Difference of two squares

When do we use *Difference of two squares* polynomials?

First we look at for a *Term* subtracting another *Term* like $x - y$.

Then we look to see if x **AND** y are squares. So does $\sqrt{x}$ **AND** $\sqrt{y}$ make something neat.

If if it does work then great. We know the solution is going to be in the form:

$$(a-b)(a+b) \tag{1}$$

where does this come from? well it comes from the expanding the brackets.

$$(a+b)(a-b) = a^2 + ab - ab - b^2 = a^2 - b^2$$

so we are really just trying to work backwards from our difference of two squares, to our factorised form.

Lets look at an example:

$$4x^2 - 9$$

Lets start by looking at the term $4x^2$, is this the square of some term?
Yes! it is, its the square of $2x$ which is to say that

$$2x \times 2x = (2x)^2 = 4x^2$$

So now that we know that the first term is a square, is the second term a square? This one is easier, $9 = 3^2$. Since there is a minus separating them, we know that this is a **difference of two squares**.

$$4x^2 - 9 = 2x^2 - 3^2 = (2x)^2 + 2x \times 3 - 2x \times 3 - 3^2 = (2x+3)(2x-3)$$

This is a lot of work to get an answer. Thankfully there is a much easier way of doing it. If we know exactly what the two squares we need to find are, we can just substitute them into what we know the solution is going to be. So for our previous example we have:

$$4x^2 - 9 = (2x+9)(2x-9) = (a+b)(a-b)$$

As long as we know a and b we and we know the problem is a perfect square we don't need anymore information, we can just substitute it into (1) So to summarise we can use the difference of two squares **IF**:

1. both terms are squares
2. they are separated by a minus sign

If it doesn't, we need a different method.

Binomial Expansions

A Binomial expansion is the process of expanding brackets with two terms in them.

there are two important expansions for us to learn.

$$(a-b)(a-b) \textbf{ or } (a-b)^2 \tag{2}$$

and

$$(a+b)(a+b) \textbf{ or } (a+b)^2 \tag{3}$$

If we take a look at $(a-b)(a-b)$ we can expand it like this:

$$(a-b)(a-b) = a^2 - 2ab + b^2$$

and similarly for $(a+b)(a+b)$ we have:

$$(a+b)(a+b) = a^2 + 2ab + b^2$$

This is stuff we know, so why is it important.

When we change small things in the terms of $a^2 + 2ab + b^2$ sometimes we can still factorise (or un-expand) it.
Let look at an examples:

$$2a^2 + 4ab + 2b^2 = 2 \times (a^2 + 2ab + b^2) = 2(a+b)(a+b)$$

All we did was multiply (3) by 2.

Next:

$$-3a^2 - 6ab - 3b^2 = -3 \times (a^2 + 2ab + b^2) = -3(a+b)(a+b)$$

Similarly we mulipied (3) by -3.

These are pretty easy examples so when does it get hard.

Remember a and b can be any numbers or variables. So we can have problems like this.

$$x^2 + 4x + 4$$

Here a is going to be x but what is b?

Well if this follows our formula from (3) then we know that $4 = b^2$, so it looks like $b = 2$ but does this make sense?

well if $b = 2$ then:

$$x^2 + 4x + 2^2 = a^2 + 2ab + b^2 = (a + b)(a + b) = (x + 2)(x + 2) = (x + 2)^2$$

Glossary

Binomial expansion The word *binomial* can be broken up into two pieces
bi meaning 2
nomial meaning term or variable.

So a Binomial expansion is just a fancy word for expanding the brackets with two terms.
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Difference of two squares Any *polynomial* that be expressed as the *difference of two squares* has a solution in the form:
$a^2 - b^2 = (a + b)(a - b)$
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Term A *term* is any letters or numbers with no addition or subtraction next to them.
$2a + b$
Here $2a$ is a term, and b is a seperate term
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