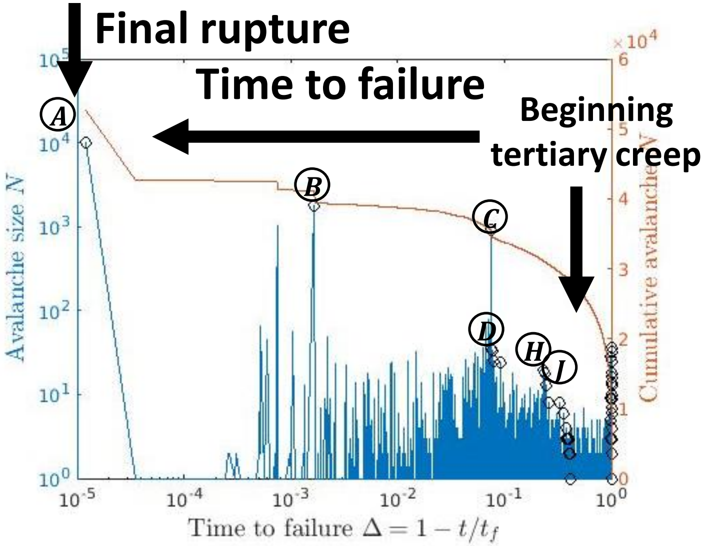


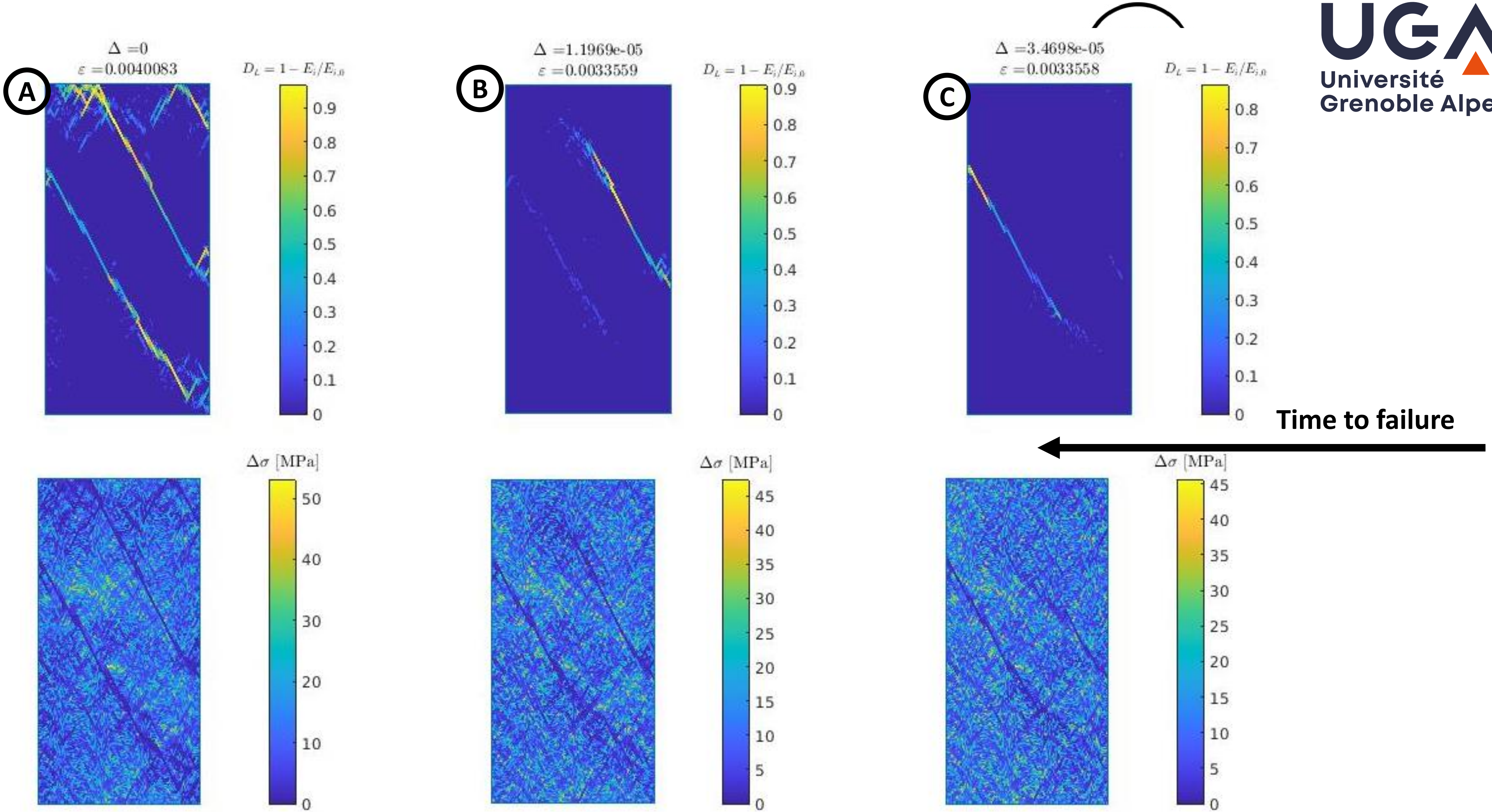
4. Results
4.4 Level of damaged during the largest avalanche size

System Sizes	$L = 64$
Number of simulations	$N = 1$
Temperature	$T = 800$ K
Stress ratio	$\sigma/\sigma_{MAX} = 0{,}95 \quad \sigma/\sigma_Y \approx 1{,}04$
Micro/Macro stress gap	$\Delta\sigma \approx 1{,}54$MPa

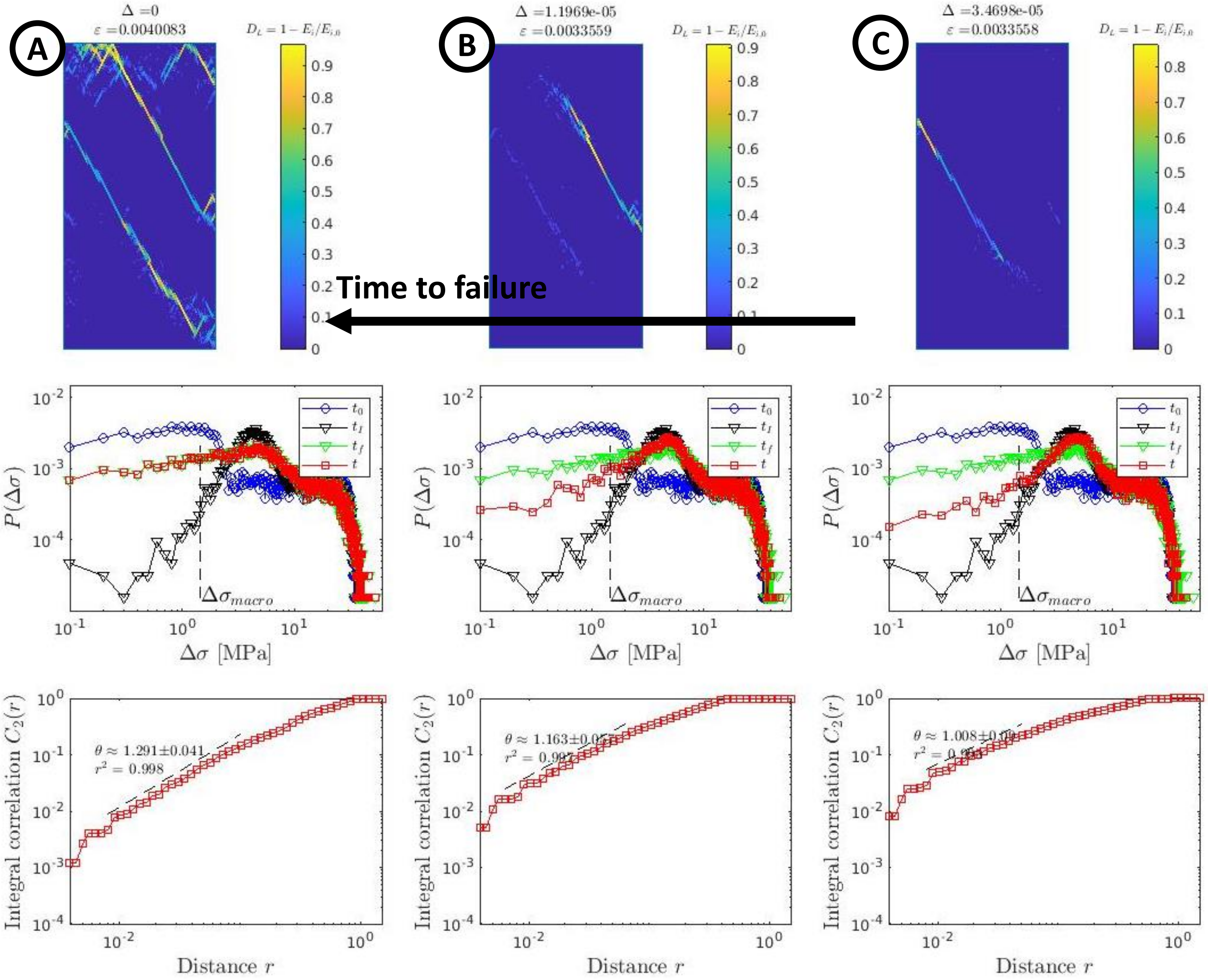


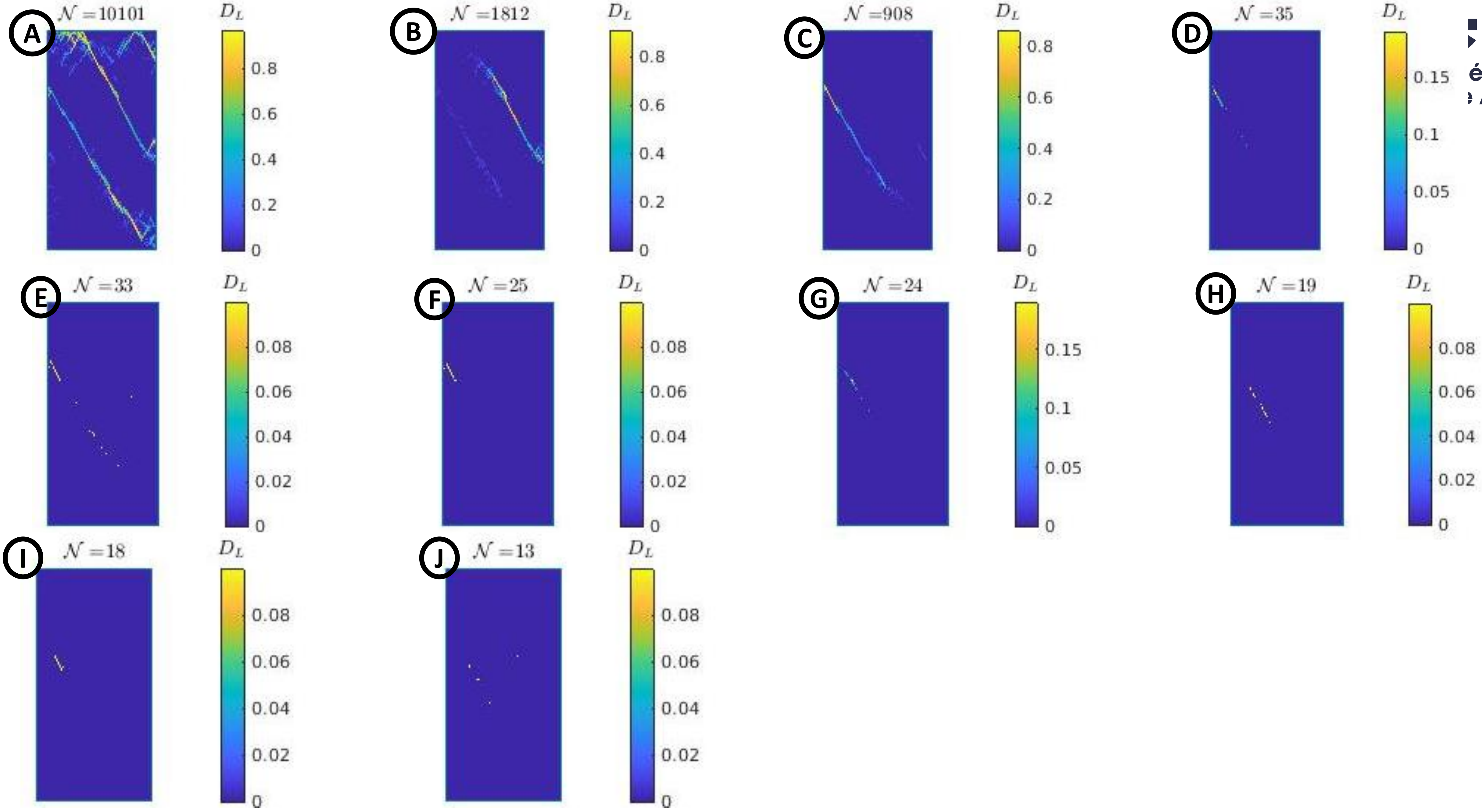






A
$\Delta = 0$
$\varepsilon = 0.0040083$
$D_L = 1 - E_i/E_{i,0}$
B
$\Delta = 1.1969e\text{-}05$
$\varepsilon = 0.0033559$
$D_L = 1 - E_i/E_{i,0}$
C
$\Delta = 3.4698e\text{-}05$
$\varepsilon = 0.0033558$
$D_L = 1 - E_i/E_{i,0}$
Time to failure
$\Delta\sigma$ [MPa]
$\Delta\sigma$ [MPa]
$\Delta\sigma$ [MPa]

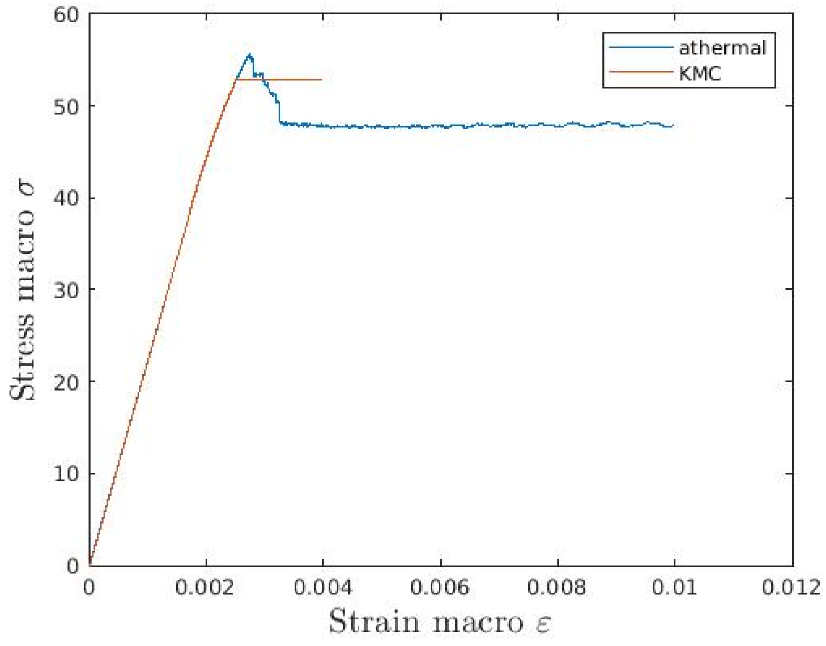

A
B
C
$\Delta = 0$
$\varepsilon = 0.0040083$
$\Delta = 1.1969e{-}05$
$\varepsilon = 0.0033559$
$\Delta = 3.4698e{-}05$
$\varepsilon = 0.0033558$
$D_L = 1 - E_i/E_{i,0}$
Time to failure
$P(\Delta\sigma)$
$\Delta\sigma$ [MPa]
$\Delta\sigma_{macro}$
t_0
t_I
t_f
t
Integral correlation $C_2(r)$
Distance r
$\theta \approx 1.291 \pm 0.041$
$r^2 = 0.998$

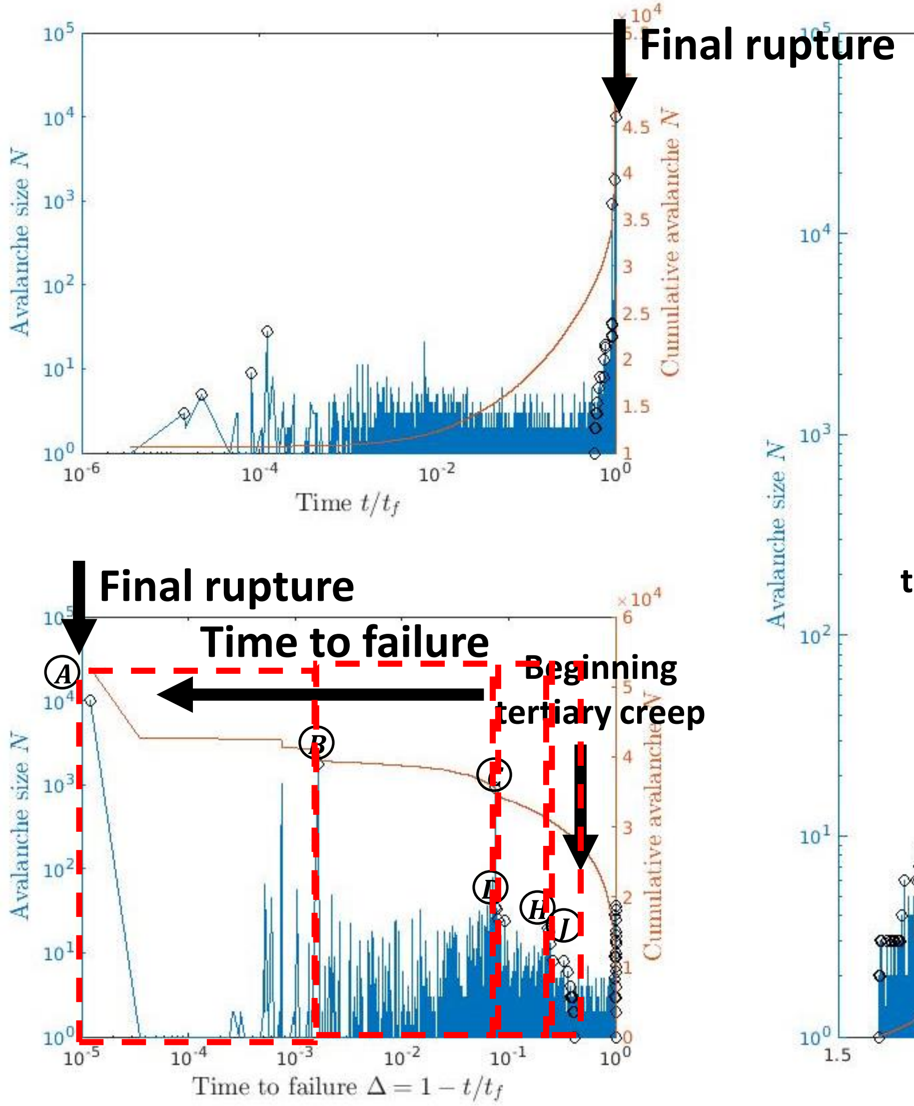

A
$\mathcal{N}=10101$
D_L
0.8
0.6
0.4
0.2
0
B
$\mathcal{N}=1812$
D_L
0.8
0.6
0.4
0.2
0
C
$\mathcal{N}=908$
D_L
0.8
0.6
0.4
0.2
0
D
$\mathcal{N}=35$
D_L
0.15
0.1
0.05
0
E
$\mathcal{N}=33$
D_L
0.08
0.06
0.04
0.02
0
F
$\mathcal{N}=25$
D_L
0.08
0.06
0.04
0.02
0
G
$\mathcal{N}=24$
D_L
0.15
0.1
0.05
0
H
$\mathcal{N}=19$
D_L
0.08
0.06
0.04
0.02
0
I
$\mathcal{N}=18$
D_L
0.08
0.06
0.04
0.02
0
J
$\mathcal{N}=13$
D_L
0.08
0.06
0.04
0.02
0

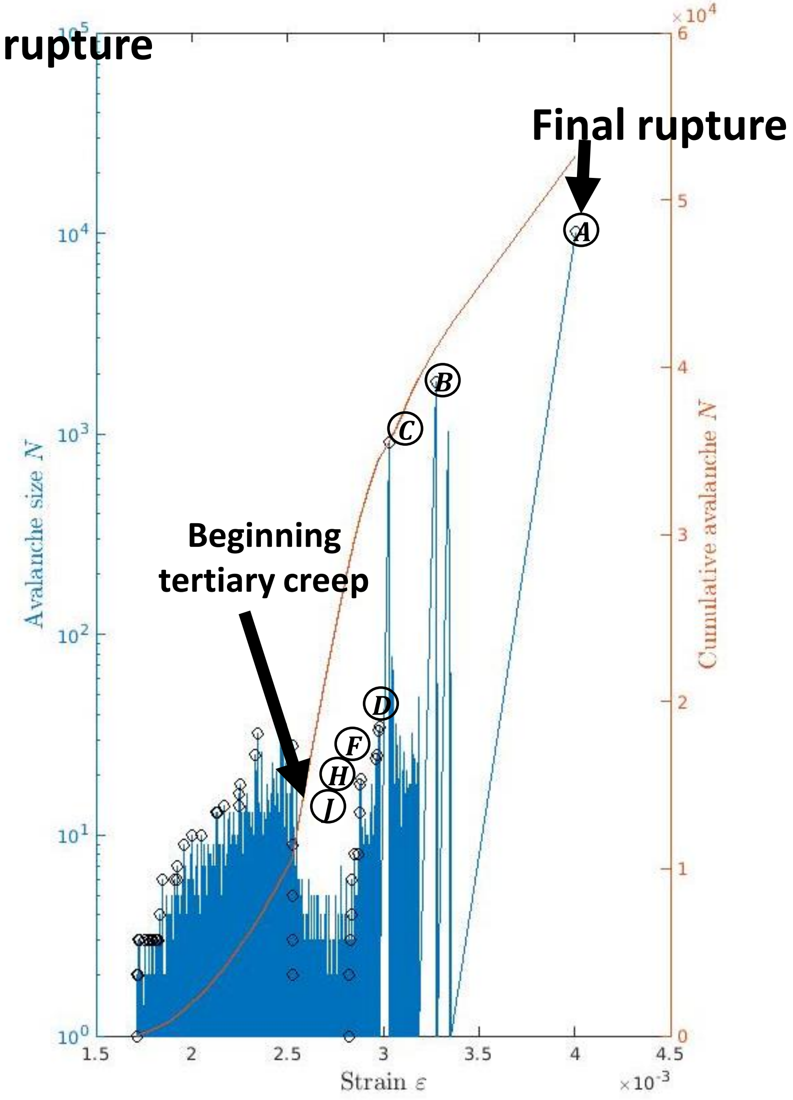
4. Results

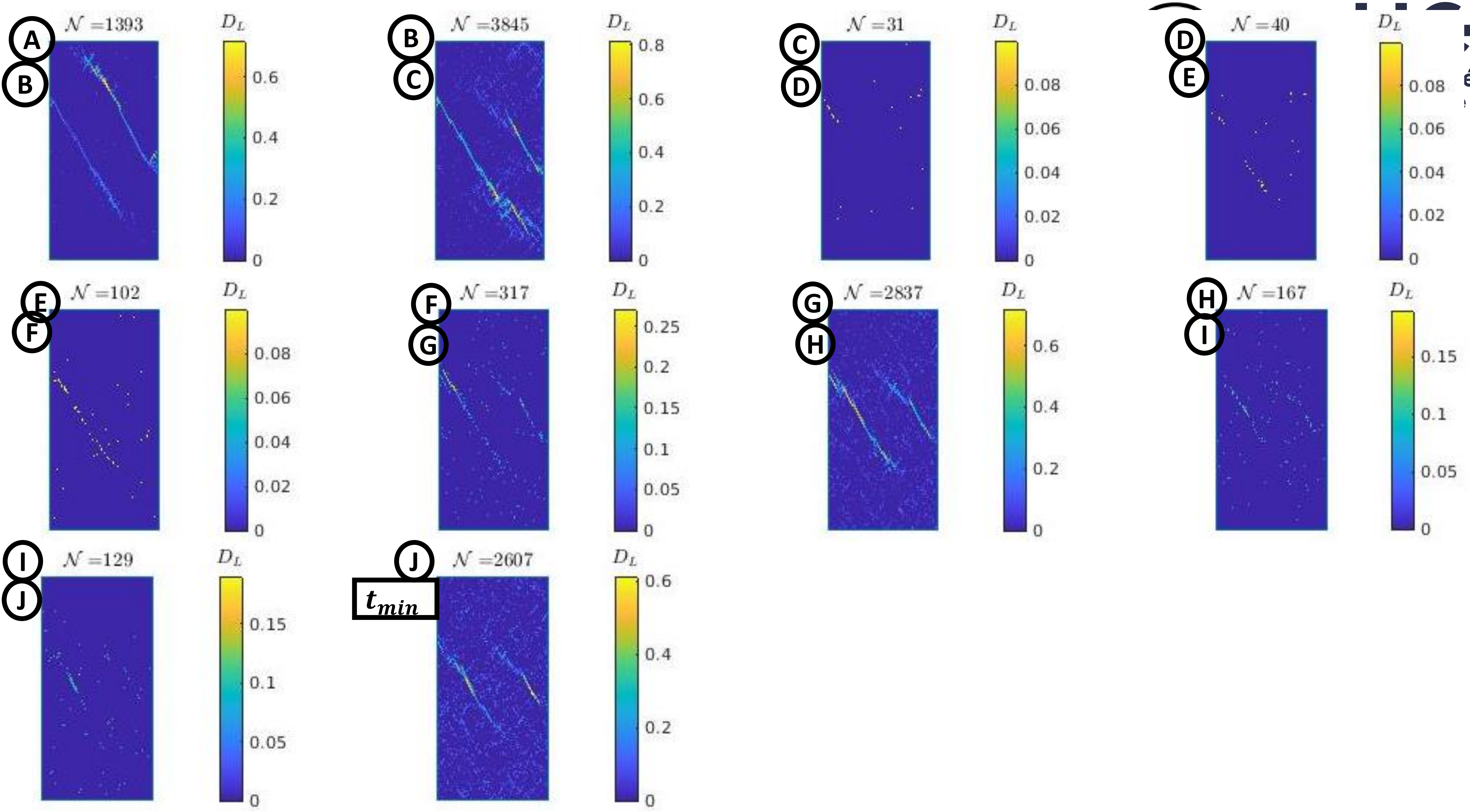
4.4 Level of damaged in different windows of time

System Sizes	$L = 64$
Number of simulations	$N = 1$
Temperature	$T = 800$ K
Stress ratio	$\sigma/\sigma_{MAX} = 0{,}95 \quad \sigma/\sigma_Y \approx 1{,}04$
Micro/Macro stress gap	$\Delta\sigma \approx 1{,}54$MPa









A
B
$\mathcal{N}$ =1393
D_L
0.6
0.4
0.2
0
B
C
$\mathcal{N}$ =3845
D_L
0.8
0.6
0.4
0.2
0
C
D
$\mathcal{N}$ =31
D_L
0.08
0.06
0.04
0.02
0
D
E
$\mathcal{N}$ =40
D_L
0.08
0.06
0.04
0.02
0
E
F
$\mathcal{N}$ =102
D_L
0.08
0.06
0.04
0.02
0
F
G
$\mathcal{N}$ =317
D_L
0.25
0.2
0.15
0.1
0.05
0
G
H
$\mathcal{N}$ =2837
D_L
0.6
0.4
0.2
0
H
I
$\mathcal{N}$ =167
D_L
0.15
0.1
0.05
0
I
J
$\mathcal{N}$ =129
D_L
0.15
0.1
0.05
0
J
t_{min}
$\mathcal{N}$ =2607
D_L
0.6
0.4
0.2
0