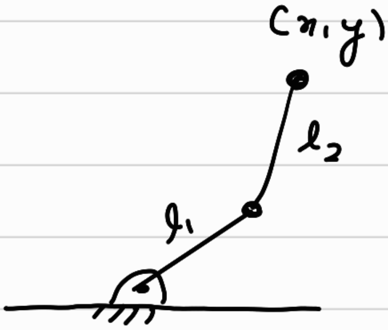
Recap :-

Inverse kinematics :- trigonometric, numerical

TRIGONOMETRIC METHOD



$$x = l_1 \cos\theta_1 + l_2 \cos\theta_{12}$$
$$y = l_1 \sin\theta_1 + l_2 \sin\theta_{12}$$

$$\cos\theta_1 = \frac{x - l_2\cos\theta_{12}}{l_1}$$

$$\sin\theta_1 = \frac{y - l_2 \sin\theta_{12}}{l_1}$$

$$\sin^2\theta_1 + \cos^2\theta_{12} = 1$$

$$\underbrace{a\cos\theta_{12}}_{^1(\theta_{12})} + \underbrace{b\sin\theta_{12}}_{^2(\theta_{12})} + c = 0$$

Method 2

$$x^2 + y^2 = l_1^2 + l_2^2 + 2l_1 l_2 \cos\theta_2$$

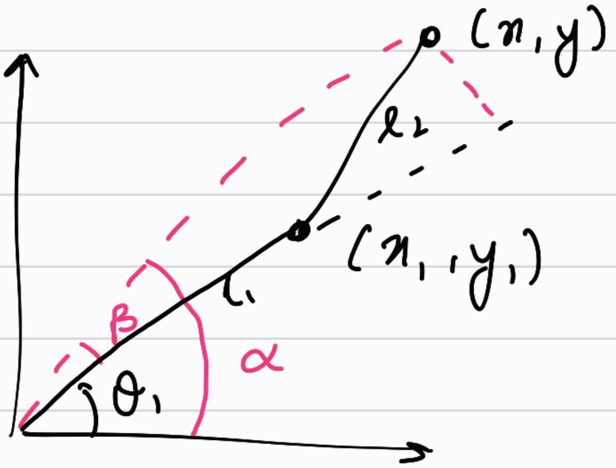
$$\theta_2 = \pm \arccos\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2}\right)$$

$$-1 \leq \frac{x^2+y^2 - l_1^2 - l_2^2}{2l_1 l_2} \leq 1$$

$$(l_1 - l_2)^2 \leq (x^2+y^2) \leq (l_1+l_2)^2$$

inner boundary — outer boundary

• as long as (x, y) is within outer & inner boundary → we will find real solutions



$$\alpha = \text{arctan2}(y, x)$$

$$\beta = \text{arctan2}(l_2 \sin\theta_2, \; l_1 + l_2\cos\theta_2)$$

$$\theta_1 = \alpha - \beta$$

Method 3

ALGEBRAIC METHOD

• if complex system – use this!

$$a c\theta_{12} + b\sin\theta_{12} + c = 0$$

using half - tangent formulas : $c\theta = \dfrac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}$

$$s\theta = \frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}$$

$$\alpha = \tan(\theta/2)$$

$$\therefore \quad a\left(\frac{1-\alpha^2}{1+\alpha^2}\right) + b\left(\frac{2\alpha}{1+\alpha^2}\right) + c = 0$$

$$a - a\alpha^2 + 2b\alpha + c + c\alpha^2 = 0$$

$$\boxed{(c-a)\alpha^2 + 2b\alpha + (a+c) = 0}$$

$$A\alpha^2 + B\alpha + C = 0$$

- how to solve this equation for 100^{th} order polynomial? → Polynomial space.

$$\Longrightarrow \alpha_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

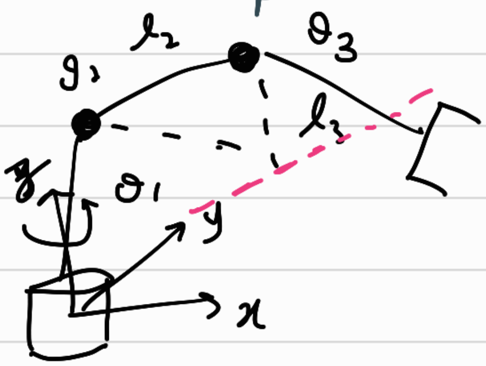
→ elbow up, elbow down

if $\Delta > 0$ → two distinct solutions (real)

$\Delta = 0$ → one solution (repeated)

$\Delta < 0$ → complex solution (inachievable sol^n)

3R manipulator



- this configuration is called orthoparallel configuration.

$$x = (l_2 c\theta_2 + l_3 c\theta_{23}) \cos\theta_1 \qquad \theta_{23} = \theta_2 + \theta_3$$

$$y = (l_2 \cos\theta_2 + l_3 \cos\theta_{23}) \sin\theta_1$$

$$z = l_1 + l_2 \sin\theta_2 + l_3 \sin\theta_{23}$$

To find θ_1, θ_2, θ_3:

$$x^2 + y^2 = (l_2 c\theta_2 + l_3 c\theta_{23})^2$$

let $r^2 = (l_2 \cos\theta_2 + l_3 \cos\theta_{23})^2$

$\Rightarrow \cos\theta_1 = \frac{x}{r}$; $\sin\theta_1 = \frac{y}{r}$

$\theta_1 = \arctan 2\left(\frac{y}{r}, \frac{x}{r}\right) \Rightarrow$ 2 solutions. (+ and −)

now, $z - l_1 = l_2 \sin\theta_2 + l_2 \sin\theta_{23}$

<u>NOTE</u>: kinematically decoupled − of position and rotation are completely disjoint.

$$\underbrace{{}^0_6[R]}_{\text{given}} = \underbrace{{}^0_3[R]}_{\substack{\text{we know this} \\ \text{because it only} \\ \text{depends on } \theta_1, \theta_2, \theta_3}} \; {}^3_6[R]$$

$${}^3_6[R] = \left({}^0_3[R]\right)^{-1} \, {}^6_0[R]$$

Using inverse transform equations,

$\theta_4 = \arctan 2\left(\frac{r_{23}}{S\theta_2}, \frac{r_{13}}{S\theta_2}\right)$

θ_5 and θ_6

→ NO OF SOLUTIONS = $\underline{\underline{8}}$

- When choosing solutions out of the 8 possible ones:
  - collision avoidance
  - low effort

• $x(t) = t x_f + (1-t) x_0$; $0 \leq t \leq 1$

$x \rightarrow$ position ; $x_0 \rightarrow$ initial ; $x_f \rightarrow$ final.

VELOCITY KINEMATICS

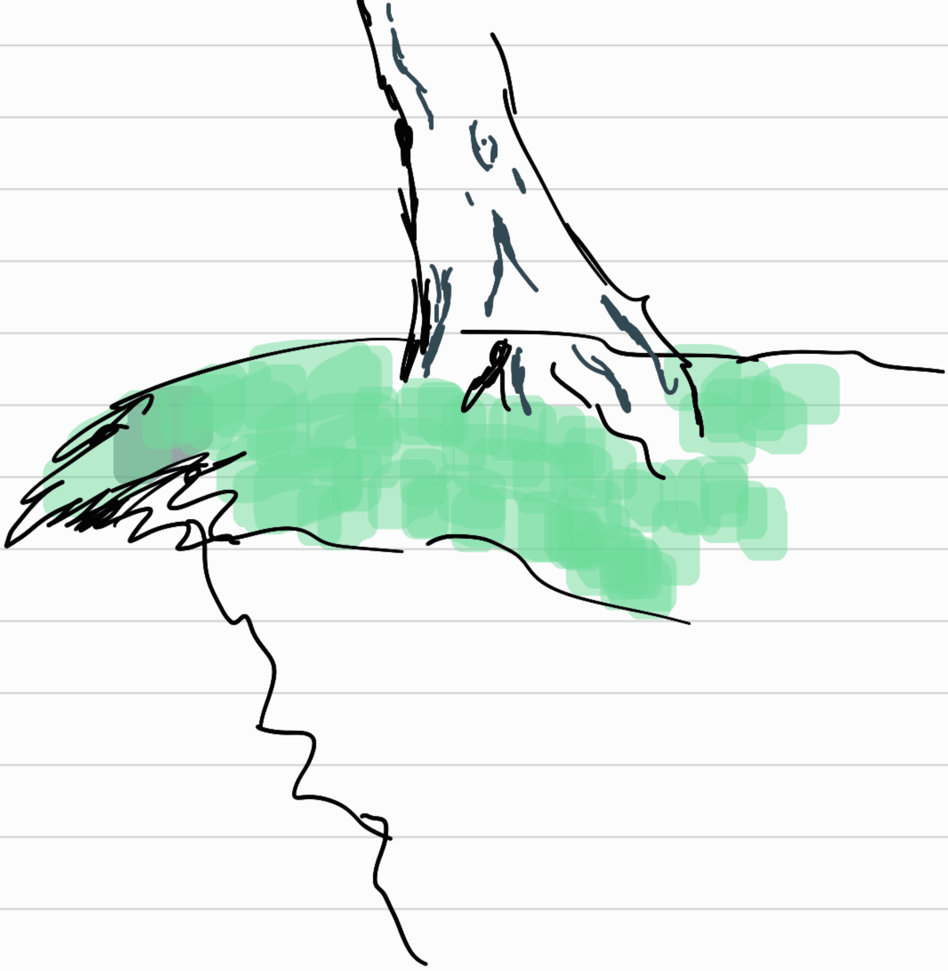
• determines how fast the pos^n kinematic is processed.
• linear velocity & angular velocity.

$$V_{0i} = \frac{d}{dt}\left({}^{0}O_i\right)$$

$$= \lim_{\Delta t \to 0} \frac{{}^{0}O_i(t + \Delta t) - {}^{0}O_i(t)}{\Delta t}$$

$\dot{i} \rightarrow$ means we would be changing direction.

eg: $r = 4\hat{i} + 2\hat{j}$
$\left(\dot{r} = 4\dot{\hat{i}} + 2\dot{\hat{j}}\right)$ magnitude is not changing but direction is changing.



$$\bullet \quad \dot{\hat{i}} = \dot{\theta}\,\hat{j} \qquad\qquad \dot{\hat{j}} = -\dot{\theta}\,\hat{i}$$

$$\dot{\theta} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

now, $A = a_1\hat{i} + a_2\hat{j}$

$V_A = -a_2\dot{\theta}\,\hat{i} + a_1\dot{\theta}\,\hat{j}$

now, $\omega = k\dot{\theta}$

$$\boxed{V_A = \omega \times a}$$

(i) prismatic joint :

$V = \dot{q}\,\hat{k}$

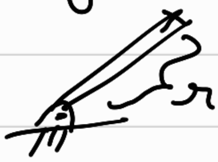
$\omega = 0$

ii) revolute joint

$v = \dot{q}\hat{k} \times r$

$\omega =$

$r \rightarrow$ length of joint



Serial manipulator