Distance and curvature

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1 Normals and curvature

There is a deep link between the evolution of the normal along a curve and the curvature.

Let C be a planar curve. To define the curvature at a point, we can consider the case of a straight line. We can admit that in this case the curvature is zero along the line. For a portion of a circle, the curvature is defined to be inversely proportional to the radius of the circle:

$$\kappa = \frac{1}{R} \tag{1}$$

More precisely, for any curve, we can locally approximate a sufficiently regular curve ($\mathcal{C}^2$ is enough) at a point by a tangent circle that best approximates it locally (see Fig. 1). This circle is called an *osculating circle*. The radius of its radius defines the curvature.

Another way to define the curvature is to consider a point along the curve C. The variation of the tangent vector along the curve defines the curvature

$$\kappa = \frac{d\mathbf{T}}{ds}, \tag{2}$$

where s is a parametrisation of the curve. Both definition of the curvature are equivalent (see Fig. 2).

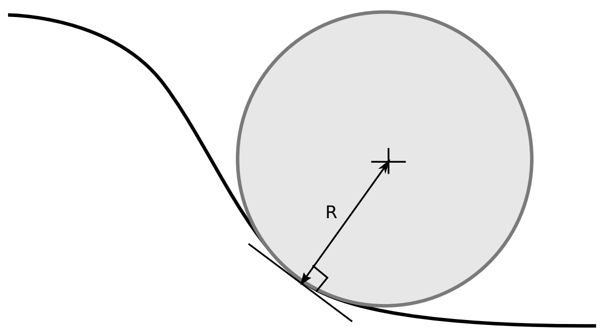

Figure 1: The notion of osculating (best local approximation) defines the curvature as the inverse of the radius of the circle.

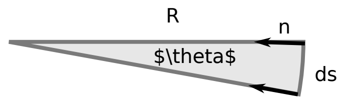

Figure 2: We can write $\sin d\theta \approx d\theta = \frac{ds}{R}$. As ds tends to zero, we have $R = \frac{d\theta}{ds} = \frac{d\mathbf{T}}{ds}$.

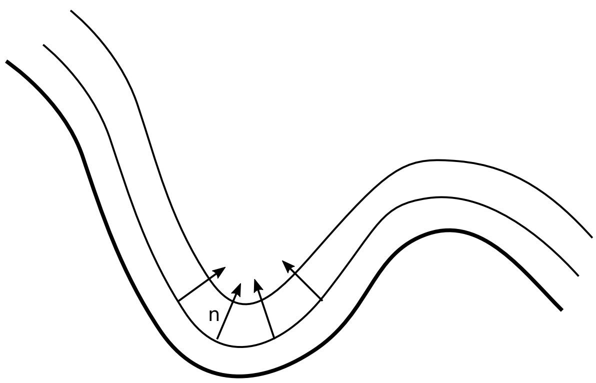

Figure 3: The faster the normal evolves along a curve, the higher the curvature.