Robotics: Dynamics and Control

Class 4

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Contents

Configuration Space

- Mechanics concerned with dynamical systems – systems whose configuration evolves in time according to some deterministic law.

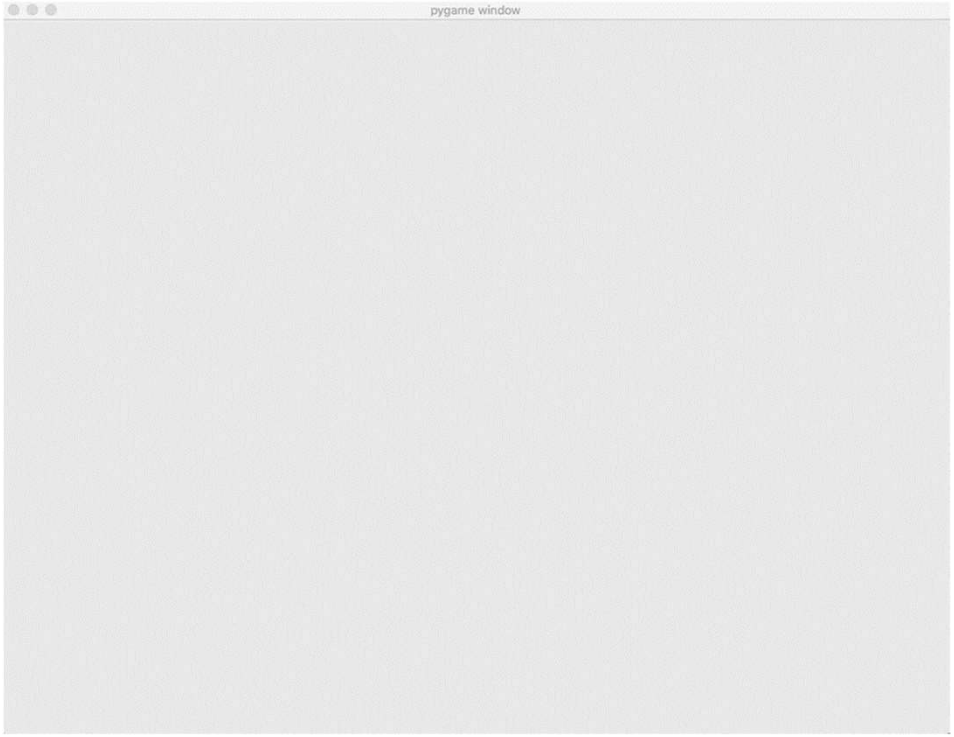
- Configuration space – which characterises what the system is doing at the given time instant.

$$\boldsymbol{r}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}} + z(t)\hat{\boldsymbol{k}}$$

Motion is restricted to $\subset \mathbb{R}^3$

Constraint is:

$$x(t)^2 + y(t)^2 + z(t)^2 = l^2$$



Important concepts

- Represent Euclidean ("flat") spaces $\mathbb{E}^n$ as $\mathbb{R}^n$.

- For curved spaces, choose

  - minimum-parameter **explicit parameterisations** (choose between **singularities** or an **atlas** of **coordinate charts**), OR

  - **implicit representation** (use more numbers subject to constraints).

What is the C-space?

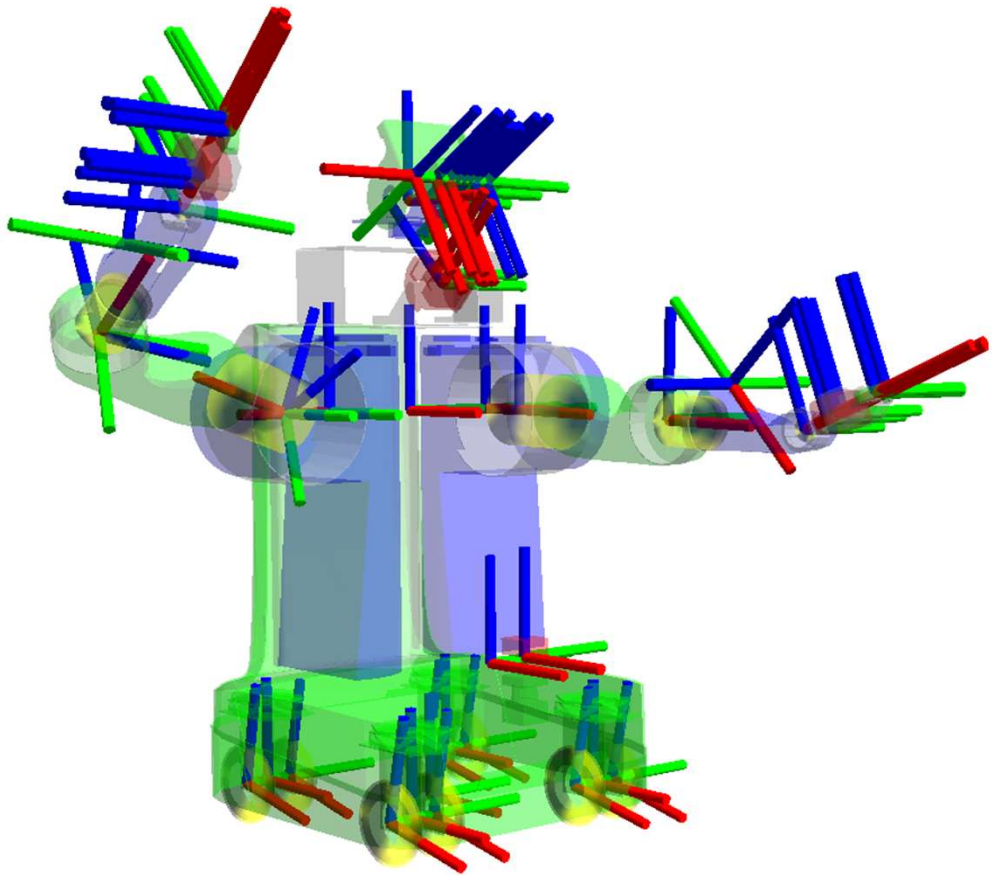
DoF	Configuration variables	
Point robot on a line:		
Point robot on the plane:		
Point robot on a Circle:		
Single-link robot with a revolute joint:		
Two-link robot with revolute joints:		
Rigid body in plane		
Rigid body in space		

Configuration Space

KUKA youBot
meccanum-wheel omnidirectional base
moving on flat ground
plus 5-DOF robot arm + gripper

C-space topology and representation?

Frame Management:



PR2 - DEGREES OF FREEDOM (DOF): 20 (Arm: 4 DoF x 2; Wrist: 3 DoF x 2; Gripper: 1 DoF x 2; Head pan/tilt: 2 DoF; Head laser tilt: 1 DoF; Telescoping spine: 1 DoF; mobile base actuators not included)

Important Definitions

A **system** is a set of points in the space X.

A **configuration** of a system is the location of every points in the system

Configuration space ($\mathcal{C}$) is a **metric space** comprising all the configurations of the system – space spanned by the configuration variables.

Degrees of freedom of a system is the dimension of $\mathcal{C}$-space – min. no. of real numbers required to specify the configuration.

Important Definitions

- Rigid body: A body that does not undergo deformation.

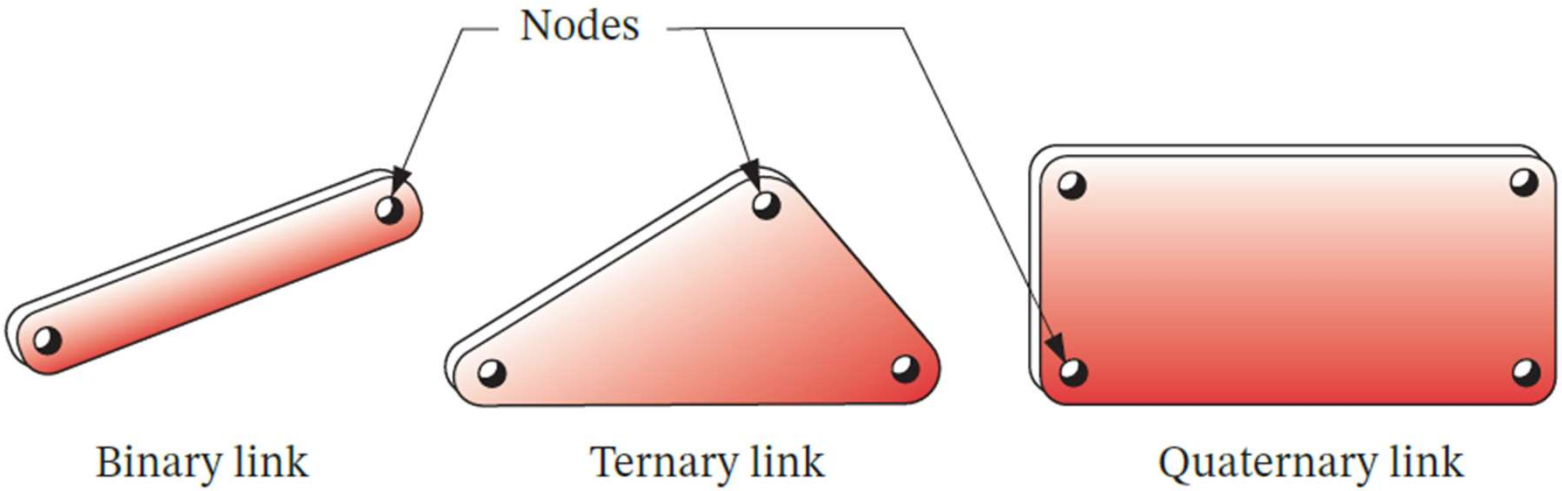
- Mechanism: A device to transform one motion to another – connection of links and joints – at least one of the link should be grounded.

- Machine: is a collection of mechanisms to transmit substantial forces.

Links

Link: rigid body that possesses at least two nodes that are points for attachment to other links.

- Binary link - one with two nodes.
- Ternary link - one with three nodes.
- Quaternary link - one with four nodes.

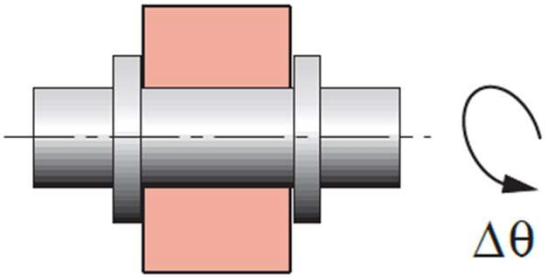


Joints

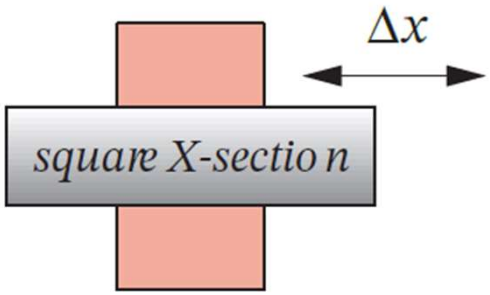
- A joint is a connection between two or more links (at their nodes), which allows some motion between the connected links.
- Joints (also called kinematic pairs)
  - Revolute (R) pair
  - prismatic (P) pair

  The R and P pairs are the basic building blocks of all other pairs.
  - The screw (H)
  - cylindric (C)
  - spherical (S)
  - flat (F) pairs

Joints



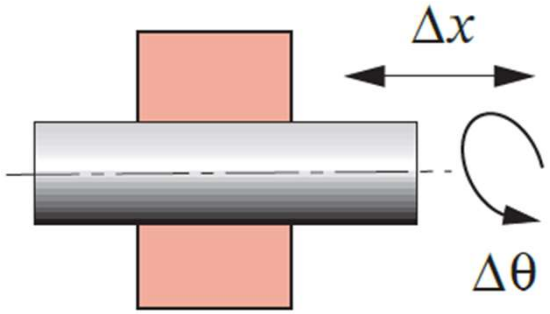
Revolute (R) joint—1 *DOF*



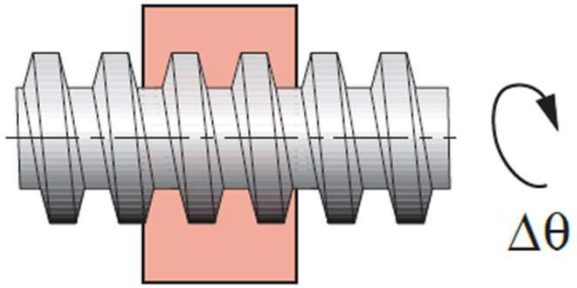
Prismatic (P) joint—1 *DOF*

Ref: L. Norton, Design of Machinery “An Introduction to the. Synthesis and Analysis of Mechanisms

Joints



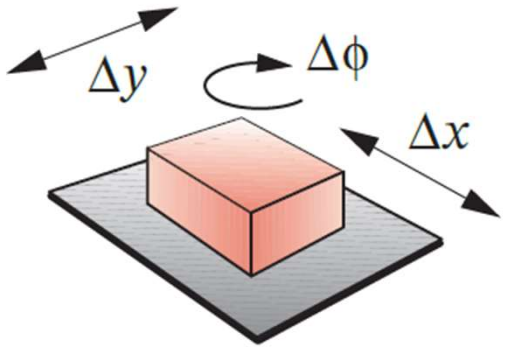
Cylindric (C) joint—2 *DOF*



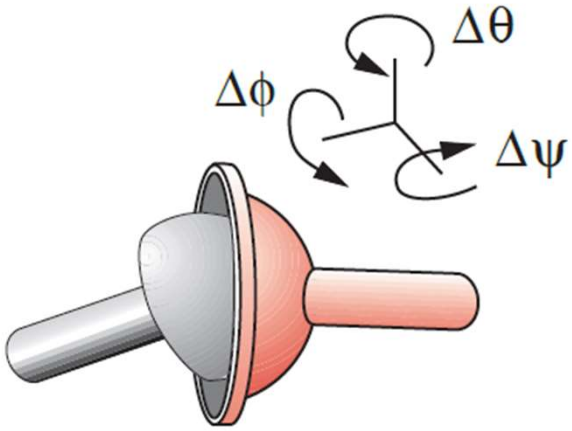
Helical (H) joint—1 *DOF*

Ref: L. Norton, Design of Machinery “An Introduction to the. Synthesis and Analysis of Mechanisms

Joints



Planar (F) joint—3 *DOF*



Spherical (S) joint—3 *DOF*

Ref: L. Norton, Design of Machinery “An Introduction to the. Synthesis and Analysis of Mechanisms

Kinematic chains

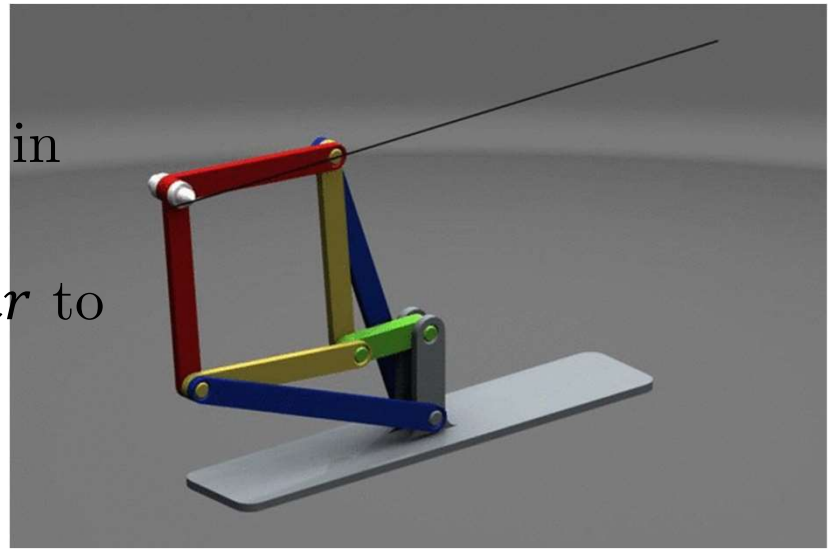
- **Kinematic chain**: an assemblage of rigid bodies/links, $l_0, l_1, l_2 \ldots l_{n-1}$ connected via joints $j_1, j_2 \ldots j_{n-1}$. Where each link l_i is attached to link l_{i+1} at joint j_{i+1}

- **Open Chain:**

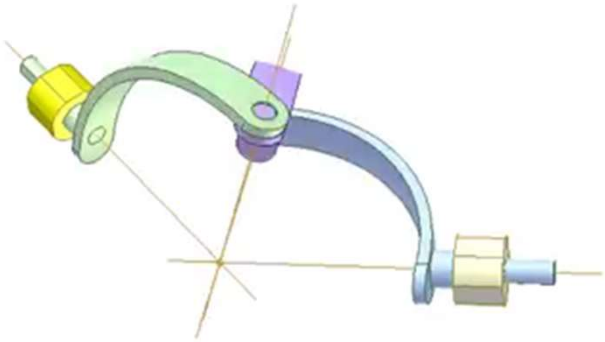
- **Closed Chain:**

Mechanism Types:

- Planar mechanism:
  - If all the points of a mechanism move in planes *parallel* to certain plane.
  - If the axes of hinges are *perpendicular* to the base plane



- Spherical mechanism:
  - The axes of all joints intersect at a point.
  - Example – Gyro.

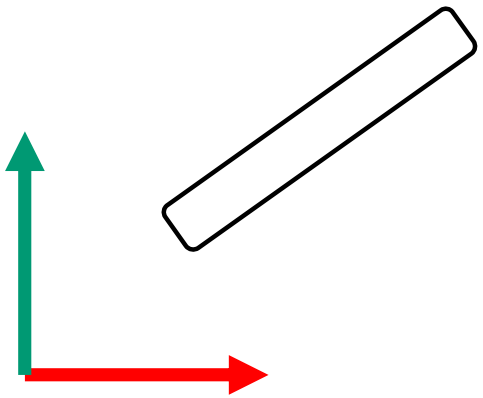


- Spatial mechanism:
  - Do not have special points or special base plane, in general, it means all the joint axes are skew to each other.

Mobility - Degrees of Freedom (DOF)

- The number of independent parameters such as joint angles and slide distances, that are needed to specify the configuration of the linkage.

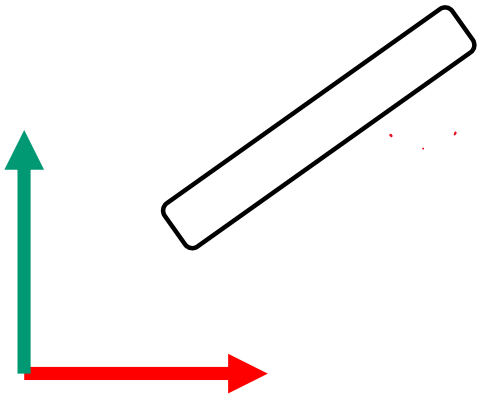
- It is known as the dimension of the system's configuration space.



Mobility Analysis

n = Number of Links (including ground link)

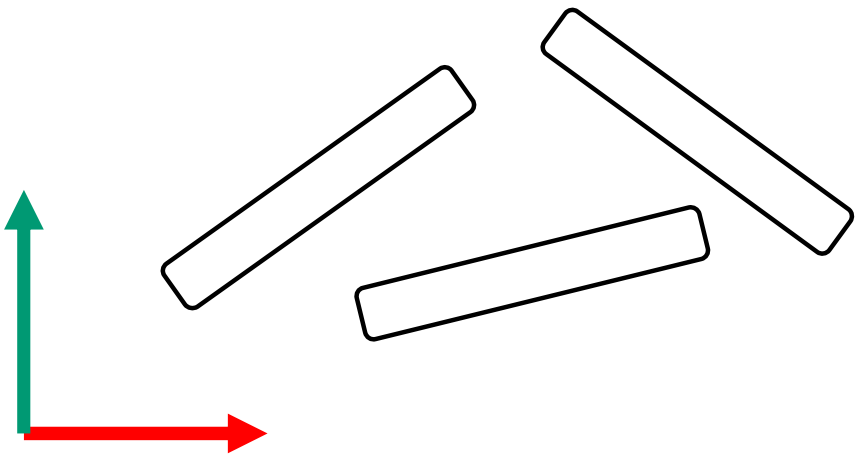
j = Number of Joints (pivot connections between links)



$DoF = 3$

$N = 1$

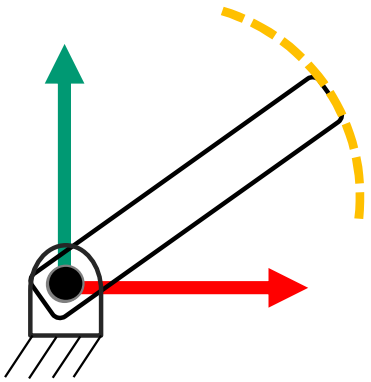
$J = 0$



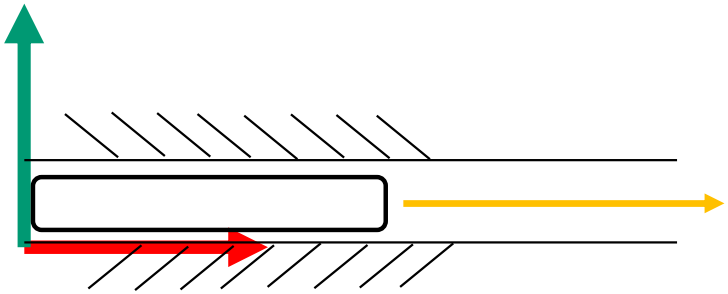
$DoF = 3N$

Degrees of Freedom (DoF)

Revolute joint (R)



Prismatic joint (P)



$$N = 2$$
$$J = 1$$
$$DoF = 3 - 2$$

$$DoF = 3(N-1) - 2j$$
$$DoF = 3(N-1) - (3 - f_i)$$

Joints

6N

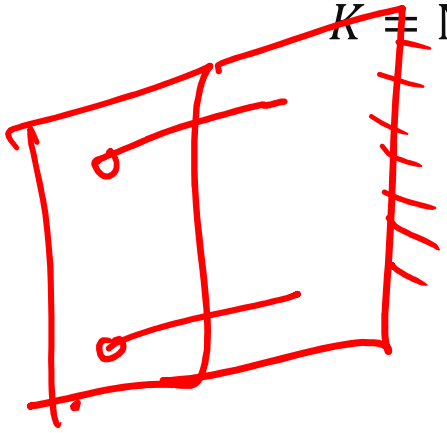
Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1		
Prismatic (P)	1		
Helical (H)	1		
Cylindrical (C)	2		
Universal (U)	2		
Spherical (S)	3		

The number of degrees of freedom f and constraints c provided by common joints.

Gruebler's Condition

the number of constraint equations imposed by j joints

$$DoF = K(N-1) - \sum_{i=1}^{j}(K - f_i)$$



K = No. of parameters required to specify a single link 3 or 6

N = no. of links

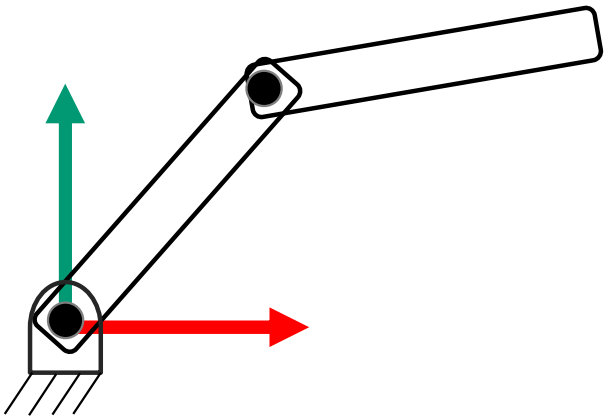
j = no. of joints

f_i = is the freedom of the jth joint

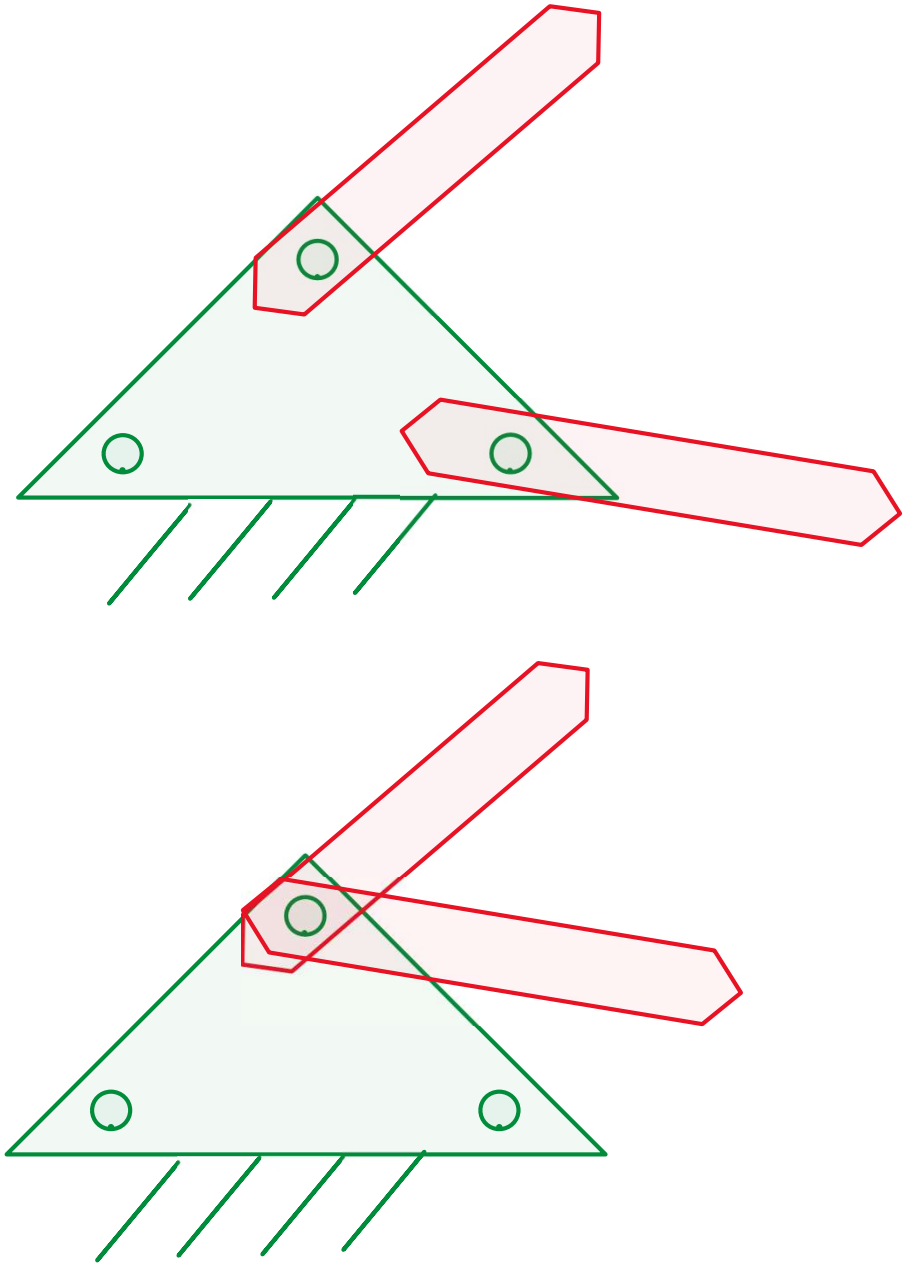
$$DoF = K(N-1-j) + \sum_{i=1}^{j} f_i$$

How to find DoF?

- The Grübler's formula correctly predicts the number of actuators needed in a regular configuration.

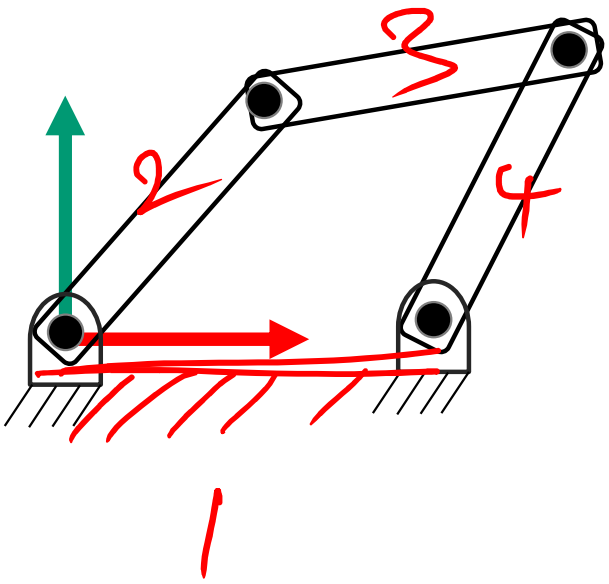


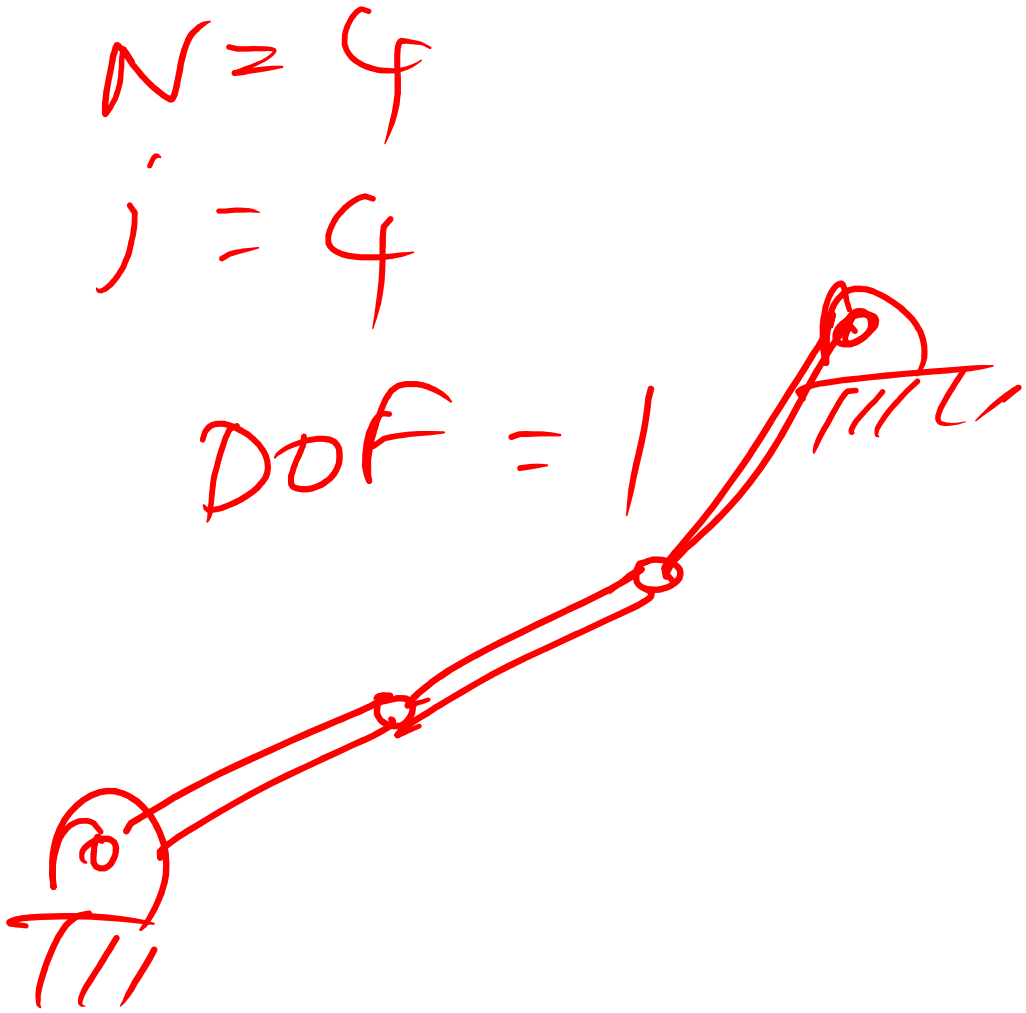
How to find DoF?



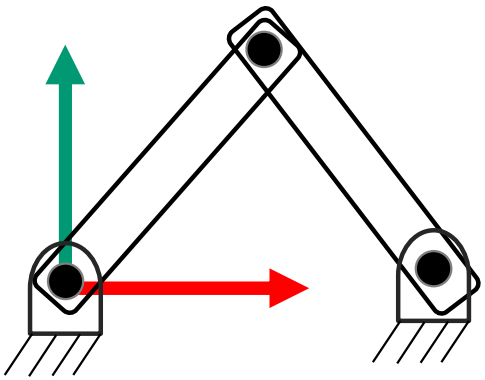
How to find DoF?

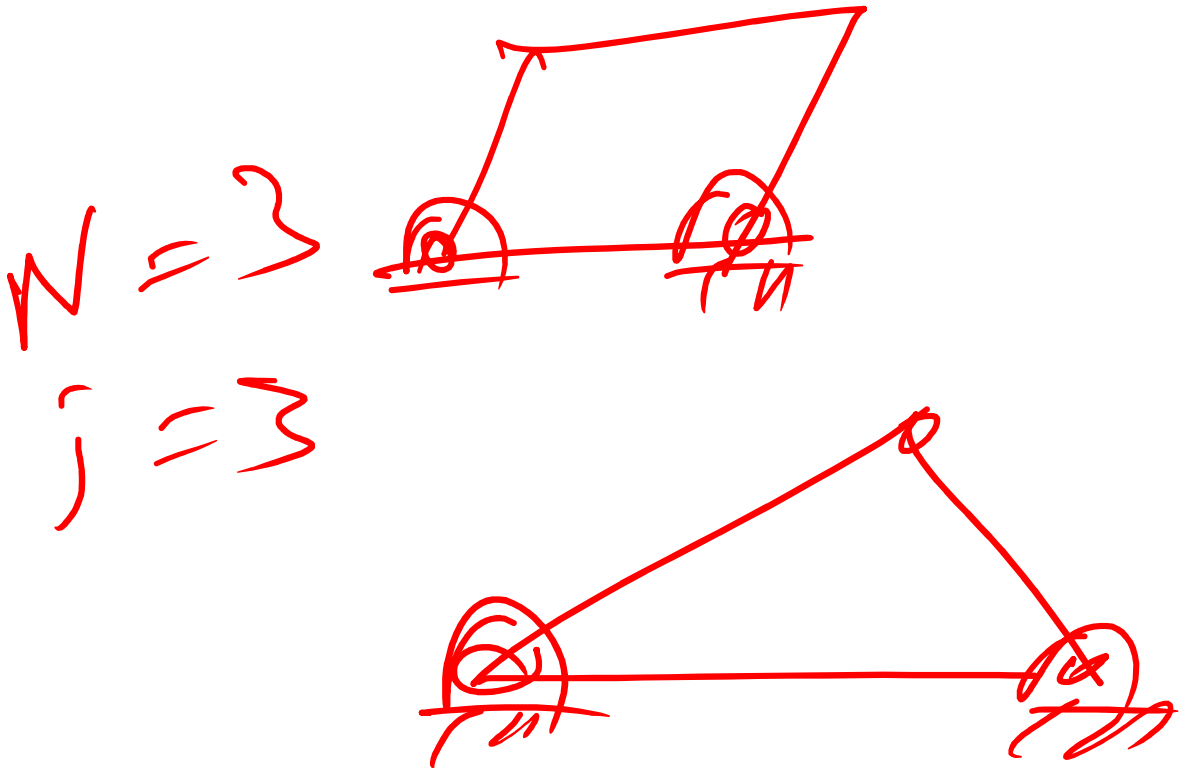
- Four links, Four Joints





How to find DoF?





- Three links, Three Joints
  - Statically Determinant Structure

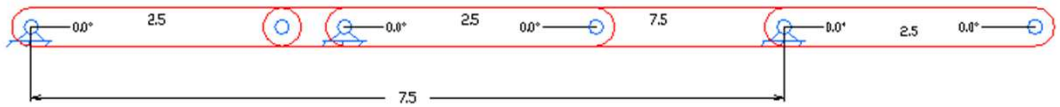
How to find DoF?

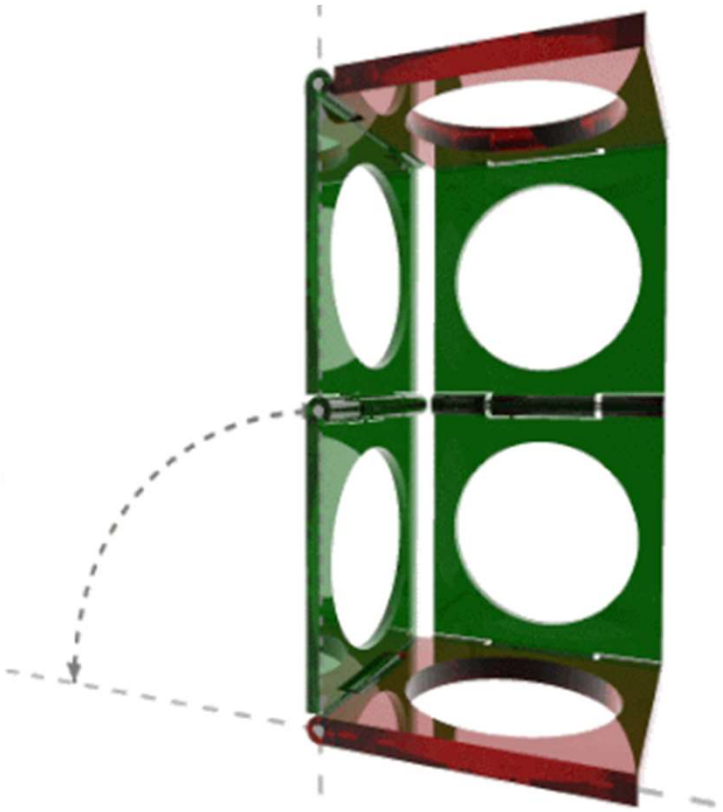
- Two links, Two Joints

Two overlapped joints – Statically Indeterminant Structure

Failure of the criterion

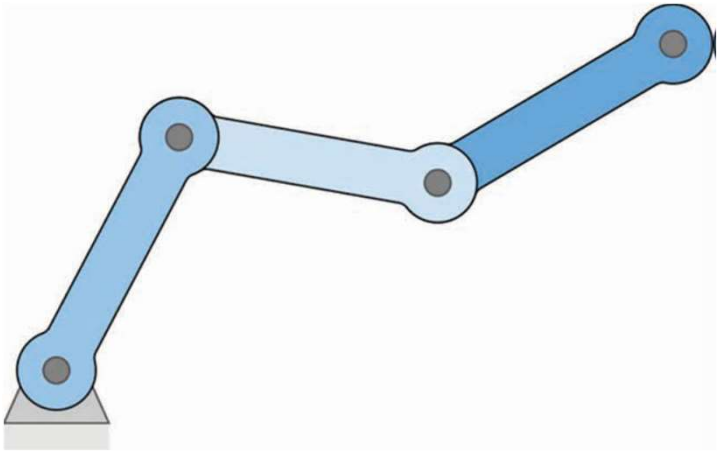
- Grubler's criteria may fail to predict the no. of actuators if the mechanism has
  - Special link lengths or geometry
  - Over constrained sub mechanism

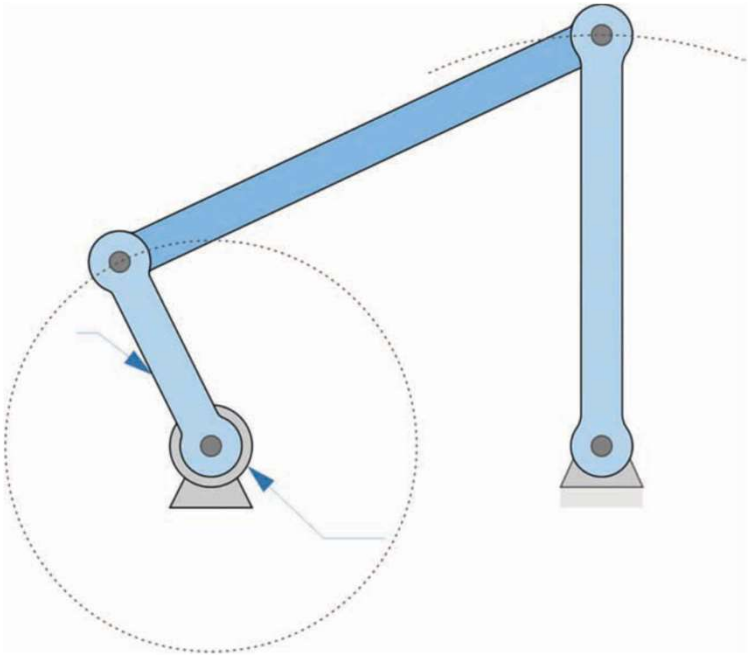


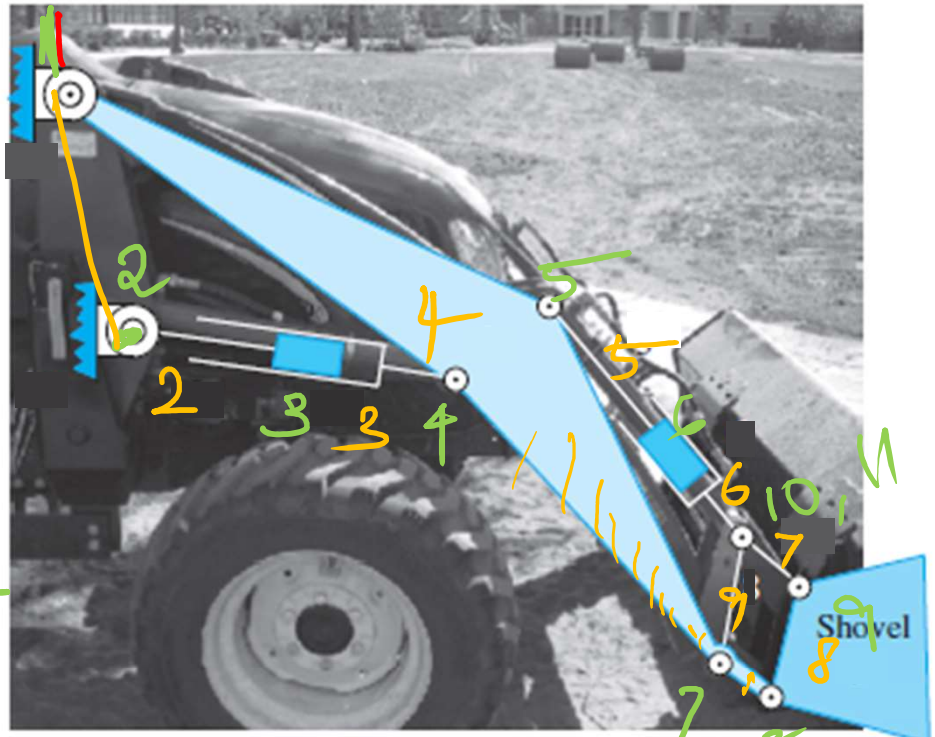


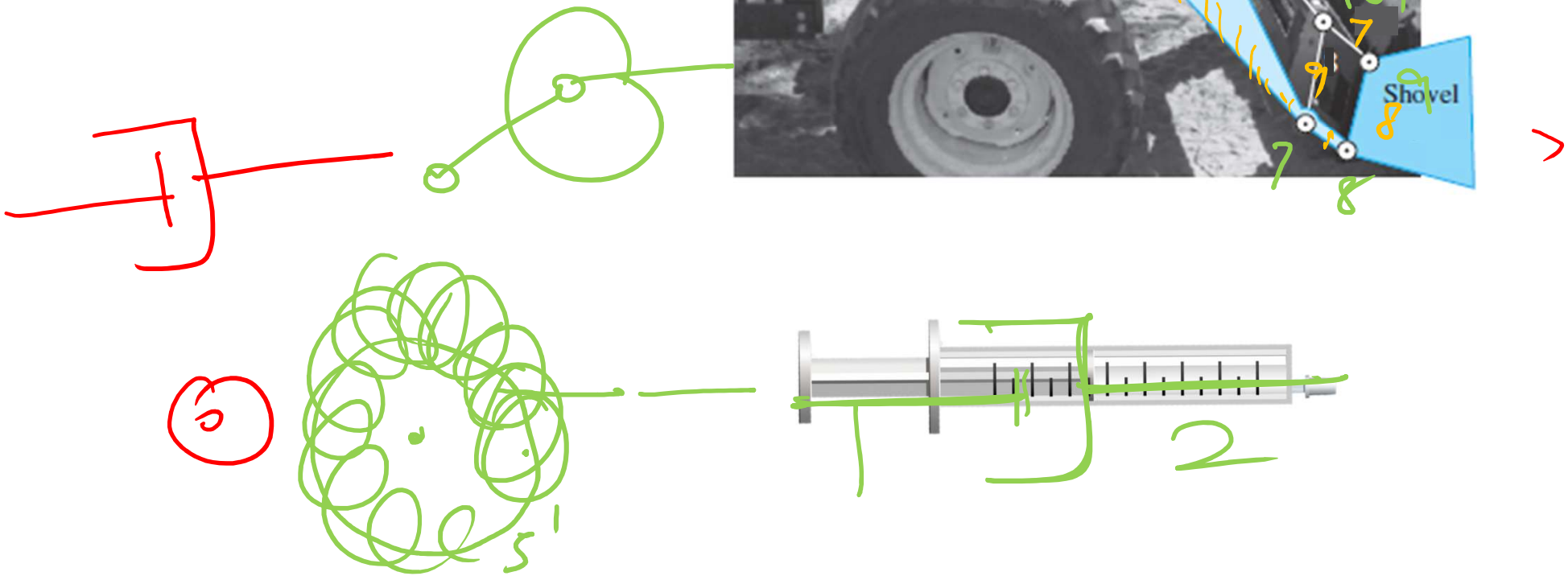
https://en.wikipedia.org/wiki/Overconstrained_mechanism

Find DoF

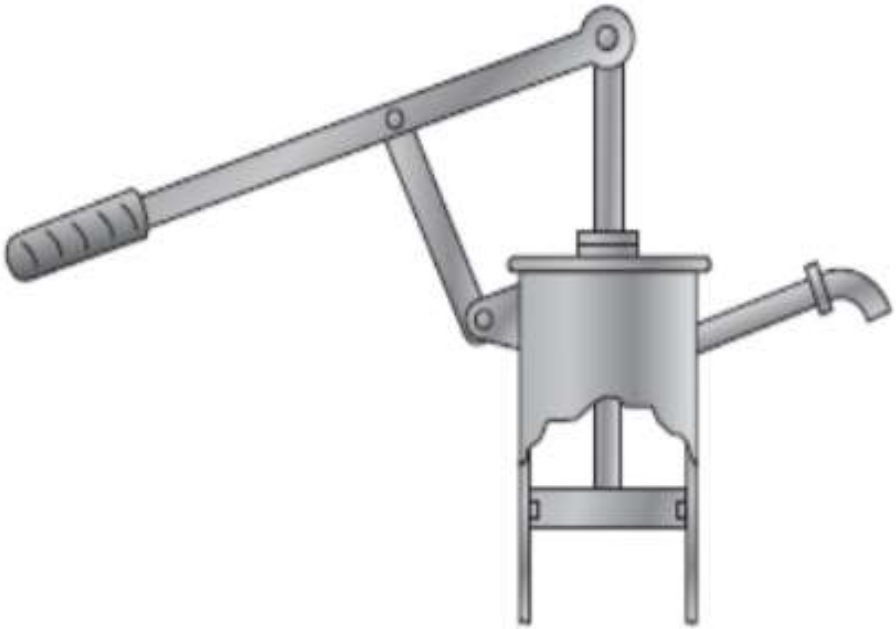




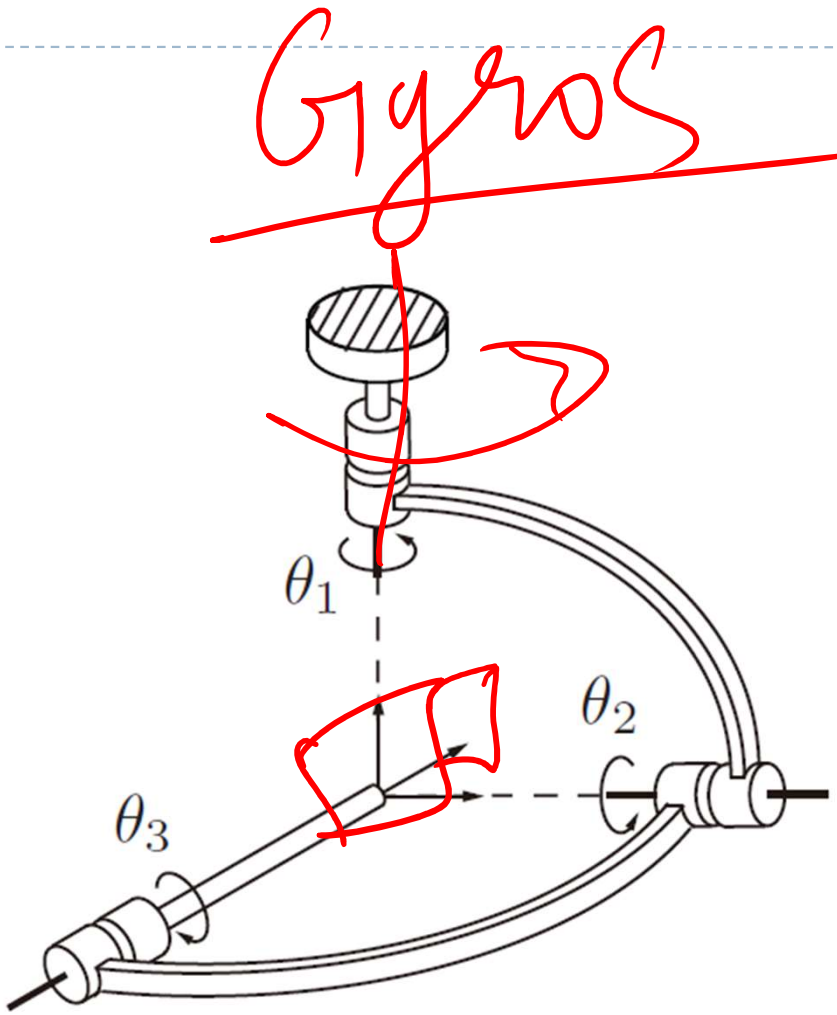

Shovel



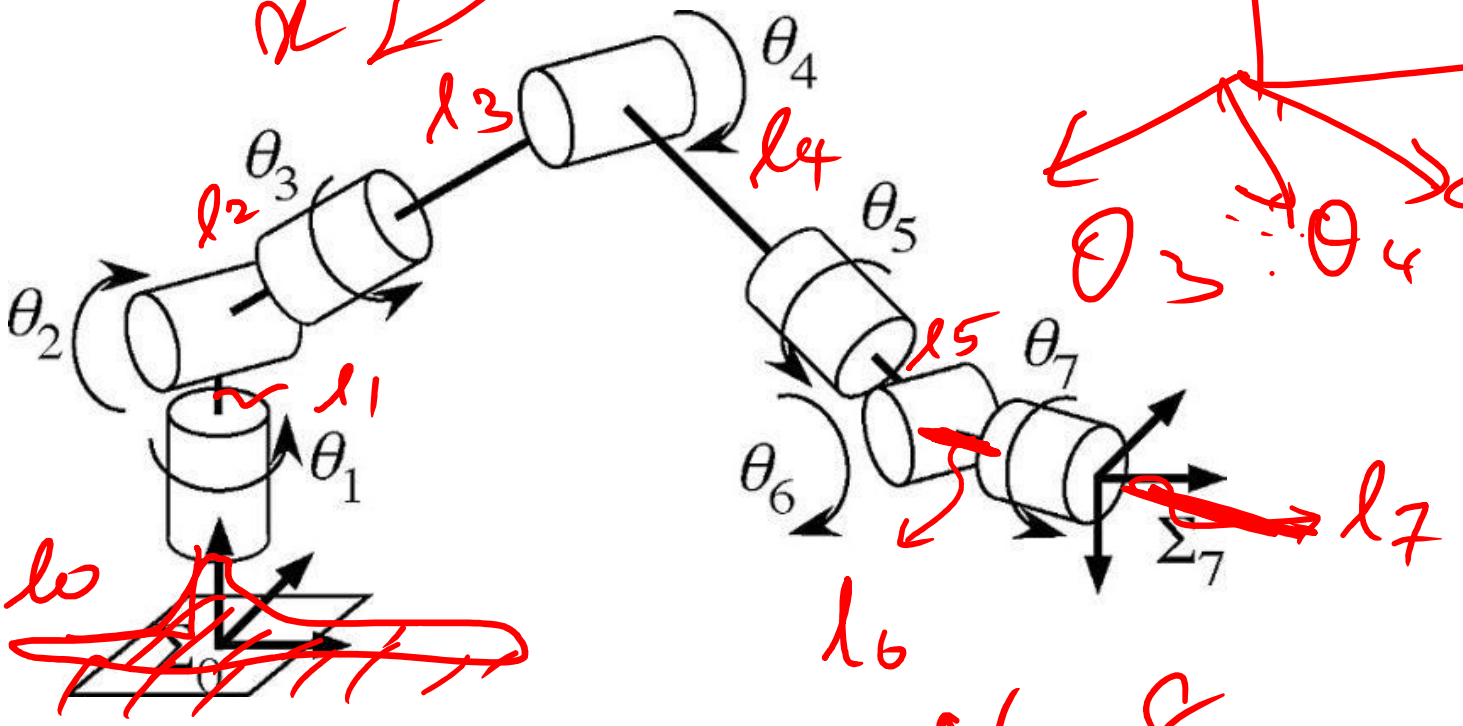
Find DoF



Find DoF



Find DoF

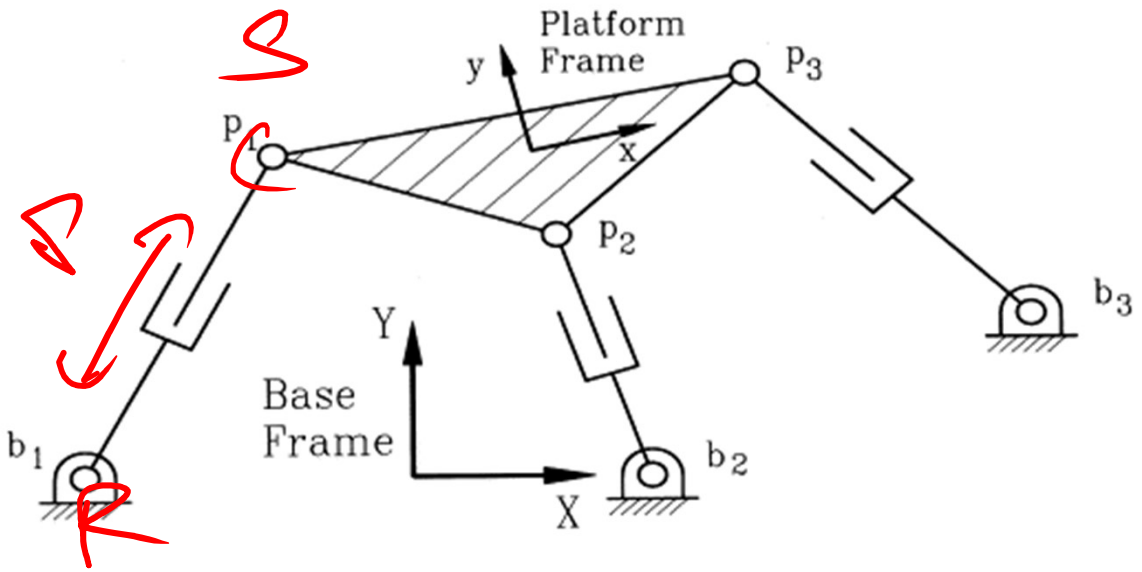


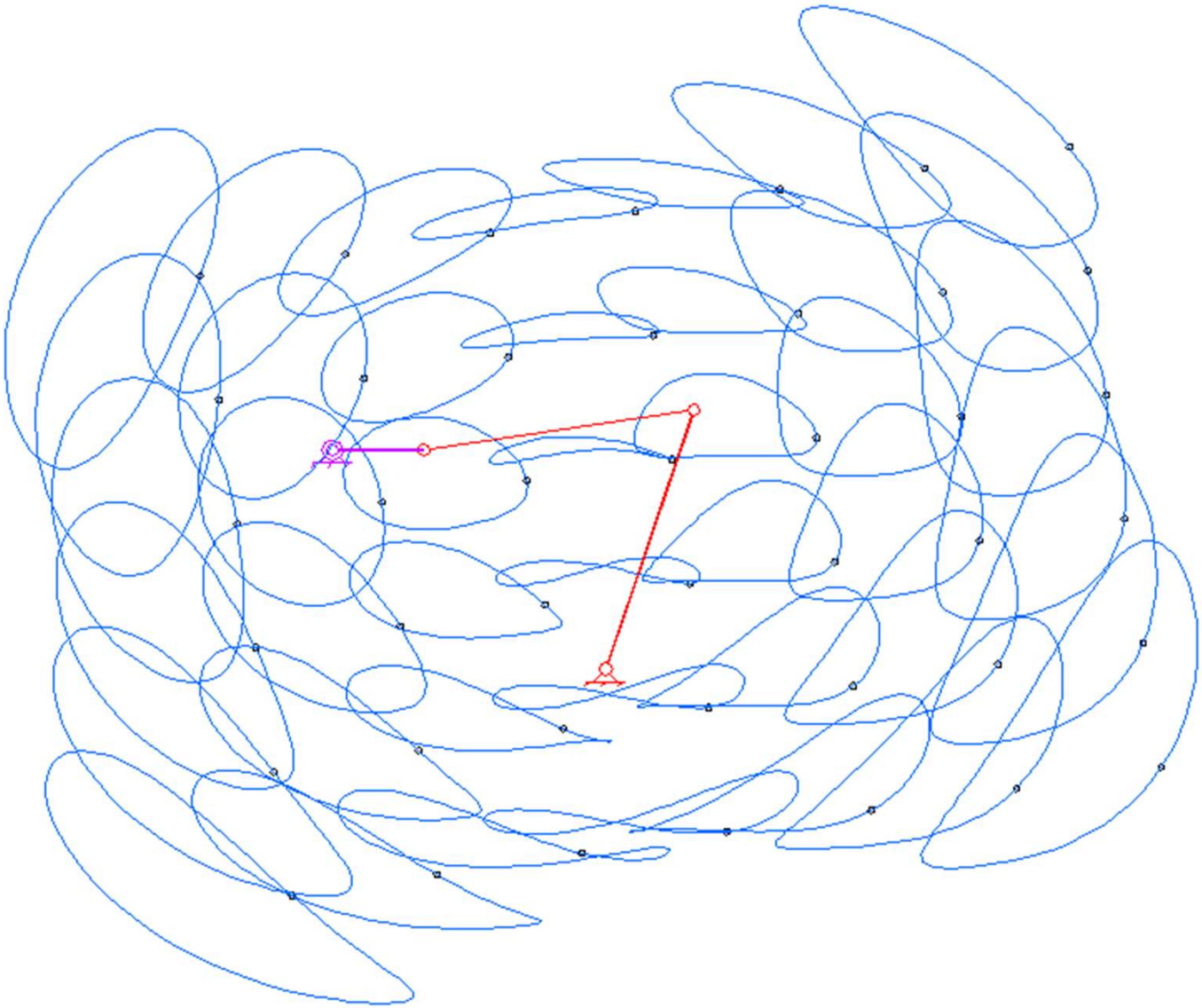
$N = 8$

$j = 7$

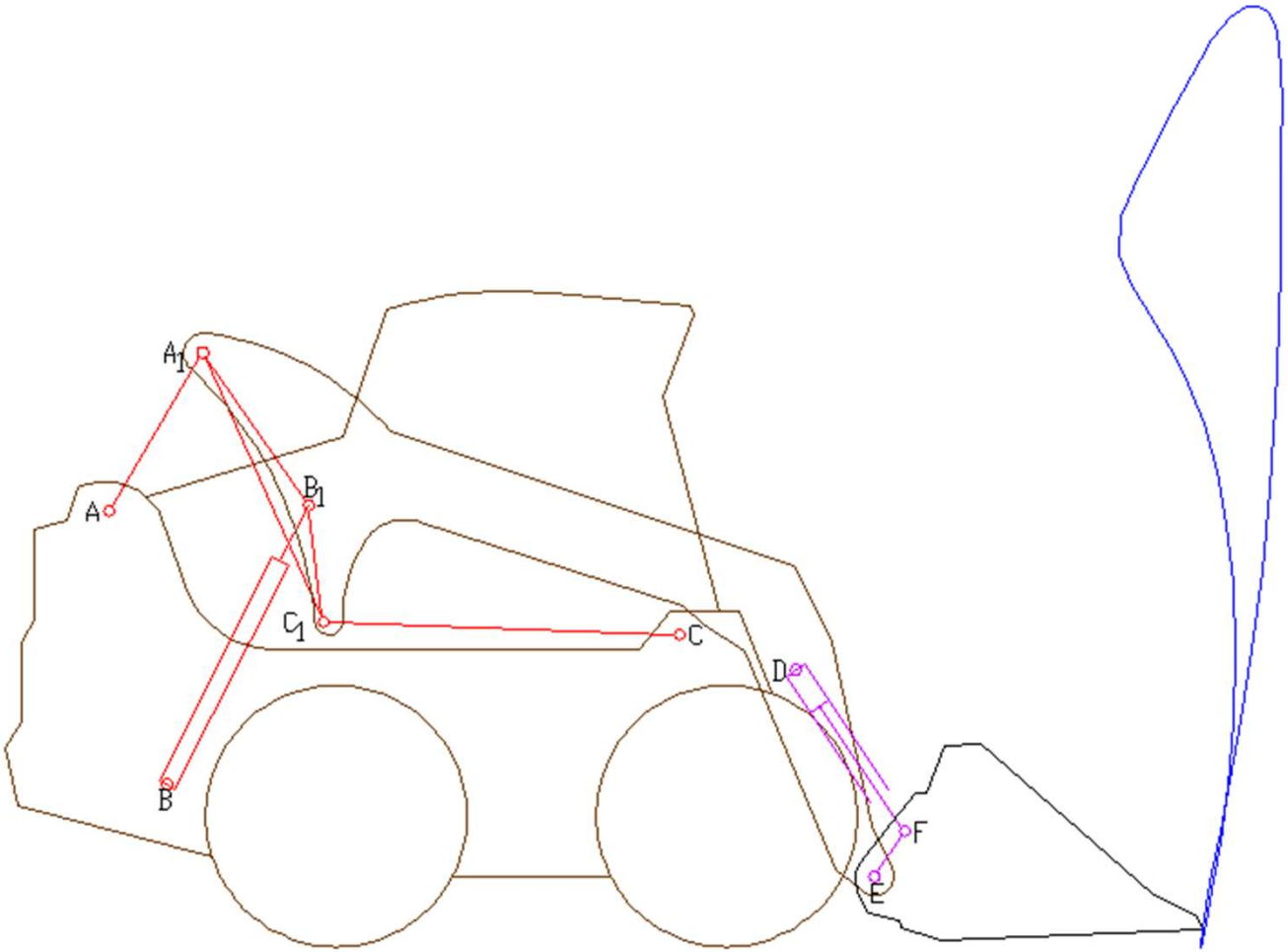
$DoF = 7$

Find DoF





Find DoF



Thank You...