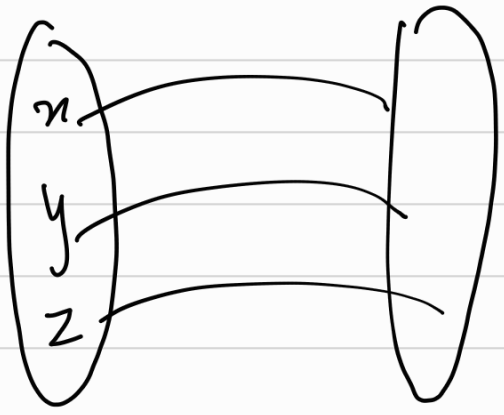
Recap: Topology, parameterization

- $S^1 : \{ p \in \mathbb{R}^2 \mid \|p\| = 1 \}$

$x = r\cos\theta$, $y = r\sin\theta$

$p = \begin{bmatrix} x \\ y \end{bmatrix}$

- a coffee mug and a donut are topologically equivalent.
- HOMOMORPHISM:



$(G_1, \cdot)$ $(G_2, *)$

$x \mapsto f(x)$
$y \mapsto f(y)$
$z \mapsto f(z)$

$x \cdot y = z$
$f(x) * f(y) = f(z)$

- open interval maps to a line

$(0, 1) \longmapsto$

$(-\pi/2, \pi/2)$
semicircle

$\xrightarrow{\tan x}$

Degrees of Freedom

DoF = (sum of freedom of points) − (no. of independent constraints)

- implicit parameterization:

$$S^n = \{ P \in \mathbb{R}^{n+1} \mid |P| = 1 \} \rightarrow \text{more constraints.}$$

// point robot on line : $x \in \mathbb{R}^1$, 1 variable

" " on plane : $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$, 2

" " on circle : $\theta \in S^1$, 1

single link robot with revolute joint : $\theta_1, \theta_2 \in S_1 \times S_1 = \mathbb{T}^2$

Rigid body in plane : $\mathbb{R}^1 \times \mathbb{R}^1 \times S^1 = \mathbb{R}^2 \times S^1$ — 3 variables

rigid body in space : $\mathbb{R}^3 \times S^1 \times S^1 \times S^1$

⟹ KUKA youBot :

manipulator → $S^1 \times S^1 \times S^1 \times S^1 \times S^1$

↳ $[0, \pi/3]^5$

5 DoF

mobile base → $\mathbb{R}^2 \times S^1$: 3 DoF

gripper → 1 DoF

∴ Total configuration space = 3 + 5 + 1

= 9 DoF

+ 4 DoF (for the wheels)

DEFINITIONS :

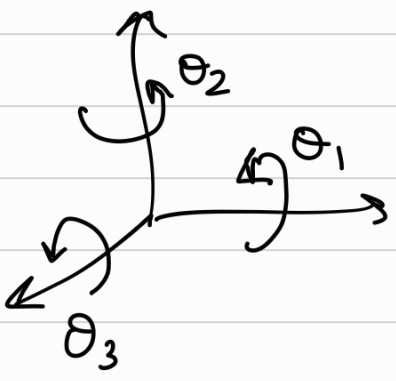
- a system is a set of points in the space X
- space X is said to be a metric space if $d(p, q) = \|p - q\|^2$
- mechanism is a device to transform one motion into another

LINKS

- Link → rigid body that possess at least two nodes
- binary link, ternary link

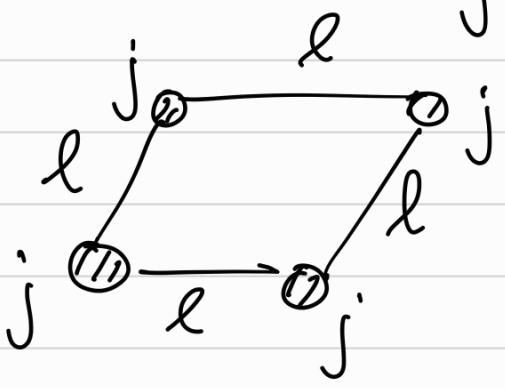
JOINTS

- connection b/w two or more links
- prismatic joint, revolute joint
- revolute joint: 1 DoF (θ)
  eg - door
- prismatic joint: 1 DoF (x)
  eg - piston
- cylindrical joint: 2 DoF (x, θ)
  [rotation and translation]
- screw joint: 1 DoF (θ)
  [rotational & translational motion are interconnected]
- planar joint: 3 DoF (θ, x, y)
  eg - blackboard & duster
  can be constructed using 2 prismatic and one revolute joint.
- spherical joint: 3 DoF (θ_1, θ_2, θ_3)



CHAINS

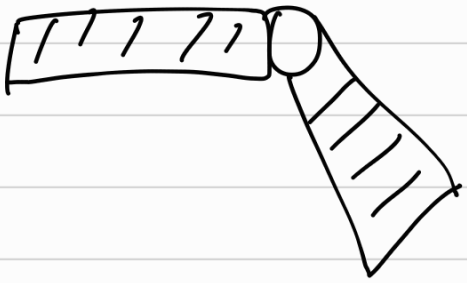
- Kinematic chains is an assembly of rigid links connected via joints



- open chain : arranged in a serial posⁿ
- closed chain : forms a closed loop.

MECHANISM

- planar mechanism : all points of a mechanism moves parallel to a certain plane
- spherical mechanism : joints of all points meet at a point.
- spatial mechanism



two links joint by a revolute joint.

$$DoF = 6 - 3 - 2$$
$$= 1$$

Formula for DoF: $3N - 3 - 2j$

$N \rightarrow$ no. of links

$j \rightarrow$ no. of joints

- revolute joint : $(3 - f_{\text{revolute joint}})$

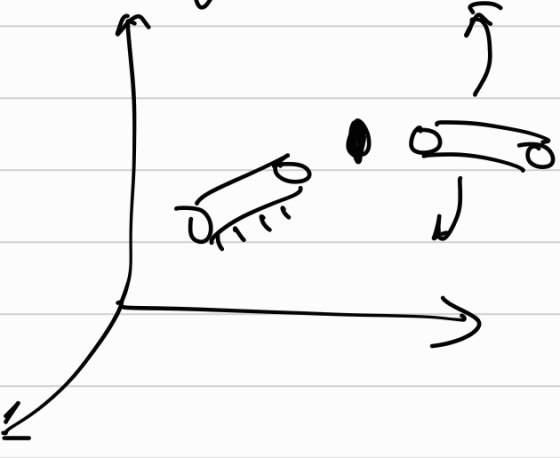
$$= \underline{\underline{2}}$$

$$\therefore DoF = 3(N-1) - (3 - f_i)$$

$f_i \rightarrow$ degrees of freedom

- no. of constraints b/w two spatial rigid bodies

$$12 - 6 - 5 = 1$$



GRUEBLER'S CONDITION :

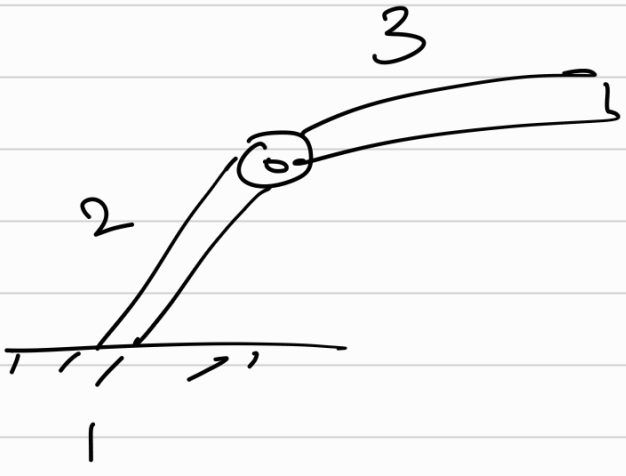
$$DoF = k(N-1) - \sum_{i=1}^{j} (k - f_i)$$

$$DoF = k(N-1-j) + \sum_{i=1}^{j} f_i$$

↳ the no. of constraints imposed by j joints.

k → no. of parameters (3/6)

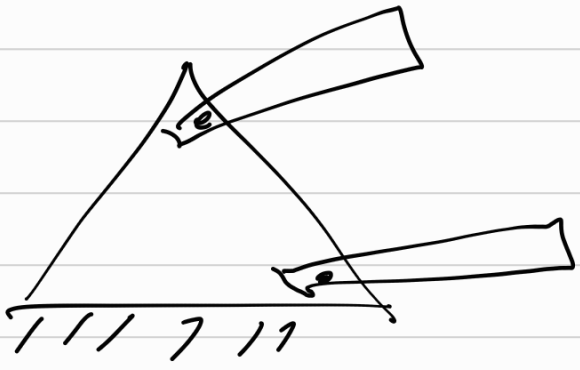
•



$N = 3$
$k = 3$
$j = 2$

$$DoF = 3(2) - (2+2)$$
$$= 2$$

•

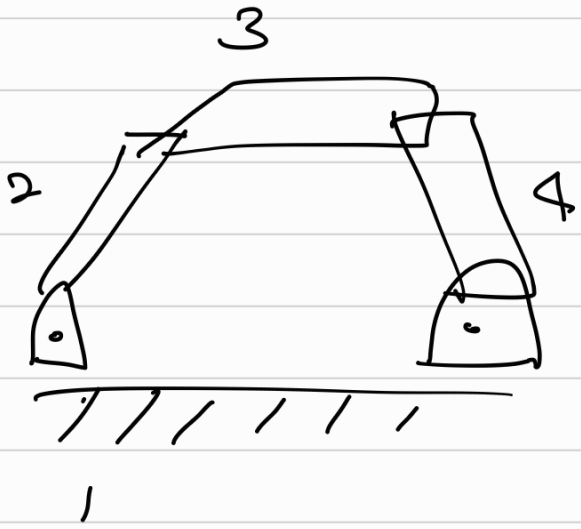


$N = 3$
$k = 3$
$j = 2$

$$DoF = 3(2) - (2 \times 2)$$
$$= 2$$

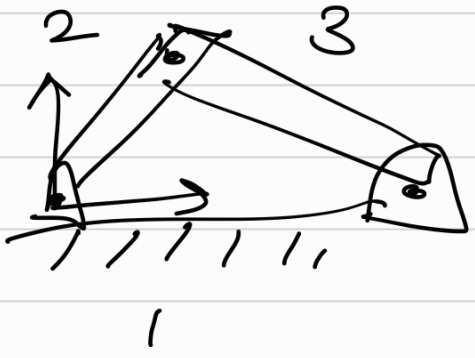
- $N = 4$
  $j = 4$

$DoF = 3(3) - 4(2)$
$= 1$



- $N = 3$
  $j = 3$

$DoF = 3(2) - 3(2)$
$= 0$



(no mobility - fixed)

- HW → find the DoF of the last system in the slides