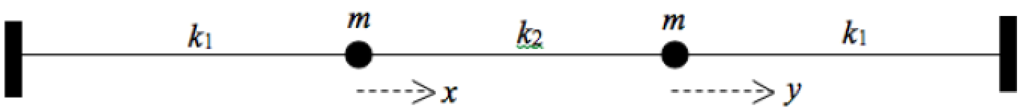
Q1. Find the
SVD of

$$A = \begin{bmatrix} \bullet & \bullet & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

a) Explain SVD in details (explain the steps). If the blue filled circles are the last two digits of your roll number (A_{11}, and A_{12}), find the SVD of this matrix.
b) Explain in which case SVD and standard diagonalization of a matrix will give you the same results (if yes).

Marks 3+2

Q2.



This is a system of two masses and three springs.

a) Find the equation of motion of the systems. Write the Lagrangian equation of motion.

Marks 3

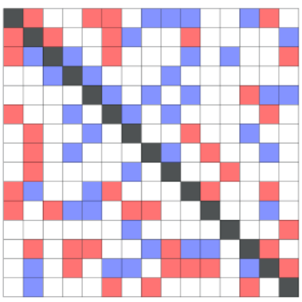
b) *The solution of this system can be mapped through eigenvalue analysis. Is it true? If yes, please explain it. Show that (in the process of deduction) the determinant can help us to find the eigenvalue. *If $k_1 = k_2 = k$, and $m_1 = m_2 = m$, then find all the frequencies.

Marks 3

c) What will happen if the initial velocities of the masses are zero, and the initial displacements are marked by (i) $x = C, y = -C$. (ii) $x = -C, y = -C$. (iii) $x = -C/2, y = -C$. Use any software to visualize the motion of the particles/masses.
C is the (last digit of your roll number + second last digit of your roll number) +1

Marks 3+3

Q3.



With probability C
M_{ij} sampled from $\mathcal{N}(0, \sigma^2)$

With probability $(1 - C)$
$M_{ij} = 0$

Diagonal entries
$M_{ii} = -d$

If the size of the matrix is S, and C is the sparsity, construct the matrix. Plot the eigenvalues (real (λ) vs imag (λ)). (i) What is the shape of the distribution? Can you give proper argument for obtaining such shape? (ii) Take S as 100, 500. (iii) For each S, take d is the last two digits of your roll number. (iv) Explain your observation. (for instance role of d, C, N,and σ).

Marks 1+3+3+3.