Generalized co-ordinates:-

Legendary Transform:-

phase space (p & q)

Liouville's theorem:-

Density doesn't change in function of time

3 ensembles

→ all equal probable.

entropy

Microcanonical ensemble → Total energy is fixed, everything have same energy

Canonical ensemble → Temperature is fixed

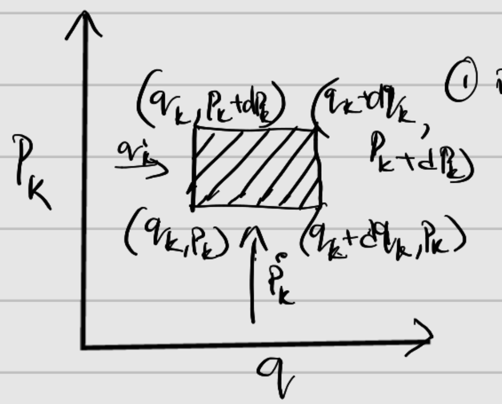
Partition function

Grand canonical ensemble → chemical Potential is fixed.

Thermal bath → canonical

Particle bath → Grand

Theorem $\left(\frac{d\rho}{dt} = 0\right)$



① Rectangle in Phase space. (Infinitesimally small Rectangle)

At time t, density is given by $\rho(\vec{q}, \vec{P}, t)$.

At time t+dt, density is given by

Flow on LHS, velocity $= \dot{q}_k$ (in horizontal direction)

flow velocity $= \dot{q}_k$

From bottom, velocity $= \dot{P}_k$

No. of particles flowing per unit time,

$= \rho \dot{q}_k dP_k$

(Taylor's Expansion) ← outflow from R.H.S. $= \rho \dot{q}_k dP_k + \frac{\partial}{\partial q_k}(\rho \dot{q}_k) dq_k dP_k$

$f(x+h) \approx f(x) + h f'(x)$

Phase density $\rho = \frac{dN}{dq_1 \ldots dq_N dp_1 \ldots dp_N}$

$$\text{Left} - \text{Right} = \underbrace{-\frac{\partial}{\partial q_k}(\rho \dot{q}_k) h q_k dP_k}_{T_1}$$

$$\text{down} - \text{up} = \underbrace{-\frac{\partial}{\partial P_k}(\rho \dot{P}_k) h P_k d q_k}_{T_2}$$

$$\text{Net flow} = \underbrace{\sum_k (T_{1k} + T_{2k})}\, dV_{phase}$$

$$\frac{d\rho}{dt} = \frac{d\rho}{dt} \cdot dV_{phase} \Rightarrow \text{volume of whole phase space}$$

Finding flows

$$= -\left[\frac{\partial \rho}{d q_k} \dot{q}_k + \rho \frac{\partial \dot{q}_k}{\partial q_k}\right] \frac{\rho\, \partial^2 H}{\partial P_k \partial q_k}$$

$$\boxed{\dot{q}_k = \frac{\partial H}{\partial P_k}} \quad \boxed{\dot{P}_k = -\frac{\partial H}{\partial q_k}}$$

$$\frac{d\rho}{dt} + \frac{\partial \rho}{\partial q_k} \dot{q}_k + \frac{\partial \rho\, \dot{P}_k}{\partial P_k} = 0$$

$$\frac{d}{dt} \rho(p, q, t) = 0 \Rightarrow \text{Because System is Hamiltonian}$$

If $\rho = \rho(H)$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial H} \cdot \sum_k \left(\frac{\partial H}{\partial q_k} q_k + \frac{\partial H}{\partial P_k} P_k\right) = 0$$

$$\Rightarrow \frac{\partial \rho}{\partial t} = 0$$

Missing Information function :- (bag example)

Entropy $= \sum -P_i \log P_i$

H tell How much info is needed to fill the micro within the macrostate.

$S(\{P_i\})$

$S(\{p\}_\gamma)$

entropy satisfies all these conditions

(i) For $P_1 = P_2 = \frac{1}{\gamma}$ S should be increasing function of γ.

(ii) S should be an continuous function of its arguments

(iii) Divide n group $(1,2 \ldots n)$

Each group contain γ_j outcomes.

(iv) prob outcome i.e. in ω_j certain group j

(v) $S(\{p\}_\gamma) = S(\{\omega\}_n)$

Particle missing information in particular state.

Macrostate

$$= \sum_{j=1}^{n} \omega_j S\left(\frac{P_{i,1}}{\omega_j}, \frac{P_{i2}}{\omega_j}\right)$$

$$\boxed{S(\{p\}_\gamma) = -k \sum_{i=1}^{\gamma} P_i \log P_i}$$

Boltzman's constant

If all p_i are equal to $1/\gamma$

Then $S(\{p\}_\gamma) = k \log(\gamma)$

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phase Space density $\rho = \dfrac{dN}{dq_1 \ldots dq_n \, dP_1 \ldots dP_n}$

part flowing in :- $\rho \dot{P}_i dq_i + \rho \dot{q}_i dP_i = \rho(\dot{q}_i dP_i + \dot{P}_i dq_i)$

part flowing out :- $dq_i \left[\rho \dot{P}_i + \frac{\partial}{\partial P_i}(\rho \dot{P}_i) dP_i\right] + dP_i \left[\rho \dot{q}_i + \frac{\partial}{\partial q_i}(\rho \dot{q}_i) dq_i\right]$

$$(\text{in} - \text{out}) \Rightarrow \underbrace{-\left[\frac{\partial}{\partial q_i}(\rho \dot{q}_i) + \frac{\partial}{\partial P_i}(\rho \dot{P}_i)\right] dq_i dP_i = \frac{\partial \rho}{\partial t} dP_i dq_i}_{\text{sum over all } i=1 \ldots n)}$$

$$= \sum_{i=1}^{n} \left(\frac{\partial \rho}{\partial q_i} \dot{q}_i + \rho \frac{\partial \dot{q}_i}{\partial q_i} + \frac{\partial \rho}{\partial P_i} \dot{P}_i + \rho \frac{\partial \dot{P}_i}{\partial P_i}\right) + \frac{\partial \rho}{\partial t} = 0$$

$$0 = \rho\left(\frac{\partial \dot q_i}{\partial q_i} + \frac{\partial \dot p_i}{\partial p_i}\right) \qquad \underbrace{\dot q_i = \frac{\partial H}{\partial p_i}}_{\frac{\partial^2 H}{\partial p_i \partial q_i}} \qquad \underbrace{\dot p_i = -\frac{\partial H}{\partial q_i}}_{-\frac{\partial^2 H}{\partial q_i \partial p_i}}$$

then

$$= \sum_{i=1}^{n}\left(\frac{\partial \rho}{\partial q_i}\dot q_i + \frac{\partial \rho}{\partial p_i}\dot p_i\right) + \frac{\partial \rho}{\partial t} = 0 \quad \text{—} \ (1)$$

$\rho = \rho(\{q\},\{p\},t) \Rightarrow$ phase space density is function of all coordinate $\{q\}$, all of the momentum $\{p\}$ and time

so eq(1) simply says

$$\boxed{\frac{d\rho}{dt} = 0}$$