VERLY
JONAH
20181279

Question 1

$$at^2 + bt + c$$

$$c = 12$$

$$\begin{cases} a\,1^2 + b\,1 + 12 = 13 \\ a\,2^2 + b\,2 + 12 = 19 \\ a\,3^2 + b\,3 + 12 = 34 \end{cases}$$

$$\Leftrightarrow \begin{cases} a + b + 12 = 13 \\ 4a + 2b + 12 = 19 \\ 9a + 3b + 12 = \cancel{}\ 34 \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 & 12 \\ 4 & 2 & 12 \\ 9 & 3 & 12 \end{pmatrix} \qquad B = \begin{pmatrix} 13 \\ 19 \\ 34 \end{pmatrix}$$

$$\Rightarrow A^T A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^T B$$

$$\begin{pmatrix} 1 & 4 & 9 \\ 1 & 2 & 3 \\ 12 & 12 & 12 \end{pmatrix} \begin{pmatrix} 1 & 1 & 12 \\ 4 & 2 & 12 \\ 9 & 3 & 12 \end{pmatrix} \begin{pmatrix} x \\ y \\ 12 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & 9 \\ 1 & 2 & 3 \\ 12 & 12 & 12 \end{pmatrix} \begin{pmatrix} 13 \\ 19 \\ 34 \end{pmatrix} =$$

$$\begin{pmatrix} 395 \\ 153 \\ 792 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 98 & 36 & 168 \\ 36 & 14 & 72 \\ 168 & 72 & 432 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 98x + 36y + 168z = 395 \\ 36x + 14y + 72z = 153 \\ 168x + 72y + 432z = 792 \end{cases}$$

$\Leftrightarrow 21x + 3$ [illegible]

VERLY
JONAH
20181279

Question 1

$\cancel{21x = 99 - 9y - 54z}$

$\cancel{x = \dfrac{99 - 9y - 54z}{21}}$

$\cancel{36\left(\dfrac{99 - 9y - 54z}{21}\right)}$

$\det A^T A = 98\left((14 \cdot 432) - (72)^2\right) - 36\left((36 \cdot 432) - (72 \cdot 168)\right) + 168\left((36 \cdot 72) - (14 \cdot 168)\right)$
$= 576$

$\Rightarrow$ unicité de solution

$$\det \begin{pmatrix} 98 & 36 & 168 \\ 36 & 14 & 72 \\ 168 & 72 & 432 \end{pmatrix}$$

$$\cancel{\begin{pmatrix} 98 & 36 & 395 \\ 36 & 14 & 153 \\ 168 & 72 & 792 \end{pmatrix} = 768}$$

$\cancel{x = \dfrac{768}{576} = 1,613}$

$\cancel{\det \ 36}$

$$\det \begin{pmatrix} 395 & 36 & 168 \\ 153 & 14 & 72 \\ 792 & 72 & 432 \end{pmatrix} = 2592$$

$a = \cancel{5\ldots}\ 4,5$

$$\det \begin{pmatrix} 98 & 395 & 168 \\ 36 & 153 & 72 \\ 168 & 792 & 432 \end{pmatrix} = -4320$$

$a = \dfrac{-4320}{576} = \cancel{\ldots}\ -7,5$

VERLY
JONAH
20181279

Question 2

$$A = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{pmatrix} \quad m_{21} = \frac{3}{4} \quad m_{31} = 3/4 \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^1 = \begin{pmatrix} 4 & 5 & 2 \\ 0 & -7/4 & 3/2 \\ 0 & -3/4 & 1/2 \end{pmatrix} \quad m_{32} = \frac{-3/4}{-7/4} = \frac{12}{28} = \frac{3}{7} \quad L^1 = \begin{pmatrix} 1 & 0 & 0 \\ 3/4 & 1 & 0 \\ 3/4 & 0 & 1 \end{pmatrix} \quad U^1 = \begin{pmatrix} 4 & 5 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 4 & 5 & 2 \\ 0 & -7/4 & 3/2 \\ 0 & 0 & -1/7 \end{pmatrix} \quad L^2 = \begin{pmatrix} 1 & 0 & 0 \\ 3/4 & 1 & 0 \\ 3/4 & 3/7 & 1 \end{pmatrix} \quad U^2 = \begin{pmatrix} 4 & 5 & 2 \\ 0 & -7/4 & 3/2 \\ 0 & 0 & -1/7 \end{pmatrix}$$

VERLY
JONAH
1279

Question 3

$$A = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix} \qquad \lambda I = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

rang $A = 2$.

$$(A - \lambda I) = \begin{pmatrix} -1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 2 \\ 0 & 2 & 2-\lambda \end{pmatrix}$$

$$\det = (-1-\lambda)\left((2-\lambda)^2 - 4\right) = \cancel{(-1-\lambda)(2-\lambda)(\ldots 4)(2-\lambda)}$$

$$= (-1-\lambda)(4 - 4\lambda + \lambda^2 - 4)$$

$$= (-1-\lambda)(4\lambda - \lambda^2)$$

$$\lambda_1 = -1$$

$$4\lambda - \lambda^2 = 0$$

$$\Delta = 16 +$$

$$\sqrt{\Delta} = 4$$

$$x_{1,2} = \frac{-4 \pm 4}{2} \begin{cases} -4 = \lambda_2 \\ 0 = \lambda_3 \end{cases}$$

$li = 1 < 2 \Rightarrow$ A n'est pas diagonalisable.

VERLY
JONAH
20181279

QUESTION 5

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

Pas 1 :

$$V_1 = (1\ 0\ 1)^T$$
$$\|V_1\| = \sqrt{2}$$
$$U_1 = \left(\tfrac{1}{\sqrt{2}}\ \ 0\ \ \tfrac{1}{\sqrt{2}}\right)^T$$

Pas 2 :

$$V_2 = (0\ \ 2\ \ 0)^T$$
$$\langle U_1, V_2 \rangle U_1 = (0\ \ 0\ \ 0)$$
$$T_2 = (0\ 2\ 0)^T \qquad \|T_2\| = \sqrt{4} = 2$$
$$U_2 = (0\ 1\ 0)^T$$

Pas 3 :

$$V_3 = (-1\ \ 0\ \ 3)^T$$
$$T_3 = V_3 - \sum_{j=1}^{2} \langle U_j, V_3 \rangle U_j$$
$$= (-1\ \ 0\ \ 3)^T - \left(\left(-\tfrac{1}{\sqrt{2}} + \tfrac{3}{\sqrt{2}}\right) \left(\tfrac{1}{\sqrt{2}}\ \ 0\ \ \tfrac{1}{\sqrt{2}}\right)^T + (0\ 0\ 0)^T \right)$$
$$= (-1\ 0\ 3)^T - \tfrac{2}{\sqrt{2}} \left(\tfrac{1}{\sqrt{2}}\ \ 0\ \ \tfrac{1}{\sqrt{2}}\right)^T$$
$$= (-1\ 0\ 3)^T - (1\ \ 0\ \ 1)^T \qquad \|T_3\| = \sqrt{8}$$
$$= (-2\ 0\ 2)^T \qquad U_3 = \tfrac{-2}{\sqrt{8}}\ \ 0\ \ \tfrac{2}{\sqrt{8}}$$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-2}{\sqrt{8}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{2}{\sqrt{8}} \end{pmatrix} \qquad R = \begin{pmatrix} \sqrt{2} & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ 0 & 0 & 2\sqrt{2} \end{pmatrix}$$

L

$$U_1 = \left(\tfrac{1}{\sqrt{2}}\ \ 0\ \ \tfrac{1}{\sqrt{2}}\right)^T$$
$$U_2 = (0\ \ 1\ \ 0)^T$$
$$U_3 = \left(-\tfrac{2}{\sqrt{8}}\ \ 0\ \ \tfrac{2}{\sqrt{8}}\right)^T$$

C

$$V_1 = (1\ \ 0\ 1)^T$$
$$V_2 = (0\ \ 2\ 0)^T$$
$$V_3 = (-1\ \ 0\ 3)^T$$

VERLY
JONAH
20181279

$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$ $\quad B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ QUESTION 5

$Ax = B$

$\Rightarrow$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$x_1 - x_3 = 1$$
$$x_2 = 2$$
$$x_1 - 3x_3 = 3$$

$\det A = 2\,(3+1) = 8$

$\det \begin{pmatrix} 1 & 0 & -1 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix} = 6 + 6 = 12$ $\quad \Rightarrow x_1 = \frac{12}{8} = \frac{3}{2}$

$\det \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix} = 2\,(3+1) = 8 \Rightarrow \frac{8}{8} = 1 = x_2$

$\det \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 3 \end{pmatrix} = 2\,(3-1) = 4 \Rightarrow \frac{4}{8} = \frac{1}{2} = x_3$