CSSE2010 / CSSE7201 – Introduction to Computer Systems
Answers to Exercises – Week One
Bits, Bytes & Binary; Intro to Logic Gates

Answers

1. Convert the following decimal numbers to binary (unsigned representation): 1984, 4000, 8192.
   11111000000, 111110100000, 10000000000000

2. What is 1001101001 (binary, unsigned) in decimal? In octal? In hexadecimal?
   617_{10}, 1151_8, 269_{16}

3. How many different positive integers can be expressed in k digits using radix r numbers?
   The number of valid strings of length k with r possibilities in each position is r^k. (The number of positive integers would be one less than this if you exclude 0.)

4. One of the earliest computers (the Manchester Mark 1, 1949) was programmed with a radix-32 number system. Suppose such a number system has digits represented by the characters 0, 1, … 9, A, B, … U, V.
   (a) Describe how a binary number can be converted into a radix-32 number.
   Bundle the bits in groups of 5 (starting from the least significant bit) and then convert each 5-bit binary number into the equivalent radix-32 digit.
   (b) Describe how a decimal number can be converted into a radix-32 number.
   There are two ways: (1) Subtract the highest power of 32 less than or equal to the number. Repeat until you reach 0. Count up the number of each of the powers of 32 which could be subtracted - these are the digits of the radix-32 representation.
   (2) Build the number up from the least significant digit by repeatedly dividing by 32. The remainder is the next digit. Repeat until the quotient reaches 0.
   (c) Convert the decimal numbers 1300 and 2300 to radix-32 representations.
   $18K_{32}$, $27S_{32}$

5. What's the largest unsigned integer that can be represented in
   (a) 10 bits
   1111111111_2 = $2^{10}-1$ = 1023_{10}
   (b) 9 decimal digits
   999999999_{10} = 10^9-1 = 999999999_{10}
   (c) 8 hexadecimal digits
   $FFFFFFFF_{16}$ = 16^8-1 = 4294967295_{10}

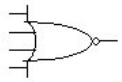
6. What's the largest number that can be counted to on ten fingers, if each finger can be considered to have two positions? Compare your answer to that in 5(a).
   You have 10 fingers and each can be in 2 positions. Hence there are 2^{10} total positions. One of these positions is for the number 0, so the largest number is $2^{10}-1 = 1023_{10}$.

7. For each of the following decimal numbers, write down the 8-bit binary representation using signed magnitude, one's complement, two's complement and excess 128 formats.

	signed magnitude	*one's complement*	*two's complement*	*excess 128*
(a) *–1*	*10000001*	*11111110*	*11111111*	*01111111*
(b) –16	*10010000*	*11101111*	*11110000*	*01110000*
(c) –99	*11100011*	*10011100*	*10011101*	*00011101*

8. What are the smallest and largest integers that can be represented in the following binary representations:
   (a) 16 bit two's complement
   smallest = –32768 largest = 32767
   (b) n bit one's complement
   smallest = $-(2^{n-1} - 1)$ largest = $2^{n-1} - 1$
   (c) excess 2^{m-1}
   smallest = -2^{m-1} largest = $2^{m-1} - 1$

9. Draw the logic symbol, write the Boolean function and write down the truth table for:
   (a) a 4 input NOR gate

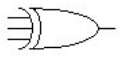


(When a gate has a "large" number of inputs, it is conventional to extend the input area as shown in this figure.)

$\overline{A+B+C+D}$

A	*B*	*C*	*D*	$\overline{A+B+C+D}$
0	*0*	*0*	*0*	*1*
0	*0*	*0*	*1*	*0*
0	*0*	*1*	*0*	*0*
0	*0*	*1*	*1*	*0*
0	*1*	*0*	*0*	*0*
0	*1*	*0*	*1*	*0*
0	*1*	*1*	*0*	*0*
0	*1*	*1*	*1*	*0*
1	*0*	*0*	*0*	*0*
1	*0*	*0*	*1*	*0*
1	*0*	*1*	*0*	*0*
1	*0*	*1*	*1*	*0*
1	*1*	*0*	*0*	*0*
1	*1*	*0*	*1*	*0*
1	*1*	*1*	*0*	*0*
1	*1*	*1*	*1*	*0*

   (b) a 3 input XOR gate (i.e. the odd function)



$A \oplus B \oplus C$

A	*B*	*C*	$A \oplus B \oplus C$
0	*0*	*0*	*0*
0	*0*	*1*	*1*
0	*1*	*0*	*1*
0	*1*	*1*	*0*
1	*0*	*0*	*1*
1	*0*	*1*	*0*
1	*1*	*0*	*0*
1	*1*	*1*	*1*