

MATH3205 Revision - 2019 Theory (4202) Practice Exam

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April 2023

1 Question 1a - Warmup

1.1 Sets

- P Players
- L Leagues
- C Clubs
- N Nationalities
- S Slots in team formulation

1.2 Data

- nat_p - Nationality of player $p \in P$
- club_p - club of player $p \in P$
- lea_p - league of player $p \in P$
- c_p - cost of player $p \in P$
- r_p - rating of player $p \in P$
- ppos_p - position of player $p \in P$
- spos_s - position required for slot $s \in S$
- TN - exact number of nationalities that must be represented in the team
- TL - exact number of leagues that must be represented in the team
- MPN - max players per nation
- MPL - max players per league
- MAR - minimum average rating of team

1.3 Variables

$x_{p,s} \in \{0,1\}$. 1 if player $p \in P$ is put in slot $s \in S$ $y_n \in \{0,1\}$. 1 if nationality $n \in N$ is represented in team
 $z_l \in \{0,1\}$. 1 if league $l \in L$ is represented in team

1.4 Objective

$$\min \left\{ \sum_{p \in P} c_p \sum_{s \in S} x_{p,s} \right\}$$

1.5 Constraints

Correct position

$$\sum_{\substack{p \in P \\ \text{ppos}_p = \text{spos}_s}} x_{p,s} = 1, \quad \forall s \in S$$

One Player per slot

$$\sum_{p \in P} x_{p,s} = 1, \quad \forall s \in S$$

Max one slot per player

$$\sum_{s \in S} x_{p,s} \leq 1, \quad \forall p \in P$$

Nationalities Set Exact

$$\begin{aligned} \sum_{n \in N} y_n &= TN \\ y_n &\leq \sum_{s \in S} \sum_{\substack{p \in P \\ \text{nat}_p = n}} x_{p,s}, \quad \forall n \in N \\ y_n &\geq \sum_{s \in S} x_{p,s}, \quad \forall n \in N, \forall p \in P | \text{nat}_p = n \end{aligned}$$

Leagues Set Exact

$$\begin{aligned} \sum_{l \in L} z_l &= TL \\ z_l &\leq \sum_{s \in S} \sum_{\substack{p \in P \\ \text{lea}_p = l}} x_{p,s}, \quad \forall l \in L \\ z_l &\geq \sum_{s \in S} x_{p,s}, \quad \forall l \in L, \forall p \in P | \text{lea}_p = l \end{aligned}$$

Nation Limit

$$\sum_{s \in S} \sum_{\substack{p \in P \\ \text{nat}_p = n}} x_{p,s} \leq \text{MPN}, \quad \forall n \in N$$

League Limit

$$\sum_{s \in S} \sum_{\substack{p \in P \\ \text{lea}_p = l}} x_{p,s} \leq \text{MPL}, \quad \forall l \in L$$

Minimum Average Rating

$$\sum_{p \in P} r_p \sum_{s \in S} x_{p,s} \geq \text{MAR} \cdot |S|$$

2 Question 1b - Match Day

2.1 Additional Sets

- E - Edges

2.2 Additional Data

- MinTotalChem - Minimum total chemistry in the team

2.3 Additional Variables

- $yc_{e,c} \in \{0, 1\} = 1$ if both players connected by edge $e \in E$ are from club $c \in C$
- $yl_{e,l} \in \{0, 1\} = 1$ if both players connected by edge $e \in E$ are in league $l \in L$
- $yn_{e,n} \in \{0, 1\} = 1$ if both players connected by edge $e \in E$ are from nation $n \in N$

2.4 Additional Constraints

Minimum Total Chemistry

$$\sum_{e \in E} \left[\sum_{c \in C} yc_{e,c} + \sum_{l \in L} yl_{e,l} + \sum_{n \in N} yn_{e,n} \right] \geq \text{MinTotalChem}$$

Club Chemistry

$$2 \cdot yc_{e,c} \leq \sum_{\substack{p \in P \\ \text{club}_p = c}} [x_{p,s_{e,1}} + x_{p,s_{e,2}}], \quad \forall e \in E, \forall c \in C$$

League Chemistry

$$2 \cdot yl_{e,l} \leq \sum_{\substack{p \in P \\ \text{lea}_p = l}} [x_{p,s_{e,1}} + x_{p,s_{e,2}}], \quad \forall e \in E, \forall l \in L$$

Nation Chemistry

$$2 \cdot yn_{e,n} \leq \sum_{\substack{p \in P \\ \text{nat}_p = n}} [x_{p,s_{e,1}} + x_{p,s_{e,2}}], \quad \forall e \in E, \forall n \in N$$