• one question from Spandan Sirs classes

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) = \tau$$

$M(q)$ → inertia matrix; $C(q,\dot{q})$ → coriolis force; $F(\dot{q})$ → dissipative forces; $G(q)$ → gravitation

• q & $\dot{q}$ → called generalized coordinates.
for mechanics, q → joint pos^n
$\dot{q}$ → joint velocity

• $q \in \mathbb{R}^n$

PROPERTIES: • mass matrix is symmetric
• $0 < \mu_1 I < M(q) \leq \mu_2 I$
mass matrix is upper and lower bounded by constants

now, $M\ddot{q} + H = \tau$ where
$H = C\dot{q} + F + G$

→ a point is called equilibrium if you can never come out of it once you're in it.
(i.e, velocity = 0)

$$\dot{x} = f(x) = 0$$

• regulation problem: get as close to equilibrium (zero) as possible
• tracking problem: what is the trajectory /path

TRACKING

- go from q to q^d (q desired)

$\implies$ $q \to q^d$
$\dot{q} \to \dot{q}^d$
$\ddot{q} \to \ddot{q}^d$

Now, $\tau = M\ddot{q}^d + H$ $\to$ dynamics to get the robot to follow desired path.

- however this will result in errors
$e = q - q^d$

- we get fine details from the acceleration of an object so we need to analyze $\ddot{e}$

$$M\ddot{q} + H = M\ddot{q}^d + H$$
$$M(\ddot{q} - \ddot{q}^d) = 0$$
$$\ddot{q} - \ddot{q}^d = 0$$
$$\boxed{\ddot{e} = 0} \longrightarrow \text{error dynamics}$$

- $q = e + q^d$

– given q^d is bounded and e is ramp then at $t \to \infty$, q is unbounded and so it is unstable

• the controller dynamic we used is open-loop (no feedback) $\rightarrow$ i.e there is no correction mechanism.

$$\ddot{x} + k_1 \dot{x} + k_2 x = 0 \qquad ; \qquad k_1, k_2 > 0$$

$$D^2 + k_1 D + k_2 = 0 \implies D_{12}$$

• poles correspond to the eigenvalues of the above equation.

Thus, we design τ such that $\ddot{e} + k_1 \dot{e} + k_2 e = 0$ which implies $q \rightarrow q^d$

now, $\ddot{e} + k_1 \dot{e} + k_2 e = 0$

$$(\ddot{q} - \ddot{q}_d) + k_1 \dot{e} + k_2 e = 0$$

$$M(\ddot{q} - \ddot{q}_d) + M(k_1 \dot{e} + k_2 e) = 0$$

$$\tau - H + M(k_1 \dot{e} + k_2 e) - M\ddot{q}^d = 0$$

$$\implies \boxed{\hat{\tau} = M\ddot{q}^d + H + M\underbrace{(k_1 \dot{e} + k_2 e)}}$$

feedback torque

feedback mechanism

• $\hat{\tau} = \bar{M}\ddot{q}^d - \bar{M}(k_1 \dot{e} + k_2 e) + \bar{H}$

$$\ddot{e} = \frac{\hat{\tau} - \bar{H} + \bar{M}(k_1 \dot{e} + k_2 e)}{\bar{M}} - \frac{\hat{\tau} - H + M(k_1 \dot{e} + k_2 e)}{M}$$

$$= \frac{\hat{\tau}}{\bar{M}} - \frac{\bar{H}}{\bar{M}} + k_1 \dot{e} + k_2 e - \frac{\hat{\tau}}{M} + \frac{H}{M} - k_1 \dot{e} - k_2 e$$

$$= \frac{\hat{\tau}}{\bar{M}} - \frac{\hat{\tau}}{M} - \frac{\bar{H}}{\bar{M}} + \frac{H}{M} + 2k_1 \dot{e} + 2k_2 e$$