1. Consider an Articulated arm shown below. The end-effector position is given by ${}^0\mathbf{P} = [x,\ y,\ z]$

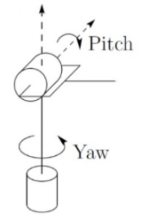


Figure 1: 2R Manipulator

(a) Find Degrees of Freedom (DoF)? **(0.5)**

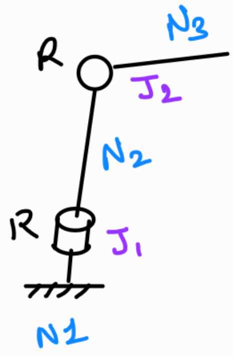
Number of Links ⇒ 3

Number of Joints ⇒ 2 Revolute

Degree of freedom of $i^{th}$ joint.

$$\text{Degree of Freedom} = m(N-J-1) + \sum_{i=1}^{J} f_i$$

↳ 6 for spacial mechanism




$$= 6(3-2-1) + 2\times 1$$

$$\boxed{\text{DoF} = 2}$$

0.5 with grublers
0 otherwise

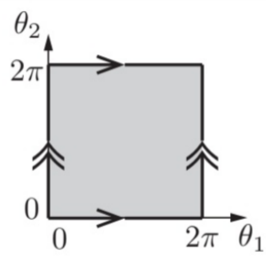
(b) Give a way to explicitly parameterize its C-Space and its Topology. **(1.5)**

As the given mechanism is a spacial 2R mechanism with 2 DoF, so its C-space can be explicitly parametrized using Joint angles $\theta_1$ & $\theta_2$ as the Configuration space variables. (0.5m)

So, the topology of C-space would be $S^1 \times S^1$ or $T^2$. (1m)

(c) Is there a point of singularity in your chosen explicit representation? If Yes, Why? How do you overcome this problem? **(2)**

1.5 [Yes, there is a point of singularity. The point of singularity is encountered in $T^2$ when either of $\theta_1$ or $\theta_2$ loops from $2\pi$ to 0.] This causes various computational issues like velocity as a time derivative of angles tending to $\infty$ at those points. Way to overcome - Implicit Parametrization (0.5)



$[0, 2\pi) \times [0, 2\pi)$

(d) Comment on implicit parameterization. **(1)**

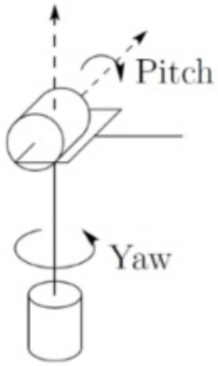



Figure 1: 2R Manipulator

The given manipulator can be implicitly parametrized using $(x, y, z)$ coordinates in 3D space while specifying the constraint as $x^2+y^2+z^2 = R^2$.

(R being the radius of sphere plotted by the end effector.)

1

2. (a) Write the properties of an $n \times n$ Rotation Matrix $\mathbf{R}$. (2)

All Rotation matrices of the order $n \times n$ belong to the special orthogonal group of order n, $SO(n)$.

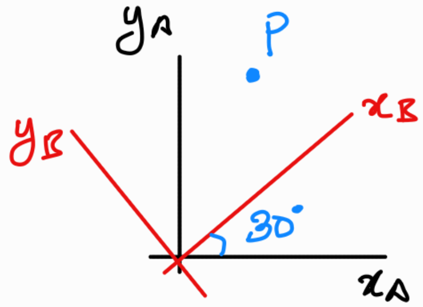
So, along with the properties of groups, an element R in $SO(n)$ has the following properties:

0.5×4

1. Orthogonality : $R^T = R^{-1}$
2. $\det(R) = +1$
3. $\|R_i\| = 1$ ; i = $i^{th}$ Column
4. All Columns are orthogonal to each other.

(b) A frame {B} is located initially coincident with a frame {A}. We rotate {B} about the $\hat{Z}_A$ by 30°. Give the rotation matrix that will change the description of vectors from ${}^B\mathbf{P}$ to ${}^A\mathbf{P}$. (1)

$$R = \begin{bmatrix} C\theta & -S\theta \\ S\theta & C\theta \end{bmatrix}$$




$$R = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$$

Now, Rotating a frame by $\theta$ about an axis is the same as rotating the point by $-\theta$ about the same axis.

0.5

${}^{A}_{B}R = Rot(Z_B, -30°)$

So,

$${}^{A}P = R^{T}\ {}^{B}P$$

$${}^{A}P = \underbrace{\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}}_{0.5} {}^{B}P$$

3. (a) List the different types of joints and their respective DoF. **(2)**

| Joint type | dof $f$ | Constraints $c$ between two planar rigid bodies | Constraints $c$ between two spatial rigid bodies |
|---|---|---|---|
| Revolute (R) | 1 | 2 | 5 |
| Prismatic (P) | 1 | 2 | 5 |
| Helical (H) | 1 | N/A | 5 |
| Cylindrical (C) | 2 | N/A | 4 |
| Universal (U) | 2 | N/A | 4 |
| Spherical (S) | 3 | N/A | 3 |

(b) What are the differences between planar and spatial mechanisms? **(1)**

In planar mechanism all the points have their trajectories confined to a single plane,

0.5x2

where as in spacial mechanisms the points on the mechanism can move in 3D space.

Or,

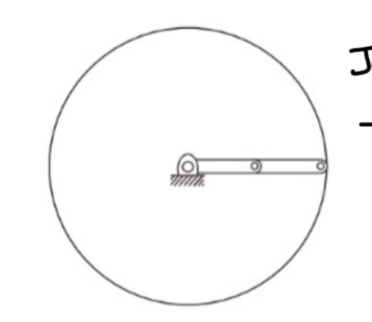
Revolute Joint

In Planar $\longrightarrow$ Joint axes of hinges are perpendicular to one plane, and axes of prismatic joints are parallel to the plane.

In Spatial $\longrightarrow$ All axes are skew to each other.

4. Give a mathematical description of the topologies of the Joint and Task spaces of the following systems. Use Cartesian product product of spaces $\mathbb{R}^k$, $\mathbb{S}^m$, $\mathbb{T}^n$ appropriately

(a) Minimum of four different types of manipulators discussed during the class. **(2)**



Joint Space : $S^1 \times S^1 = T^2$

Task Space : $R^2$

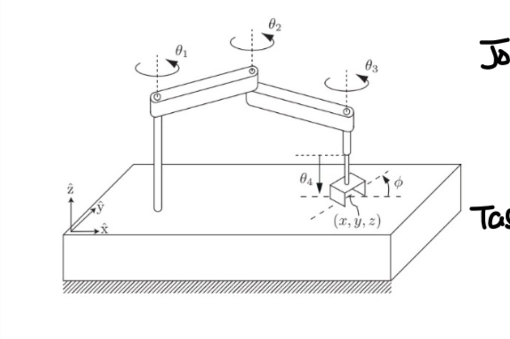
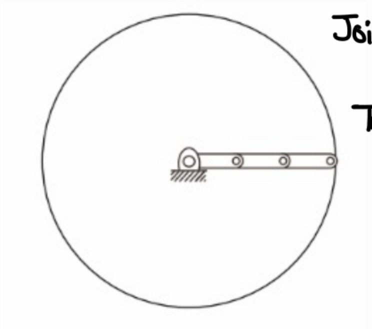


**Figure 2.13:** SCARA robot.

Joint Space : $S^1 \times S^1 \times S^1 \times R^1$
$R^1 \times T^3$

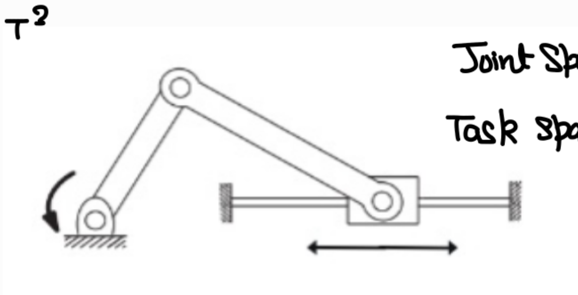
Task Space : $R^3 \times S^1$

(0.5 x4)



Joint Space : $S^1 \times S^1 \times S^1 = T^3$

Task Space : $R^2$



Joint Space : $S^1 \times S^2 \times S^3 \times R^1$

Task Space : $R^1$

(b) The car-like mobile robot (chassis only) on an infinite plane with an RRPR robot arm mounted on it. **(1)**

$$\text{Joint Space} = \overbrace{R^2 \times S^1}^{\text{Chassis}} \times \overbrace{S^1 \times S^1 \times R^1 \times S^1}^{\text{Arm}}$$

$$= R^3 \times T^4$$

0 if not considered the chassis.

$$\text{Task Space} = R^3 \times T^3$$

0.5 for foregetting its Orientation.

↳ Assuming no joint limits the Task space is all of 3D space.

5. The dual-arm robot is rigidly grasping a box as shown in Figure. The box can only slide on the table; the bottom face of the box must always be in contact with the table.

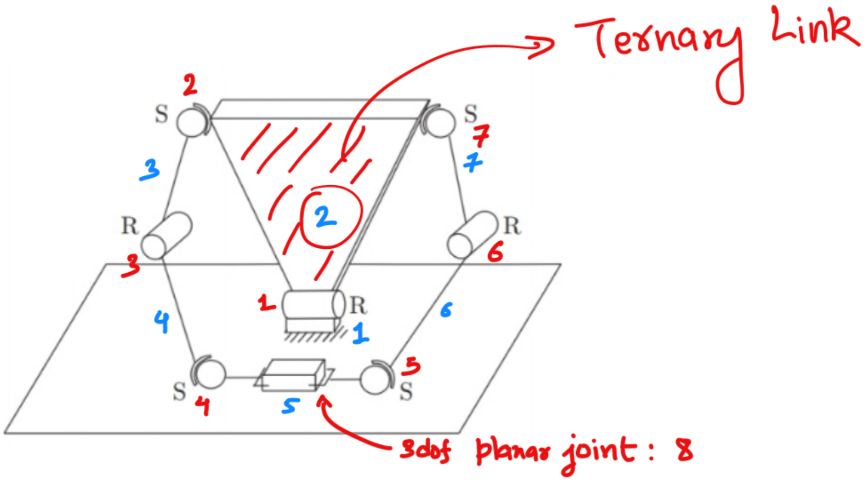



Figure 2: Dual-arm robot

(a) Find the No. of Links and Joints (1)

(b) How many degrees of freedom does this system have? (2)

(a) No. of Links = 7 Links 0.5

No. of Joints = 3R + 4S + 1 Planar = 8 Joints 0.5

(b)

$$\text{degree of freedom} = m(N - J - 1) + \sum_{i=1}^{J} f_i$$

$$= 6(7 - 8 - 1) + (3\times1 + 4\times3 + 1\times3)$$

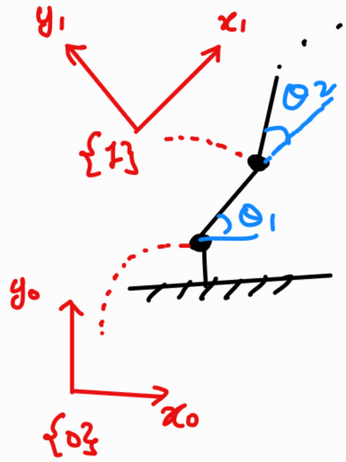
$$= -12 + 18$$

$$\boxed{\text{dof} = 6}$$ 2

6. (a) Assume $n$ number of revolute joints are connected to form a planar serial manipulator. Write a rotation matrix in a compact way that describes the orientation of the end-effector with respect to the fixed frame.

(1)

Rotation matrix from frame {0} to {1} is given as.




$$R(\theta_1) = \begin{bmatrix} c\theta_1 & -s\theta_1 \\ s\theta_1 & c\theta_1 \end{bmatrix}$$

1m with derivation

0.5 for direct answer

Similarly, $R(\theta_2) = \begin{bmatrix} c\theta_2 & -s\theta_2 \\ s\theta_2 & c\theta_2 \end{bmatrix}$

Now, orientation of the end effector if it was placed after $2^{nd}$ joint would be.

$$R' = R(\theta_2)\, R(\theta_1) = \begin{bmatrix} c\theta_2 & -s\theta_2 \\ s\theta_2 & c\theta_2 \end{bmatrix} \begin{bmatrix} c\theta_1 & -s\theta_1 \\ s\theta_1 & c\theta_1 \end{bmatrix}$$

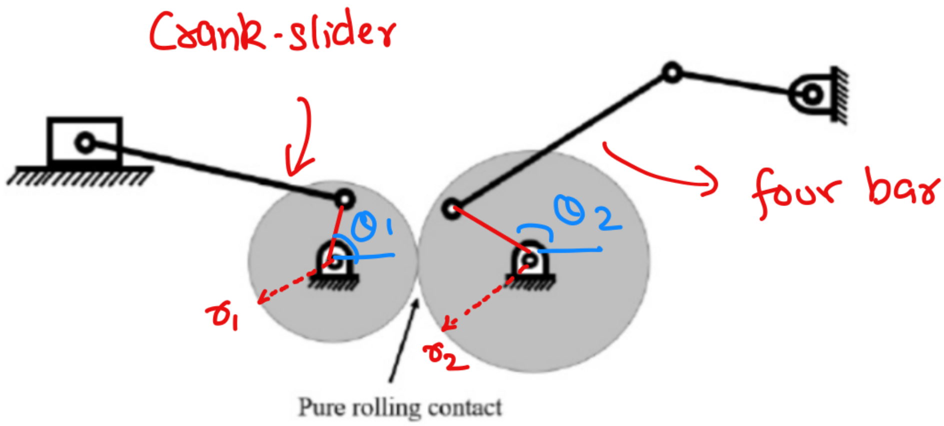
$$= \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) \end{bmatrix}$$

Similarly solving from $\theta_1$ to $\theta_n$

$$R_{endeff} = \begin{bmatrix} c(\Sigma\theta_i) & -s(\Sigma\theta_i) \\ s(\Sigma\theta_i) & c(\Sigma\theta_i) \end{bmatrix}$$

☆ Angles are measured from previous link after $1^{st}$ joint

(b) Find DoF (2)




Considering left and right halves separately

It can be easily found from grublers that dof for both crank-slider and four bar are 1. 0.5m

So, dof without pure rolling contact = 2

0.5m

Now, the rolling contact imposes one constraint

$$\left( \frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} \right)$$

So, net dof of the combined system is:

$$\boxed{dof^{new} = 2 - 1 = 1}$$