

## Bus scheduling

Collection of timetabled trips as described by the data below.

Minimise the fixed cost of the buses required plus the cost of travel between trips and other trips or the depot, subject to having a bus operate every trip.

For example, one solution assigns a different bus to each trip, and has cost:

$$\sum_{t \in T} BusCost + \min_{d \in Dep_t} (DTTrav_{dt} + TDTrav_{td})$$

## Sets

$T$  set of timetabled trips

$L$  set of locations

$D$  set of depots

$Dep_t$  subset of depots which can operate trip  $t \in T$

$Succ_{td}$  set of trips that can follow trip  $t \in T$  if it is operated by a bus from depot  $d \in D$

$Pred_{td}$  set of trips that can come before trip  $t \in T$  if it is operated by a bus from depot  $d \in D$

## Data

$TStart_t, TEnd_t$  start and end times for trip  $t \in T$

$LStart_t, LEnd_t$  start and end location for trip  $t \in T$

$TTime_{l_1 l_2}$  travel time between two locations

$TTTrav_{t_1 t_2}$  travel time from trip  $t_1$  to trip  $t_2$

$TDTrav_{td}$  travel time from trip  $t$  to depot  $d$

$DTTrav_{dt}$  travel time from depot  $d$  to trip  $t$

$BusCost$  fixed cost of a bus

## Variables

$x_{ijd} \in \{0,1\}$  1 if we travel from trip  $i$  to trip  $j$  with a bus from depot  $d$ ,  $\forall i \in T, d \in Dep_i, j \in Succ_{id}$

$ys_{id} \in \{0,1\}$  1 if we start from depot  $d$  and travel straight to trip  $i$ ,  $\forall i \in T, d \in Dep_i$

$ye_{id} \in \{0,1\}$  1 if we end at depot  $d$  and straight after trip  $i$ ,  $\forall i \in T, d \in Dep_i$

### Objective

$$\min \sum_{\substack{i \in T \\ d \in Dep_i}} (BusCost + DTTrav_{di}) y_{s_{id}} + TDTrav_{id} y_{e_{id}} + \sum_{\substack{i \in T \\ d \in Dep_i \\ j \in Succ_{id}}} TTTrav_{ij} x_{ijd}$$

### Constraints

Serve every trip

$$\sum_{d \in Dep_i} y_{s_{id}} + \sum_{\substack{d \in Dep_i \\ j \in Pred_{id}}} x_{jid} = 1 \quad \forall i \in T$$

Conserve depot flow

$$y_{s_{id}} + \sum_{j \in Pred_{id}} x_{jid} = y_{e_{id}} + \sum_{j \in Succ_{id}} x_{ijd} \quad \forall i \in T, d \in Dep_i$$

## **Driver Scheduling**

Maximum shift duration – 8 hours

Maximum time without a break – 4:30

Break length is 30 minutes and can be taken anywhere

End at the depot they started from

Fixed cost per shift – minimize the number of shifts

Approximate solution if we use bus formulation, don't connect trips if it would give a shift > 8 hours.

Make sure the average duration for each depot  $\leq$  8 hours.

We can add lazy constraints to the approximate formulation.

OR we can do column generation.