Signal Processing - LAB 3 Report

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In this lab, we built simple discrete time (DT) systems that performs various tasks. We characterised this system using pole-zero plots, ROC's and impulse responses.

QUESTION 1

a) The Moving Average (MA) system is used to observe trends in the input signal. It finds the average of the signal, over the past few samples, depending on the window size.

b) The system is tested with the discrete unit signal as input.

c) The trend of the given input signal is plotted using the system we implemented.

d) We notice that as we increase the value of N, the output signal becomes more accurate. This makes sense because when N increases, we're taking more samples of the signal into consideration when finding the average, rendering the output signal to be more accurate.

▼ QUESTION 1.1

We also construct the same system using the impulse response and convoluting it with the given input signal. The result is the same in both the cases.

We notice that the method of convolution to construct the system is better because we don't need to have the system equation to get the output. All we need is the impulse response. Convoluting the impulse response with any arbitrary input signal will give us the corresponding output signal.

QUESTION 2

a) An upsampler is a system that increases the length of a given sequence. The unknown values in the new output signal is found using various interpolation techniques. The two we use in the question are zero order interpolation (we have the same value until we hit a new known sample) and linear interpolation (two consecutive known samples are connected with a straight line and the unknown values are found using the line equation).

b) The two given input signals are used to test our system and we also change the value of M to see how the output sequence changes.

NOTE:

- In file q2_a → The input signal in ‘q2_1.mat’ is upsampled using zero order interpolation and M=2 and M=3.
- In file q2_b → The input signal in ‘q2_2.mat’ is upsampled using linear interpolation and M=2 and M=3.

QUESTION 3

a) An Amplitude Modulation (AM) system is used to shift the frequency of a given signal in a communication system. This is done using the formula:

$$y[n] = (cos\omega_0 n)x[n]$$

where x[n] → message signal and $\cos(w_0 n)$ → carrier signal

When the input signal with the specified parameters are given to the system, the output signal is amplitude modulated

b) H(z) is found by finding the z-transform of the impulse response h[n]

$$y[n] = (\cos \omega_0 n)\, x[n]$$

when $x[n] = \delta[n]$,

$$y[n] = (\cos \omega_0 n)\, \delta[n]$$

now, $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise.} \end{cases}$

$$\Rightarrow h[0] = (\cos(0)) \times 1$$

$$= \underline{\underline{1}}.$$

so, $H(z) = \underline{\underline{1}}$.

c) Since H(z) = 1, this implies that the system has no poles or zeros.

d) Since the transfer function has no poles or zeros, and H(z) is a constant, the z-transform of the signal converges to a finite always. Hence, the ROC of the system would be the whole z-plane.

QUESTION 4

a) To use the *zplane* function, we need the coefficients of z inverse

$$H(z) = \frac{z^2 - (2\cos\theta)z + 1}{z^2 - (2r\cos\theta)z + r^2}$$

$$= \frac{1 - (2\cos\theta)z^{-1} + z^{-2}}{1 - (2r\cos\theta)z^{-1} + z^{-2}r^2}$$

Using $r = 0.95$ and $\theta = \pi/3$,

$$H(z) = \frac{1 - z^{-1} + z^{-2}}{1 - 0.95z^{-1} + (0.95)^2}$$

Using the coefficients found, the pole-zero plot of the given system is graphed.

b) To find the inverse z-transform we use partial fractions and standard z-transforms.

3.4) $$H(z) = \frac{z^2 - (2\cos\theta)\, z + 1}{z^2 - (2r\cos\theta)\, z + r^2}$$

Numerator $\rightarrow$ $z^2 - (2\cos\theta)\, z + 1$

$$z = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$$= \frac{2\cos\theta \pm 2\sqrt{\cos^2\theta - 1}}{2}$$

$$= \cos\theta \pm \sqrt{\cos^2\theta - 1}$$

$$\therefore H(z) = \frac{(z-e^{j\pi/3})(z-e^{-j\pi/3})}{(z-0.95e^{j\pi/3})(z-0.95e^{-j\pi/3})}$$

Applying partial fractions,

$$\frac{(z-e^{j\pi/3})(z-e^{-j\pi/3})}{(z-0.95e^{j\pi/3})(z-0.95e^{-j\pi/3})} = \frac{A}{(z-0.95e^{j\pi/3})} + \frac{B}{(z-0.95e^{-j\pi/3})}$$

$$(z-e^{j\pi/3})(z-e^{-j\pi/3}) = A(z-0.95e^{-j\pi/3}) + B(z-0.95e^{j\pi/3})$$

$$z^2 + (-e^{j\pi/3} - e^{-j\pi/3})z + e^{j2\pi/3} = A(z-0.95e^{-j\pi/3}) + B(z-0.95e^{j\pi/3})$$

Comparing coefficients,

$$A+B = -e^{j\pi/3} - e^{-j\pi/3}$$

$$e^{j2\pi/3} = -A(0.95e^{-j\pi/3}) + B(0.95e^{j\pi/3})$$

$$\therefore A+B = -2 \times 1/2 = \underline{\underline{-1}}. \implies A = -1 - B$$

$$-\frac{1}{2} + j\frac{\sqrt{3}}{2} = -0.95A\left(1/2 - j\frac{\sqrt{3}}{2}\right) - 0.95B\left(1/2 + j\sqrt{3}/2\right)$$

$$-\frac{1}{2} + j\frac{\sqrt{3}}{2} = (+0.95 + 0.95B)(1/2 - j\sqrt{3}/2) - 0.95B(1/2 + j\sqrt{3}/2)$$

$$-\frac{1}{2} + j\frac{\sqrt{3}}{2} = 0.95/2 - j0.95\sqrt{3}/2 + \cancel{0.95B/2} - 0.95Bj\sqrt{3}/2$$
$$- \cancel{0.95B/2} - 0.95j\sqrt{3}/2B.$$

$$-\frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{0.95}{2} + j\frac{0.95\sqrt{3}}{2} = -1.90j\frac{\sqrt{3}}{2}B.$$

$$\therefore B = \frac{(-1.95 + 1.95\sqrt{3}j)\cancel{2}}{\cancel{2} \times -1.9\sqrt{3}j}$$