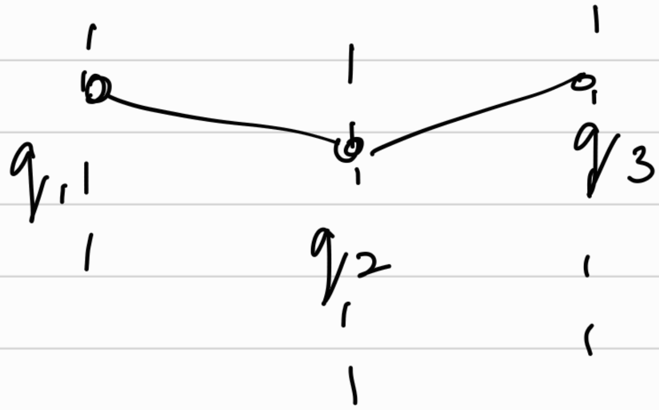
RECAP:

- planar mechanism :-



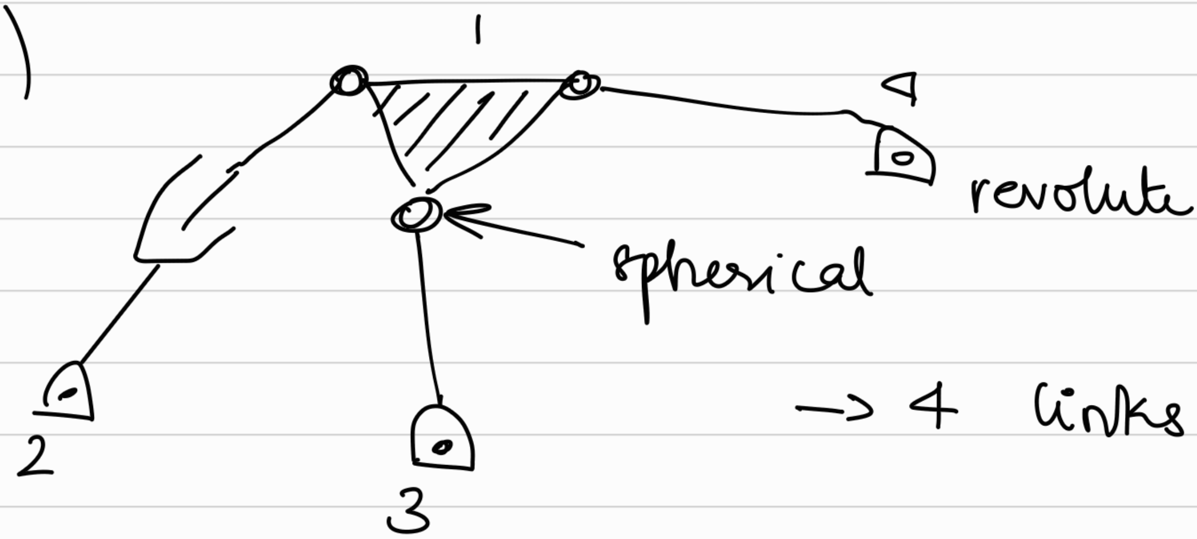
$q_1 \parallel q_2 \parallel q_3$ → parallel axis

- spatial mechanism :-

$q_1 \nparallel q_2$

* ———————— *

Q)



→ 4 links

17/08/23

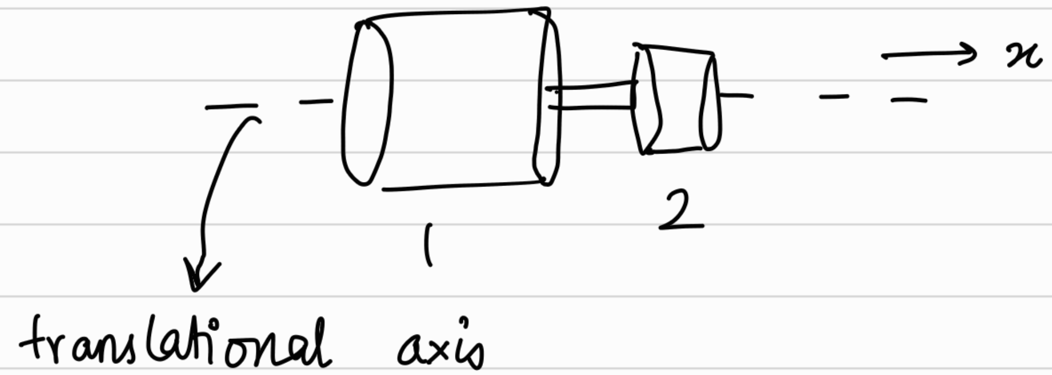
- $S^1 \times S^1 \neq S^2 = \mathbb{T}^2$

in general, $S^1 \times S^1 \times \cdots \times S^n = \mathbb{T}^n$

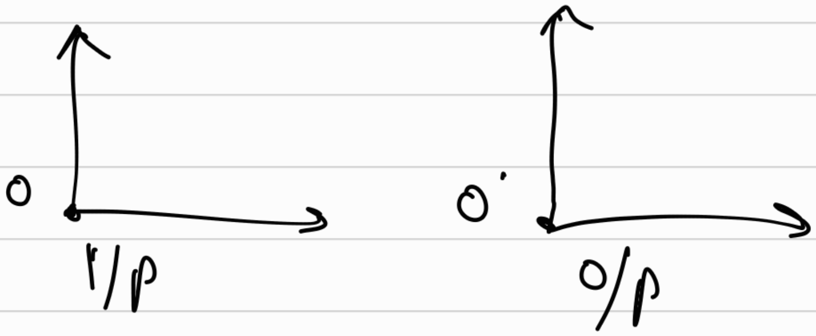
$\mathbb{R}^1 \times \mathbb{R}^1 \times \cdots \times \mathbb{R}^n = \mathbb{R}^n$

$\mathbb{R}^1 \times \mathbb{R}^1 \times S^1 = \mathbb{R}^2 \times S^1$

→ PRISMATIC JOINT : $\mathbb{R}^1$ space ; 2 links

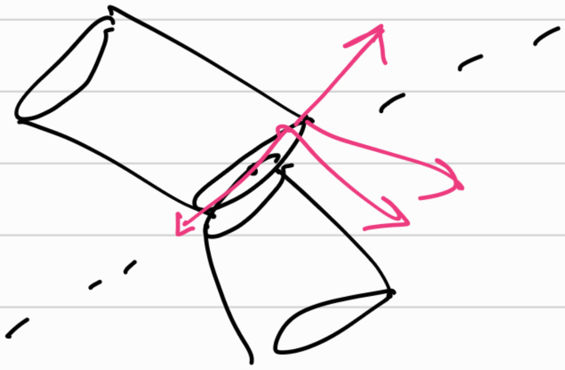


- assign the two links different frames and then we can find the relative position b/w the two based on the difference b/w origin.

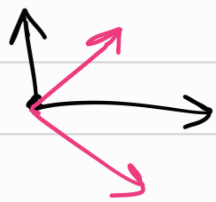


- all points move parallel to each other.

→ REVOLUTE JOINT : S^1 space ; 2 links



- two frames : one at the joint itself
- there is a rotational axis



JOINT SPACE & WORK SPACE

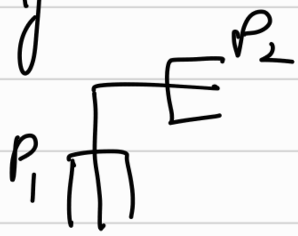
- <u>Joint space</u> : entire space that the robot works in.

- <u>Task space</u> : space in which tasks are naturally expressed.
  - $\mathbb{R}^2$ or $\mathbb{R}^3$

- <u>Work space</u> : reachable configuration of the robot's end effector.

usually subset of $\mathbb{R}^2 / \mathbb{R}^3$.

- For two prismatic joints P_1 & P_2: maximum area is covered if -

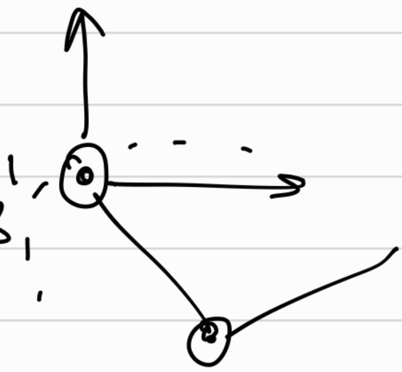


$P_1 \perp P_2$

Task space - $\mathbb{R}^2$
Work space - subset of $\mathbb{R}^2$
Joint space - $\mathbb{R}^1 \times \mathbb{R}^1 = \mathbb{R}^2$

$\Rightarrow$ if Joint space and work space is the same $\rightarrow$ one to one mapping

Q)



$q_1 \in S^1$

$q_2 \in S^1$

Task space $= \mathbb{R}^2$
Work space $=$ annulus (concentric circles)
$W \subset \mathbb{R}^2$

Joint space $= S^1 \times S^1$
$\approx T^2$

donut (do + nut)
$\downarrow$
what your mum asked me to do last night

Q) 1 prismastic + 1 revolute

Workspace : subset of $\mathbb{R}^2$

Joint space : $\mathbb{R}^1 \times S^1$

- non - Smooth curve because it is not different-iable at the corners.

Q)

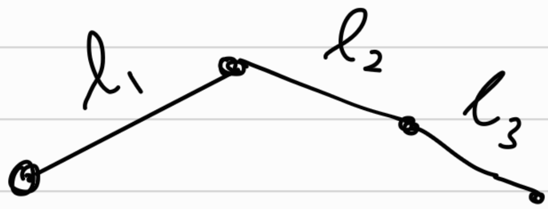
1 revolute + 1 prismatic

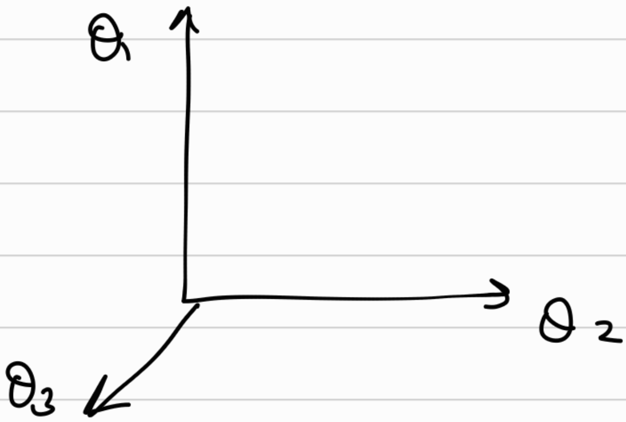
Workspace : subset of $\mathbb{R}^2$

Joint space : $\mathbb{R}^1 \times S^1$

3-R Manipulator

- Task space - $\mathbb{R}^2$
  Joint space - $S^1 \times S^1 \times S^1$

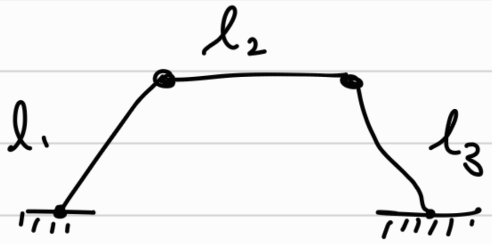




- For 3R, we will have infinite number of solutions.

(For 2R, we had 2 sets of solutions)

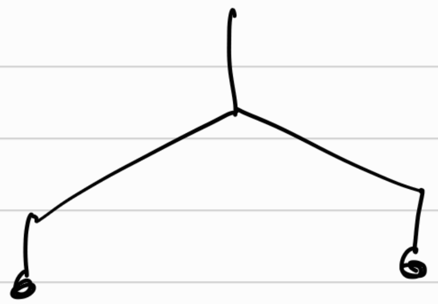
- If we constrain the end pos for a 3R configuration:



– θ_2 and θ_3 depend on θ_1

TYPES OF MANIPULATORS

Serial	Parallel
• large workspace • error accumulates • less carrying capacity	• limited workspace • error averaging • high payload carrying capacity



i. Cartesian Robot :

Workspace = Joint space = $\mathbb{R}^3$

ii. Cylinderical Robot :

DoF = 3

Workspace = $S^1 \times S^1$

iii. Polar robot :

DoF = 3

Workspace = subset of two concentric spheres

iv. SCARA

DoF = 4

→ articulated arm : Six DoF

$(x, y, z, \alpha, \beta, \gamma)$

- the joints have an <u>orthoparallel</u> configuration.

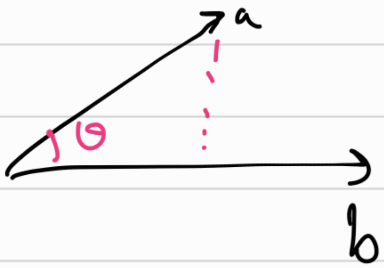
robo ANALyzer

RIGID BODY TRANSFORMATIONS

- linear body transformations – $A(cx_1) = cA(x_1)$
  - reflection, scaling, rotation (all satisfied by linear bodies)

Scalar product :

- commutative ; distributive ; not associative
- $a \cdot b = |a||b| \cos\theta$
- orthogonal vectors ; $a \cdot b = 0$
  co-directional ; $a \cdot b = |a||b|$
- component of $\vec{a}$ along $\vec{b}$: $\left(\frac{a \cdot b}{|b|}\right)\hat{b}$



Cross Product :

- direction of $(\vec{a} \times \vec{b})$ is $\perp$ to $\vec{a}$ & $\vec{b}$.
- not commutative or associative
- co-directional : $a \times b = 0$

$\Longrightarrow$ a mapping $g : \mathbb{R}^3 \to \mathbb{R}^3$ is a rigid body transformation if :-

note :-

ⓐ length is preserved
$\|p - q\| = \|g(p) - g(q)\|$

(b) cross product is preserved (orientation is preserved)

$$g_*(v \times w) = g_*(v) \times g_*(w)$$

- inner product is also preserved