18/09/23

- DH convention is used for forward kinematics.
- $a_{i-1} \rightarrow$ distance b/w z_{i-1} and z_i along x_{i-1}.

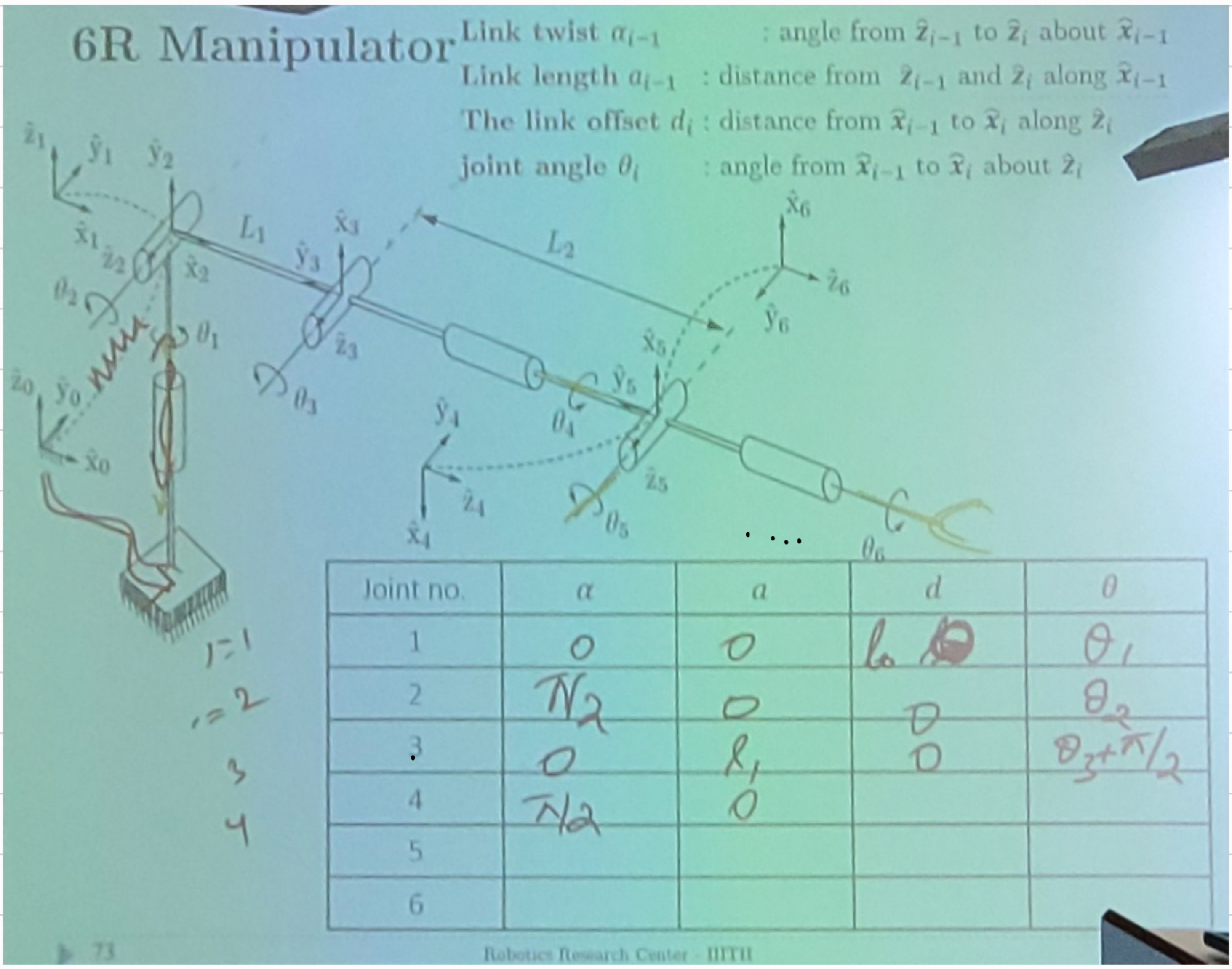
Joint no.	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	l_1	0	θ_2
3	90°	l_2	0	0

joint angles

$\hookrightarrow$ because end effector frame doesn't change.

$$ {}^{0}_{3}[T] = {}^{0}_{1}[T]\,{}^{1}_{2}[T]\,{}^{2}_{3}[T] $$

- So we make the transformation matrices based on this.



Joint no.	α	a	d	θ
1	0	0	l_0	θ_1
2	$\pi/2$	0	0	θ_2
3	0	l_1	0	$\theta_3+\pi/2$
4	$\pi/2$	0		
5				
6				

6-R manipulator

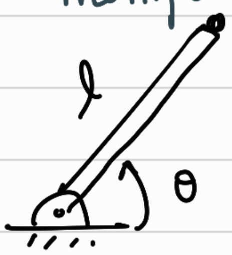
JOINT No.	α	a	d	θ
1	0	0	l_0	θ_1
2	$\pi/2$	0	0	θ_2
3	0	l_1	0	$\theta_3 + \pi/2$
4	$\pi/2$	0	l_2	$\pi + \theta_4$
5	$\pi/2$	0	0	$\theta_5 + \pi$
6	$\pi/2$	0	0	θ_6

- we then substitute these values in the transformation matrix.

INVERSE KINEMATICS

- given end effector position we need to find joint angles
- several methods: trigonometric, algebraic, geometric & numerical method.

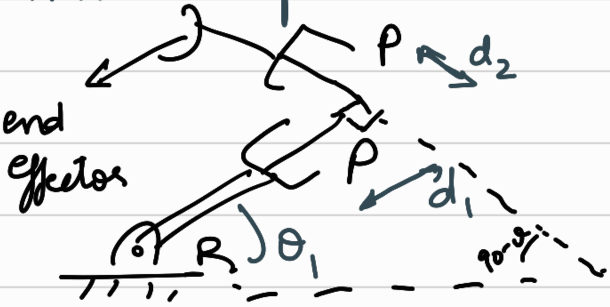
1R manipulator



$$x = l\cos\theta$$
$$y = l\sin\theta$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$= \operatorname{arctan2}\left(\frac{y}{l}, \frac{x}{l}\right)$$

R PP manipulator



$$d_1 \cos\theta - d_2 \sin\theta = x$$

$$d_1 \sin\theta + d_2 \cos\theta = y$$

(i) Trigonometric method

$$e_x = l_1 \cos\theta_1 + l_2 \cos\theta_{12}$$
$$e_y = l_1 \sin\theta_1 + l_2 \sin\theta_{12}$$

now $$c\theta_1 = \left(\frac{e_x - l_2 c\theta_{12}}{l_1} \right)$$

$$s\theta_1 = \left(\frac{e_y - l_2 \sin\theta_{12}}{l_1} \right)$$

using $s\theta_1^2 + c\theta_1^2 = 1$,

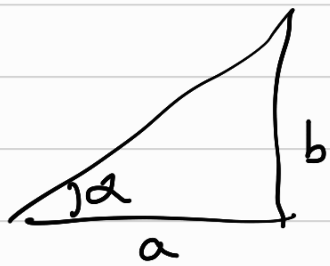
$$a \cos\theta_{12} + b \sin\theta_{12} + c = 0$$

$\therefore$ $a = 2 l_2 e_x$; $b = 2 l_2 y_2$; $c = l_1^2 - l_2^2 - e_x^2 - e_y^2$.

Divide by $\sqrt{a^2+b^2}$,

$$\frac{a}{\sqrt{a^2+b^2}} \cos\theta_{12} + \frac{b}{\sqrt{a^2+b^2}} \sin\theta_{12} + \frac{c}{\sqrt{a^2+b^2}} = 0$$

$$\cos\alpha \cos\theta_{12} + \sin\alpha \sin\theta_{12} = \frac{-c}{\sqrt{a^2+b^2}}$$



$$\cos(\theta_{12} - \alpha) = \frac{-c}{\sqrt{a^2+b^2}}$$

$$\theta_{12} - \alpha = \cos^{-1}\left(\frac{-c}{\sqrt{a^2+b^2}}\right)$$

$$\therefore \theta_{12} = \alpha + \cos^{-1}\left(\frac{-c}{\sqrt{a^2+b^2}}\right)$$

$$= \tan^{-1}\left(\frac{b}{a}\right) + \cos^{-1}\left(\frac{-c}{\sqrt{a^2+b^2}}\right)$$

assuming $(a^2+b^2 \geq c^2)$ [this ensures real sol^n]

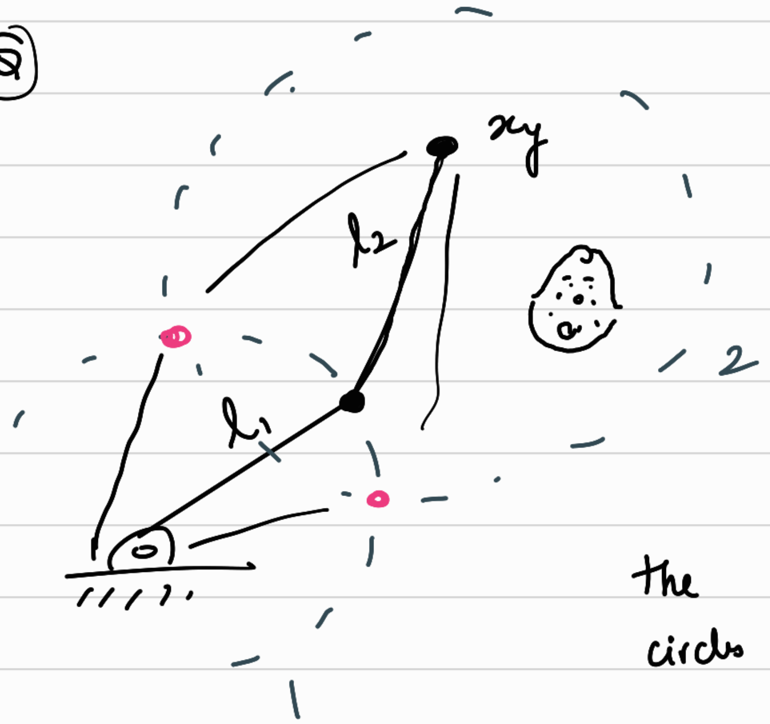
$$\boxed{\theta_{12} = \pm \arccos\left(\frac{-c}{\sqrt{a^2+b^2}}\right) + \arctan\left(\frac{b}{a}\right)}$$

- two solutions for θ_{12} will give us the end effector position.

To find θ_1 and θ_2 : $\theta_{12} = \theta_1 + \theta_2$

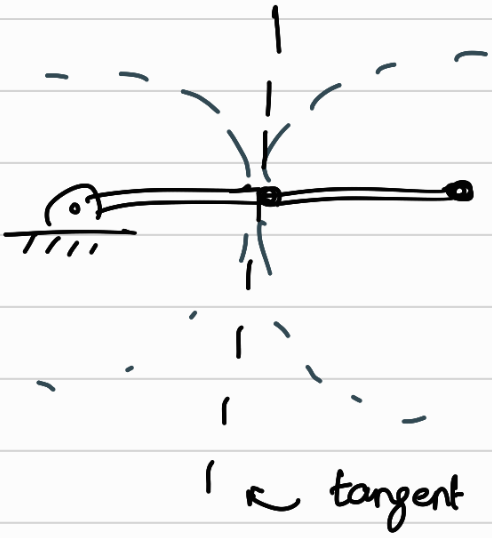
$\theta_1 = \arctan(s\theta_{12}, c\theta_{12})$

Q



1 is centered at pivot
2 is centered at endeffector.

the intersection of the two circles are two solutions.

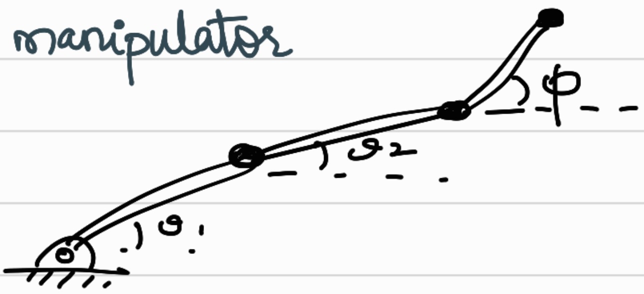


at this extreme pos^n:
only one point of intersection.

- no intersection
  so no solution.
  (imaginary solution).

$\implies$ Singularity is the loss of DoF.
Mathematically, it is where the function cannot be defined (undefined)

3R manipulator



$(x, y; \phi) \rightarrow$ given

$$\phi = \theta_1 + \theta_2 + \theta_3$$

- this will also have 2 solutions (elbow down and elbow down configuration)
- if ϕ is not given $\rightarrow$ we would get infinitely many solutions.

now, $e_x = l_1 \cos\theta_1 + l_2 \cos\theta_{12}$

$e_y = l_1 \sin\theta_1 + l_2 \sin\theta_{12}$

squaring & adding,

$e_x^2 + e_y^2 = l_1^2 + l_2^2 + 2l_1 l_2 \cos\theta_2$

$$\Rightarrow \theta_2 = \pm \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1 l_2}\right)$$

now $\left|\frac{x^2+y^2-l_1^2-l_2^2}{2l_1 l_2}\right| \le 1$

$$l_1^2+l_2^2-2l_1l_2 \le x^2+y^2 \le l_1^2+l_2^2+2l_1l_2$$

$$(l_1-l_2)^2 \le x^2+y^2 \le (l_1+l_2)^2$$

$\therefore$ as long as (x,y) within this circle $\rightarrow$ real solution.