29/03/2022

Information & communication

Course Evaluation

2 Quizzes → 10% each.

Mid Sem → 20%

End Sem → 35%

(4-5) Assignments → 25%

Communication system (CS)

Transmitter → Medium → Receiver

Medium → Air/water/wires/optical fibres/space.

Medium is more offenly called as Channel.

Channel

→ A channel always has noise.
Noise is an ~~ofstald~~ obstackle in CS.

Noise

→ Disturbance to CS.

↳ Low Noise (Eg. wired) → more no. of messages can be sent

↳ High Noise (Eg. wireless). → less no. of messages can be sent.

relative to signal power.

Source of a signal → convert it into digital form → Source compression

(stare/ communication)

Sampling Theorem → basis for converting any signal to digital signal.

Source compression → Lossless compression
→ Loss compression.

always adds noise

Channel capacity ← Channel ← Chanel coding

adding retendency/adding bits so that we are sure that we receive the correct information (error correction.)

Modulated symbols + Carrier

+ Noise

(air/sonar/wire).

Demodulation → Remove the carrier, map signals

→ channel Decoding

Source decompression

Receiver

The signal received at the receiver need not be the same as the source, due to noise.

→ More is the noise in the channel, the more retendency we should add.

→ If the noise is less, then the retendency needed/added is less.

1/04/2022

Signals.

Signal → function of independent variables

$t \to$ time.

$x \to$ space.

(2-D) → (t, x).

Eg. of signals → speech, audio, images, videos, communication signals.

* continuous signal vs discrete-time signal.

$x(t)$; $t \in R$ — $x[n]$, $n \in Z$

Eg. waves, electro-magnetic waves, (functions of both time & space). all waves(t, x) — Eg. images.

* Analog signal vs. Digital signal.

Analog signal ↓ continuous time & continuous amplitude signal.

Digital signal ↓ discrete in both time and amplitude signal.

* periodic signal. v.s. Aperiodic signal.

$\exists T < \infty$.

$x(t+T) = x(t) \; \forall t$

Smallest such T will be period. → for continuous signal.

The signal repeats itself after a certain period of time (called period).

$\exists N < \infty$

$x[n+N] = x[n] \; \forall n$

↑ for discrete signal.

Aperiodic signal ↓ A signal does not repeat itself after a certain period of time.

Energy signal
↓
$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty.$$

$$\text{Power} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt.$$

(1-D signals).

More examples of signals + their Acquisitions.

→ speech → Microphones.
(air pressure variations converted to electrical).

→ ECG (Electronic cardiogram) → Potential difference (between different parts of the body).

→ Seismograph. → vibrations sensors.
(~~Richter scale~~)
(Richter scale)

→ Financial data (stocks → Eg. shares)

→ natural signals.

Ideal Signals:

→ Sinusoids. – Alternating current (tone in music).

→ Impulse – sudden high voltages, electric sparks, clock pulses, ~~etc~~ force in elastic collision, movie clap

→ Step Function

In communication systems, carrier signal is a sinusoid. There will be oscillators to generate this carrier signals.

→ switch OFF to ON.

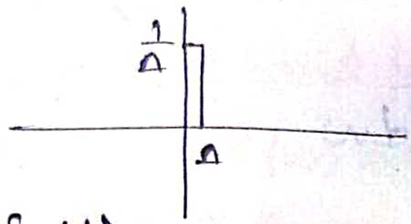
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

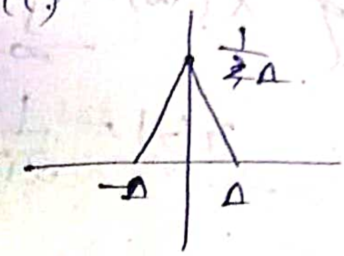
Impulse function → $\delta(t) = 0 \quad , t \neq 0$

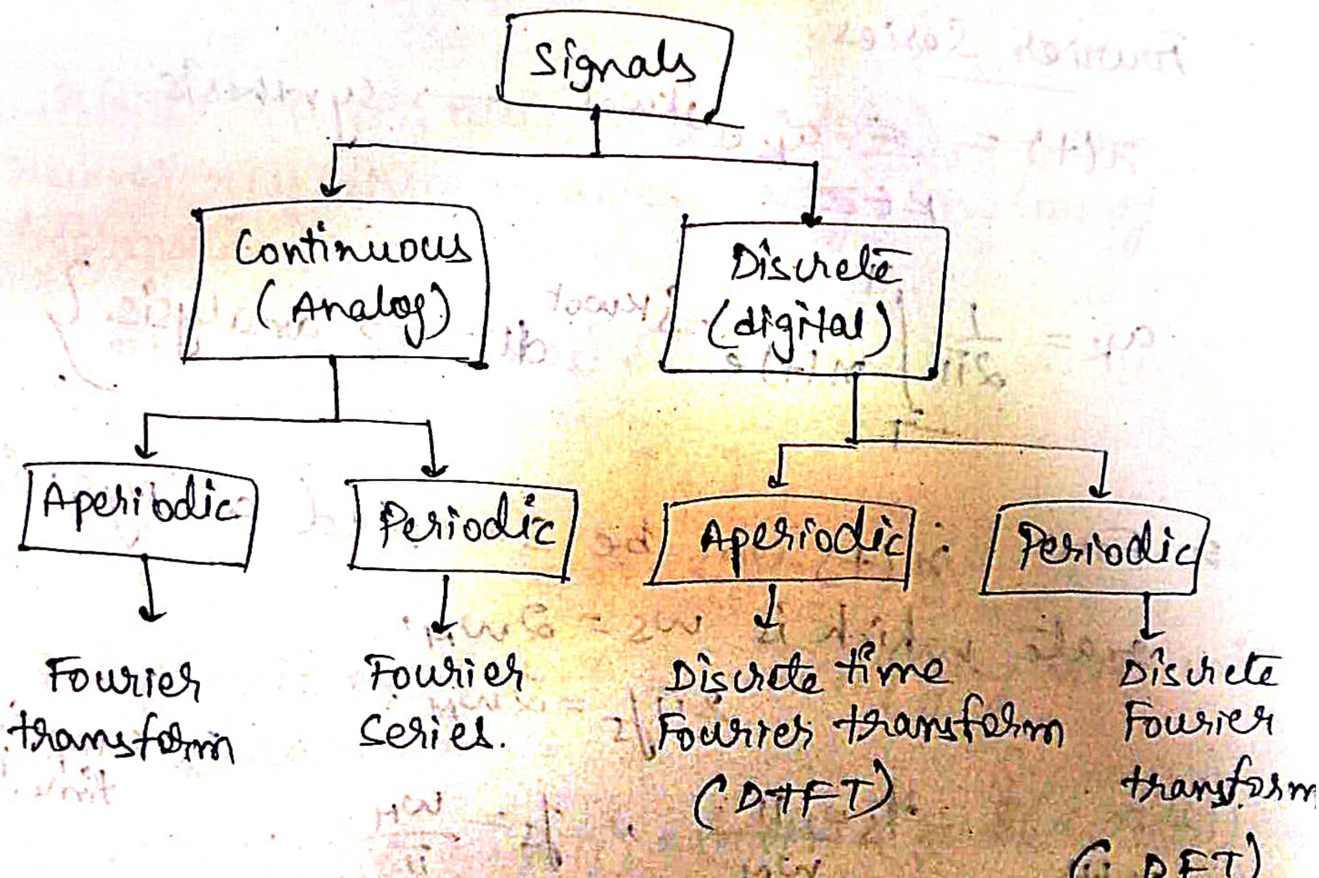
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Impulse fn is a limiting function.

$$\delta(t) = \lim_{\Delta \to 0} \delta_{\Delta}(t)$$



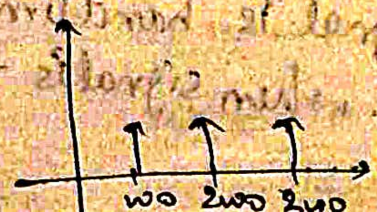




Difference between Fourier series & Fourier Transform.

there is no fundamental frequency as it is aperiodic.

fundament frequency w_0 & all its harmonics.



Sampling Theorem

If a signal $x(t)$ is bandlimited,

$x(t) \xrightarrow{\text{Fourier transform}} X(\omega)$

outside the band, It should be zero

$X(\omega) = 0, \ |\omega| \geq \omega_M.$

Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega.$$

Fourier Series.

$$x(t) = \sum_{k \in \mathbb{Z}} a_k e^{jk\omega_0 t} \longrightarrow \text{synthesis.}$$

$$a_k = \frac{1}{2\pi} \int_{-T}^{T} x(t) e^{-jk\omega_0 t} dt \longrightarrow \text{analysis.}$$

~~x Fow~~ $x(t)$ can be sampled at Nyquist rate which is $\omega_s = 2\omega_M$.

$$2\pi f_s = 2\omega_M$$

$$t_s = \frac{\pi}{\omega_M}, \quad f_s = \frac{\omega_M}{\pi}$$

T_s - sampling time

$x(t)$

$x(0), x(T_s), x(2T_s).$

$$\frac{1}{f_s} = \frac{\pi}{\omega_M} = t_s$$

given the samples, we can determine $x(t)$ when signal is bandlimited ($X(\omega) = 0$ for $|\omega| \geq \omega_M$) i.e. when signal is not arbitrary.

sampling time $(T_S) \leq \frac{\pi}{\omega_M}$

when $T_S \leq \frac{\pi}{\omega_M}$

If $T_S \geq \frac{\pi}{\omega_M}$, we won't get back correct $x(t)$.

Sampling Theorem

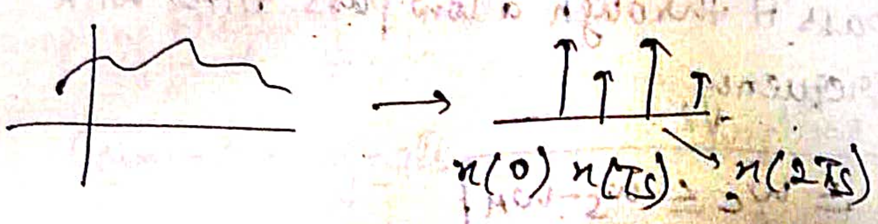
If $x(t)$ is a band limited signal,

$$X(\omega) = 0 \quad , \quad |\omega| \geq \omega_M \quad , \quad \text{Let } \omega_s \geq 2\omega_M$$

then $T_S \leq \frac{2\pi}{\omega_S}$

$\ldots, x(-2T_S), x(-T_S), x(0), x(T_S), x(2T_S)$; we can recover $x(t)$ from samples exactly. (no loss of information.)

$$x(t) \longrightarrow \sum x(nT_S)\,\delta(t - nT_S)$$



Sampling is equivalent to multiplying $x(t)$ with an impulse train.

$$\sum_n x(nT_S)\,\delta(t - nT_S) = (x(t)) \cdot \left(\sum_n \delta(t - nT_S)\right)$$

multiplication in time domain is equivalent to convolution in the frequency domain.

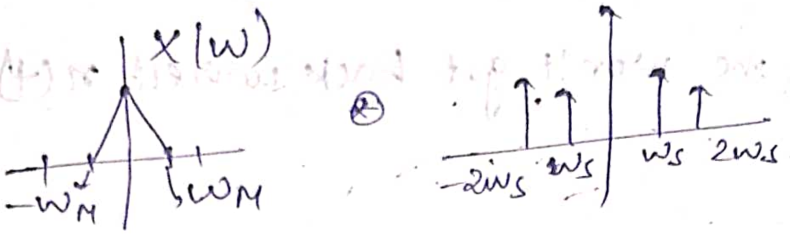
$$x(t) \longrightarrow X(\omega)$$

$$\sum_n \delta(t - nT_S) \longrightarrow c \sum_K \delta(\omega - K\omega_S)$$

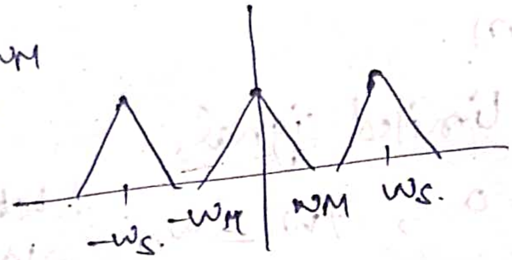
$$\omega_S = \frac{2\pi}{T_S}$$

Consider the FT of the sampled signal.

$$x(t)\sum_n \delta(t-nT_s) \xrightarrow{FT} X(\omega) \circledast c\left(\sum_K \delta(\omega - k\omega_s)\right)$$



If $w_s \geq 2w_M$ then there is no overlap.



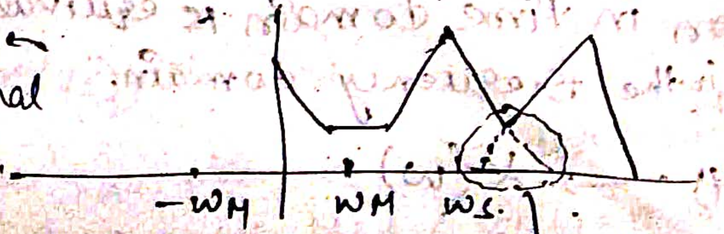
If $w_s \geq 2w_M$, then fourier transform of the sampled signal has non overlapping copies of the Fourier Transform of the original signal $x(t)$.

→ To recover $x(t)$ from the sampled signal, processing that needs to be done is to suppress all the copies ~~exactly~~ except for the main one. For this, pass it through a low pass filter with cut off frequency,

$$\boxed{w_M \leq w_c \leq w_s - w_M}$$

If $w_s < 2w_M$, then the FT of the sampled signal will be.

Digital to Analog signal conversion.



there is decreasing frequency and the other is increasing freq, both of them add up to give a constant frequency, so the information here is being lost.

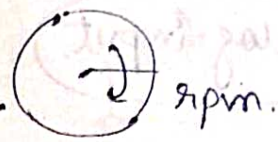
→ In some regions, FT of the original signal is distorted, so x(t) cannot be recovered.

This overlapping of copies is called <u>Aliasing</u>.

Aliasing – Phenomenon of overlap is termed as aliasing.

<u>Examples of aliasing</u>

1.) <u>Strobe Effect</u>



Illuminating on and off with some freq

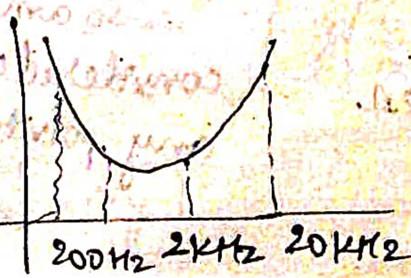
If the illumination freq is less than 2rpm, then the direction appears to be opposite (reverse direction). and the movement is at a much slower rate.

2.) Oscilloscopes.

<u>Sampling Freqs. of Signals</u>

<u>Common Signals</u>.	<u>Sampling Freq</u>.
Telephone std.	8 kHz
Digital Radio	32 kHz
Audio CDs.	44.1 kHz
Android Devices.	8, 16, 24, 44.1, 48 kHz
Common Pixel Sampling Rate	13.5 MHz
High Definition.	96 kHz.

<u>Sensitivity of the ear</u>. Intensity – dB



20Hz to 20kHz → able to hear properly

Above 20kHz → ultrasonic sound

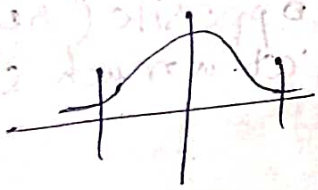
From designing system perspective,

$$\boxed{\text{Anti Alias Filter}} \rightarrow \boxed{\text{Sampling}} \rightarrow \boxed{\text{Quantization \& converting into bits}}$$

← Analog filter

Analog input

Only ideal signals have band limited FT, for a natural signal

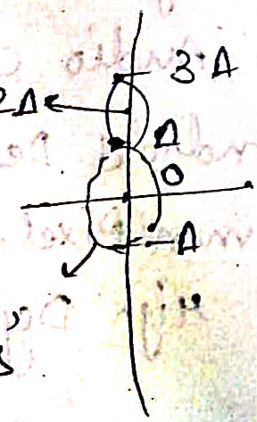


→ signal is passed through a low-pass filter to make it bandlimited, to ~~find~~ get the exact signal that we want.

The filter used for digital to analog & analog to digital conversion, is different.

Continuous Amplitude → even after sampling, the signal might have continuous amplitude, so we quantize the signal.

In quantisation, depending on the number of bits used, the entire real line is divided into some finite number of intervals.



quantisation makes continuous amplitude into discrete amplitude.

for any point between Δ to 3Δ, the amplitude is considered as 2Δ

midpoint is 0, so amplitude is considered as 0 for any point between Δ, -Δ.

converting to bits.

analog amplitude is mapped to digital bits.

Eg. $0 \longrightarrow 000$

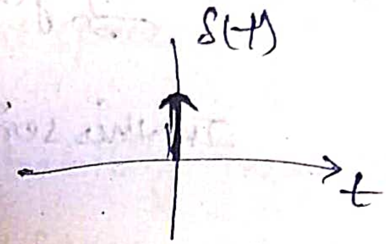
$2 \longrightarrow 001$ ·· so on.

This is called ADC (Analog to Digital Converter)

quantisation & conversion to bits.

quantisation is an irreversible process, which always leads to loss in data.

Tut-1.



①. $\delta(t) = \begin{cases} +\infty, & t=0 \\ 0, & \text{otherwise.} \end{cases}$ $\longrightarrow *$

generalised function.

Eq(*) is one way to define the unit impulse function. [Dirac delta function].

a) Equivalently, $\delta(t)$ is the "generalised" function $v(t)$ that obeys

$$f(0) = \int_{-\infty}^{\infty} f(x)\, v(x)\, dx,$$

for "most" functions $f(\cdot)$

$$\delta(at) = \frac{\delta(t)}{|a|}, \ \forall t, \ a \in R. \longrightarrow ?$$

Heuristic proof:

For $a > 0$,

Evaluating $\int_{-\infty}^{\infty} f(x)\, \delta(ax)\, dx$

$ax \rightarrow y$. $\quad = \int_{-\infty}^{\infty} f\left(\frac{y}{a}\right) \delta(y) \frac{dy}{a}$

$a\,dx \rightarrow dy$.

[set $g(y) = f(y/a), \ \forall y$]

$$= \frac{1}{a}\int_{-\infty}^{\infty} g(y)\,\delta(y)\,dy = \frac{g(0)}{a} = \frac{f(0)}{a}.$$

For $a<0$,

$$\int_{-\infty}^{\infty} f(x)\cancel{f(o}\,\delta(ax)\,dx = \frac{f(0)}{-a} = \frac{f(0)}{|a|}$$

For any $a \in \mathbb{R}\setminus\{0\}$,

$$\int_{-\infty}^{\infty} f(x)\,\delta(ax)\,dx = \int_{-\infty}^{\infty} f(x)\,\frac{\delta(x)}{|a|}\,dx.$$

In this sense, we can get $\delta(ax) = \frac{\delta(x)}{|a|}, \forall x$

b.) For some $\epsilon > 0$

Plot $g_\epsilon(x) = \frac{1}{2\sqrt{\pi\epsilon}}\, e^{-\frac{x^2}{4\epsilon}}$, for $x \in [-1,1]$

as the limit $\epsilon \to 0^+$, $g_\epsilon(x) \simeq \cancel{f}\,\delta(x)$.

c) $\delta(x) \longrightarrow$ Dirac delta function.

Probability Theorem.

In quantisation, to know how much information is cost, we use propbability.

Approach.

Relative frequency ~~Probability~~ → counting the number of trials & finding probability.

Axiomatic Approach. → using axioms defined to find the probability.

Probability space. (S, F, P)

S → ~~Sa~~ sample space. (set of outcomes).

F → Event space.

P → Probability measure.

Experiment

- Physical
- Thought experiment.

Egs for sample space ~~is~~

① one coin toss.

$S = \{H, T\}$

② Three coin tosses.

$S = \{HHH, HHT, \ldots, TTT\}$.

③ Tossing a coin until first head appears.

$S = \{H, TH, TTH, \ldots\}$ → infinite terms.

Sample space ~~is tou~~ has countabley infinite terms.

④ Throwing darts & $[0, r]$ → radius of dart.

$S = [0, r]$ → uncountably infinite terms.

F → Event space → F is a collection of subsets of S for which probability is defined.

$A \in F$, $P(A)$ is defined, $P(A^c)$ is also defined.
$1 - P(A)$.

If $A \in F$, then $A^c \in F$.
If A belongs to event space, then A^c must also belong to event space.

Axioms of Event Space.

(1) $\phi \in F$; $S \in F$.
(2) $A \in F$; $A^c \in F$

Ex. Die Throwing.
$S = \{1, 2, 3, 4, 5, 6\}$.
$F = \{ \phi, S, \underbrace{\{1,2\}}_{A}, \underbrace{\{3,4\}}_{B}, \{3,4,5,6\}, \{1,2,5,6\}\}$

$P(A) = P(\{1,2\})$
$P(B) = P(\{3,4\})$
$P(\{1,2,3,4\}) = P(A) + P(B)$

(3) $A_1, A_2, \ldots A_n \in F$, $\bigcup_{i=1}^{n} A_i \in F$
(union)

Prove that if $A_1, A_2, \ldots A_n \in F$,
$\bigcap_{i=1}^{n} A_i \in F$ (intersection).

Proof. $\bigcap_{i=1}^{n} A_i \Leftrightarrow \bigcup_{i=1}^{n} A_i^c$ because $A_i^c \in F$.
complement of each other

$\therefore A_1, A_2, \ldots \in F$, $\bigcup_{i=1}^{\infty} A_i \in F$.

Question.

Q1) Axiom 3: $A_1, A_2, \dots, \in F$; $\bigcup_{i=1}^{\infty} A_i \in F$.

For any given n.

Property: $A_1, A_2, \dots A_n \in F$, then $\bigcup_{i=1}^{n} A_i \in F$.

Prove that Axiom 3 can be proved from property.

For that, we can either repeat $A_1, A_2, \dots$ ∞ times, or keep adding ∞ null sets to the $A_1, A_2, \dots A_n$ to get infinite sets $A_1, A_2, \dots$

Q2.) Complete the following event space F.

$S = \{1, 2, 3, 4, 5, 6\}$.

$F = \{\phi, S, \{1,2,3\}, \{3,5,6\}, \{4,5,6\}, \{1,2,4\}, \{1,2,3,5,6\}, \{4\}, \{3\}, \{1,2,4,5,6\}, \dots\}$.

$S = [0,1]$

$F = \{\phi, S, [0, \frac{1}{4}], [\frac{1}{6}, \frac{2}{3}]\}$.

Axioms of Probability (P → Probability measure).

$P: F \to [0,1]$

Probability is a function from event space to $[0,1]$ satisfying the following axioms:-

① $P(S) = 1$

② If $A_1 \dots A_n \in F$ are disjoint events,

$$\boxed{P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)}$$ → this is true for, $n \to \infty$.

Derived Properties of Probability.

① . $P(A^c) = 1 - P(A)$.

A, A^c are disjoint sets.

$A \cup A^c = S$.

$P(A \cup A^c) = P(S) = 1 = P(A) + P(A^c)$

$\Rightarrow P(A^c) = 1 - P(A)$.

Disclaimer: Venn Diagrams are only means of visualization, they are not means of proof.

② $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Proof: $P(A \cup B) = P(A \cap B) + P(A \cap B^c) + P(A^c \cap B)$

$= P(A \cap B) + P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A \cap B)$

$(\because P(A \cap B^c) = P(A) - P(A \cap B)$

$A = (A \cap B) \cup (A \cap B^c)$.

Conditional Probability $\rightarrow$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Experiment $\rightarrow$ (S, F, P)

Additional info that event B has occurred.

$B \in F$, $\boxed{P(B) > 0.}$

Now the probability space is changes to

(B, F_B, P_B)

F = Power set of S.

$B = \{2, 4, 6\}$. $\{1, 3\} \in F, \notin B$.

If $A \in F$;

If $A \in F$;

$F_B = \{ A \cap B \mid A \in F \}$.

$\{1, 3\}$

$A \cap B' = \{1, 3\} \cap \{2, 4, 6\} = \phi$.

For any $B \in F_B$, $P_B(A \cap B) = \dfrac{P(A \cap B)}{P(B)}$, $P(B) > 0$

not applicable when $P(B) = 0$.

But, for the probability space to be valid, ~~we~~ we need to prove that F_B is an event space.

To prove that F_B is an event space,

① $D \in F_B$, then prove $D^c \in F_B$.

$D = (A \cap B) \in F$.

Some ~~eler~~set of $(A \cap B)$.

$(A \cap B) \in F$, then $(A^c \cap B) \in F$; $\cancel{(A \cap B^c) \in F}$.

$D \in F_B$, then $D^c \in F_B$

as. $A^c \cap B = D^c$ (or) $\cancel{D^c = A \cap B^c}$.

$F_B = \{ A \cap B \mid A \in F \}$.

$D \in F_B$; $D = A \cap B$ for some $A \in F$.

②. $D_1, D_2, \ldots \in F_B$.

$A_1 \cap B$, $A_2 \cap B$, …… such that

$A_1, A_2, \ldots \in F$.

$\bigcup_{i=1}^{n} (A_i \cap B)$

<u>Recap</u>: Probability space (S, F, P)

S → Sample space

F → Event space

P → Probability measure

Set of all outcomes of an experiment

collection of subsets of S - $\phi \in F$, closure under complement & countably infinite unions.

① Positive
② $P(S) = 1$
③ disjoint unions – $A_1, A_2, \ldots$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Derived Properties

① $P(A^c) = 1 - P(A)$

② $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

③ $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$ → from inclusion–exclusion

conditional probability → (B, F_B, P_B)

~~Pro~~ $B \in F$ s.t. $P(B) > 0$

$(A \cap B)$

$P_B(A \cap B) = \dfrac{P(A \cap B)}{P(B)}$

Directly conditional probability is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Independent Events

S → Sample space.

$A, B \in F$.

$A \cap B \in F$.

Two events A & B are said to be independent if

$P(A \cap B) = P(A) \cdot P(B)$.

$P(A|B) = P(A)$ if $P(B) > 0$

Examples of independent / Non-independent events

Mutually exclusive sets need not necessarily be independent.

1.) A^c and A are mutually exclusive but not independent (as if one occurs, the other will definitely not occur.)

2.) If A & B are disjoint. A & B are dependent

$P(A/B) = 0$. but $P(A) > 0$, $P(B) > 0$.

3.) we are throwing 2 fair dice. A = sum of outcomes is 6.

B = first die has 4.

$P(A) = 5/36$, $P(B) = 4/6$, $P(A \cap B) = \frac{1}{36}$.

A = sum of the ~~ab~~ two dice $= 7$.

B = first die has 4.

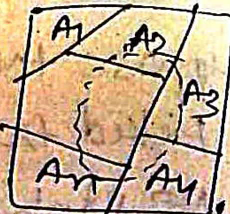
$P(A) = 4/6$, $P(B) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{36}$.

$P(A/B) = P(A) = 1/6$.

They are independent

Total Probability Theorem

$A_1, A_2, \dots A_n$ form a partition of the sample space S.



$A_i \cap A_j = \phi$

$\bigcup_{i=1}^{n} A_i = S$.

Pick any arbitrary event B.

$$\boxed{P(B) = \sum_{i=1}^{n} P(B/A_i) \cdot P(A_i)}$$

$B = \bigcup_{i=1}^{n} (B \cap A_i)$

← disjoint union.

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i)$$

$$= \sum_{i=1}^{n} P(B/A_i) \cdot P(A_i)$$

<u>Baye's Theorem</u>

$$P(A_i/B) = \frac{P(B \cap A_i)}{P(B)}$$

$$\boxed{P(A_i/B) = \frac{P(B/A_i) \cdot P(A_i)}{\sum_{i=1}^{n} P(B/A_i) \cdot P(A_i)}}$$

<u>Ex. 1</u>

A diagnostic test has a probability 0.95 of giving +ve result when applied to a person suffering from a certain disease & a probability of 0.1 of giving a (false) +ve when applied to non-sufferer. It is estimated that 0.5% of population are sufferers. Suppose the test is administered on a random person, picked from the population. Calculate the following ~~P~~ probability.

(a). Test result is +ve.

A_1 = People who are suffering with disease
A_2 = People who are not suffering
B = test came +ve.

$P(A_1) = 0.005$
$P(A_2) = 0.995$
$P(B/A_1) = 0.95$, $P(B/A_2) = 0.1$

$$P(B) = P(B/A_1) \times P(A_1) + P(B/A_2) \cdot P(A_2)$$

$$= 0.95 \times 0.005 + 0.1 \times 0.995$$

$$= 0.10425$$

(b). Given a +ve result, probability that person is sufferer.

$$\Rightarrow P(A_1/B)$$

$$= \frac{P(B/A_1) \cdot P(A_1)}{P(B)}$$

$$= \frac{0.095 \times 0.005}{P(B)} = 0.0456.$$

(c) Given a negative result, the person is a sufferer.

$P(A_1) = 0.005$

$P(A_2) = 0.995$

$$\Rightarrow P(A_1/B^c) = \frac{P(A_1 \cap B^c)}{P(B^c)}$$

$$\Rightarrow P(A_1/B_c) = \frac{P(A_1) - P(A_1 \cap B)}{P(B^c)}$$

$$P(A_1 \cap B) = P(B/A_1) \times P(\cancel{B}A_1)$$

$$= 0.95 \times 0.005$$

$$\Rightarrow P(A_1/B^c) = \frac{0.005 \times 0.05}{1 - 0.10425} = 0.00028$$