1/11/2023 2021 Sem 2 Practice Theory Exam.

Scheduling Greenhouse Gas Mitigation.

Sets : ~~P~~ T Time Periods.

R Resources

$IR \subseteq R$ Inventory Resources.

P Projects.

D Project Dependencies

Data :

a_{rt} newly available resource $r \in R$ $t \in T$ appearing in time period ~~p~~ t.

m_r max inventory for $r \in R$.

0 for resources which cannot be stored.

$cost_p$ cost of project $p \in P$ \$ million

dur_p duration in time periods of project $p \in P$.

$CO2MC_p$ CO_2 mitigated (millions of tonnes) by project $p \in P$ upon completion

$resp_{rpt}$ resource $r \in R$ required by project $p \in P$ in time period $t \in T$

~~after project completion~~ $CO2M\phi P_p$ CO_2 mitigated (in millions of of tonnes) by project $p \in P$ per time period after completing project

$respac_{pr}$ ~~resptp~~ resource $r \in R$ produced from project $p \in P$ after completion

$respptp_{pr}$ resource $r \in R$ produced from project $p \in P$ per time period after completion.

B Budget over planning horizon.

Project Dependencies.

dep_{ia} project a from dependency $i \in D$

dep_{ib} " b " "

$nat_{ia} \in \{0,1\}$ nature a for dependency $i \in D$.

nat_{ib} " nat i " b " "

1/11/2023   2021. Paper Theory (continued)

~~Dec~~ Varriables.

Assuming you don't have to do all projects but each project can only be done once.

h_{rt} inventory amount for resource $r \in R$ in period $t \in T$

$x_{pt} \in \{0,1\}$ project $p \in P$ starts in time period $t \in T$.

$w_{pt} \in \{0,1\}$ 1 if project $p \in P$ active in period $t \in T$

$y_{pt} \in \{0,1\}$ 1 if project $p \in P$ finishes in time period $t \in T$.

$z_{pt} \in \{0,1\}$ 1 if project $p \in P$ has previously been completed before $t \in T$.

Objective:

CO_2 mitigation on completion of project

$$\max \left\{ \sum_{t \in T} \sum_{p \in P} CO2ME_p \cdot y_{pt} + \sum_{p \in P} \sum_{t \in T} z_{pt} \cdot CO2MtP_p \right\}$$

= CO_2 mitigation per time period after project completed

Constraints

One project at most started/completed once.

$$\sum_{t \in T} x_{pt} \leq 1. \qquad \forall p \in P.$$

$$\sum_{t \in T} y_{pt} \leq 1 \qquad \forall p \in P.$$

Completed at duration after starting

$$y_{p,(t+dur_p)} = x_{pt}. \qquad \forall t \in T \quad \forall p \in P.$$

Z off ~~after y is on.~~ ~~and at where~~ before y is on.

$$z_{pt} \leq \sum_{\substack{t' \in T \\ t' < t}} y_{pt'}. \qquad \forall t \in T \quad \forall p \in P.$$

Budget Limit.

$$\sum_{t \in T} \sum_{p \in P} x_{pt} \cdot cost_p \leq B.$$

Resource requirements. Assume 0 starting inventory. Also assume if a resource becomes available in a time period that it can be used in that time period as well as after if it can be stored

1/11/2022   2022   Review (continued)

Constraints (continued).

Starting Inventory is 0* for all

Inventory h only defined for $r \in IR$

[redacted] $h_{r,0} = 0.$ $\forall r \in IR.$

[redacted]

In all time periods after the first resources in inventory equal to previous period inventory − used in prev per. + produced in curr. on completion + made available in curr.

$$h_{rt} = h_{r,t-1} - \sum_{p \in P} w_{p(t-1)} \cdot resr_{rp(t-1)} \quad \forall r \in IR. \ \forall t \in T \setminus \{T_0\}.$$

$$+ \underbrace{\sum_{p \in P} \big(y_{pt} \cdot respa_{pr}}_{\text{on completion}} + \underbrace{z_{pt} \cdot resppt_{pr}\big)}_{\text{after completion}}$$

made avail → $+ \ a_{rt}.$

Set w. → On only when project is being completed.
Only need to bound below.

$$w_{pt} \geq \sum_{\substack{t' \in T \\ t' > t}} (y_{pt'} - x_{pt'}). \quad \forall p \in P. \ \forall t \in T$$

[redacted] kind of limit

resource minimum of 0. inventory −

$h_{rt} \geq 0 \quad \forall r \in IR \quad \forall t \in T.$

$h_{rt} \leq m_r \quad \forall r \in IR \quad \forall t \in T.$

Non stored resources also can't be negative

$$\Big(a_{rt} - \sum_{p \in P} w_{pt} \cdot resr_{rpt} + \sum_{p \in P} (y_{pt} \cdot respa_{pr} + z_{pt} \cdot resppt_{pr})\Big) \geq 0. \quad \forall r \in R \setminus IR. \ \forall t \in T.$$

This in turn limits which projects can be started.

2021 Practice (continued).

Project Dependencies.

B can't start before A.

$$x_{dep_{i,b},t} \leq \sum_{\substack{t' \in T \\ t' \leq t}} x_{dep_{i,a},t'} \quad \forall i \in D \mid nat_{ia} = 1 \quad \forall t \in T$$

B cannot start until A ends

$$x_{dep_{i,b},t} \leq \sum_{\substack{t' \in T \\ t' < t}} y(dep_{i,a}, t') \quad \forall i \in D \mid nat_{ib} = 1. \quad \forall t \in T.$$

B and A in-compatible.

$$\sum_{t \in T} \left(x_{dep_{i,a},t} + x_{dep_{i,b},t} \right) \leq 1. \quad \forall i \in D \mid nat_{ic} = 1.$$

B must be done immediately after A ends or not at all.

$$y_{dep_{i,a},t-1} = x_{dep_{i,b},t} \quad \forall i \in D \mid nat_{id} = 1 \quad \forall t \in T$$

$$\cancel{+ 2 \sum_{\substack{t' \in T \\ t' \neq t}} x_{dep_{i,b},t}} + \log \prod_{\substack{t' \in T \\ t' \neq t}} (1 - x_{dep_{i,b},t'}).$$

take log

0 if project a started anywhere else

$$\log y_{dep_{i,a},t-1} = \log x_{dep_{i,b},t} + \sum_{\substack{t' \in T \\ t' \neq t}} \log(1 - x_{dep_{i,b},t'})$$

$$\forall i \in D \mid nat_{id} = 1 \quad \forall t \in T.$$

since $\log \prod \cdots = \sum \log \cdots$

2/1/2023 2021 Revision (cont.)

Questions 2 & 3 are for prac.

Question 4. Stochastic.

Project Cost Uncertain's new:

$$cost_p \sim N(\underline{\mu}, \underline{\Sigma}).$$

$\underline{\mu}$ is $1 \times p$.

$\underline{\Sigma}$ is $p \times p$. covariance

[redacted]

Question 5.

[redacted]

For Correlated Data:

[redacted]

Extra Variable

$\gamma_p \in \mathbb{B}$ if project p is done:

[redacted]

$$\left[\sum_{p \in P} \mu_p \gamma_p\right]^2 + \left[F^{-1}(0.95)\right]^2 \underline{\gamma}^T \underline{\Sigma} \, \underline{\gamma} \le B^2$$