Signal Processing - LAB 2 Report

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In today's lab, we focussed on analysing LTI systems using the Z-transform. We used various MATLAB tools to visualise the system in the z-domain.

QUESTION 1

a) The function *roc_cs(p)* takes a vector of poles as the input. We then write the function to find the number of possible ROC's, and whether each ROC is causal or stable.

For causal systems, the ROC should be right-handed and for stable systems, the ROC should contain the unit circle, |z| = 1.

[**NOTE:** The function file is named as the function itself and not 'q1_a.m' because of MATLAB convention to name the function script the same as the function name.]

b) The output of the function is verified for different values of poles. Verifying the answer for p = [0, j, -j] :

1. b) $p = [0, j, -j]$

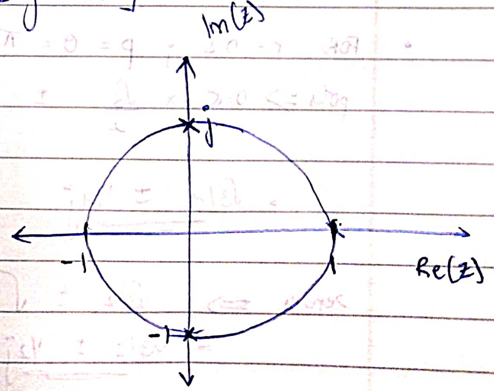
Number of possible ROC's = 2

Range of ROC's :- $(0, j)$ $= \begin{bmatrix} 0 & j \\ j & \infty \end{bmatrix}$

(j, ∞)

Causal vector, $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Stability vector, $S = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



QUESTION 2

a) *zplane(b,a)* is used to plot the pole-zero plot where b and a are the coefficient vectors of the numerator and denominator respectively.

b) *freqz(b,a,n)* is used to plot the frequency response (magnitude and phase plot) given the same coefficient vectors b and a, and n, the number of points.

c) *impz(b,a)* plots the impulse response of the filter based on the same inputs, a and b.

There is only one possible impulse response to the system, which occurs when the unit impulse function is given as input.

d) We observe that as the value of p decreases, the impulse response becomes more even since for p = 0.1, we see that the response has some negative components, while this is not the case for p = -0.8.

Also, the frequency plot of -0.8 has a negative magnitude while that of 0.1 has a positive amplitude.

e) To input the correct b and a vectors into the function, we need to write the transfer function in terms of z inverse, as shown :

2. e) $H(z) = \dfrac{z - p^{-1}}{z - p}$

$$= \frac{1 - p^{-1}/z}{1 - p/z}$$

$$= \frac{1 - p^{-1}z^{-1}}{1 - pz^{-1}}$$

$\therefore$ $b = [1, -p^{-1}]$ and $a = [1, -p]$.

QUESTION 3

a) The poles and zeroes of the system for the pair of values (r = 0.5 and p = pi/6) and (r=0.25 and p = pi/2) are calculated as below:

3. a) $H(z) = \dfrac{z^2 - (2\cos\theta)z + 1}{z^2 - (2r\cos\theta)z + r^2}$

POLES: $z^2 - (2r\cos\theta)z + r^2 = 0$

$\therefore\ z = \dfrac{2r\cos\theta \pm \sqrt{4r^2\cos^2\theta - 4r^2}}{2}$

$= r\cos\theta \pm r\sqrt{\cos^2\theta - 1}$

ZEROES: $z^2 - (2\cos\theta)z + 1 = 0$

$\therefore\ z = \dfrac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

$= \cos\theta \pm \sqrt{\cos^2\theta - 1}$

- FOR $r = 0.5$; $p = \theta = \pi/6$,

poles $\Rightarrow 0.5 \times \dfrac{\sqrt{3}}{2} \pm 0.5 \times \sqrt{-1/4}$

$= \underline{\sqrt{3}/4 \pm 1/4j}$

zeroes $\Rightarrow \sqrt{3}/2 \pm \sqrt{-1/4}$

$= \underline{\sqrt{3}/2 \pm 1/2j}$

- FOR $r = 0.25$; $p = \theta = \pi/2$,

poles $\Rightarrow 0.25 \times 0 \pm 0.25\sqrt{-1}$

$= \underline{0.25j}$

zeroes $\Rightarrow 0 \pm \sqrt{0-1}$

$= \underline{j}$

b) No, the system can never be stable because the condition for stability will never be satisfied. The condition states that the ROC should contain the unit circle. However, this will never be possible because we notice that the poles of the system are always within the unit circle (in the given range of r and p), implying that the ROC can never contain the circle.

The system can be causal if the ROC is right-handed.

c) I varied p in steps of pi/10 from 0 to pi for the fixed value of r = 0.95, to observe the behaviour of the system.

d) I varied r in steps of 0.01 from 0.1 to 0.9 for the fixed value of p = pi/3, to observe the behaviour of the system.

QUESTION 4

a) From the frequency plot of the transfer function, we can tell that the system will have 6 poles and they they will be present in the form of conjugate pairs (negative of each other).

b) The pole-zero plot obtained is in agreement with our previously stated observation.