Recap:

- $p = f(\theta_1, \theta_2)$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{J_v} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

- dimensions of J_v : $(3 \times n)$
  $n \rightarrow$ degrees of freedom.

- differentiate position $\dot{p} = J_v \dot{\theta}$ to get velocity Jacobian.

- If singular configuration,
  $\det(J) = 0$

$\Rightarrow \dot{\theta} = J^{-1}\dot{p}$
$\Rightarrow$ infinity.

SINGULARITIES

(i) wrist singularity
(ii) elbow singularity
(iii) shoulder singularity

→ in singularity, joint space velocity is ∞ theoretically

STATICS

- study of forces acting on the body without any motion (or acceleration)

- torque : $\tau = r \times F$ (rotational equivalent of force)

- mechanical power : $P = \frac{W}{t}$

for linear motion, $P = \frac{Fd}{t} = \underline{\underline{F \cdot v}}$

PRINCIPLE OF CONSERVATION OF POWERS:
power at joints = power to move robot + (power at end effector)

Because of statics, power to move robot = 0

$\Longrightarrow$ power at joints = power at end-effector

now, power at joints = $\tau^T \omega$
power at end-effector = $F^T v$

Thus, $\tau^T \omega = F^T v$

$\tau^T \dot{\theta} = F^T J_v \dot{\theta}$ $\quad [v = J_v \dot{\theta}]$

$\tau^T = J_v F^T$

$\boxed{\tau = J_v^T F}$

*if it jiggles when she walks,
i listen when she talks.*

Thus, $\tau = J_v^T F$

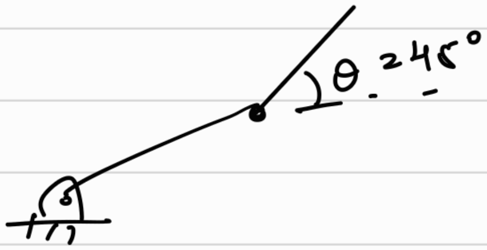
Test 1
Test 2

The quick brown fox jumped over the lazy dog

- humanoids have non-singular trajectories for walking because of manipulability ellipsoid.

eg -

① 2R planar



$\theta_1 = 0°, \quad \theta_2 = 45°,$

$F_y = -1N$

$L_1 = L_2 = 1$

$F = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad \tau = J^T F$

now $J_v = \begin{bmatrix} -l_1 s\theta_1 - l_2 s\theta_{12} & -l_2 s\theta_{12} \\ l_1 c\theta_1 + l_2 c\theta_{12} & l_2 c\theta_{12} \end{bmatrix}$

$= \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ (1 + 1/\sqrt{2}) & 1/\sqrt{2} \end{bmatrix}$

$J_v^T = \begin{bmatrix} -1/\sqrt{2} & (1+1/\sqrt{2}) \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

Thus, $\tau = J_v^T F$

→ $\tau = 0$ (singular configuration)

- For a spatial manipulator, use a full Jacobian matrix.

$$\boxed{F = J^{-T} \tau}$$

- velocity ellipsoid is $\perp$ to configuration while force ellipsoid is in the direction.

→ <u>Trajectory</u>: path parameterized wrt time t.

<u>Motion planning</u>: finding robot's motion from a start position to an end position.

<u>Path planning</u>: finding a collision - free path from initial to final configuration.

$\theta(s) : [0,1] \rightarrow \Theta$

when $s=0 \Rightarrow \theta(0) : \theta_{initial}$

$s=1 \Rightarrow \theta(1) : \theta_{final}$

- linear motion in task space will have non-linear motion in joint space (and vice-versa)

- if we take linear motion in task space $\Longrightarrow$ velocity and acceleration will be non-linear

- to have smooth velocity & acceleration $\Rightarrow$ cubic polynomial.

Let : $p_0 = (x_0, y_0)$
$p_f = (x_f, y_f)$

$p_s = (x(s), y(s))$

$\Longrightarrow p(s) = (1-s) p_0 + s p_f$

- When doing linear configuration, the intermediate points may be out of the reachable workspace!

- To satisfy n constraints, we need to consider 'n' independent coefficients.

Say, $s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$
$\dot{s}(t) = a_1 + 2a_2 t + 3a_3 t^2$

$s(t_0) = x$; $s(t_f) = x_f$; $\dot{s}(t_0) = 0$; $\dot{s}(t_f) = 0$

Now, $s(0) = a_0$ (initial pos^n)
$\dot{s}(0) = a_1$

$s(t_f) =$; $\dot{s}(t_f) =$

- no. of constraints $= 6$ (considering acceleration & velocity constraints)

$\Rightarrow$ order of polynomial $= 6 - 1 = 5$

you physically only feel acceleration
that's why you want to keep it smooth.
But past that your soul shudders.

- $\det(M) = -4(t_f - t_0)^9$

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