For now we see through a glass, darkly, but then . . .

Paul of Tarsus

Every researcher in the applied sciences who has analyzed data has practiced inverse theory. Inverse theory is simply the set of methods used to extract useful inferences about the world from physical measurements. The fitting of a straight line to data involves a simple application of inverse theory. Tomography, popularized by the physician's CT and MRI scanners, uses it on a more sophisticated level.

The study of inverse theory, however, is more than the cataloging of methods of data analysis. It is an attempt to organize these techniques, to bring out their underlying similarities and pin down their differences, and to deal with the fundamental question of the limits of information that can be gleaned from any given data set.

Physical properties fall into two general classes: those that can be described by discrete parameters (e.g., the mass of the earth or the position of the atoms in a protein molecule) and those that must be described by continuous functions (e.g., temperature over the face of the earth or electric field intensity in a capacitor). Inverse theory employs different mathematical techniques for these two classes of parameters: the theory of matrix equations for discrete parameters and the theory of integral equations for continuous functions.

Being introductory in nature, this book deals mainly with "discrete inverse theory," that is, the part of the theory concerned with parameters that either are truly discrete or can be adequately approximated as discrete. By adhering to these limitations, inverse theory can be presented on a level that is accessible to most first-year graduate students and many college seniors in the applied sciences. The only mathematics that is presumed is a working knowledge of the calculus and linear algebra and some familiarity with general concepts from probability theory and statistics.

Nevertheless, the treatment in this book is in no sense simplified. Realistic examples, drawn from the scientific literature, are used to illustrate the various techniques. Since in practice the solutions to most inverse problems require substantial computational effort, attention is given to how realistic problems can be solved.

The treatment of inverse theory in this book is divided into four parts. Chapters 1 and 2 provide a general background, explaining what inverse problems are and what constitutes their solution as well as reviewing some of the basic concepts from linear algebra and probability theory that will be applied throughout the text. Chapters 3–7 discuss the solution of the canonical inverse problem: the linear problem with Gaussian statistics. This is the best understood of all inverse problems, and it is here that the fundamental notions of uncertainty, uniqueness, and resolution can be most clearly developed. Chapters 8–11 extend the discussion to problems that are non-Gaussian, non-linear, and continuous. Chapters 12–13 provide examples of the use of inverse theory and a discussion of the steps that must be taken to solve inverse problems on a computer.

MatLab scripts are used throughout the book as a means of communicating how the formulas of inverse theory can be used in computer-based data processing scenarios. *MatLab* is a commercial software product of *The MathWorks, Inc.* and is widely used in university settings as an environment for scientific computing. All of the book's examples, its recommended homework problems, and the case studies of Chapter 12 use *MatLab* extensively. Further, all the *MatLab* scripts used in the book are made available to readers through the book's Web site. The book is self-contained; it can be read straight through, and profitably, even by someone with no access to *MatLab*. But it is meant to be used in a setting where students are actively using *MatLab* both as an aid to studying (that is, by reproducing the examples and case studies described in the book) and as a tool for completing the recommended homework problems.

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REFERENCE

Menke, W., Menke, J., 2011. Environmental Data Analysis with MatLab. Academic Press, Elsevier Inc, Oxford UK, 263pp.

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