Dynamics :

Two formulations : Newton - Euler formulation.

&

Euler - Lagrangian formulation

- We use Euler - Langrangian.
  It is an energy based formulation.

- Statics is useful for the study of equilibrium.

→ dynamics is concerned with the study of forces and their effects on motion.

inverse dynamics : given a → find F

$F = ma$

- For a multibody system,

$$\tau = M(q)\ddot{q} + \underbrace{C(q,\dot{q})}_{\text{coriolis + centripetal}} + G(q)$$

$\tau \rightarrow$ joint torque

$\ddot{q} \rightarrow$ joint acceleration.

forward dynamics : given F → find a

$$\ddot{q} = M^{-1}(q)\left(\tau - C(q,\dot{q}) - G(q)\right)$$

Linear momentum :

$$\vec{p} = mv$$
$$= m\dot{r}$$
$$= m\frac{dr}{dt}$$

- $\vec{p} = \vec{p_1} + \vec{p_2}$

Newton's Law :-

Force $\propto$ acceleration

$F = ma$

- comes from the principle of Linear momentum.

$$p = mv$$
$$\dot{p} = F$$
$$= m\dot{v} + \cancel{\dot{m}}v$$
$$= \underline{\underline{ma}}$$ (w.r.t. inertial frame)

- a system of particles has a center of mass.

$$r_{cm} = \sum \frac{m_1r_1 + m_2r_2 + m_3r_3}{m_1 + m_2 + m_3}$$

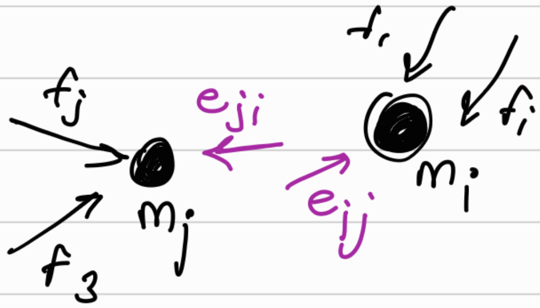
$$= \sum \frac{m_ir_i}{M}$$

- Thus for a rigid body,

$$p = M\dot{r}_{cm}$$

$\dot{r}_{cm} \rightarrow$ velocity of COM

- $\dot{p} = \sum_i m_i \frac{d^2}{dt^2} r_i$



$$= \sum_i \left(f_i + \sum_j e_{ij} \right)$$

now $\sum_i \sum_j e_{ij} = 0$ (equal & opposite)

$$\therefore \dot{p} = \sum_i f_i$$

$$= \underline{\underline{F}}$$

Angular velocity : ω

Angular momentum : $H = I\omega$

↳ moment of inertia

- $H = r \times p$

$$= r \times m\dot{r}$$

- Now, $H = \sum_i r_i \times p_i$

$$= \sum_i r_i \times (m_i \dot{r}_i)$$

$$= \sum_i r_i \times m_i (\omega \times r_i)$$

$$= \sum m_i r_i \times (-r_i \times \omega)$$

$$= \underline{I\omega}$$

- principle of angular momentum.

ENERGY

- $K = \sum_i \frac{1}{2} m_i v_i^2$ — (1)

$P = \sum_i m_i g h_i$

$$r_i = r_{cm} + \Delta r_i$$

$$\Downarrow$$

$$v_i = v_{cm} + \Delta \dot{r}_i$$

$$\downarrow$$

$$\omega \times \Delta r_i$$

$$v_i = v_{cm} + \omega \times \Delta r_i \quad \text{- (2)}$$

(2) in (1)

$$K = \sum \frac{1}{2} m_i \, v_i \cdot v_i$$

$$\boxed{K = \frac{1}{2} m (v_{cm}^T \cdot v_{cm}) + \frac{1}{2} \omega^T I \omega}$$

Like a bad relationship you can keep it alive but someone else has the key.

Q

$F_x = 0$

$F_y = -mg$

now to find pos^n and velocity, we integrate

$\cancel{m}\ddot{y} = -\cancel{m}g$

$\boxed{\ddot{y} = -g}$

→ Λ : set of all motions

$m : t \to q(t)$

- $\Lambda \to$ trying to assign a real number for motion

- For maxima & minima, derivate $= 0$

  " derivate > 0 (minima)

Shortest distance :-

$ds = \sqrt{dx^2 + dy^2}$

$$\int_{x_1}^{x_1} ds = \int_{x_1}^{x_2} \sqrt{(1+y')^2}\, dx$$

$y' = dy/dx$

$$\int_{1}^{x_1} ds = \int_{x_1}^{x_2} \mathcal{L}\left(y'(x),\ y(x),\ x\right)\ dx$$

$\hookrightarrow$ $\boxed{\text{functional.}}$

$$\cdot\ \frac{d}{dx}\left(\frac{d\mathcal{L}}{d\dot{y}}\right) - \frac{d\mathcal{L}}{dy} = 0$$

now, $\alpha = \sqrt{1+y'^2}$

$$\frac{\partial L}{\partial \dot{y}} = \frac{y'}{\sqrt{1+y'^2}}$$

$$\frac{\partial L}{\partial y} = 0$$

$$\frac{d}{dx}\left(\frac{\partial L}{\partial \dot{y}}\right) = 0$$

$\Rightarrow \dfrac{d\mathcal{L}}{\partial \dot{y}}$ is a constant.

$$\therefore C = \frac{y'}{\sqrt{1+y'^2}}$$

$$C(1+y'^2) = y'^2$$

$$c = (1-c)\, y'^2$$

$$y'^2 = \frac{c}{1-c}$$

$$y' = M \quad \text{(constant)}$$

$$\frac{dy}{dx} = M$$

$$dy = M\,dx$$

$$\Rightarrow \quad y = mx + c \qquad \text{(straight line equation)}$$