Signal Processing - LAB 1 Report

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In this lab, we numerically studied the Fourier series (FS) analysis and synthesis. We wrote a function to compute the FS coefficients of a periodic signal, and another function that reconstructs the signal, given its FS coefficients. We then analysed the properties of FS and looked into the Gibbs Phenomenon.

QUESTION 1

a) The function *fourierCoeff(t,xt,T,t1,t2,N)* takes the following as inputs:

- t - symbolic variable
- xt - periodic signal (whose FS coefficients are to be determmined). This signal is a function of the symbolic variable.
- T - time period of the signal
- t1, t2 - limits of integration where the signal is valid
- N - number of FS coefficients to compute

The function computes the FS coefficients and returns it as a vector F.

The FS coefficients for the given sinusoidal xt are plotted.

b) The FS coefficients are also computed for the periodic square wave described by

$$x(t) = \begin{cases} 1, & -T_1 \le t \le T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

QUESTION 2

a) The function *partialfouriersum(F,T,time_grid)* takes the following as inputs:

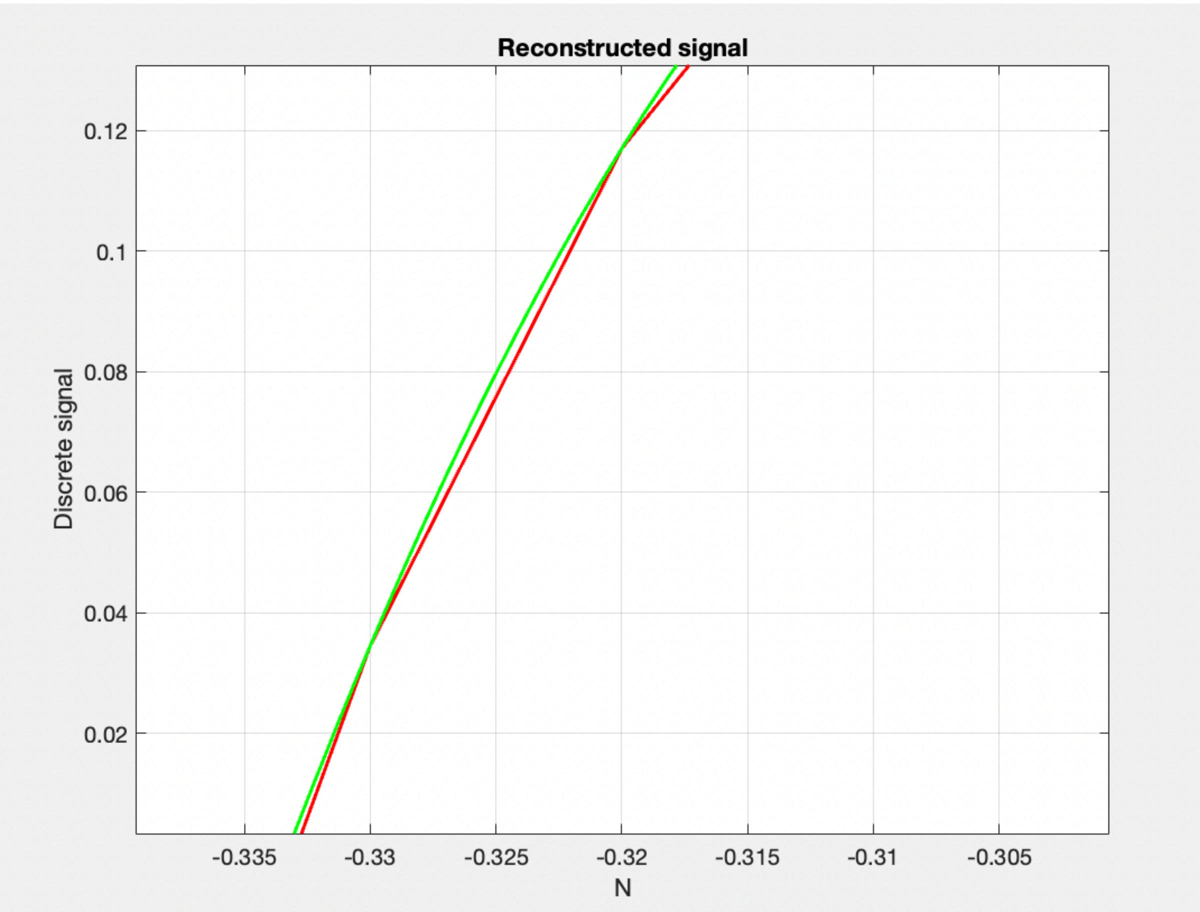
- F - vector of the FS coefficients of the signal
- T - period of the signal
- time_grid - time vector

The function returns the vector y, which is the reconstruction of the original signal.

The FS coefficients from the previous part are given as input and the reconstructed signal is plotted.

b) Both the reconstructed signal and the original signal are plotted on the same graph using the *hold on* function.

c) We observe that there is a slight error between the two signals.



We characterise this difference between the two signals by:

1. Maximum Absolute Error (MAE) → The maximum absolute error is the absolute difference between the measured value and the actual value.
2. Root Mean Squared (RMS) error → The root mean square error is given by the formula

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

QUESTION 3

a) The FS coefficients for the square wave is computed as below:

$T=1$ and the signal is only valid for $-T_1 = -0.1$ to $T_1 = 0.1$.

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} x(t)\, e^{-j\omega_0 k t}\, dt$$

For $k=1$,

$$a_1 = \frac{1}{1}\int_{-0.1}^{0.1} 1 \times e^{-j\omega_0 t}\, dt$$

$$= \int_{-0.1}^{0.1} e^{-j\omega_0 t}\, dt$$

$$= \left[\frac{e^{-j\omega_0 t}}{-j\omega_0}\right]_{-0.1}^{0.1}$$

$$= \left[\frac{e^{-j\omega_0(0.1)} - e^{-j\omega_0(-0.1)}}{-j\omega_0}\right]$$

$$= \left[\frac{e^{j\omega_0(0.1)} - e^{-j\omega_0(0.1)}}{+j\omega_0}\right]$$

$$= \left[\frac{e^{j2\pi(0.1)} - e^{-j2\pi(0.1)}}{+j2\pi}\right] \qquad [\omega_0 = 2\pi/T = 2\pi].$$

$$= \frac{\sin(2\pi(0.1))}{\pi}$$

$$\approx \underline{0.187}$$

b) The FS coefficients for the signal are scaled and then plotted. From the plots we observe that as the FS coefficients are scaled, the reconstructed signal is also scaled.

c) The square wave is reconstructed using the *partialfouriersum* function.

It is evident that the reconstruction that there is a sharp discontinuity in the reconstructed signal, which when superimposed with the original signal, shows us that the sharp upshoots are in line with the original graph.

QUESTION 4

a) The FS coefficients of the first and second signal are found using the function and are then plotted on the graph.

b) We see that for the first signal, the plot is mirrored about the y-axis, just like the original signal.

For the second signal, half the FS coefficients are positive and the rest are negative, again following signal behaviour.

Hence, the FS coefficient plots follow signal behaviour.