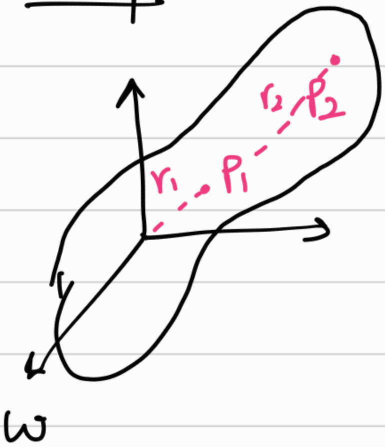
Recap:



- as we move outwards towards P_2 → velocity increases

- at axis of rotation, linear velocity is zero

at $P_1 \rightarrow \omega \times r_1 = \vec{v_1}$

$P_2 \rightarrow v_2 = \omega \times r_2$

- vector field for P_1 : circle of radius r_1

vector field for P_2 : circle of radius r_2

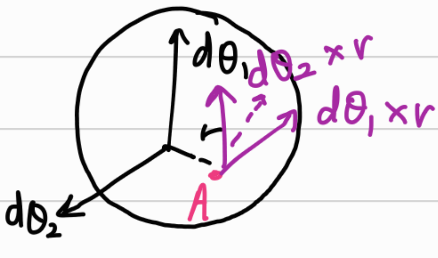
Total vector field : linear combination of the two fields.

Angular velocity at a fixed point

- $[\alpha, \beta, \gamma]$ is an improper vector] can't be added

- $\underbrace{[\dot{\alpha}, \dot{\beta}, \dot{\gamma}]}_{\text{angular velocity}}$ is a proper vector

- consider a rigid body with pure rotation :



[linear velocity direction is ⊥ to both axis]

$\therefore$ Total rotation of $\boxed{A = (d\theta_1 + d\theta_2) \times r}$

- Transformation matrix, $T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$

to get linear velocity, just differentiate P.

$\Longrightarrow$ TO GET ANGULAR VELOCITY :-

$${}^0_i[R] \, {}^0_i[R]^T = [I]$$

$$\dot{R}R^T + R\dot{R}^T = 0 \qquad \text{(differentiate wrt T)}$$

(which is the form of a skew symmetric matrix)

$$\longrightarrow \boxed{{}^0_i[\dot{R}] \, {}^0_i[R]^T = {}^0_i[\omega^S]}$$

now, $R = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\dot{R}R^T = \begin{bmatrix} 0 & -\dot{\theta} & 0 \\ \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \nearrow \text{(angular velocity)}$$

- ${}^{0}_{i}[\omega_s]p = {}^{0}\omega_i^s \times {}^{0}p$

$\omega_s \rightarrow$ space fixed frame

${}^{0}\omega_i^s \rightarrow$ describes velocity of {i} wrt {0}

- ${}^{0}_{i}[\omega_b] = {}^{0}_{i}[R]^T \, {}^{0}_{i}[\dot{R}]$

  $\hookrightarrow$ body fixed vector.

Relation between space & body-fixed frame

$${}^{i}_{0}R^T \, {}^{0}_{i}[\omega^s] = {}^{0}_{i}[\dot{R}] \, {}^{0}_{i}[R]^T$$

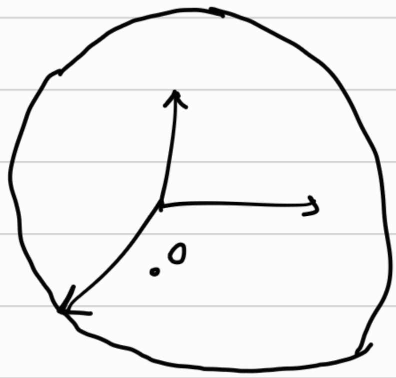
$${}^{0}_{i}[\omega^b] = {}^{0}_{i}[R]^T \, {}^{0}_{i}[\dot{R}]$$

$$\Longrightarrow \boxed{{}^{0}\omega_i^s = {}^{0}_{i}[R] \, {}^{0}\omega_i^b}$$

Velocity Kinematics

object under rotation :-

${}^{0}p = {}^{0}_{i}[R] \, {}^{i}p$



object under translation & rotation :-

$$^0p = {}^0_i[R]\,{}^ip + {}^0O_i$$

differentiate :- $$^0v_p = {}^0\dot{O}_i + {}^0_i[\dot{R}]\,p + \underbrace{{}^0_i[R]\,{}^i\dot{p}}_{\text{included if } {}^ip \text{ is moving wrt } i^{th} \text{ frame.}}$$

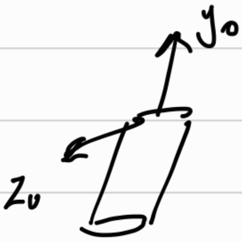
$$\boxed{{}^0v_p = {}^0v_{O_i} + {}^0\omega_i \times {}^0_i[R]\,{}^ip + {}^0_i[R]\,{}^iv_p}$$

zero
one
two
three
four
five
six
seven
eight
nine
ten

$$^iv_i = {}^i_{i-1}[R]$$

$\longrightarrow$ $$^0\omega_i = {}^0\omega_{i-1} + {}^0\hat{k}_i\,\dot{\theta}_i$$

eg :



we know,

$$^i\omega_i = {}^i_{i-1}[R]\left({}^{i-1}\omega_{i-1}\right) + \hat{k}_i\,\dot{\theta}_i$$

and

$$ {}^{i}v_i = {}^{i}_{i-1}[R] \left({}^{i-1}v_{i-1} + {}^{i-1}\omega_{i-1} \times {}^{i-1}O_i \right) + d_i \hat{k}_i $$

i) $\underline{i=1}$:-

$$ {}^{1}\omega_1 = \cancel{{}^{1}_{0}R \; ({}^{0}\omega_0)}^{\,0} + \dot{\theta}_1 [0 \quad 0 \quad 1]^T $$

$$ = [0 \quad 0 \quad \dot{\theta}_1]^T $$

$$ {}^{1}v_1 = {}^{1}_{0}R \left(\cancel{{}^{0}v_0}^{\,0} + \cancel{{}^{0}\omega_0}^{\,0} \times O \right) + d_0 \hat{k}_0 $$

$$ = 0\hat{k}_0 = \underline{\underline{0}} $$

(Thus instantaneous velocity of 1^{st} frame wrt 1^{st} frame is zero)

ii) $\underline{i=2}$:-

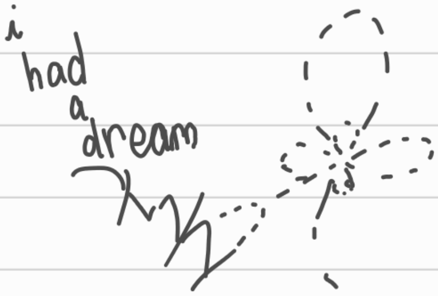
$$ {}^{2}\omega_1 = {}^{2}_{1}R \; {}^{1}\omega_1 + \dot{\theta}_2 [0 \quad 0 \quad 1]^T $$

$$ = \begin{pmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} $$

• NOTE :- for prismatic joints : $d_i \hat{k}_i = 0$.
it is non-zero for revolute joint.

if $i = 3$:-

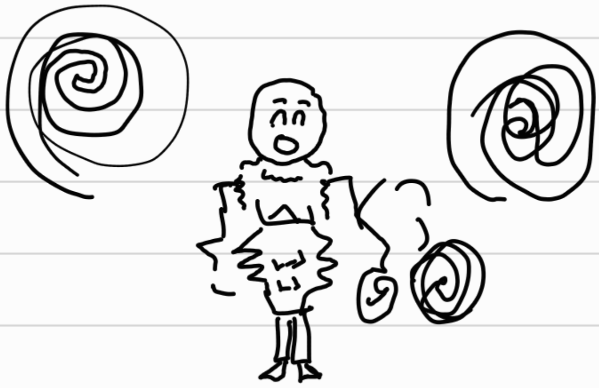
$$^3\omega_3 = {}^3_2[R]$$



NOTE :-

• given $^3v_3$ and j we want $^0v_3$

$$\Longrightarrow {}^0_3[R]\ {}^3v_3$$



$$\Longrightarrow$$

$$^j\left({}^0v_{0i}\right) = {}^j_0R\left({}^0v_{0i}\right)$$

omg

joker

$$\left({}^0v_0\right)$$

$\Downarrow$

How i feel

$$^i\left({}^0J\dot{\theta}\right) = {}^j_0R\ {}^0J$$

$$\boxed{{}^BJ = {}^B_AR\ {}^AJ}$$

$$\det\left({}^BJ\right) = \det(R)\det\left({}^AJ\right)$$