Robotics: Dynamics and Control - Project Report

Team Name: Joints

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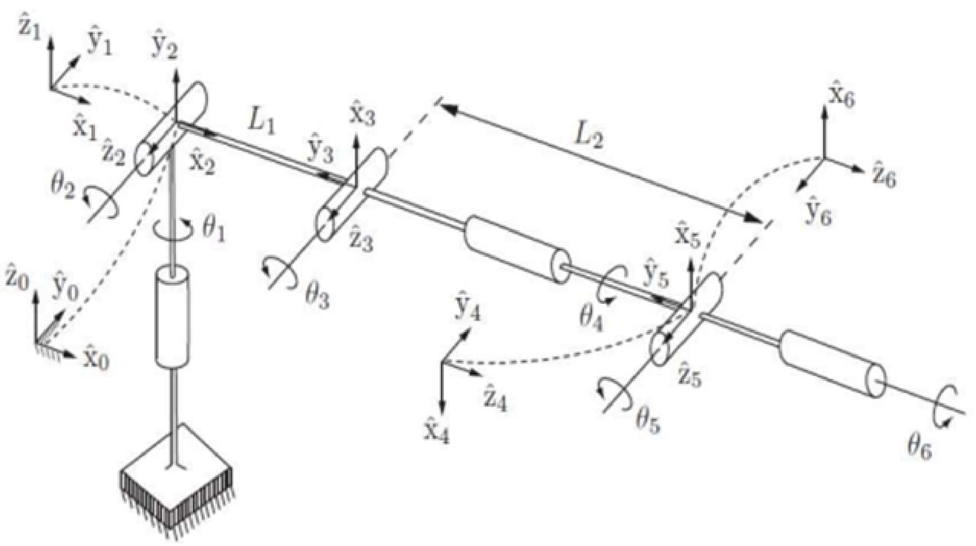
1. Problem Statement

The problem statement consists of a 6R manipulator with the first three links in the ortho-parallel configuration and the other three forming a spherical wrist.

Two of the link lengths are given: L1 = 1.5 and L2 = 1



We are asked to analyse the complete kinematics of the manipulator by deriving the:

- **Forward Kinematics:** compute end effector pose given joint angles.

- **Inverse Kinematics:** compute the joint angles given the end effector position (x,y,z)
- **Velocity Kinematics:** compute the linear and angular velocities at each of the joints and the end effector

We have also been asked to track the trajectory of a ball within the workspace of the manipulator:

- Plot the trajectory of the ball
- Compute the inverse kinematics of points on the trajectory to get the joint angles of the robot
- Simulate the manipulator pose based on the computed joint angles.

2. Scope of the problem

a) *Defense:* Track missiles or drones and deploy defense mechanisms in response. This involves the same problem of trajectory tracking that is also present in this project.

b) *Sports Tracking*: Can be used to track the ball/object with camera for sports telecasting.

3. Methodology

1) Forward Kinematics -

The problem consisted of going from joint angles to end effector position. We used the modified DH parameters to go from frame i to (i+1).

Frame #1	Link length (a)	Link twist (α)	Link offset (d)	Joint angle (θ)
1	0	0	0	θ1
2	0	$\pi/2$	0	θ2
3	L1	0	0	$\pi/2$ + θ3
4	0	$\pi/2$	L2	π + θ4
5	0	$\pi/2$	0	π + θ5
6	0	$\pi/2$	0	θ6

The transformation matrix going from i to (i+1) is given by:

$$ {}^{i-1}_{i}[\mathbf{T}] = \begin{pmatrix} \mathrm{Cos}[\theta_i] & -\mathrm{Sin}[\theta_i] & 0 & a_{i-1} \\ \mathrm{Cos}[\alpha_{i-1}]\mathrm{Sin}[\theta_i] & \mathrm{Cos}[\alpha_{i-1}]\mathrm{Cos}[\theta_i] & -\mathrm{Sin}[\alpha_{i-1}] & -\mathrm{Sin}[\alpha_{i-1}]d_i \\ \mathrm{Sin}[\alpha_{i-1}]\mathrm{Sin}[\theta_i] & \mathrm{Cos}[\theta_i]\mathrm{Sin}[\alpha_{i-1}] & \mathrm{Cos}[\alpha_{i-1}] & \mathrm{Cos}[\alpha_{i-1}]d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

We then post-multiply by all the transformation matrices to get the transformation matrix that represents the end-effector pose.

$$ x = \cos\theta_1 \left[\cos\theta_2\, l_1 + \cos(\theta_2 + \theta_3)\, l_2 \right] $$
$$ y = \sin\theta_1 \left[\cos\theta_2\, l_1 + \cos(\theta_2 + \theta_3)\, l_2 \right] $$
$$ z = \sin\theta_2\, l_1 + \sin(\theta_2 + \theta_3)\, l_2 $$

2) Inverse Kinematics -

In this problem, we are required to go from end-effector position to joint angles. The equations that have been used to derive the first three joint angles are shown below:

The position of the end effector is dependent only on $\theta_1, \theta_2, \theta_3$. The orientation of the end effector is dependent only on $\theta_4, \theta_5, \theta_6$.

The manipulator is kinematically decoupled.

To solve for $\theta_1, \theta_2, \theta_3$.

We use the forward kinematics equations.

Let us assume

$$p = \cos\theta_2\, l_1 + \cos(\theta_2 + \theta_3)\, l_2$$

$$q = \sin\theta_2\, l_1 + \sin(\theta_2 + \theta_3)\, l_2$$

Now $x = \cos\theta_1 \, p$

$y = \sin\theta_1 \, p$

we can see that

$$\tan\theta_1 = y/x$$

$$\boxed{\theta_1 = \tan^{-1}(y/x)}$$

Further,

$$p = \sqrt{x^2 + y^2}$$

$$q = z$$

$$p^2 + q^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \left[\cos\theta_2 \cos(\theta_2 + \theta_3) + \sin\theta_2 \sin(\theta_2 + \theta_3)\right]$$

Using trigonometric transformations,

$$p^2 + q^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_3$$

$$\boxed{\theta_3 = \pm\cos^{-1}\left((p^2 + q^2 - l_1^2 - l_2^2) / 2 l_1 l_2\right)}$$

Now, consider

$$p = \cos\theta_2\, l_1 + \cos(\theta_2+\theta_3)\, l_2$$

$$q = \sin\theta_2\, l_1 + \sin(\theta_2+\theta_3)\, l_2$$

Using trigonometric formulas,

$$p = \cos\theta_2\, l_1 + \cos\theta_2\cos\theta_3\, l_2 - \sin\theta_2\sin\theta_3\, l_2$$

$$= \cos\theta_2\left[l_1 + \cos\theta_3\, l_2\right] - \sin\theta_2\left(\sin\theta_3\, l_2\right)$$

$$q = \sin\theta_2\, l_1 + \sin\theta_2\cos\theta_3\, l_2 + \cos\theta_2\sin\theta_3\, l_2$$

$$= \sin\theta_2\left[l_1 + \cos\theta_3\, l_2\right] + \cos\theta_2\left(\sin\theta_3\, l_2\right)$$

Assume

$$\tan\phi = \frac{l_2\sin\theta_3}{l_1 + l_2\cos\theta_3}$$

$$\phi = \tan^{-1}\frac{l_2\sin\theta_3}{l_1 + l_2\cos\theta_3}$$

Now, consider

$$p = \cos\theta_2\, l_1 + \cos(\theta_2+\theta_3)\, l_2$$

$$q = \sin\theta_2\, l_1 + \sin(\theta_2+\theta_3)\, l_2$$

Using trigonometric formulas,

$$p = \cos\theta_2\, l_1 + \cos\theta_2 \cos\theta_3\, l_2 - \sin\theta_2 \sin\theta_3\, l_2$$

$$= \cos\theta_2 \left[l_1 + \cos\theta_3\, l_2 \right] - \sin\theta_2 (\sin\theta_3\, l_2)$$

$$q = \sin\theta_2\, l_1 + \sin\theta_2 \cos\theta_3\, l_2 + \cos\theta_2 \sin\theta_3\, l_2$$

$$= \sin\theta_2 \left[l_1 + \cos\theta_3\, l_2 \right] + \cos\theta_2 (\sin\theta_3\, l_2)$$

Assume

$$\tan\phi = \frac{l_2 \sin\theta_3}{l_1 + l_2\cos\theta_3}$$

$$\phi = \tan^{-1} \frac{l_2 \sin\theta_3}{l_1 + l_2\cos\theta_3}$$

Now, we know that

$$\tan\gamma = q/p$$

$$\gamma = \tan^{-1}(q/p)$$

$$\boxed{\theta_1 = \gamma - (\pm\beta)}$$

Now, consider

$$p = \cos\theta_2 \, l_1 + \cos(\theta_2+\theta_3)\, l_2$$

$$q = \sin\theta_2 \, l_1 + \sin(\theta_2+\theta_3)\, l_2$$

Using trigonometric formulas,

$$p = \cos\theta_2 \, l_1 + \cos\theta_2 \cos\theta_3 \, l_2 - \sin\theta_2 \sin\theta_3 \, l_2$$

$$= \cos\theta_2 \left[l_1 + \cos\theta_3 \, l_2 \right] - \sin\theta_2 \left(\sin\theta_3 \, l_2 \right)$$

$$q = \sin\theta_2 \, l_1 + \sin\theta_2 \cos\theta_3 \, l_2 + \cos\theta_2 \sin\theta_3 \, l_2$$

$$= \sin\theta_2 \left[l_1 + \cos\theta_3 \, l_2 \right] + \cos\theta_2 \left(\sin\theta_3 \, l_2 \right)$$

Assume

$$\tan\phi = \frac{l_2 \sin\theta_3}{l_1 + l_2\cos\theta_3}$$

$$\phi = \tan^{-1} \frac{l_2 \sin \theta_3}{l_1 + l_2 \cos \theta_3}$$

Now, we know that

$$\tan \gamma = q/p$$

$$\gamma = \tan^{-1}(q/p)$$

$$\boxed{\theta_1 = \gamma - (\pm \beta)}$$

For the problem of tracking the trajectory of the ball: given the initial conditions of **u = 8 units/sec** , starting at position **(5,0,0)** , and thrown at an angle of **135°** with horizontal.

- The parabolic trajectory of the ball is obtained using the equation:

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x = x0 + u * cos(launch_angle_rad) * t;

y = y0 + u * sin(launch_angle_rad) * t - 0.5 * g * t2;

z = z0;
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(assuming ball thrown in XY plane)

- The reachable workspace of the manipulator is: **(L1 - L2 , L1 + L2)**

  This represents the annulus formed by the two links of length L1 and L2.

3) Velocity Kinematics -

- The linear velocities of the joints are obtained by differentiating their positions.

  The linear velocity Jacobian J_v gives the relation between the velocities and their positions.

[-(sin(t1)*(2*cos(t2 + t3) + 3*cos(t2)))/2, -(cos(t1)*(2*sin(t2 + t3) + 3*sin(t2)))/2, -sin(t2 + t3)*cos(t1), 0, 0, 0]

[(cos(t1)*(2*cos(t2 + t3) + 3*cos(t2)))/2, -(sin(t1)*(2*sin(t2 + t3) + 3*sin(t2)))/2, -sin(t2 + t3)*sin(t1), 0, 0, 0]

[0, cos(t2 + t3) + (3*cos(t2))/2, cos(t2 + t3), 0, 0, 0]

- The recursive formula is used to obtain linear velocities at every joint:

$$^{i}\mathbf{v}_i = {}_{i-1}^{\;\;i}[\mathbf{R}]\left({}^{\mathrm{i-1}}\mathbf{v}_{i-1} + {}^{\mathrm{i-1}}\boldsymbol{\omega}_{i-1} \times {}^{i-1}\boldsymbol{O}_i\right) + d_i\hat{\boldsymbol{k}}_i$$

- Similarly, the recursive formula is also used to compute the angular velocities from the rotation matrices:

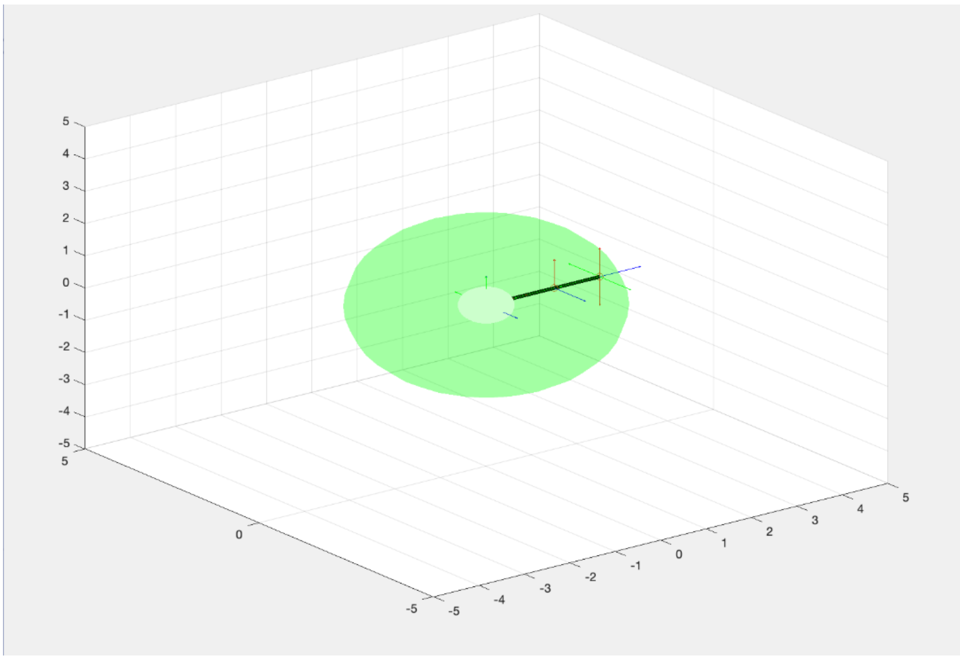
$$ {}^{i}\boldsymbol{\omega}_i = {}_{i-1}^{\;\;i}[\mathbf{R}]\left({}^{i-1}\boldsymbol{\omega}_{i-1}\right) + \hat{\boldsymbol{k}}_i\dot{\theta}_i $$

4. Results

The forward kinematics function gives us the end effector transformation matrix. The value of this matrix at home position (when joint angles are all zero):

0.0000	-0.0000	1.0000	2.5000
0.0000	-1.0000	-0.0000	-0.0000
1.0000	0.0000	-0.0000	-0.0000
0	0	0	1.0000

The simulation of the manipulator at home configuration is shown below. The dark green circle shows us the dexterous workspace of the manipulator.



The working of the ball-tracking manipulator has been shown in the video file that has been submitted along with the report.

5. Discussion

- Key takeaways: The complete kinematics (forward, inverse and velocity) of a robot would be required to make the robot move and perform tasks.
- We were successfully able to track the ball whenever it was in the reachable workspace of the robot and move the manipulator to follow the ball’s trajectory.
- Challenges faced: Simulating the manipulator’s configuration and finding all possible solutions for inverse kinematics
- Further improvements: Better simulation of manipulator configuration