14/09/23

Workspace: if Q is the configuration space of a manipulator and $g: Q \rightarrow SE(3)$

$$W = \{ g(\theta) \mid \theta \in Q \} \subset SE(3)$$

- reachable workspace is the set of positions that can be reached.

$$W_R = \{ g(\theta) \mid \theta \in Q \}$$

→ 2R manipulator :

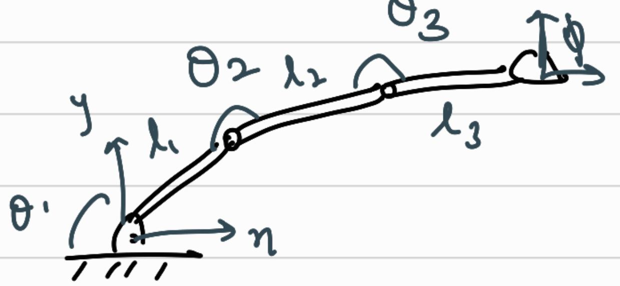
configuration space – (θ_1, θ_2)

workspace – (x, y)

- Inverse kinematics : $(x, y) \rightarrow (\theta_1, \theta_2)$

  Forward kinematics : $(\theta_1, \theta_2) \rightarrow (x, y)$

→ 3R planar manipulator :



$$x = l_1 \cos\theta_1 + l_2 \cos\theta_{12} + l_3 \cos\theta_{123}$$

$$y = l_1 \sin\theta_1 + l_2 \sin\theta_{12} + l_3 \sin\theta_{123}$$

$\phi = \theta_{123}$ (summation)

- infinite number of solutions for $\theta_1, \theta_2, \theta_3$.

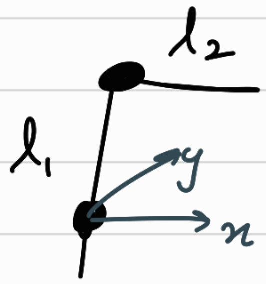
DEXTEROUS WORKSPACE

- W_R does not consider the ability to arbitrarily orient the end-effector.
- can reach any arbitrary orientation.

$$W_D = \{ p \in \mathbb{R}^3 \mid \forall R \in SO(3) \} \subset \mathbb{R}^3$$

- 2R manipulator : $W_D = W_R$. (only for 2R)

(i) RR spatial manipulator



sphere

- workspace :

2R $\rightarrow$ $(l_1 + l_2)$ outer
$(l_1 - l_2)$ inner boundary

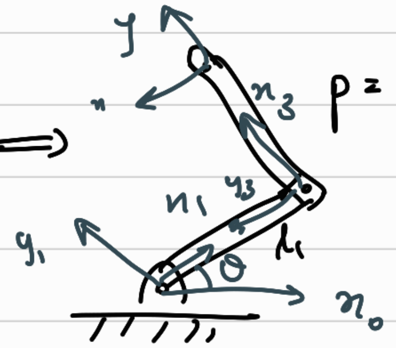
3R $\rightarrow$ $(l_1 + l_2 + l_3)$ outer
$(l_1 - l_2 - l_3)$ inner boundary

- radius of dexterous workspace :

$$(l_1 - l_2 + l_3 \;,\; l_1 + l_2 - l_3) = W_D$$

- Thus,

$$x = l_2 \cos\theta_2 \cos\theta_1$$
$$y = l_2 \cos\theta_2 \sin\theta_1$$
$$z = l_1 + l_2 \sin\theta_2$$



$\Longrightarrow$ $p = [x, y]$

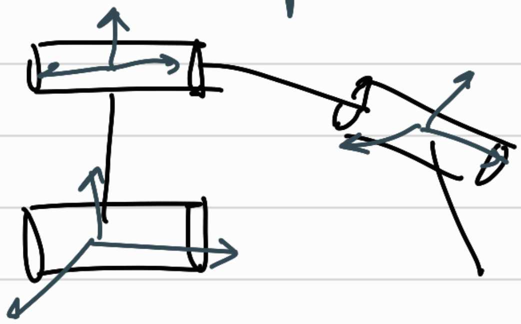
- rotate base frame by θ
- scale by l_1
- rotate by θ_2
- translate by l_2.

$\Rightarrow$ rotate by z : (wrt body fixed) $\rightarrow$ POST MULTIPLY

$$\boxed{\therefore T = R(\hat{z}, \theta_1)\, T(\hat{x}, l_1)\, R(\hat{z}, \theta_2)\, T(\hat{x}, l_2)}$$

For the first rotation,
$T(\hat{z}, 0)\ R(\hat{z}, \theta_1)$ $\Longrightarrow$ pure rotation

(iii) 3R Spatial manipulator



1) translate about z
2)

- the joint axis is always z-axis
- to check the correctness of your transformation matrix $\rightarrow$ substitute $\theta_1, \theta_2, \theta_3 = 0$
this is called home position.
(pos^n of end effector frame wrt base frame)

DENAVIT - HARTENBERG CONVENTION

- we usually need 6 parameters to describe adjacent link frames
- however with this convention, we need only 4.

RULES:

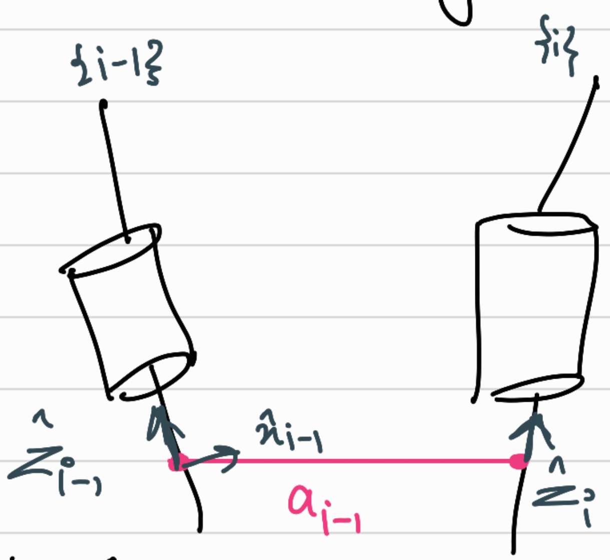
- two link parameters : link length & link twist
- two joint parameters : joint angle & link offset.

n = no. of joints

$(n+1)$ = no. of links

- $\hat{z}$ axis should coincide with joint axes.

→ Find a line segment a_{i-1} that mutually intersects $\hat{z}_i$ and $\hat{z}_{i-1}$



- $\hat{x}_{i-1}$ is chosen to be in the direction of the mutually perpendicular pointing from $\hat{z}_{i-1}$ and z_i

- $\hat{y}_{i-1}$ is determined using RH rule. (perpendicular to $\hat{z}_{i-1}$ & $\hat{x}_{i-1}$)
$\hat{z} \times \hat{x}$

LINK LENGTH : a_{i-1} or the length b/w $\hat{z}_{i-1}$ and $\hat{z}_i$ along $\hat{x}_{i-1}$.

- LINK TWIST (α_{i-1}) is the angle from $\hat{z}_{i-1}$ to $\hat{z}_i$ measured about $\hat{x}_{i-1}$

LINK OFFSET (d_i) is the distance from the intersection of $\hat{x}_{i-1}$ and z_i to the origin of the link {i} frame.

JOINT ANGLE

$$ {}^{i-1}_{i}[T] = R(\hat{x}_{i-1}, \alpha_{i-1})\, T(\hat{x}_{i-1}, a_{i-1}) $$

CASE 1: $z_i \perp z_{i-1}$

link length $= 0$

α_{i-1} could be -ve.

CASE 2: $\hat{z}_i \parallel \hat{z}_{i-1}$