Signal Processing - LAB 8 Report

Sreeja Guduri (2021102007)

In this lab, we try to understand the concept of filters by designing a few FIR and IIR filters and use the *filterDesigner* tool in MATLAB.

8.1 - Low-pass FIR filter using windows

a) We define the signal h[n] by computing the inverse DFT of the given impulse response, $H_d(w)$ and thus finding $h_d[n]$.

Then, we calculate h[n] by multiplying $h_d[n]$ with a rectangular window w[n] of length 51.

$$h_D[n] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_D(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{-j\omega n_c} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{-j\omega(-n+n_c)}\, d\omega$$

Now, n_c for $N = 51$: $n_c = \frac{N-1}{2} = 25$

$$\therefore h_D[n] = \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{-j\omega(-n+25)}\, d\omega$$

$$= \frac{1}{2\pi}\left(\frac{e^{-j\omega(25-n)}}{-j(25-n)}\right)_{-\pi/6}^{\pi/6}$$

$$= \frac{1}{\pi(25-n)}\left[\frac{e^{-j\pi/6\,(25-n)} - e^{j\pi/6\,(25-n)}}{-2j}\right] = \frac{1}{\pi(25-n)}\left[\sin\left(\pi/6\,(25-n)\right)\right]$$

computing inverse DFT

b) Using the *fft* function in MATLAB, we compute the Fourier Transform of h[n]. We then plot the magnitude and phase spectrums.

We notice that the phase spectrum is in fact linear, implying a linear phase filter.

c) Now instead of w[n] being a rectangular window, we set it to be a Blackmann window of length 51.

d) We notice that the transition bands and the side-lobes are more flat and sharper in the case of the Blackmann filter, when compared to the rectangular window filter.

e) We generate 201 sample of the signal

$$x[n] = cos(pi * n/16) + 0.25 * sin(pi * n/16)$$

and plot the signal and the filter outputs.

Similarly, we compute filter outputs for the signal

$x[n] = cos(pi * n/16) + 0.25 * randn(pi * n/16)$

f) We notice that even if the impulse response is changed like given, the magnitude spectrum remains the same.

8.2 - Notch Filter

a) A notch filter is one that passes most frequencies but has zero frequency response at $w = w_0$.

For the given H(z), we find b_0 using the relation that H(1) = 1.

Thus, we get b_0 to be 1.706.

We then find the coefficients of the numerator and denominator terms and use the *freqz( )* function to plot the frequency resposne.

$$H(z) = b_0 (1 - e^{j\omega_0} z^{-1})(1 - e^{-j\omega_0} z^{-1})$$

Now, $H(z) = 1$ when $z = 1$.

$$1 = b_0 (1 - e^{j\omega_0})(1 - e^{-j\omega_0})$$

Now, $\omega_0 = \frac{\pi}{4}$

$$\therefore\ 1 = b_0 (1 - e^{j\pi/4})(1 - e^{-j\pi/4})$$

$$1 = b_0 (1 - e^{j\pi/4} - e^{-j\pi/4} + e^{0})$$

$$1 = b_0 (2 - 2\cos \pi/4)$$

$$\therefore b_0 = \frac{1}{2 - 2 \times 1/\sqrt{2}}$$

$$= \frac{1}{2 - \sqrt{2}}$$

$$\text{Now, } H(z) = b_0 (1 - e^{j\omega_0} z^{-1} - e^{-j\omega_0} z^{-1} + z^{-2})$$

$$= b_0 (1 - (e^{j\omega_0} + e^{-j\omega_0}) z^{-1} + z^{-2})$$

$$= b_0 (1 - 2\cos(\pi/4) z^{-1} + z^{-2})$$

$$= b_0 (1 - \sqrt{2} z^{-1} + z^{-2})$$

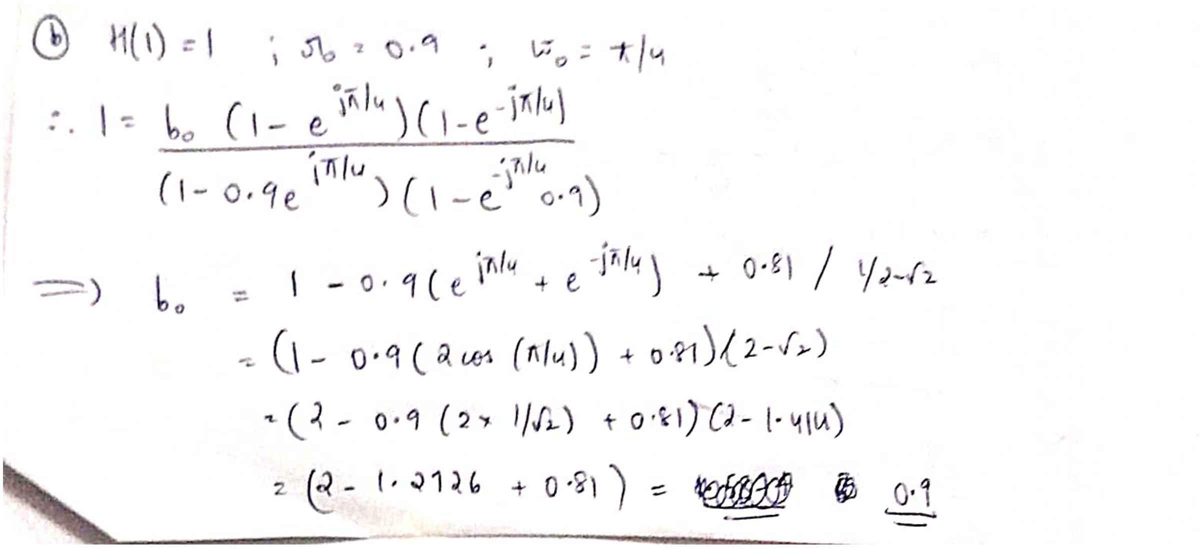
$$= b_0 - \sqrt{2} b_0 z^{-1} + b_0 z^{-2}$$

computing b0 and finding the coefficients of H(z)

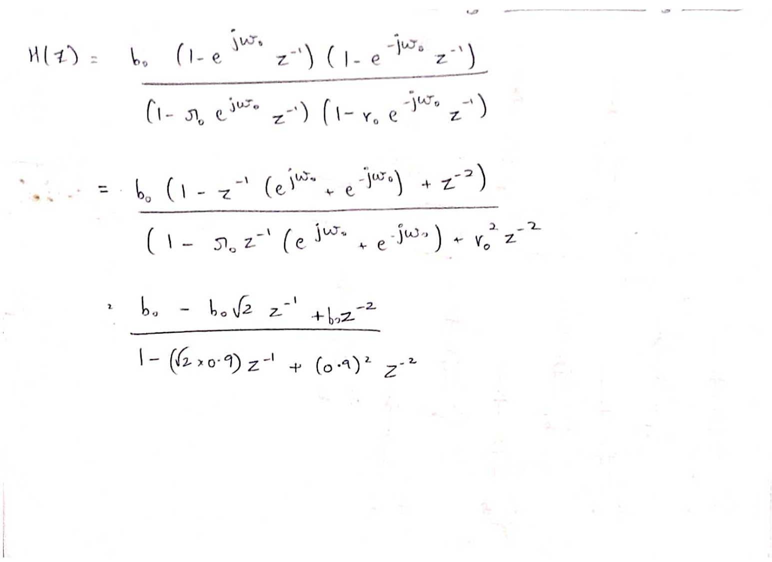
b) Similar to (a) we find b_0 using the relation that H(1) = 1.

Thus, we get b_0 to be 0.9.

We then the *freqz( )* function to plot the frequency resposne.



computing b0



finding the coefficients of H(z)

c) The first filter is not stable because it’s poles and zeroes are not bounded by the unit circle (as is eveident from the *zplane* plot).

The second filter however is both stable and causal.

d) We use the function *fvtool( )* to visualize the various components of the notch filters.

e) We notice that the first filter doesn’t produce a very clear output while the output of the second filter is crisp and clear.

f) We plot the first 100 samples of the filter and verify that they work as expected.

8.3 - *filterDesigner* function

We familiarise ourselves with the function that is used to design various types of filters.