Communication System

Analog | Digital

Analog Communication System

Analog source

→ modulation / modulated

with some form of analog modulation technique

Analog Modulation

→ Amplitude modulation

→ Frequency modulation

→ Phase modulation

Analog → continuous in both time & amplitude

$s(t)$

Modulating signal + carrier signal = Modulated signal

Modulating:

signal source → Baseband signal

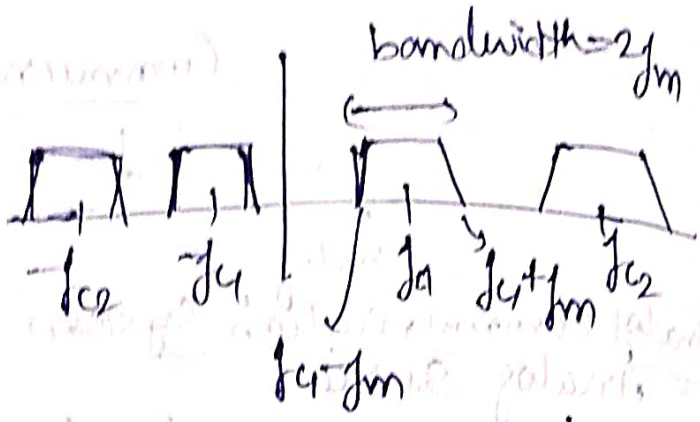
Baseband signal → Non-zero frequency around '0'.

Pass band → Non-zero frequency are centered around some carrier freq f_c.

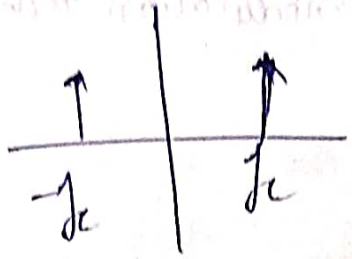
If in baseband → overlap of signals takes place causing loss of information. So, we center frequencies around diff carrier freq. to get greater bandwidth.

$m(t)$ → Message signal.

Earlier original signal



after modulation



2 diff non-overlapping signal can be obtained & we can tune into which station we want, through modulation.

<u>Carrier signal:</u> $A\cos(2\pi f_c t)$

<u>Modulated signal:</u> $u(t) = A m(t) \cos(2\pi f_c t)$

<u>Eg.</u> $m_1(t) = A_1 m_1(t) \cos(2\pi f_{c_1} t)$

$m_2(t) = A_2 m_2(t) \cos(2\pi f_{c_2} t)$

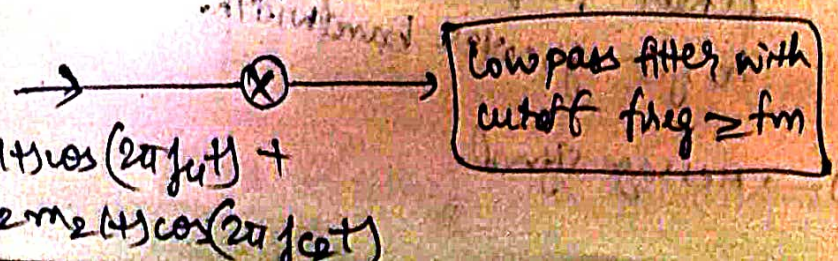
They superimpose in air to give:

$$A_1 m_1(t) \cos(2\pi f_{c_1} t) + A_2 m_2(t) \cos(2\pi f_{c_2} t)$$

Now, when we want to tune to channel 1/~~ch~~ 2, we must get only one of them, so we must demodulate.

<u>Demodulation of Amplitude Demodulated Signal</u>

we are interested in $m_1(t)$



(x) → multiply input signal with $\cos(2\pi f_{c1} t)$.

⇓

$$\text{input} = A_1 m_1(t)\cos^2(2\pi f_{c1} t) + A_2 m_2(t)\cos(2\pi f_{c1} t)\cos(2\pi f_{c2} t)$$

$$= \underbrace{A_1 m_1(t)\left(\frac{1+\cos 4\pi f_{c1} t}{2}\right)}_{\text{centered around '0'}} + \underbrace{\frac{A_2 m_2(t)\left(\cos 2\pi(f_{c1}+f_{c2})t + \cos 2\pi(f_{c1}-f_{c2})t\right)}{2}}_{\text{centered around } f_{c1}}$$

we must make sure modulated signal is not centered around '0'.

Then we use a low pass filter to make sure extra overlapping things of all don't interfere. & just upto a bandwidth of f_m, it will be received and f_{c1}/f_{c2} and all too don't change what we receive.

Frequency Modulation

$$A(t)\cos(2\pi f_c t + \phi(t))$$

$\phi(t)$ → phase

Amplitude, frequency, phase

Phase Modulation

$$A\cos(2\pi f_c t + k m(t))$$

$\Rightarrow \phi(t) = k m(t)$

$\theta(t)$

In Freq modulation, phase is not changed with message signal but freq changes with message signal.

Carrier signal : $A\cos(2\pi f_c t + \theta(t))$

Instantaneous frequency as a function of an angle

$$\boxed{f(t) = \frac{1}{2\pi}\left(\frac{d}{dt}\theta(t)\right)_t \text{ fc.}}$$

$\cos(2\pi f_c t)$

$\theta(t) = 2\pi f_c t$

$$f_c = \left(\frac{d}{dt}\theta(t)\right) \times \frac{1}{2\pi}$$

In Freq modulation:

$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = km(t)$$

Freq modulated signal

$$A\cos(2\pi f_c t + \theta(t))$$

$$\theta(t) = \left(2\pi\left(\int_0^t (km(t)\,dt)\right) + \theta_0\right)$$

$$u(t) = A\cos\left(2\pi f_c t + 2\pi\left(\int_0^t km(t)\,dt\right) + \theta_0\right)$$

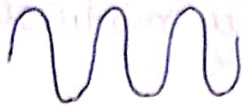
Frequency Demodulation

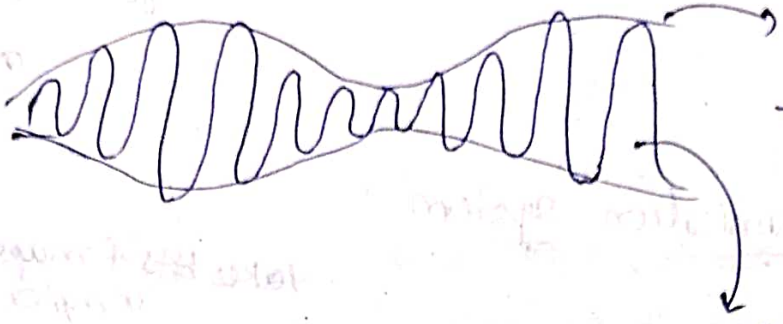
Modulated signal: $u(t) = A\cos\left(2\pi f_c t + 2\pi k\left(\int_0^t m(t)\right) + \theta_0\right)$

$$\frac{d\,u(t)}{dt} = -A(2\pi f_c + 2\pi k m(t)) \times \sin\left(2\pi f_c t + 2\pi k\left(\int_0^t (m(t)) + \theta_0\right)\right)$$

Envelope Detection → ~~to~~ Done by simple RC circuit.

$A_m(t)\cos(2\pi f_c t)$





this captures the m(t) frequency

we get diff phase shifted frequencies due to internal integration, so we do envelope detection to get frequency of m(t).

Then we need to suppress the DC component, then f_c to get m(t)

Bandwidth of this is not clear but is somewhat higher than AM signal.

AM	FM
→ less Bandwidth	→ Higher bandwidth
→ Bad noise performance	→ Better noise performance
↓	↓
more susceptible to noise because amplitude is not constant & noise directly adds up easily	less susceptible to noise as freq is changing, amplitude is constant & so, noise added to freq is very less.

Recap.

Analog communication system

m(t) – analog signal.

$A\cos 2\pi f_c t$ – carrier signal.

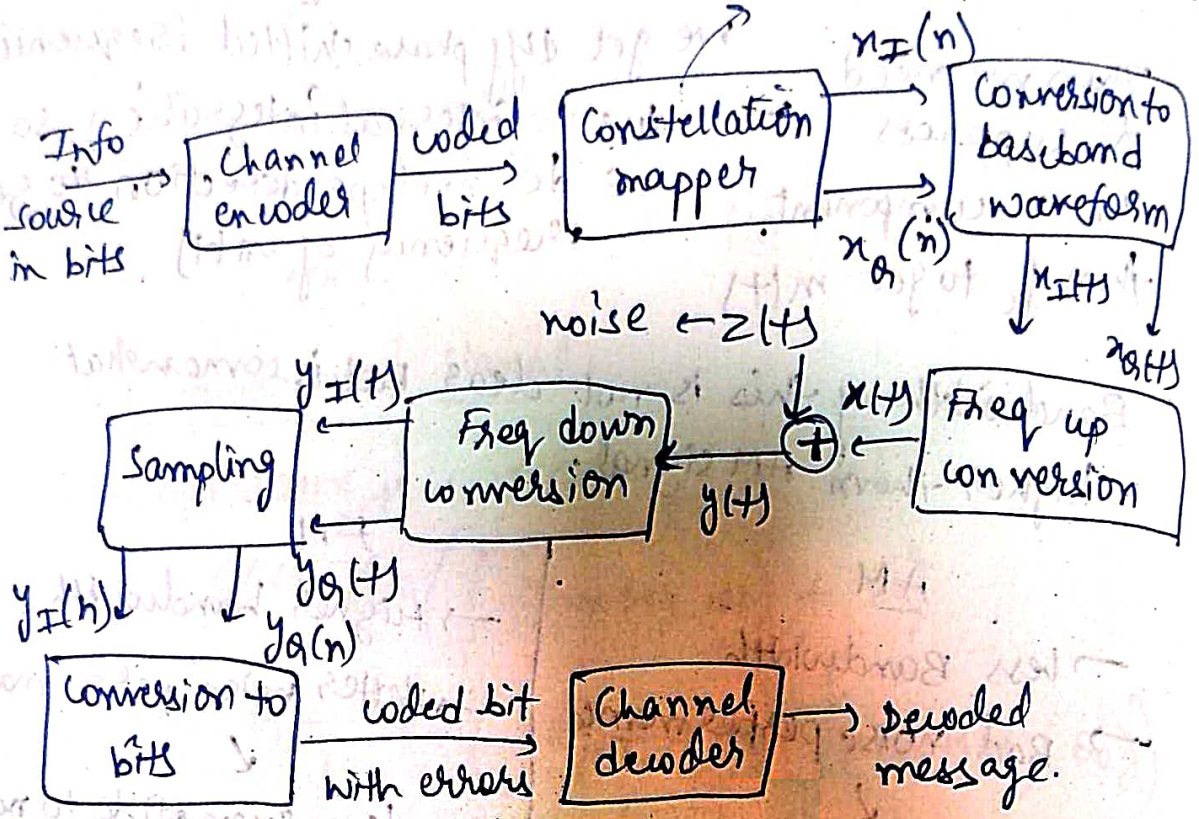
Amplitude modulated signal – $A_m(t)\cos 2\pi f_c t$

noise ← performance

Frequency modulated signal

$\hookrightarrow A\cos\left(2\pi f_c t + k\int_0^t m(t)\,dt + \theta_0\right)$

Digital Communication system



Constellation Mapper

$\hookrightarrow$ takes sequence of bits as input and output is a complex sequence.

$x_I(n) \rightarrow$ Inphase component

$x_Q(n) \rightarrow$ Quadrature phase component.

$$\boxed{x_I(n) + j x_Q(n)}$$

higher

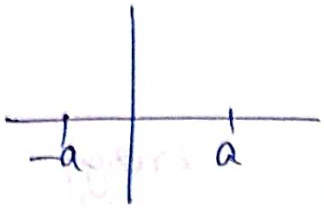
BPSK $\rightarrow$ Binary phase shift keying

QPSK $\rightarrow$ Quadra ternary phase shift keying

16 – QAM $\rightarrow$ 16 – Quadrature Amplitude Modulation

64-QAM → ~~Quad~~ 64- Quadralare Amplitude Modulation

BPSK.



$0 \to a$, $1 \to -a$

1010110001100 1

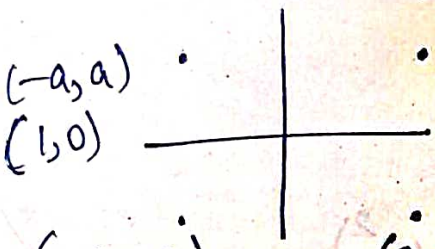
$-a, a, -a, a, -a, -a, a, a, a,$
$-a, -a, a, a, -a$

a is decided based on the energy of the constellation mapper.

a is some real number
↓
we are not using the imaginary part.

QPSK

$a + j(a)$



1010110001100 1

input signal for which we want to do QSPK mapping

$\to x_I(n)$

$(-a, a)$,

$x_I(n) \to -a \; -a \; -a \; a \; a \; -a \; a$

$x_Q(n) \to a \; a \; -a \; a \; -a \; a \; -a$

$\boxed{x_I(n) + j\, x_Q(n)}$

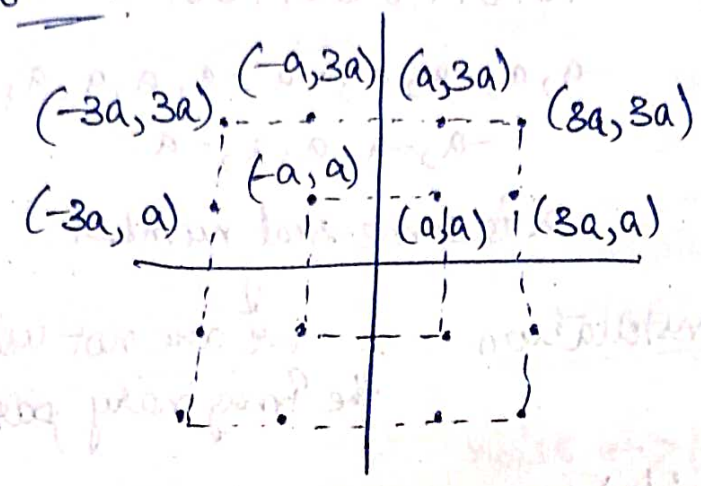
If there are odd no. of bits, then we add a dummy bit or 0 (as it contains no information) and repeat the above procedure.

(Complex plane is best plane to represent 2-D signals)

There is also a feedback from receiver to the transmitter which tells for this input, there is less/more noise, so accordingly the transmitter

Sends the signals using higher / lower constellations used (for less noise) (for high noise)

16 — QAM



~~Energy~~

Energy

Energy per bit of a constellation

(amplitude)^2) , in complex plane

((complex amplitude)^2)

Let's say there are M points in the constellation :

$$E_b = \frac{|s_1|^2 + |s_2|^2 + \cdots + |s_M|^2}{M \log_2 M}$$

E_b — energy per bit.

$s_1, s_2, \ldots s_M$ are points

$|s_1|, |s_2|$ — the complex values.

BSPK

(all the points are of the same energy)

$$E_b = \frac{a^2 + a^2}{2 \log_2 2} = a^2 \Rightarrow \boxed{a = \sqrt{E_b}}$$

QSPK

QSPK

$$E_b = \frac{\overbrace{2a^2 * 4}^{(a^2+a^2)\times 4}}{4\log_2 4} = a^2$$

(all the points are of the same energy.)

16 – QAM

$$E_b = \frac{(2a^2 \times 4) + (2 \times 9a^2 \times 4) + (a^2 + 9a^2) \times 8}{16 \log_2 16}$$

$$= \frac{\overset{2}{\cancel{8}}a^2 + \overset{18}{\cancel{72}}a^2 + \overset{20}{\cancel{80}}a^2}{16 \times \cancel{4}} = \frac{\cancel{110}a^2}{16 \times 4} = \frac{40a^2}{16} = \frac{5a^2}{2}$$

Discrete sequence

$$\begin{matrix} x_I(n) \longrightarrow \\ x_Q(n) \longrightarrow \end{matrix} \begin{pmatrix} x_I(t) \\ x_Q(t) \end{pmatrix}$$

baseband signal ($x_I(t)$ should have freq within some range)

$$\boxed{x_I(t) = \sum_n x_I(n)\, p(t - nT_s)}$$

Frequency Up Conversion (Adding a carrier)

$$x(t) = \underbrace{x_I(t)}_{\text{baseband signal}} \cos 2\pi f_c t + \underbrace{x_Q(t)}_{\text{baseband signal}} \sin 2\pi f_c t$$

One component is multiplied by cos and the other is multiplied by ~~sin~~ $\cos(90-\theta)$, so it is called inphase and quadrature.

$x(t) \longrightarrow \boxed{\text{channel}}$

noise is always modelled as additive.

$$x(t) \longrightarrow \oplus \longrightarrow y(t) = x(t) + z(t)$$

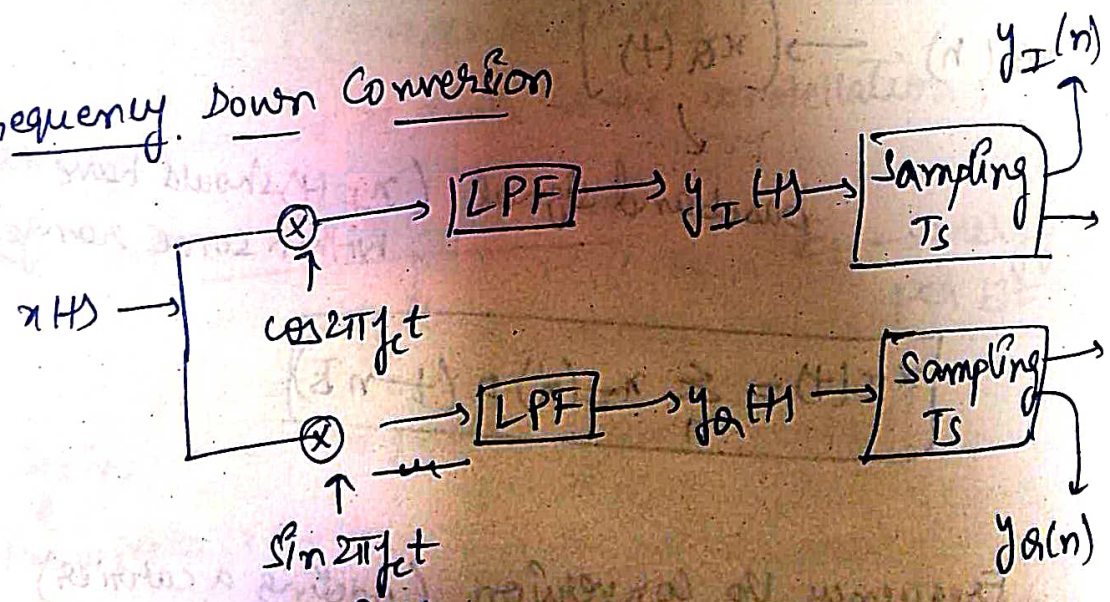
$z(t)$ — thermal noise at the receiver.

$$\underbrace{\frac{x_I(t)}{2}}_{\text{centered at 0}} + \left(\frac{x_I(t)}{2}\cos 4\pi f_c t + \frac{x_Q(t)}{2}\sin 4\pi f_c t\right)$$

$$x(t)\cos 2\pi f_c t$$

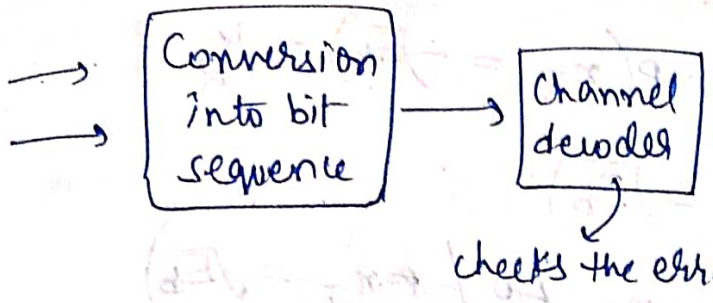
$$= \underbrace{x_I(t)\left(\frac{1+\cos 4\pi f_c t}{2}\right)}_{\text{centered at 0}} + \underbrace{x_Q(t)\left(\frac{\sin 4\pi f_c t}{2}\right)}_{\text{centered at } f_c}$$

Frequency Down Conversion



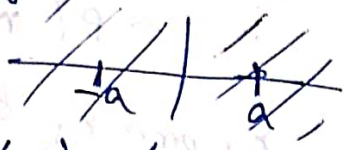
LPF → Low pass filter.

$y_I(n)$, $y_Q(n)$ is the noisy version of the constellation mapper output.



$y_I(n)$, $y_Q(n)$ could be any point in the complex plane.

So we can take +ve part as (a) and negative part as (-a) (taking BSPK)



Usually, noise is gaussian and for gaussian r.v, this process works.

For QSPK → we can take quadrants.

We divide the whole complex plane into regions ~~and~~ such that the point $(y_I(n) + j y_Q(n))$ is closest to that region. / or the points in the region are closest to $(y_I(n) + j y_Q(n))$.

Eg. If $y_I(n), y_Q(n) > 0$ then map to a. (BPSK)

Probability of Error Calculation for BPSK

$$y_I(n) = x_I(n) + n_I(n)$$

$$y_Q(n) = x_Q(n) + n_Q(n)$$

noise

BSPK: If $y_I \geq 0$ then map to $\sqrt{E_b}$

& if $y_I < 0$, then map to $-\sqrt{E_b}$.

$$P_E = P(y_I < 0 \mid x_I = \sqrt{E_b})\, P(x_I = \sqrt{E_b}) + P(y_I \geq 0 \mid x_I = -\sqrt{E_b})\, P(x_I = -\sqrt{E_b})$$

P_E ← prob of error

$$P(x_I = \sqrt{E_b}) = P(x_I = -\sqrt{E_b}) = 1/2$$

$$P(y_I < 0 \,/\, x_I = \sqrt{E_b})$$

$$= P(n_I + x_I < 0 \,/\, x_I = \sqrt{E_b})$$

$$= P(n_I < -\sqrt{E_b} \,/\, x_I = \sqrt{E_b}) = P(n_I < -\sqrt{E_b})$$

n_I and x_I are independent

Gaussian is symmetric about origin.

$$\Rightarrow P(n_I < -\sqrt{E_b}) = \int_{-\infty}^{-\sqrt{E_b}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-n_I^2}{2\sigma^2}} dn_I$$

Tut. - PS6.

1.) $m(t) = e^{-\alpha t} u(t)$

$$e^{-\alpha t} u(t) \xrightarrow[FT]{} \frac{1}{\alpha + j\omega}$$

$$\max(FT(e^{-\alpha t} u(t))) = \max\left(\frac{1}{|\alpha + j\omega|}\right) = \frac{1}{\sqrt{\alpha^2}} = \frac{1}{\alpha}$$

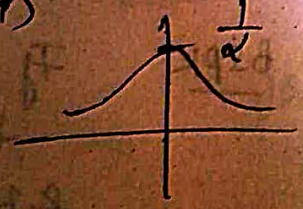
$$\frac{1}{\alpha} \times 0.95 \le \frac{1}{\sqrt{\alpha^2 + \omega^2}} \Rightarrow \boxed{\omega \le 0.33\alpha}$$

bandwidth $\boxed{B = 2(f_m + f)}$ → for FM signal

$$f_i(t) = f_c + f_A \int_0^t m(t)\,dt \qquad m(t) \to e^{-\alpha t} u(t)$$

$$f_i = f_c + f_A \int_0^t$$

modulated signal



$$\Rightarrow f_i(t) = f_c + \int_0^t e^{-\alpha t} u(t)\,dt$$

$$= f_c + \left(\frac{e^{-\alpha t}}{-\alpha}\right)\Big|_0^t$$

$$= f_c + \left(\frac{1}{-\alpha}\right)\left(e^{-\alpha t} - 1\right)$$

$$= f_c + \left(\frac{1-e^{-\alpha t}}{\alpha}\right)$$

$$A \quad A_c\cos(2\pi f_c + 2\pi f(t))$$

$$= A_c\cos\left(2\pi f_c + 2\pi\left(\frac{1-e^{-\alpha t}}{\alpha}\right)\right)$$

max $= \frac{1}{\alpha}$

$$\cancel{\frac{d}{dt}\left(\frac{1-e^{-\alpha t}}{\alpha}\right) = \frac{e^{-\alpha t}}{\alpha^2}}$$

$$B = 2\left(\frac{0.33\alpha}{2\pi} + \frac{1}{\alpha}\right)$$

$$\text{Bandwidth} = 2\left(f_m + f_i(\max)\right)$$

$$= 2\left(\frac{0.33\alpha}{2\pi} + \frac{1}{\alpha}\right)$$

2) $m(t) = \frac{\sin(2\pi f_m t)}{2\pi}$, $f_m = 10\,kHz$

$c(t) = A_c\sin(2\pi f_c t)$, $A_c = 100$. $A = 10$.

carrier signal

$f_m = 10\,kHz$

$f_c = 100\,kHz$

AM,

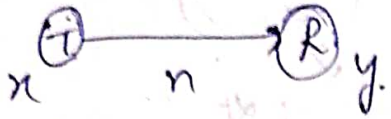
$$s(t) = (A_c + m(t))\sin(2\pi f_c t)$$

$$= \cancel{A_c m(t)\sin(2\pi f_c t)}$$

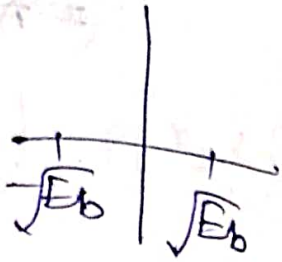
$$= \left(A_c + \frac{A\sin(2\pi f_m(t))}{2\pi}\right)\sin(2\pi f_c t)$$

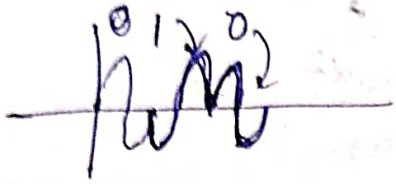
3.) $\mathcal{X} = \{-\sqrt{E_b}, \sqrt{E_b}\} \rightarrow$ BSPK signalling

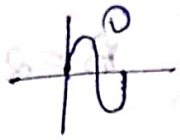
$$\hat{x}_{MAP} = \arg\max_{x \in \mathcal{X}} f_{X/Y}(x/y)$$

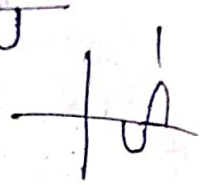


$x + n = y$





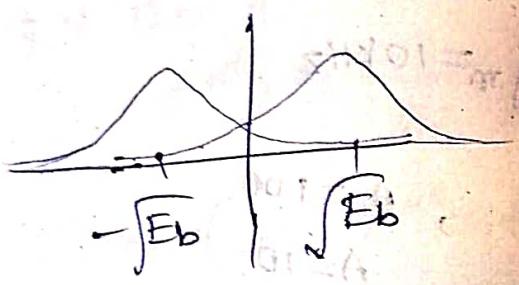




$$\hat{x}_{MAP} = \arg\max_{x \in \mathcal{X}} f_{X/Y}(x/y)$$

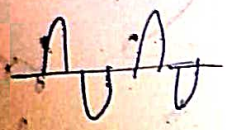
$$\mathcal{X} = \{-\sqrt{E_b}, \sqrt{E_b}\}$$

$$P(X = \sqrt{E_b}/a+ib), \; P(X = -\sqrt{E_b}/a+ib)$$



Recap

$\rightarrow$ 0101... $\rightarrow$ $\boxed{\text{Constellation mapper}}$ $\rightarrow$ sequence of baseband pulses $\rightarrow$



$\boxed{\text{BPSK}}$

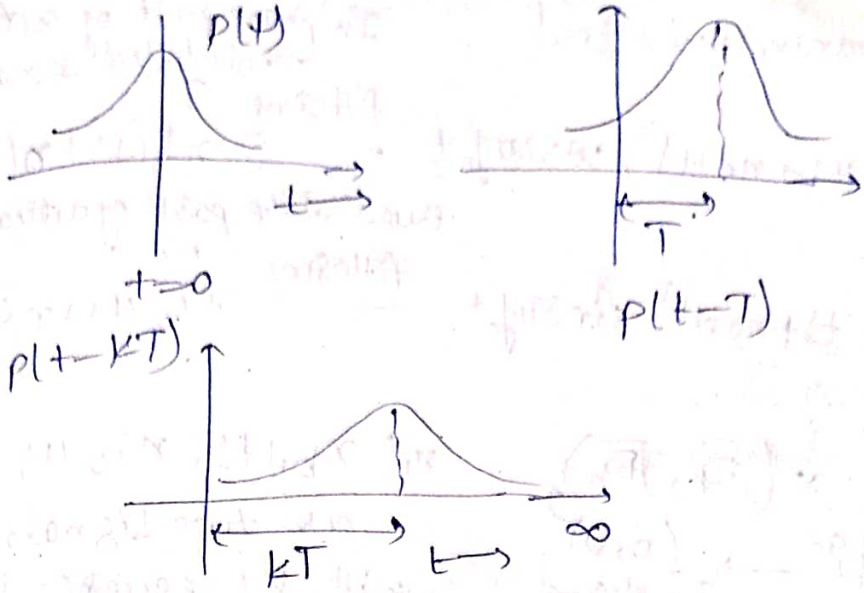
$0 \rightarrow +\sqrt{E_b}$

$1 \rightarrow -\sqrt{E_b}$

$$\underset{b_0\, b_1\, b_2\, b_3\, b_4\, b_5}{0\,1\,0\,1\,0\,0} \xrightarrow{\text{BPSK}} +\sqrt{E_b}\,p(t) - \sqrt{E_b}\,p(t-T) + \sqrt{E_b}\,p(t-2T)$$

BPSK
(Baseband signal)

$$\boxed{x_b(t) = \sum_{k=0}^{\infty} (-1)^{b_k} \sqrt{E_b}\, p(t-kT)}$$



The pulse should be easily detectable and one pulse should not overlap with another pulse

Characteristics of pulse to take

should have time constraints eg. energy should be constrained within some time.

This baseband signal is transmitted, after carrier modulation through the channel.

→ At the receiver end, the received signal is 'demodulated' back to the base band.

Received baseband signal

↳ $r_b(t) = x_b(t) + n(t)$

$n(t)$ is the noise ~~of~~ at baseband.

QPSK

$x_{b1}(t) \longrightarrow x_1(t)$ $\cos 2\pi f_c t$ (In phase carrier)

can be diff signals.

$x_{b2}(t) \longrightarrow x_2(t)$ $\sin 2\pi f_c t$ (Quadrature carrier)

Transmitted signal → $x_1(t) + x_2(t)$

they are orthogonal

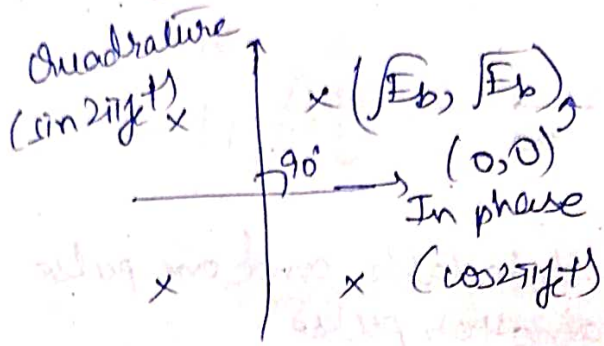
So the transmitted signal

In phase part of actual modulated signal

$$\int (x_1(t)+x_2(t)) \cos 2\pi f_c t \xrightarrow{\text{filtering}} x_{b_1}(t)+n(t)$$

Quadrature part of actual

$$\int (x_1(t)+x_2(t)) \sin 2\pi f_c t \xrightarrow{\text{filtering}} x_{b_2}(t)+n(t)$$



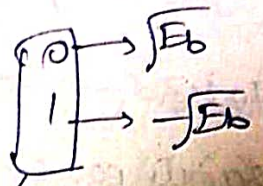
If $x_{b_1}(t)$, $x_{b_2}(t)$ are two signals with bit sequence b_1, b_2

$b_1 \rightarrow$ 0 0 1 0 1

$b_2 \rightarrow$ 1 0 1 1 0

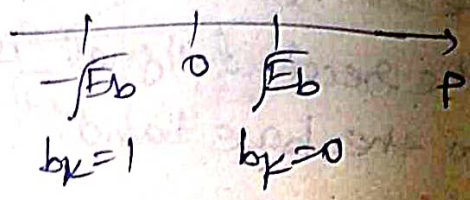
bit sequence 1

bit sequence 2



$(\sqrt{E_b}, -\sqrt{E_b})$.

$(0,1) \Rightarrow (\sqrt{E_b}, -\sqrt{E_b})$



$r_b(t) = x_b(t) + n(t)$ → sample this at k^{th} time constant

$r_k = (-1)^{b_k}\sqrt{E_b} + n_k$ → sampled received signal value at k^{th} time instant

Received has this & it wants to decide where actual $b_k = 0$ or 1

(or) $+\sqrt{E_b}$ or $-\sqrt{E_b}$.

Assuming that n_k is Gaussian noise, $N(0, \frac{N_0}{2})$ (mean, variance)

then the detector is basically (MAP/ML rule)

Min Euclidean distance rule

If r_k is closer to $+\sqrt{E_b}$, then receiver fixes $+\sqrt{E_b}$

is the actual transmitted signal, if r_k is close to $-\sqrt{E_b}$ then it fixes $-\sqrt{E_b}$ as t.s

$$P(\text{error}) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

gaussian.

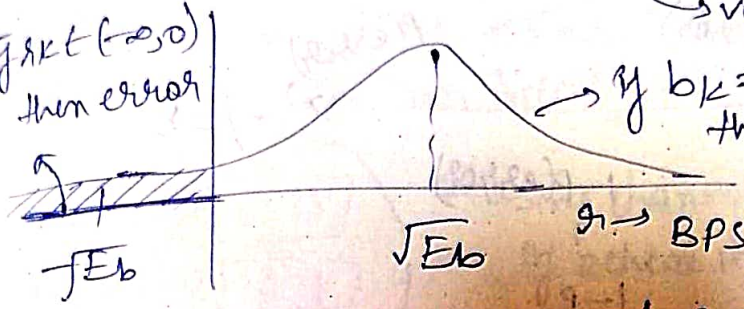
$r_k = (-1)^{b_k}\sqrt{E_b} + n_k \rightarrow$ Gaussian shifted by $(-1)^{b_k}\sqrt{E_b}$

(only mean is shifted, variance is not shifted.)

$$r_k \sim N\left((-1)^{b_k}\sqrt{E_b}, \frac{N_0}{2}\right)$$

$\rightarrow$ variance.

If $r_k \in (-\infty, 0)$ then error



$\rightarrow$ If $b_k = 0$ then r_k is distributed like this

$$f_{r_k}(r) = \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{\left(r - (-1)^{b_k}\sqrt{E_b}\right)^2}{2\left(\frac{N_0}{2}\right)}}$$

Gaussian distribution

$$P(\text{error} \mid \text{Transmitted signal} = +\sqrt{E_b}) = P(r_k \in (-\infty, 0))$$

Tr_x

$$= \int_{-\infty}^{0} f_{r_k}(r)\,dr$$

area under the curve from $(-\infty, 0)$

$$\text{area} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \text{prob of error when } \sqrt{E_b} \text{ is}$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

~~where~~ where

$$\boxed{Q(a) = \frac{1}{2\pi}\int_{a}^{\infty} e^{-x^2/2}\,dx}$$

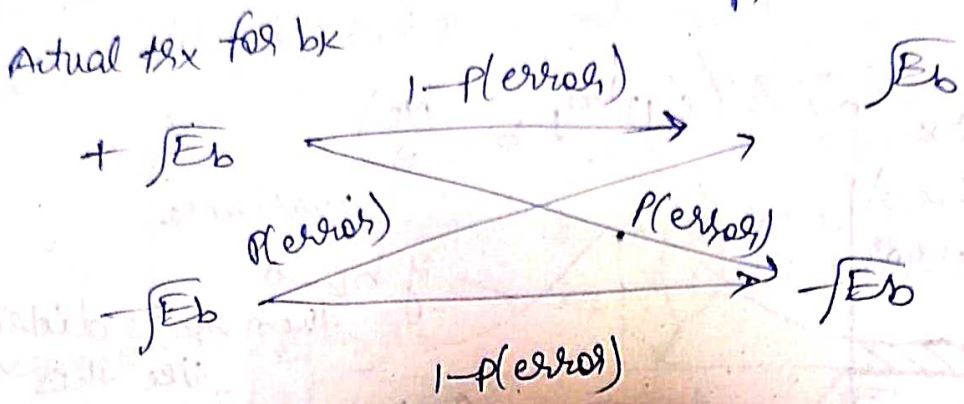
$$P(\text{error} \mid -\sqrt{E_b} \text{ was } tr_x)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

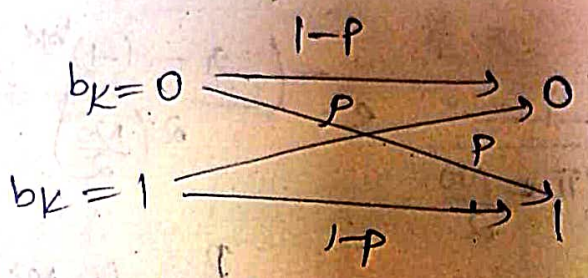
Average probability of error $= P(+\sqrt{E_b}$ was trx$)$

$$Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + P(-\sqrt{E_b} \text{ was trx}) \times Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

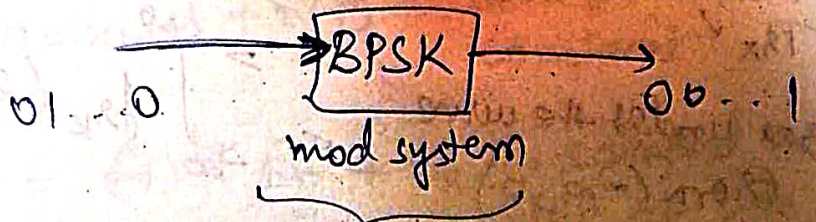
$$\boxed{\text{BPSK } P(\text{error}) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$



Eg



$P = P(\text{error})$



Binary symmetric channel: Input alphabet (all possible inputs $\rightarrow \{0,1\}$)

two possible inputs

Output Alphabet (all possible outputs $\rightarrow \{0,1\}$)

Probability of input $= x$

Probability of output $= y$

$$P(y/x) = \begin{cases} p & \text{if } y \neq x \\ 1-p & \text{if } y = x \end{cases}$$

Symmetric → the behavior of the channel is the same for any input (0 or 1)

In the BSC channel with $p < 0.5$ $(p = 0.1)$
what is the way to communicate reliably?
(without bit flips)

Inspite of bitflips, we communicate what we want to transmit

bit flips happen (cannot avoid)
but we embed ~~over~~ our actual message we want to trx into the bit stream, in an intelligent way.

codeword corresponding to msg 1

Msg 1 — 0 0 0
Msg 2 — 1 1 1
code

↙ using channel thrice, send 1 bit of actual information.

Rate of this scheme $= \frac{1}{3}$ (average information bits communicated per channel use)

3 → codelength / blocklength

every ~~one~~ time one bit is communicated through the channel.

$\frac{\text{no. of bits comm}}{\text{total no. of bits}}$

Decoding

Error: When message 1 is trx & the decoder estimates that message 2 was transmitted.

msg 2 trx → estimated as msg 1

Error across message
actual transmitted & estimated message are different.

Decoder → Min Hamming distance decoder.

Decoder declares 000 if $d_H(r, 000) \leq d_H(r, 111)$

111 if $d_H(r, 000) > d_H(r, 111)$

$r \in \{000, 001, \ldots 111\} \rightarrow (\{0,1\}^3)$

$d_H(r, 000)$ = no. of indices where $\underline{r}$ differs from 000.

In general,

$d_H(\underline{r}, \underline{c})$ = no. of indices where $\underline{r}$ differs from $\underline{c}$ (eg. $d_H(101, 000) = 2$, $d_H(101, 111) = 1$)

the minimum flips required to get either of the codeword gives ~~the~~ ~~the d~~ that codeword as transmitted ~~as the~~ ~~recd~~ ~~received signal~~. is declared (closest to the codeword)

signal 101 $\xrightarrow{\text{is mapped to}}$ 111 (flip=1)

101 $\longrightarrow$ 000 (flips=2)

P(decoding error/ msg 1 was the actual message trx)

= P(decoding error/actual codeword=000)

$= P(\underline{r} \in \{101, 011, 110, 111\} / \text{actual codeword} = 000)$

$= p^2(1-p) + p^2(1-p) + p^2(1-p) + p^3$ (events are disjoint, we can get either ~~are~~ of them)

$P(\underline{r} = 101 / \underline{c} = 000)$

$= 3p^2(1-p) + p^3$.

prob. of decoding error when message 1 was the actual message transmitted.

P(decoding error/ msg 2 = actual trx)

$= 3p^2(1-p) + p^3$.

P(decoding error) $= 3p^2(1-p) + p^3$

Compare this with the scheme: msg 1 → 0

msg 2 → 1

P(decoding error) = p. $\longrightarrow$ (0→1, 1→0.)

for $p = 0.1$ (for $p < 0.5$)

$3p^2(1-p) + p^3 < p$

Repetition scheme (coding) $\rightarrow$ Rate $= \frac{1}{3}$

No coding : Rate $= 1$

Prob of error of coding scheme < prob of error of no-coding scheme

So, Increased the reliability of communication by sacrificing on the rate

To increase the reliability $\Rightarrow$ decrease of $P(\text{de error})$ we can map messages to longer codewords, but rate decreases.

we want high rate + very low $P(\text{de error})$.

what is the exact tradeoff between these?

$P(\text{error}) \rightarrow 0$ as ~~block~~ blocklength $\rightarrow \infty$

but rate also goes to ~~rate~~ 0, rate $\rightarrow 0$

Can we get some +ve rate (non zero rate) with $P(\text{error}) \rightarrow 0$ as $n \rightarrow \infty$?

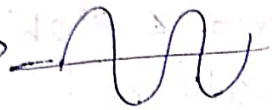
Not possible using a repetition code/some other coding scheme.

$$\text{Rate} = \frac{k}{n} \left(\frac{\text{No. of info bits}}{\text{length of code}} \right)$$

Messgs 2^k of them $\longrightarrow$

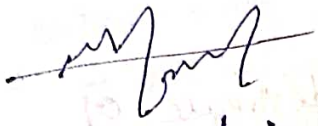
Modulation

High frequency periodic signal can be used to transmit a ~~ban~~ baseband signal



carrier signal.

Signal → Amplitude
Frequency
Phase

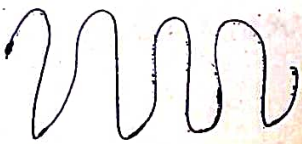


Baseband signal

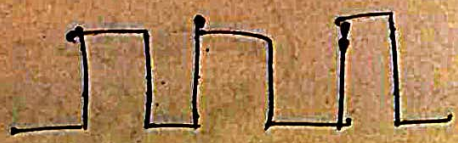
Modulation is the process where one of the signal properties of carrier signal is changed according to the baseband signal.

Modulation
- Analog
- Digital

Analog Modulation
- Continuous wave modulation
- Pulse Modulation



Carrier signal

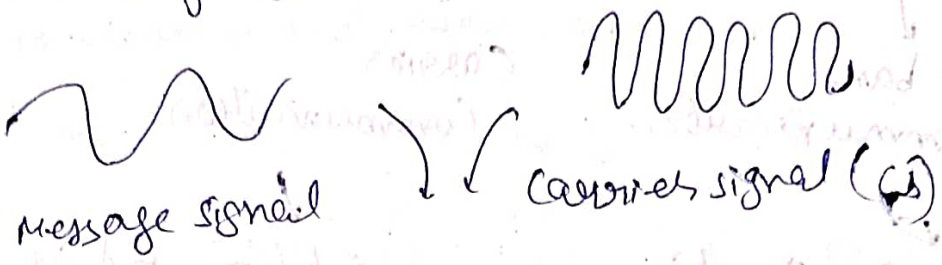


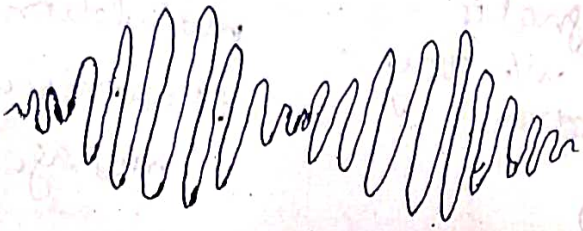
Carrier signal

Continuous Modulation
- Amplitude modulation
- Frequency modulation
- Phase modulation

Amplitude modulation (AM)

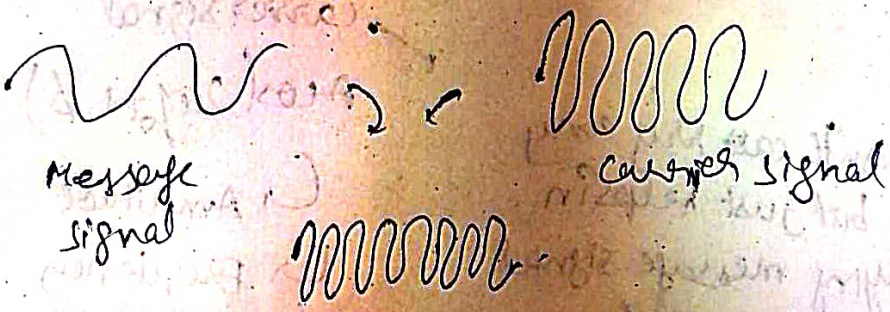
- The amplitude of carrier wave changes according to the message signal.





Modulated signal

Frequency Modulation (FM)

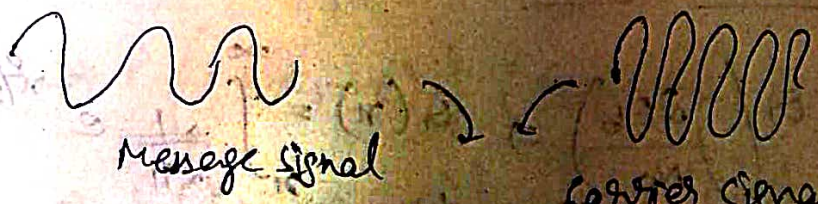


Modulated signal

With the increase in the amplitude of the message signal, the frequency of the modulated signal also increases.

Phase Modulation

The phase of CS changes according to message signal.



Modulated signal

Amplitude modulation

Types of Communication

- Base band communication
  - message signal is transmitted w/o any modulation.
- Carrier Communication
  - modulation to shift the frequency spectra of the message

$m(t)$

Baseband signal

Carrier signal

$A\cos(2\pi f_0 t + \phi)$

- ↳ Amplitude
- ↳ Frequency
- ↳ phase

doesn't carry any info but just helps in carrying message signal so it acts as envelope

Properties are changed according to message signal.

Prob error for BPSK

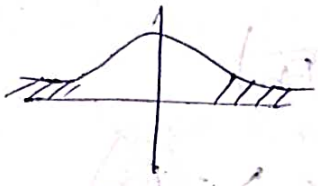
$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \Rightarrow Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx.$$

(Q function is the tail of (zero mean, ~~with~~ unit variance, gaussian r.v))

$$P_e = P(n_I < -\sqrt{E_b}) = \int_{-\infty}^{-\sqrt{E_b}} \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{\frac{-n_I^2}{2\frac{N_0}{2}}} dn_I$$

n_I is a Gaussian r.v with mean 0 and variance $\frac{N_0}{2}$.

$n_I + jn_Q \rightarrow n_I, n_Q$ identical & independent



$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$= \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{2E_b}{N_0 \times 2}} d\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$=$

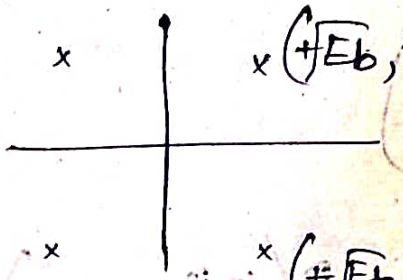
Errata

Digital Comm.

$$E_b = \frac{|s_1|^2 + \cdots |s_M|^2}{M \log_2 M}$$

$$E(|n^2|) = E(n_I^2 + n_Q^2)$$

Prob error for QPSK



$$P_e = \left((y_I, y_Q) \in \text{one of the 3 other quadrants} \;\middle|\; n \in (\sqrt{E_b}, \sqrt{E_b}) \right)$$

$$= \left(\begin{matrix} -ve, +ve \\ -ve, -ve \\ +ve, -ve \end{matrix} \;\middle|\; (\sqrt{E_b}, \sqrt{E_b}) \right)$$

$$P\left((-ve, +ve) \;\middle|\; (\sqrt{E_b}, \sqrt{E_b}) \right)$$

$$= + P\left((+ve, -ve) \;\middle|\; (\sqrt{E_b}, \sqrt{E_b}) \right)$$

$$+ P\left((-ve, -ve) \;\middle|\; (\sqrt{E_b}, \sqrt{E_b}) \right)$$

$$= P\left(n_I < -\sqrt{E_b},\ n_Q > -\sqrt{E_b}\right)$$
$$+ P\left(n_I > -\sqrt{E_b},\ n_Q < -\sqrt{E_b}\right)$$
$$+ P\left(n_I < -\sqrt{E_b},\ n_Q < -\sqrt{E_b}\right)$$

Because n_I and n_Q are independent

$$P\left(n_I < -\sqrt{E_b},\ n_Q > -\sqrt{E_b}\right)$$
$$= P\left(n_I < -\sqrt{E_b}\right) P\left(n_Q > -\sqrt{E_b}\right)$$
$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left(1 - Q\sqrt{\frac{2E_b}{N_0}}\right)$$

neglecting this term as $x^2 \ll 1$

$$\simeq Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

(~~for~~ slightly less error than BPSK)

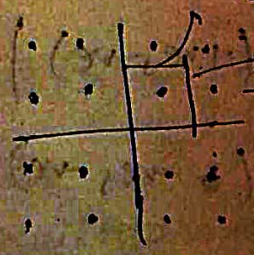
$$\Rightarrow P_e = 2\,Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left(1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right)$$
$$+ \left(1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right)\left(1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right)$$
$$\simeq 2\,Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P(e) = P\left(\text{Error} \,/\, (\sqrt{E_b}, \sqrt{E_b})\right) \times P\left(r \text{ is } (\sqrt{E_b}, \sqrt{E_b})\right) + \ldots$$

$\left(\frac{1}{4}\right)$

bisector.

For any constellation, the region closest to a certain point is obtained by drawing bisectors of that point



with every other point and take its intersection.

For gaussian it is always the closest point rule.

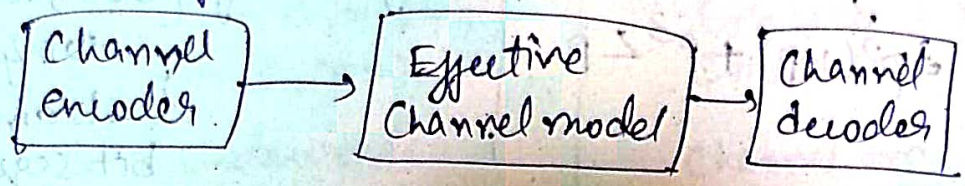
* Noise is always taken as gaussian r.v.

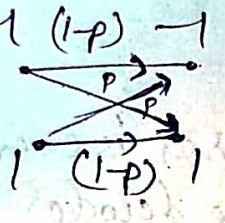
$$e^{-\frac{n_I^2 + n_Q^2}{2}}$$

minimum distance rule

Effective Channel model

(as seen by ~~en~~ channel encoder and decoder)





$$p = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

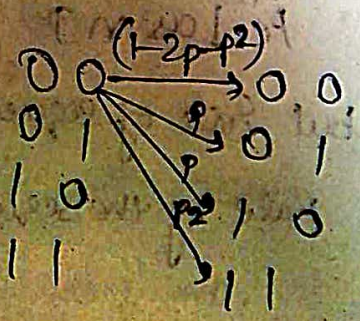
Binary symmetric channel.

For BPSK.

For QPSK

Effective channel model upto symbol level

$$p = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

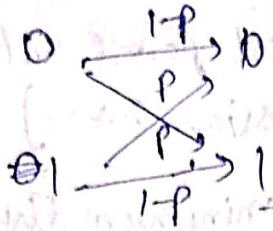


Source Coding → compress the source to its entropy.

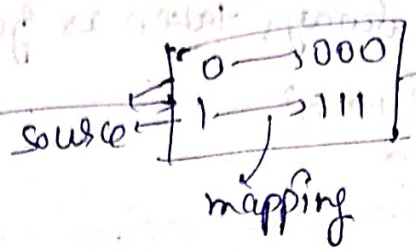
Channel Capacity

Discrete Memoryless Channel (DMC)

Consider a BSC (binary symmetric channel)



$P_e = p$ (when no coding is used)



3bit sequence → channel decoder

gives 0 if {000, 001, 010, 100} is obtained as bit sequence.

P_e for 3 repetition is p

$000 \rightarrow 110, 101, 011, 111$

$3p^2(1-p) + p^3 < p$

$0 \rightarrow \underbrace{000}_{\text{3 repetition}}$

$0 \rightarrow \underbrace{00000}_{\text{5 repetition}}$ (prob of error is even less, as in the decoder side ~~each~~ obtained sequence is ~~more~~ mapped to either ~~one~~ of the basket, i.e. each basket {} has more numbers.)

(odd repetitions is preferred)

So more ~~nos~~ map to 0 or 1.

$n \rightarrow$ repetition factor

$P_e^{(n)} \downarrow$ as $n \uparrow$

but rate of sequence $\downarrow$

Rate of the sequence of repetition code $= \frac{1}{n}$

as $n \uparrow$ rate $\rightarrow 0$

$$\text{rate} = \frac{\text{no. of information bits}}{\text{no. of coded bits}}$$

$0 \rightarrow \underbrace{000}_{\text{no. of coded bits} = 3}$

no. of info bits = 1

Shannon told.

$P_e^{(n)} \downarrow \approx 0$

and rate $\leq$ constant

channel capacity

Shannon told probability of error can be made as ~~to~~ close to zero and rate as close to a constant (not zero) called channel capacity.

<u>Codebook</u> → set of codewords which we are transmitting.

Codebook $\propto n$.

Let's say channel capacity is 0.75 bits/channel, use

rate $= R \approx 0.75$
$= 0.7499$.

$n = 3$, $n = 5$, $n = 7$

2^{nR} → codewords

$$\frac{\log 2^{nR}}{n} = \frac{nR}{n} = R$$

Consider a random source.

Channel ~~ed~~ encoder has → 00, 01, 10, 11

00 → 000
01 → 010
10 → 110
11 → 111

→ 4 coded words

rate $= 2/3$

If this is not there we can still find the rate

↓

no. of info bits $= \log_2(4)$
$= 2$.

There exist a sequence ~~codewords~~ with size 2^{nR} books

as n is increasing, such that $P_e^{(n)} \downarrow \to 0$.