UNIVERSE ACROSS SCALES - Assignment 1

(1) A lightyear is the distance that light travels in a year. $= 9.46 \times 10^{12}$ km

(i) No. of meters in a light year $= 9.46 \times 10^{15}$ km

1 light year = 365 light days

$\Rightarrow$ No. of meters in a light day $= \dfrac{9.46 \times 10^{15}}{365}$

$= 0.0259 \times 10^{15}$

$= \underline{2.59 \times 10^{13}\ m}$

(ii) No. of km in a light year $= 9.46 \times 10^{12}$ km

No. of light microseconds in a light year =

$365 \times 24 \times 60 \times 60 \times 10^{6}$

$= 3.15 \times 10^{13}$

$\Rightarrow$ No. of km in a light microsecond $= \dfrac{9.46 \times 10^{12}}{3.15 \times 10^{13}}$

$= 3 \times 10^{-1}$

$= \underline{0.3003}$ km

(iii) 1 light year $= 9.46 \times 10^{15}$ m

1 light passec $= 3.09 \times 10^{16}$ m

$\Rightarrow$ No. of light years in a light passec

$$= \frac{3.09 \times 10^{16}}{9.46 \times 10^{15}}$$

$$= \underline{\underline{3.266}} \text{ years}$$

(iv) No. of m in a light second $= 2.99 \times 10^{8}$ m

1 light fermi $= 10^{-15}$ m

$\Rightarrow$ No. of light seconds in light fermi $= \dfrac{10^{-15}}{2.99 \times 10^{8}}$

$$= 0.334 \times 10^{-23}$$

$$= \underline{\underline{3.34 \times 10^{-24}}} \text{ s}$$

2 $(x, y, z, t) = (100\text{ km}, 10\text{ km}, 55\text{ km}, 2\text{ ms})$

(a) $v = 0.95c$

Using Lorentz transformation,

$$x' = \frac{1}{\sqrt{1 - v^2/c^2}}(x - vt)$$

$$= \frac{1}{\sqrt{1-(0.95)^2}}\left(10^5 - (0.95c)(2\times10^{-3})\right)$$

$$= \frac{1}{0.312}\left(10^5 - 5.7\times10^5\right)$$

$$= -15.06\times10^5$$

$$= -1.506\times10^4 \text{ m}$$

$$y' = y = 10\text{ km} = 10^4 \text{ m}$$

$$z' = z = 55\text{ km} = 55\times10^3 \text{ m}$$

$$t' = \frac{1}{\sqrt{1 - v^2/c^2}}\left(t - \frac{xv}{c^2}\right)$$

$$= \frac{1}{0.312}\left(2\times10^{-3} - \frac{(100\times10^3)(0.95c)}{c^2}\right)$$

$$= \frac{1}{0.312}\left(0.002 - 0.316\times10^{-3}\right)$$

$$= 5.39\times10^{-3} \text{ s}$$

Thus, the coordinates in the O' frame are :-

$(-1.506 \times 10^4 \text{ m}, \quad 10^4 \text{ m}, \quad 55 \times 10^3 \text{ m}, \quad 5.39 \times 10^{-3} \text{ s})$

(b) Using the inverse Lorentz transforms,

$$x = \gamma(x' + vt')$$

$$= \frac{1}{0.312}\left(-1.506 \times 10^4 + (0.95c)(5.39 \times 10^{-3})\right)$$

$$= \frac{1}{0.312}\left(-1.506 \times 10^4 + 15.38 \times 10^5\right)$$

$$= \frac{1}{0.312}\left(0.32 \times 10^5\right)$$

$$= \underline{\underline{1.02 \times 10^5 \text{ m}}} \qquad (\approx 100 \text{ km})$$

$$y = y' = 10 \text{ km} = \underline{\underline{10^5 \text{ m}}}$$

$$z = z' = 55 \text{ km} = \underline{\underline{55 \times 10^3 \text{ m}}}$$

$$t = \gamma\left(t' + \frac{x'v}{c^2}\right)$$

$$= \frac{1}{0.312}\left(5.39 \times 10^{-3} + \frac{-(1.506 \times 10^4)(0.95c)}{9 \times 10^{16}}\right)$$

$$= \frac{1}{0.312}\left(5.39 \times 10^{-3} - 0.047 \times 10^{-3}\right)$$

7:00

$$= 18.8 \times 10^{-3}$$

$$\approx \underline{\underline{2\,ms}}$$

8:00

Hence, the inverse Lorentz transformation gives roughly

9:00

the same answer as well.

10:00

V:..

A:..

3 $T_{\text{half-life}} = 1.56\ \mu s$

Speed of muons, $v = 0.98c$

distance of atmosphere to surface $= 10\text{km}.$

(a) $v = \dfrac{d}{t}$

$\therefore t = \dfrac{d}{v}$

$= \dfrac{10 \times 10^{3}}{0.98 \times 3 \times 10^{8}}$

$= \underline{\underline{3.4 \times 10^{-5}}}\ s$

$\therefore$ No. of half-lives $= \dfrac{t}{T_{\text{half}}} = \dfrac{3.4 \times 10^{-5}}{1.56 \times 10^{-6}}$

7:00

$$\approx 2.18 \times 10$$

$$\approx \underline{\underline{22}}$$

8:00

(b) we know, the number of half lives $= 22$

$N_0 = 10^6$

10:00

$$\Rightarrow \frac{N}{N_0} = 2^{-t}$$

11:00

$$N = 2^{-22} \times 10^6$$

12:00

$$= \underline{\underline{0.238}}$$

(c) Lorentz factor, $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

14:00

$$= \frac{1}{\sqrt{1 - (0.98)^2}}$$

15:00

$$= \frac{1}{\sqrt{1 - 0.9604}}$$

16:00

$$= \frac{1}{\sqrt{0.0396}}$$

17:00

$$= \frac{1}{0.198}$$

18:00

$$= \underline{\underline{5.05}}$$

19:00

(d) Half life of a muon as measured on Earth,

20:00 $t' = \gamma t$

$= 5.05 \times 1.56 \times 10^{-6}$

$= \underline{\underline{7.87 \times 10^{-6}\ s}}$

9(e) Numbers of half-lives, $t = \dfrac{3.4 \times 10^{-5}}{7.87 \times 10^{-6}}$

$= \underline{\underline{4.32}}$

(calculated with new half-life)

Thus, numbers of muons reaching the surface of earth : $\dfrac{N}{N_0} = 2^{-t}$

$\Rightarrow N = 10^{6} \times 2^{-(4.32)}$

$= 10^{6} \times \dfrac{1}{19.9}$

$\approx \underline{\underline{50{,}066}}$

(f) Due to time dilation and length contraction, the muon experiences time more slowly (because it's speed is close to the speed of light). Thus, an observer in the muon's reference frame will experience time moving more slowly and distances becoming shorter.
This is what enables muons to reach the earth's surface, despite their short half-life.

7(PP)

(a) According to classical physics,

$KE = eV$

where, $KE \rightarrow$ kinetic energy

$e \rightarrow$ charge of an e^- $[1.602 \times 10^{-19}\ C]$

$V \rightarrow$ potential difference

we also know $KE = \frac{1}{2} m_e c^2$

$\Rightarrow \frac{1}{2} m_e c^2 = eV$

$\Rightarrow V = \frac{1}{2} \frac{m_e c^2}{e}$

$= \frac{1}{2} \times \frac{9.10 \times 10^{-31} \times 9 \times 10^{16}}{1.602 \times 10^{-19}}$

$= 25.56 \times 10^4$

$= \underline{2.556 \times 10^5\ V}$

(H)

(b) To find the actual velocity that the e^- attains we consider relativistic effects,

$$KE = (\gamma - 1)\, mc^2$$

we also know that $KE = eV$

$$\therefore \; eV = (\gamma - 1)\, mc^2$$

$$\frac{eV}{mc^2} = \gamma - 1$$

$$\gamma = 1 + \frac{eV}{mc^2}$$

Energy required to accelerate a particle to the speed of light $= \frac{1}{2}mc^2$

$$\Longrightarrow \gamma = 1 + \frac{\frac{1}{2}\cancel{mc^2}}{\cancel{mc^2}}$$

Squaring,

$$\gamma^2 = \left(1 + \frac{1}{2}\right)^2$$

$$\gamma^2 = \frac{9}{4}$$

$$\frac{1}{1 - v^2/c^2} = \frac{9}{4}$$

$$1 - \frac{v^2}{c^2} = \frac{4}{9}$$

$$\frac{v^2}{c^2} = 1 - \frac{4}{9}$$

$$= \frac{5}{9}$$

$$v = \sqrt{\frac{5c^2}{9}}$$

$$= \underline{\underline{0.74c}} \text{ m/s}$$

Thus, the actual velocity obtained by the e^- is $\underline{\underline{0.74}}$ c m/s

(5) $v = 0.95c$

$l_0 = 3\,m$

8:00 Length contraction : $l' = l_0 \sqrt{1 - v^2/c^2}$

9:00 $= 3\sqrt{1-(0.95)^2}$

10:00 $= 3\,(0.312)$

$= \underline{\underline{0.936\ m}}$

11:00

Thus, the vaccum tube appears to be 0.936m for the electron.

(6) $v = 0.6c$

The Lorenty factor, $\gamma = \left(\sqrt{1 - v^2/c^2}\right)^{-1}$

$= \left(\sqrt{1-(0.6)^2}\right)^{-1}$

15:00

$= \left(\sqrt{0.64}\right)^{-1}$

16:00

$= (0.8)^{-1} = \underline{\underline{1.25}}$

17:00

(6)

16:00 (b) In rest frame, time read by clock

$$t = \frac{240\,m}{0.6c}$$

17:00

18:00 Using Lorentz transformation, $t' = \gamma\left(t - \frac{xv}{c^2}\right)$

19:00 $t' = \left(\frac{240}{0.6c} - \frac{240(0.6c)}{c^2}\right)1.25$

20:00

7:00

$$= \left(\frac{240\,(1 - 0.36)}{0.6c} \right) 1.25$$

8:00

$$= 106.6 \times 10^{-8}$$

$$= 1.066 \times 10^{-6}\ s$$

9:00

7 Length of the aeroplane $L = 40m$

$v = 630 \, m/s$

By length contraction, $L' = L\sqrt{1 - v^2/c^2}$

$\frac{L'}{L} = \sqrt{1 - v^2/c^2}$

$= \sqrt{1 - (630/3\times10^8)^2}$

$= \sqrt{1 - (210\times 10^{-8})^2}$

$= \sqrt{0.999}$

$= \underline{\underline{0.99}}$

Thus length contraction, $\frac{L - L'}{L} = 1 - \frac{L'}{L}$

$= 1 - 0.99$

$= \underline{\underline{0.01}}$

(B) The time dilation is given by $t' = \frac{t}{\gamma}$

(using Lorenty equations)

we want, $t - t' = 1 \mu s$

$$t - \frac{t}{\gamma} = 1 \times 10^{-6}$$

$$t = \frac{\gamma \times 10^{-6}}{\gamma - 1}$$

$$= \frac{1.01 \times 10^{-6}}{0.01}$$

$$= 101 \times 10^{-6}$$

$$= \underline{\underline{1.01 \times 10^{-4}}} \text{ s}$$

(8) In frame O, $x_1 = 1200m$ and $x_2 = 480m$

$\therefore \Delta x = x_2 - x_1$

$= -1200 + 480$

$= \underline{-720m}$

It is given that in frame O', the flashes occur at the same coordinate.

$\therefore \Delta x' = 0$

(a) Using Lorentz equations,

$\Delta x' = \gamma(\Delta x - vt)$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$0 = \frac{1}{\sqrt{1 - v^2/c^2}}\left(-720 - v(5\times10^{-6})\right)$$

$720 + 5v\times10^{-6} = 0$

$5v\times10^{-6} = -720$

$5v = -720\times10^{6}$

$$v = -\frac{720\times10^{6}}{5}$$

$= -144\times10^{6}$

$= \underline{-1.44\times10^{8}}$ m/s

Thus, O' is moving along the x-axis with a speed

(-1.44×10^{8}) m/s.

(b) The -ve sign implies that O' is moving in the negative x-direction.

(c) To find which flash occurs first, we calculate the time difference $\Delta t'$ b/w the two flashes in frame O'.

If $\Delta t'$ is positive $\Rightarrow$ same sequence as O.

$$\Delta t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

$$= \frac{1}{\sqrt{1-(0.48)^2}}\left(5\times10^{-6} - \frac{(-720)(-1.44\times10^{8})}{9\times10^{16}}\right)$$

$$= \frac{1}{\sqrt{0.7696}}\left(5\times10^{-6} - \frac{(-720)(-1.44\times10^{8})}{9\times10^{16}}\right)$$

$$= \frac{1}{0.877}\left(5\times10^{-6} - 1.152\times10^{-6}\right)$$

$$= \frac{1}{0.877} \times 3.848\times10^{-6}$$

$$= \underline{\underline{4.38\times10^{-6}}}\ \text{s}$$

Since the time difference is positive $\Longrightarrow$ the flashes occur in the same sequence as in frame O.

8:00

(d) As calculated in part (c), the time interval between the flashes in O' is $\underline{\underline{4.38\,\mu s}}$.

10:00

(9) $v' = 0.40c$

$v^{\circ} = ?$

$V = 0.60c$ (velocity of O' wrt O)

Using velocity addition,

$$v = \frac{V + v'}{1 + Vv'/c^2}$$

$$= \frac{0.60c + 0.40c}{1 + \frac{(0.60\cancel{c})(0.40\cancel{c})}{\cancel{c^2}}}$$

$$= \frac{1.00c}{1 + (0.24)}$$

$$= \frac{c}{1.24}$$

$$= \underline{0.806c} \text{ m/s}$$

Thus, the velocity of the particle as observed by O $= \underline{0.806c}$ m/s

(18) $E = 14\,nJ = 14 \times 10^{-9}\,J$

width $= 21\,cm$

(a) To find the length we use the formula for length contraction,

$$L = \frac{L_0}{\gamma}$$

Relativistic energy, $E = \gamma m_p c^2$

$$\gamma = \frac{E}{m_p c^2}$$

$$= \frac{14 \times 10^{-9}}{1.67 \times 10^{-27} \times (3 \times 10^8)^2}$$

$$= 0.931 \times 10^2$$

$$= \underline{93.14}$$

Thus, $L = \frac{21\,cm}{93.14} \times 10^{-2}$

$$= 0.225 \times 10^{-2}$$

$$= \underline{2.25 \times 10^{-3}\,m}$$

(b) We know, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}$$

$\therefore \dfrac{v^2}{c^2} = 1 - \dfrac{1}{\gamma^2}$

$= 1 - \dfrac{1}{(93.14)^2}$

$= 0.9998$

$\Rightarrow v^2 = 0.9998\,(3\times 10^8)^2$

$= 8.99\times 10^{16}$

$\therefore v = 2.99 \times 10^8$

$\approx \underline{0.99c}$

Thus, time taken in our reference frame t,

$\Delta t = \dfrac{\text{distance}}{\text{speed}}$

$= \dfrac{21\,\text{cm} \times 10^{-2}}{0.99c}$

$= \underline{7.07 \times 10^{-10}}$ s

(c) Using the formula for time dilation,

$\Delta t = \gamma \Delta t'$

Where, $\Delta t \rightarrow$ time observed by observer outside frame of reference

$\Delta t' \rightarrow$ time observed by observer in the frame of reference

7:00 $\Longrightarrow$ $\Delta t' = \dfrac{\Delta t}{\gamma}$

8:00 $= \dfrac{7.07 \times 10^{-10}}{93.14}$

9:00 $= 0.075 \times 10^{-10}$

$= 7.59 \times 10^{-12}$ s

10:00

(ii)

(a) Relativistic energy, $E = T + mc^2$

where $T \rightarrow$ kinetic energy

Squaring both sides, $E^2 = (T + mc^2)^2$

$$E^2 = m^2c^4 + T^2 + 2mc^2T \quad -(1)$$

we also know, $E^2 = m^2c^4 + p^2c^2 \quad -(2)$

Comparing (1) and (2) $\Rightarrow \cancel{m^2c^4} + p^2c^2 = \cancel{m^2c^4} + T^2 + 2mc^2T$

$$2c^2Tm = p^2c^2 - T^2$$

7:00

$$m = \frac{p^2c^2 - T^2}{2Tc^2}$$

8:00

$$= \frac{(pc)^2 - T^2}{2Tc^2}$$

Hence, proved

(b) From part (a) $\longrightarrow$ $M = \dfrac{(pc)^2 - T^2}{2Tc^2}$

As $v \to 0$, we apply the Newtonian formulas where $p = mv$ and $KE = \frac{1}{2}mv^2$

20:00

Thus, $v \to 0$ for $\dfrac{(mvc)^2 - (1/2\, mv^2)^2}{2(1/2\, mv^2)c^2}$

Since $v <<< c \Rightarrow$

$$\frac{(mvc)^2}{(mv^2c^2)} = \frac{m^2 \cancel{v^2c^2}}{m \cancel{v^2c^2}}$$

$$= \underline{\underline{m}}$$

Thus, as $v \to 0$, the right-hand side becomes m.

$T = 55$ MeV

$p = 121$ MeV/c

$M_e = 0.510$ MeV/c^2

$$m = \frac{(pc)^2 - T^2}{2Tc^2}$$

$$= \frac{(121)^2 - (55)^2}{2(55)c^2}$$

$$= \underline{\underline{105.6}} \text{ MeV}/c^2$$

Thus, $\dfrac{m}{M_e} = \dfrac{105.6}{0.510}$

$$= \underline{\underline{207}}$$

Thus, the mass of this particle is 207 times the mass of an electron.

SPACETIME DIAGRAM

① If a particle has a slanted worldline, then it has a constant velocity v.

In particular, the slope of the object's worldline is

$$\tan\theta = \text{slope} = \frac{c}{v} \;//$$

Thus, the greater the slope of the line, the smaller the velocity of the particle as compared to c.

② Since, $m = \frac{c}{c} = 1$ the line that make 45° with the x-axis is light world line.

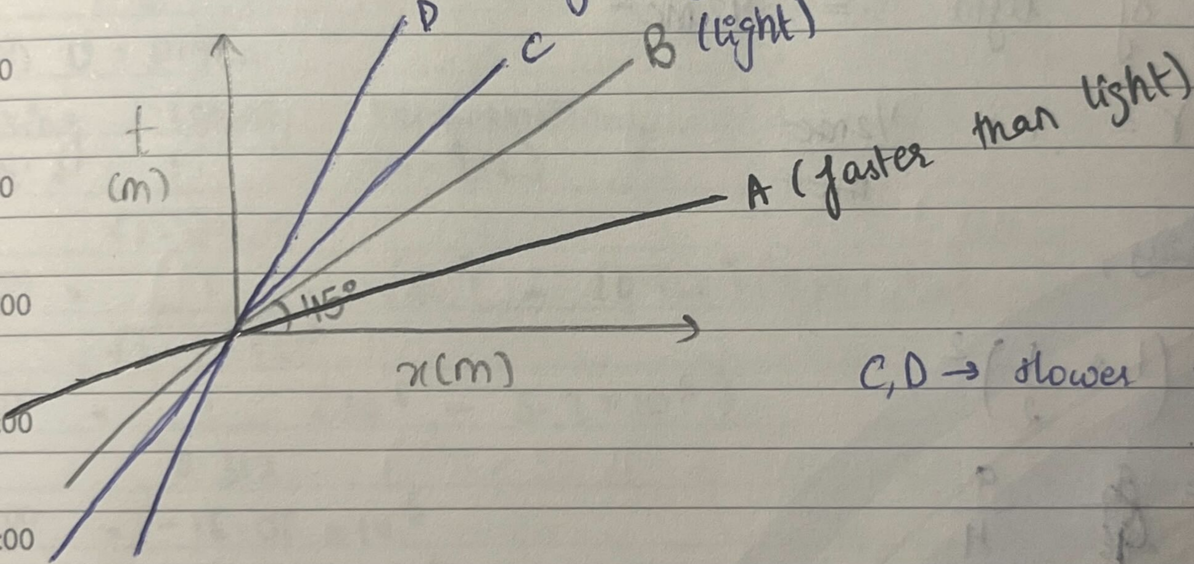
Thus, A → faster than light.

$$\frac{c}{v} << 1$$

C, D → slower than light

$$\frac{c}{v} >> 1$$

B → world line of light.



C, D → slower than light.