The Universe Across Scales

SC1.308 - Spring 2023-2024

Assignment 2: Quantum Mechanics

Submission Deadline: 6 February, 2024

1. At time t = 0 a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} Ax/a, & \text{if } 0 \le x \le a \\ A(b-x)/(b-a), & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

where A, a, and b are constants.

   (a) Normalize Ψ (that is, find A in terms of a and b).

   (b) Sketch $\Psi(x,0)$ as a function of x.

   (c) Where is the particle most likely to be found, at $t = 0$?

   (d) What is the probability of finding the particle to the left of a? Check your result in the limiting cases $b = a$ and $b = 2a$.

2. Show that when you add a constant V_0 to the potential energy (constant means independent of x and t) the wave function picks up a time-dependant phase factor: exp $(-iV_0t/\hbar)$.

3. Show that for a 1D infinite potential well

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 \, dx = 0$$

for any two solutions to the Schrodinger equation.

4. At time t = 0 a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} A(a^2 - x^2), & \text{if } -a \le x \le a \\ 0, & \text{otherwise} \end{cases}$$

where A, a, are constants.

   (a) Normalize Ψ .

   (b) What is the expectation value of x?

   (c) What is the expectation value of p?

   (d) What is the expectation value of x^2?

   (e) What is the expectation value of p^2?

   (f) Find the uncertainty in x.

   (g) Find the uncertainty in p.

   (h) Check that your result is consistent with the uncertainty principle.

5. Show that there is no acceptable solution to the time-independent Schrodinger equation for the infinite square well with E=0.

6. A particle of mass m in the infinite square well (of width a) starts out in the state

$$\Psi(x,0) = \begin{cases} A, & \text{if } 0 \le x \le a/2 \\ 0, & \text{if } a/2 \le x \le a \end{cases}$$

for some constant A, so it is (at t=0) equally likely to be found at any point in the left half of the well. What is the probability that a measurement of the energy (at some later point t) would yield the value $\pi^2\hbar^2/2ma^2$?

7. Show that the wave function of a particle in the infinite square well returns to its original form after time $T = 4ma^2/\pi\hbar$. That is $\Psi(x,T) = \Psi(x,0)$ for any state (not just a stationary state).

8. Explain why you can copy a single quantum state but cannot copy a linear combination of states.

9. For a spin-half system in a Stern-Gerlach experiment with an angle θ between the two magnets, what angle makes spin-up with respect to the second magnet twice as likely as spin-down?

10. A photon-entanglement experiment takes a millisecond to make a measurement of each pair. How far apart would you need to put the stations to make sure there isn't some causal influence between Alice's measurement and Bob's?

11. Estimate the uncertainty in the position of

   (a) a neutron moving at 10^6 ms^{-1}.

   (b) a 50 kg person moving at 2 ms^{-1}.

12. Explain why an electron cannot reside inside the nucleus.