- wrist decoupled is the same as kinematically decoupled manipulator

- $$T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$$

$$v = \frac{dP}{dt}$$ (differentiate P wrt time)

VELOCITY KINEMATICS

$$\dot{p} = J(\theta)\,\dot{\theta}$$

- $J(\theta)$: Jacobian matrix maps from $R^n \rightarrow R^m$
  $\dot{p} \in R^m$, $\dot{\theta} \in R^n$

For a 2R manipulator,

Pos^n :
$$\begin{bmatrix} c\theta_1 l_1 + c(\theta_1+\theta_2)\, l_2 \\ c\theta_2 l_2 + s(\theta_1+\theta_2)\, l_2 \end{bmatrix}$$

orientation : $\theta_1 + \theta_2$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_1(\theta) \\ f_2(\theta) \end{bmatrix}$$

$$\dot{x} = \frac{\partial}{\partial \theta_1} f_1(\theta)\,\dot{\theta}_1 + \frac{\partial}{\partial \theta_2} f_1(\theta)\,\dot{\theta}_2$$

now, $J_v = \begin{bmatrix} \dfrac{\partial}{\partial \theta_1} f(\theta) & \dfrac{\partial}{\partial \theta_2} f_1(\theta) \\ \dfrac{\partial}{\partial \theta_2} f_1(\theta) & \dfrac{\partial}{\partial \theta_2} f_2(\theta) \end{bmatrix}$

linear velocity Jacobian

Angular velocity Jacobian : $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Combining both Jacobians,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} -s\theta_1 l_1 - s\theta_{12} l_2 & -s\theta_{12} l_2 \\ c\theta_1 l_1 + c\theta_{12} l_2 & c\theta_{12} l_2 \\ 1 & 1 \end{bmatrix}}_{\text{Full Jacobian Matrix}} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

but this is dimensionally non-homogeneous matrix

- condition number → ratio of maximum eigen value to minimum eigen value
→ it is a measure of how close a matrix is to being singular

(NOTE : singular value is root of eigen value)

$\Longrightarrow$ if condition value is small : we say the matrix is well defined. (or well conditioned)

eg: $\begin{bmatrix} 1000 & 0 \\ 0 & 1 \end{bmatrix}$ condition no $= \frac{1000}{1}$

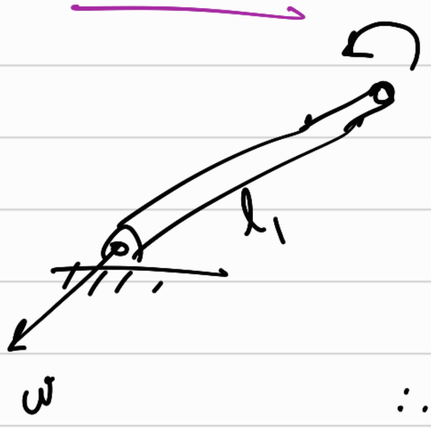
$= \underline{\underline{1000}}$

Taking only J_v,

$J_v = \begin{pmatrix} J_1(\theta) & J_2(\theta) \end{pmatrix}$

$v_{tip} = \begin{pmatrix} J_1(\theta) & J_2(\theta) \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$

<u>1-R robot</u>



$v = \omega \times r$

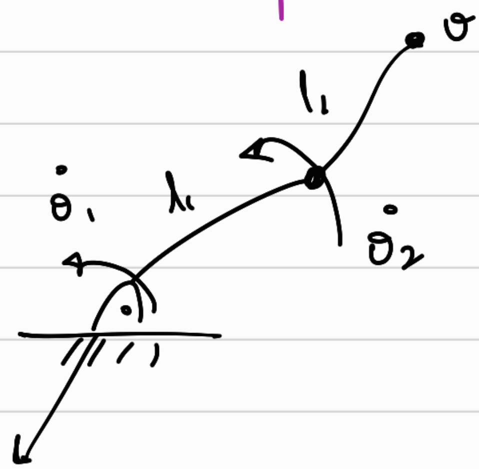
here, $\omega = \dot{\theta}\hat{k}$

$r = l_1$

$\therefore v = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \begin{bmatrix} l_1 \cos\theta \\ l_1 \sin\theta \\ 0 \end{bmatrix}$

$= \begin{bmatrix} -l_1(\sin\theta)\dot{\theta} \\ -l_1(\cos\theta)\dot{\theta} \\ 0 \end{bmatrix}$

2-R manipulator



$$v_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} \begin{bmatrix} l_2 c\theta_{12} \\ l_2 s\theta_{12} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 s\theta_{12} \\ l_2 c\theta_{12} \\ 0 \end{bmatrix} \dot{\theta}_2$$

considering first joint,

$$v_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \begin{bmatrix} l_1 c\theta_1 + l_2 \cos\theta_{12} \\ l_1 s\theta_1 + l_2 \sin\theta_{12} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin\theta_{12} \\ l_1 \cos\theta_1 + l_2 \cos\theta_{12} \\ 0 \end{bmatrix} \dot{\theta}_1$$

$$\Longrightarrow \boxed{\begin{aligned} v_{tip} &= \vec{V_1} + \vec{V_2} \\ &= J_1 \dot{\theta}_1 + J_2 \dot{\theta}_2 \end{aligned}}$$

- $\vec{V_2}$ is perpendicular to second link
  $\vec{v_1}$ is perpendicular to line joining joint 1 and tip.

Thus, tool tip velocity is linear combination of joint velocities.

(ii) When $l_1 = l_2 = 1$ and $\theta_2 = 0$

$$J_v = \begin{bmatrix} -2\sin(\theta_1) & -\sin\theta_1 \\ 2\cos\theta_1 & \cos\theta_1 \end{bmatrix}$$

- this is a singular matrix [det = 0]
- so inverse not possible.

(iii) When $l_1 = l_2 = 1$ and $\theta_2 = \pi$

$$\Rightarrow \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} = J_v^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

- if the eigen value of your Jacobian approaches zero then your manipulator is singular.

Redundant Manipulator

$x = f(\theta_{123})$

$y = f(\theta_{23})$

- dimension of n-joint planar manipulator : $\underline{\underline{2 \times n}}$

- Rank of Jacobian = $\underline{\underline{2}}$

- $\dot{p}_{(2\times1)} = J_{(2\times3)} \; \dot{\theta}_{(3\times1)}$

$\hookrightarrow$ 3R manipulator.

- if DoF > rank (J) ⟹ redundant manipulator

if DoF > 6 (for spatial) → redundant manipulator
DoF > 3 (for planar) → redundant ..

↳ [3 because full Jacobian - $x, y, \dot{\theta}$]

class end when?
me wants class to end now

SAME 

- $\dot{p} = J(\theta)\, \dot{\theta}$

$\dot{\theta} = J^{-1}(\theta)\, \dot{p}$

To take inverse of rectangular matrix :-
we have either full row rank or full column rank → then we use pseudoinverse

- if we don't have row or column rank → SVD (singular value decomposition)

SVD : $A = S \Lambda S^{-1}$ } EVD

↳ eigen vectors of A

- if A is symmetric: $A = Q \Lambda Q^T$
because its eigen vectors will be orthogonal.

→ now for SVD: $A = U \Sigma V^T$
where, $U = AA^T$ (symmetric) (orthogonal)
Σ = diagonal matrix

$$\therefore \dot{\theta} = J^{-1} \dot{p}$$
$$= (U \Sigma V^T)^{-1} \dot{p}$$
$$= V \Sigma^{-1} U^T \dot{p}$$
$$\boxed{V^T \dot{\theta} = \Sigma^{-1} U^T \dot{p}}$$
$$\dot{\theta}^* = \Sigma^{-1} \dot{p}^*$$

$\dot{\theta}^*$ → rotated version of $\dot{\theta}$
$\dot{p}^*$ → rotated version of p
(because U and V are orthogonal matrices.

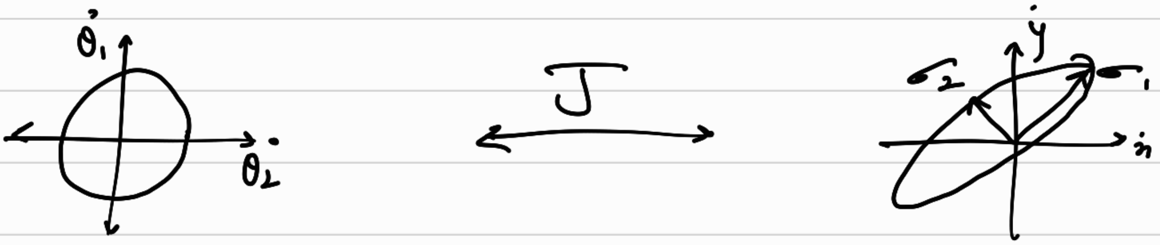
$$\dot{\theta}^* = \Sigma^{-1} \dot{p}^*$$
$$\dot{\theta}^* = \frac{1}{\sigma_i} \dot{p}^*$$

Manipulability ellipsoid

constraint $\|\dot{\theta}^*\| \leq 1$

$$\frac{\dot{p}_1^{*2}}{\sigma_1^2} + \frac{\dot{p}_2^{*2}}{\sigma_2^2} + \cdots \qquad \kappa \qquad \leq 1$$

(this takes the form of an ellipsoid)



maps circle in joint space to ellipse in cartesian space

- n-dimensional unit sphere $\xleftrightarrow{J}$ m-dimensional hyperellipsoid.

- if singularity $\rightarrow$ ellipsoid becomes a line.
  - if volume is smaller $\Rightarrow$ closer to singularity.

(*) Through this we notice that we need to choose l_1 & l_2 such that volume is maximum.

isotropy: if $l_1 = \sqrt{2}\, l_2$

- ellipse becomes circles
- move in any direction with equal effort.