ROBOTICS : DYNAMICS & CONTROL

Assignment-1

(1)

a. Rigid body transformation → a single mapping $g: O \rightarrow \mathbb{R}^3$ which maps the coordinates of points in the rigid body from initial to final configuration.
Here, O is the collection of points of the rigid body.

b. Configuration space → the n-dimensional space containing all possible configurations of all the points of a robot.

c. workspace → the reachable configuration of robot's end effector.

d. taskspace → space in which the tasks are naturally expressed ($\mathbb{R}^2$ or $\mathbb{R}^3$)

e. Degree of freedom → minimum number of real-valued coordinates required to represent the configuration of the robot.

August

S	1	8	15	22	29
M	2	9	16	23	30
T	3	10	17	24	31
W	4	11	18	25	
T	5	12	19	26	
F	6	13	20	27	
S	7	14	21	28	

September

S		5	12	19	26
M		6	13	20	27
T		7	14	21	28
W	1	8	15	22	29
T	2	9	16	23	30
F	3	10	17	24	
S	4	11	18	25	

October

S		3	10	17	24	31
M		4	11	18	25	
T		5	12	19	26	
W		6	13	20	27	
T		7	14	21	28	
F	1	8	15	22	29	
S	2	9	16	23	30	

a. Explicit parameterization involves using n-coordinates or parameters to represent an n-dimensional space.

Implicit parameterization views the n-dimensional space as embedded in a Euclidean space of more than n-dimensions, but subjects these coordinates to constraints (so that DoF is reduced).

Eg: for a sphere, using implicit parameterization, we assume it to be embedded in 3-D space (x, y, z) such that $x^2 + y^2 + z^2 = 1$

EXPLICIT PARAMETERIZATION	IMPLICIT PARAMETERIZATION
→ advantage: minimum number of coordinates to represent the configuration space.	→ advantage: fixes the problem of singularities, or changing coordinates.
→ disadvantage: due to the difference in topology b/w a curved space and a euclidian space, coordinate singularities arise.	→ disadvantage: more number of coordinates to represent the same space resulting in higher complexity.

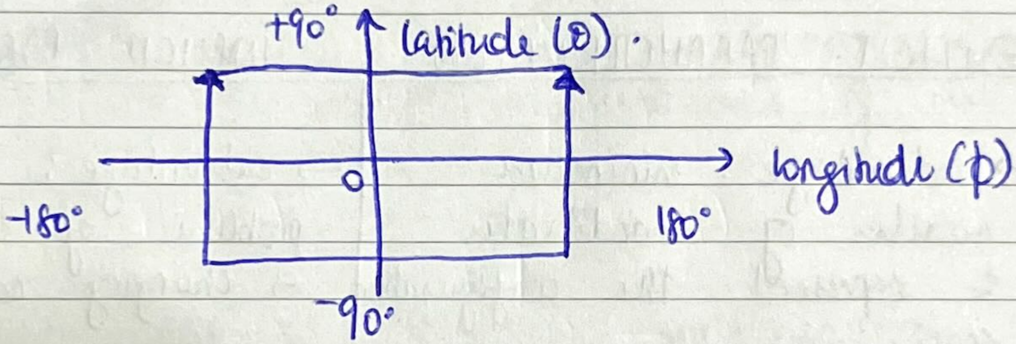
D. Coordinate singularity occurs when we represent a curved space using explicit parameterization (minimum number of coordinates) due to the difference in topology.

It happens because of a discontinuity or rapidly changing coordinates in a coordinate frame.

EXAMPLE:

Let us consider the example of a sphere.

In explicit parameterization, we use two coordinates (θ, ϕ) for the latitude and longitude.



The singularity occurs near the pole of the sphere because the longitude changes rapidly near the poles than when compared to the equator.

Also, as we step over the pole there is a 180° shift in the longitude leading to a discontinuity.

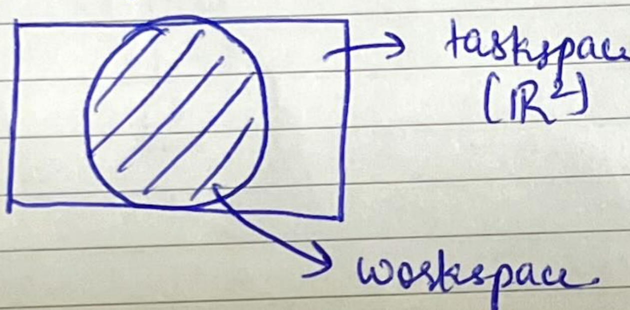
This causes a coordinate singularity (at the pole multiple longitudes correspond to one latitude)

3 a

Joint space variables : $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$

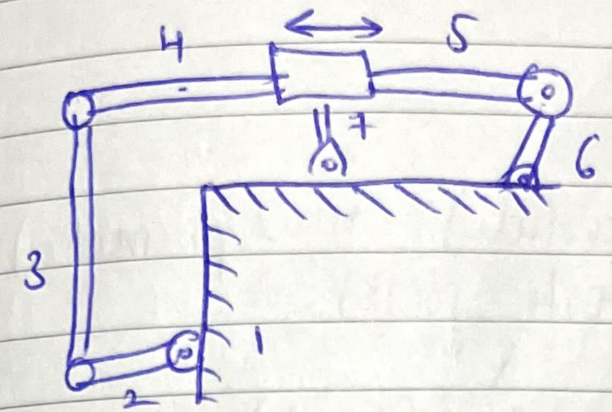
b Since the task is the clean a borad, the task space would be $\mathbb{R}^2$ with variable (x, y).
Thus the topological representation would be $\mathbb{R}^2 \times S^1$ (considering the orientation of end effector)

c The workspace of the manipulator would be a circle in the task space of $\mathbb{R}^2$ of length l (length of joints).



12:00 ١٢:٠٠

(4) i-



13:00 ١٣:٠٠

14:00 ١٤:٠٠

15:00 ١٥:٠٠

$\#$ of links = 7

16:00 ١٦:٠٠

$\#$ of joints = 6 revolute + 1 prismatic

17:00 ١٧:٠٠

∴ Using Greubler's formula,

18:00 ١٨:٠٠

$$DoF = k(N-1) - \sum_{i=0}^{j} (k - f_i)$$

19:00 ١٩:٠٠

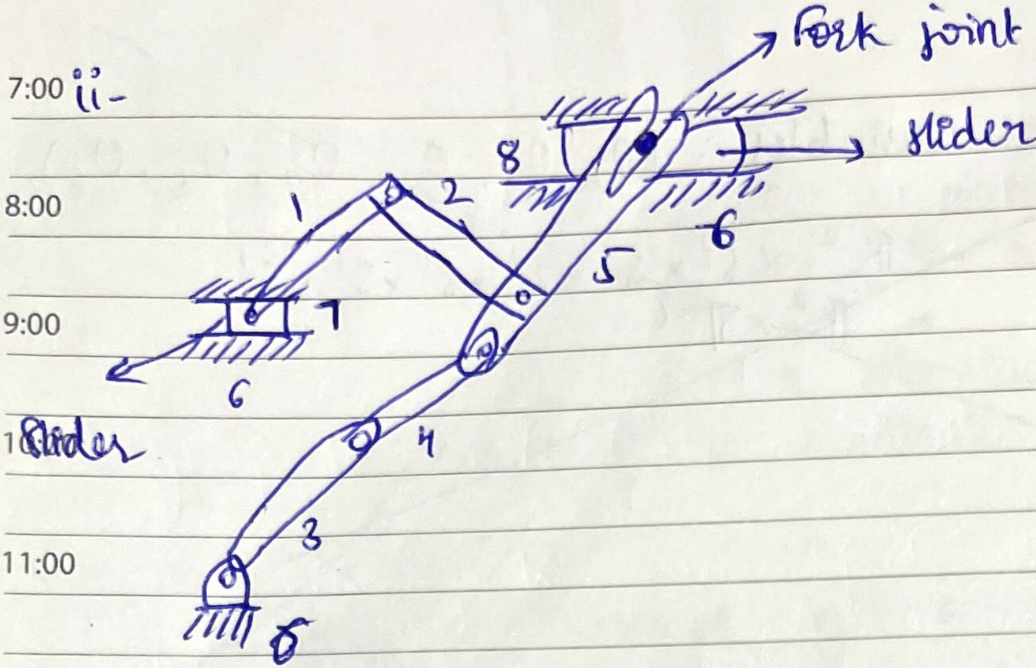
$$= 3(7-1) - 6(3-1) - 1(3-1)$$

20:00 ٢٠:٠٠

$$= 18 - 12 - 2$$

$$= \underline{\underline{4}}$$

ii-



# of links = 8

# of joints = 5 (revolute) + 2 (prismatic) + 1 (fork joint)

$$DoF = k(N-1) - \sum_{i=1}^{j} (k - f_i)$$

$$= 3(8-1) - 5(3-1) - 2(3-1) - 1(3-2)$$

$$= 21 - 10 - 4 - 1$$

$$= \underline{\underline{6}}$$

17:00

5

18:00

a. The 4R (four bar) linkage has one DoF.

19:00

\# of links = 14

20:00

\# of joints = 4 x 4 = 16 (1 DoF)

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October						
S		3	10	17	24	31
M		4	11	18	25	
T		5	12	19	26	

7:00 V:••

Using Greubler's formula,

8:00 Λ:••

$$DoF = K(N-1) + \sum_{j=1}^{j} (K - f_i)$$

9:00 ٩:••

$$= 3(14-1) + (3-1)16$$

$$= 39 - 32$$

10:00 ١٠:••

$$= \underline{\underline{7}}$$

11:00 ١١:••

b. If there were n legs:

12:00 ١٢:••

# of links = $3n + 2$

13:00 ١٣:••

# of joints = $4n$

14:00 ١٤:••

$\Rightarrow$ $DoF = 3(3n+2-1) - 4n(3-1)$

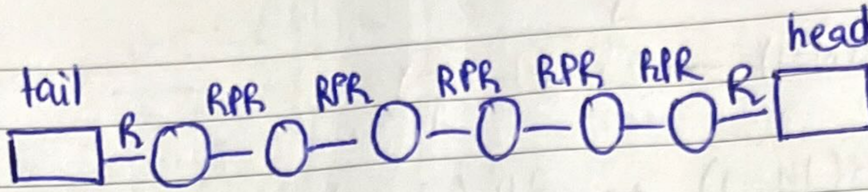
$$= 9n + 3 - 12n + 4n$$

15:00 ١٥:••

$$= \underline{\underline{n+3}}$$

16:00

a.



\# of links = 8

\# of joints = 7 (2R + 5 RPR joints)

For the RPR joints, the joint DoF is 3

Using Greubler's formula,

$$DoF = K(N-1) - \sum_{i=1}^{j} (K - f_i)$$

$$= 3(8-1) - 5(3-3) - 2(3-1)$$

$$= 21 - 0 - 4$$

$$= \underline{\underline{17}}$$

Hence, the DoF = $\underline{\underline{17}}$.

b. When all body links make contact with the leg:

\# of links = 8

\# of joints = 5 RPR + 2R.

In this case, the degree of freedom of the RPR joints will reduce to 2 from 3.

$$\Rightarrow DoF = 3(8-1) - 5(3-2) - 2(3-1)$$
$$= 21 - 5 - 4$$
$$= \underline{\underline{12}}$$

In this case, the head and tail of the caterpillar make contact with the leaf. There is no restriction on their movement, so '1' DoF from head and tail each. The body links give 2 DoF (one prismatic & one revolute)

$$\therefore DoF = 2(1) + 5(2)$$
$$= 2 + 10$$
$$= \underline{\underline{12}}$$