The Universe Across Scales
IIIT-H, Semester: Winter 24, Module 2, Assignment 1

Submission deadline: March 31, 2024

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1. With respect to an observer at earth, the co-moving coordinates of a galaxy is $(r, 0, 0)$. Two consecutive waves of a light ray emitted from the galaxy at t_1 and $t_1 + dt_1$ are observed at earth at t_0 and $t_0 + dt_0$.

   (a) Show that $dt_1/a(t_1) = dt_0/a(t_0)$, where $a(t)$ is the scale factor.

   (b) Show that the shift in wavelength observed by the observer is $z = (a(t_0)/a(t_1)) - 1$.

2. The expansion of the universe at the present time t_0 is given by the Hubble parameter $H(t_0)$. To measure measure the expansion at some earlier time t_1, one needs to measure the rate of change of red-shift over some time. Using the result from 1(a) & 1(b), show that

$$H(t_1) = H(t_0)(1 + z) - \frac{dz}{dt_0}$$

3. The ionization fraction X during the recombination is given as (see lecture notes uploaded)

$$X = \frac{-1 + \sqrt{1 + 4S}}{2S}. \tag{1}$$

   where the function S is

$$S = 3.84\eta \left(\frac{k_B T}{m_e c^2}\right)^{3/2} \exp\left[\frac{Q}{k_B T}\right]. \tag{2}$$

   where the symbols have their usual meanings (see lecture).

   (a) Plot X as a function of temperature. (Find the values of the parameters yourself).

   (b) Find the temperature when $X = 0.8$.