MATH3090/7039: Financial mathematics
Assignment 1
Semester I 2024

Due Tuesday March 19 1pm

MATH3090 total marks **24 marks**

MATH7039 total marks **30 marks**

Submission:

- Submit onto Blackboard softcopy (i.e. scanned copy) of (i) your assignment solutions, as well as (ii) Matlab/Python code for Problem 3. Hardcopies are not required.
- Include all your answers, numerical outputs, figures, tables and comments as required into one single PDF file.
- You also need to upload all Matlab/Python files onto Blackboard.

General coding instructions:

- You are allowed to reuse any code provided/developed in lectures and tutorials.

Notation: "Lx.y" refers to [Lecture x, Slide y]

Assignment questions - all students

1. (6 marks) a. (3 marks) Suppose a company issues a zero coupon bond with face value $\$10,000$ and which matures in 20 years. Calculate the price given

(i) an 8% discrete compound annual yield, compounded annually,

(ii) an 8% continuous annual yield,

(iii) a nonconstant yield of $y(t) = 0.06 + 0.2te^{-t^2}$.

b. (3 marks) A 10 year $\$10,000$ government bond has a coupon rate of 5% payable quarterly and yields 7%. Calculate the price.

2. (6 marks) Consider the cash flow

$$C_0 = -3x, \quad C_1 = 5, \quad C_2 = x$$

(at periods $0, 1, 2$ respectively) for some $x > 0$.

a. (3 marks) Apply the discount process $d(k) = (1 + r)^{-k}$ so that the present value is

$$P = \sum_{k=0}^{2} d(k) C_k.$$

What is the range of x such that $P > 0$ when $r = 5\%$?

b. (3 marks) The IRR (internal rate of return) is r such that $P = 0$. For what range of x, will there be a unique, strictly positive IRR?

Cashflows (C_i)	Times (t_i)
2.3	1.0
2.9	2.0
3.0	3.0
3.2	4.0
4.0	5.0
3.8	6.0
4.2	7.0
4.8	8.0
5.5	9.0
105	10.0

Table 1: Bond cashflows

3. (8 marks) In this question, consider a bond with the set of cashflows given in Table 1. Here, note that the face value F is already included in the last cashflow. Let y be the yield to maturity, t_i be the time of the i^{th} cashflow C_i, and $PV = 100$ be the market price of the bond at $t = 0$. Assume continuous compounding. Then, y solves

$$PV = \sum_i C_i e^{-y t_i} \ . \tag{1}$$

a. (3 marks) Write out the Newton iteration to compute y_{n+1} from y_n (see L2.49). Specifically, clearly indicate the functions $f(y)$ and $f'(y)$.

b. (5 marks) Implement the above Newton iteration in Matlab using the stopping criteria

$$|y_{n+1} - y_n| < 10^{-8}.$$

Fill in Table 2 for $y_0 = 0.05$ (add rows as necessary).

In addition, try with larger values for y_0 and observe the accuracy and convergence speed. How does the performance change?

n	y_n	$\lvert y_n - y_{n-1}\rvert$
0	...	N/A
1	...	...
2	...	...
3	...	...
$\vdots$	$\vdots$	$\vdots$

Table 2: Output

4. (4 marks) In the Constant Growth DDM model, the present value of the share is

$$PV = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}, \tag{2}$$

where $D_1, D_2, \ldots$ are (non-random) dividends and $k > 0$ is the required rate of return. Suppose $D_0 > 0$, $k > 0$ and $g > 0$.

Derive the formula for the present value (2) when

$$D_t = D_0(1+g)^{\lceil t/2 \rceil}, \quad t = 1, 2, \ldots,$$

where $\lceil x \rceil$ is the smallest integer greater than or equal to x. What is the condition of g so that the PV is finite? To get full marks, you need to write an explicit expression (without summation).

Assignment questions - MATH7039 students only

6. (3 marks) In Q4, derive the formula for the present value (2) if

$$D_t = D_0(1+g)^{\max(t,10)}, \quad t = 1, 2, \ldots.$$

What is the condition of g so that the PV is finite? To get full marks, you need to write an explicit expression (without summation).

7. (3 marks) Recall that the discount rate corresponding to a simple interest rate r when maturity is T is given by

$$d(T) = \frac{r}{1+rT}.$$

Suppose $r = 3\%$. Let

$$f(T) = d(0) + Td'(0) + \frac{T^2}{2}d''(0)$$

be the second-order (Taylor) approximation and

$$\varepsilon(T) = \ln\left(\frac{d(T) - f(T)}{T^3}\right)$$

be a (log) normalised error. Complete the following table:

T	$d(T)$	$f(T)$	$\varepsilon(T)$
10	$\cdots$	$\cdots$	$\cdots$
5	$\cdots$	$\cdots$	$\cdots$
1	$\cdots$	$\cdots$	$\cdots$

You can use Matlab but you do not need to submit the code for this problem.