1.5 Olber's Paradox:

Until the early 20^{th} century it was believed that the universe is infinitely large, eternally old, and static. These assumptions lead to a paradox. The night sky is dark with thousands of stars and galaxies randomly scattered. If the universe is infinite, the stars and galaxies are also randomly distributed to infinity. Furthermore, the universe is eternally old and so would be the stars and the galaxies. In a clear night sky if our line of sight is extended to infinity, it would end in a star or a galaxy. So there will be no dark patch of sky to be seen. In fact, if we assume that all stars are as bright as the sun, calculations show that the sky should be as bright as the surface of the sun. But this is not what we observe. This paradoxical observation is called Olber's paradox.

One or more of the assumptions must be wrong. Firstly, I assumed that our line of sight can be infinitely extended before it ends at a star – this may not be true. Non-luminous objects may block the line of sight. The assumption of an eternally old universe may be untrue. In a universe that had a beginning, the stars have existed for a finite time. As the speed of light is finite, the light from far away stars have not reached us since they started to sign. Finally, the assumption of infinitely large universe may be untrue. For a universe that have a finite size, only a fraction of the sky is covered by stars leaving a vast area that looks dark. Olber's paradox can be resolved in many ways. As it stands, the correct explanation is that the universe has finite age, *i.e.,* it had a beginning.

1.6 Isotropy and homogeneity

The study of the universe rests on the *cosmological principle* which states that at sufficiently large scale the universe is homogeneous and isotropic. Homogeneous means that there is no unique location in the universe – the universe looks the same from any location. Isotropy means all directions from a point in the universe are identical. There is no proof of the principle. But there are experimental observations, for example the cosmic microwave spectrum spectrum, that supports it. Furthermore, N-body simulation of the universe based on ΛCDM model (we will learn this) gives rise to a isotropic and homogeneous universe.

1.7 Hubble's Law

In 1929 Edwin Hubble observed that light coming from distant galaxies are redshifted and the redshift is proportional to the distance to the galaxy. Red shift is define as

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}\,, \tag{14}$$

where λ_{em} is the wavelength of emitted light and λ_{obs} the wavelength observed at earth. If r is the distance to a galaxy, then according to Hubble's observation

$$z \propto r\,.$$

Red-shift occurs due to the Doppler effect, a change in the wavelength due to relative velocity between source and observer. The relative change in the wavelength is proportional to the relative velocity

$$z \propto v\,.$$

The preceding equation imply

$$v \propto r\,, \quad \text{or} \quad \boxed{v = H_0 r} \tag{15}$$

where the constant of proportionality H_0 is called the Hubble constant. The equation states that galaxies are receding from us with velocities that is proportional to the distance from the earth. This is called the Hubble's law. In figure 5 receding velocities are plotted against the distance.

Hubbles' law combined with the cosmological principle implies that the universe is expanding. To understand how it works, consider any three points $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ in the universe forming a triangle. The relative distances between the points are $r_{12} = |\vec{r}_1 - \vec{r}_2|$, $r_{23} = |\vec{r}_2 - \vec{r}_3|$, $r_{31} = |\vec{r}_3 - \vec{r}_1|$. According to the cosmological principle, the points recede away from each other in a homogenous fashion such that the shape of the triangle remain the same. So the relative distances at time t is related to those at t_0 as

$$r_{12}(t) = a(t)r_{12}(t_0)\,, \quad r_{23}(t) = a(t)r_{23}(t_0)\,, \quad r_{31}(t) = a(t)r_{31}(t_0)\,.$$

So the speed of receding are

$$v_{12}(t) = \frac{\dot{a}}{a}r_{12}(t)\,, \quad r_{23}(t) = \frac{\dot{a}}{a}r_{23}(t)\,, \quad r_{31}(t) = \frac{\dot{a}}{a}r_{31}(t)\,. \tag{16}$$

So, any two points in the universe is receding away from each other with speed that is proportional to their distance of separation and the proportionality constant is

$$H_0 = \frac{\dot{a}}{a}\,.$$

This was the first direct evidence of the expansion of the universe.

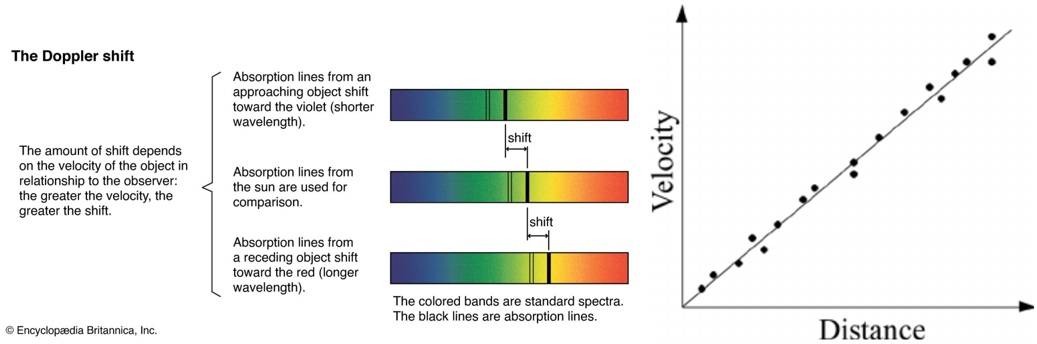

Figure 5: Hubble's observation: galaxies receding at velocities proportional to distance.

The Hubble constant H_0 has a dimension of t^{-1}. Hubble's own measurement of this constant was about seven times larger than present measurements which are more accurate. At present H_0 has been measured by two methods – using CMB data and using Supernova (explosion of a star) data. Though they yield slightly different values we will use

$$H_0 = (72 \pm 7)\ km\ s^{-1}Mpc^{-1}\,. \tag{17}$$

In literature a dimensionless Hubble constant h is more often used

$$H_0 = 100h\ km\ s^{-1}Mpc^{-1} \tag{18}$$

where $h = 0.72 \pm 0.007$.

We will soon see that the Hubble constant is not really a constant in time – throughout the different stages of the universe it had different values. The value quoted above is the one at the present epoch – and thats why I have put 0 in the suffix.

As H_0 has the dimension of inverse time, H_0^{-1} is approximately the age of the universe. If the universe is expanding any two galaxies were closer in the past. Let's assume that there is no force in the universe to accelerate of decelerate the relative motion of galaxies and they recede at constant speed. So, two galaxies have taken a time

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1}\,,$$

to be separated by distance r. This is approximately the age of the universe. With $H_0 = 72km/s/Mpc$ one gets $H_0^{-1} \approx 14$ billion years as the universe's age.