MATH3205 - Report

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Contents

1 Benders Formulation

1.1 Master Problem (BMP)

The master problem will assign events to time periods (with no consideration of how events will be assigned to rooms within each time period).

Sets

- E the set of events
- P the set of periods
- $PA_e \subseteq P$ the set of available periods for event $e \in E$
- $PD \subseteq E$ the set of all events with period preferences.
- $PU \subseteq E$ the set of all events with undesired periods.
- $PB \subseteq P$ the set of all periods which are undesired (for all events).
- $P1$ the set of all unordered pairs s of events $s = \{e_1, e_2\}$ for which there exists a curriculum with courses $c_1 \neq c_2$, not both primary courses, such that e_1 and e_2 are examination events for c_1 and c_2 respectively.
- $P2 \subseteq E$ the set of all events which have undesired rooms.
- $P3$ the set of all unordered pairs s of events $s = \{e_1, e_2\}$ which satisfy any of:
  - events from the same (written and oral) examination,
  - events of *consecutive* examinations from the same course, or
  - there exists a curriculum with courses $c_1 \neq c_2$ such that e_1 and e_2 are examination events for c_1 and c_2 respectively.

Data

- RA_e the set of rooms to which the event $e \in E$ could be assigned.
- PA_e the set of periods to which the event $e \in E$ could be assigned.
- SOFT_CONFLICT$_s$ the $S1$ penalty for the unordered event pair $s \in P1$.
- UNDESIRED_PERIOD the fixed penalty for assigning an event $e \in PU$ to an undesired period.
- NOT_PREFERRED the fixed penalty for assigning an event $e \in PD$ to a period other than that which is preferred.
- UNDESIRED_ROOM the fixed penalty for assigning an event $e \in P2$ to an undesired room.
- UP_e the set of undesired periods for $e \in PU$.
- PP_e the set of preferred periods for $e \in PD$.
- $UR_e \subset RA_e$ the set of undesired rooms for $e \in P2$.

Variables

- $y_{e,p} \in \{0, 1\}$ the decision variable assigning events e to periods p.
- $S1_s$ the penalty for a soft conflict between the events in the unordered pair events s, for all $s \in P1$.
- $S2_{ep}$ the estimate of the penalty for assigning the event $e \in P2$ to an undesired room in period $p \in P$.
- $S3_s$ the estimate of the penalty for non-ideal distances between the events in the unordered pair s, for all $s \in P3$.

Objective

$$\min \quad \sum_{s\in P1} S1_s \quad + \quad \text{UNDESIRED\_PERIOD}\left(\sum_{p\in PB}\sum_{e\in E} y_{e,PB} + \sum_{e\in PU}\sum_{p\in UP_e} y_{e,p}\right)$$
$$+ \quad \text{NOT\_PREFERRED}\sum_{e\in PD}\sum_{p\in PP_e}(1-y_{ep}) \quad + \quad \sum_{e\in P2}\sum_{p\in P} S2_{ep} \quad + \quad \sum_{s\in P3} S3_s$$

Constraints

Each event must be scheduled to exactly one time period (BSP (§1.2) will allocate events to rooms once the period assignment has been fixed here):

$$\sum_{p\in P} y_{e,p} = 1 \qquad \forall e\in E \tag{1}$$

We need to make sure that $S1$ is exactly the penalty for a soft conflict (two events from a common curriculum being scheduled at the same time). To do this, we will ensure that $S1_s \geq 0$ (which can be implemented by setting the lower bound for the variable in the Gurboi API) and that it achieves the cost if and only if two events are in soft conflict:

$$S1_s \geq 0 \qquad \forall s \in P1 \tag{2}$$
$$S1_{\{e_1,e_2\}} \geq \text{SOFT\_CONFLICT}_{\{e_1,e_2\}}(y_{e_1,p} + y_{e_2,p} - 1) \qquad \forall\{e_1,e_2\}\in P1,\ \forall p\in P \tag{3}$$

Lazy Constraints

We write $(\cdot)^*$ to denote the value of a variable in some fixed solution to BMP. For each period $p \in P$, let $E_p := \{e \in E \mid y^*_{e,p} = 1\}$, the set of events allocated to period p, and do the following:

- Solve BSP to obtain a room allocation for the events of E_p to the rooms available in p which minimizes the number of events allocated to undesired rooms. Let RM_e be the room assigned to event $e \in E_p$.
- If BSP is infeasible, add feasibility cuts TODO.
- If BSP has a non-zero objective value, there does not exist any way to allocate each of the events scheduled in the current period such that no event is in an undesired room. Indeed, the number of undesired room allocations has been minimized. Let $UR \subset E_p$ be the set of events allocated to an undesired room and, for each $e \in UR$ define $K_e := \{e' \in E_p \cap P2 \mid RM_{e'} \in RA_e\}$. Observe that $e \in K_e$ and, whenever BMP schedules any set of events $\mathcal{E}$ with $K_e \subseteq \mathcal{E}$ to the current period p, the penalty $S2_{ep}$ *cannot* decrease. Therefore we add the following optimality cut:

  $$S2_{e,p} \geq \text{UNDESIRED\_ROOM}\left(\sum_{e'\in K_e} y_{e',p} - |K_e| + 1\right) \qquad \forall e \in UR \tag{4}$$

  Notice that we cannot generalize this to every period because the rooms available is *not* constant across periods.

We also need to calculate the $S3$ penalties at the current solution and add optimality cuts. TODO

1.2 Sub-problem (BSP)

The sub-problem will allocate events to rooms within a time period once they have been scheduled by the master problem. In addition to the sets, data and variables defined in the master problem, we fix $p \in P$ (and hence E_p) and define the following.

Data

Let R_p be the set of rooms available in the current period.

Variables

Let $x_{e,r}$ be the decision variable assigning events $e \in E_p$ to rooms $r \in RA_e \cap R_p$.

Objective

$$\min \qquad \sum_{e \in E_p \cap P2} \sum_{r \in UR_e \cap R_p} x_{e,r}$$

Constraints

Our only constraints are that each event is allocated exactly one room, and no two events are allocated the same room:

$$\sum_{r \in RA_e \cap R_p} x_{e,r} = 1 \qquad \forall e \in E_p$$

$$\sum_{e \in E_p \,:\, r \in RA_e} x_{e,r} \leq 1 \qquad \forall r \in R_p$$