Logik

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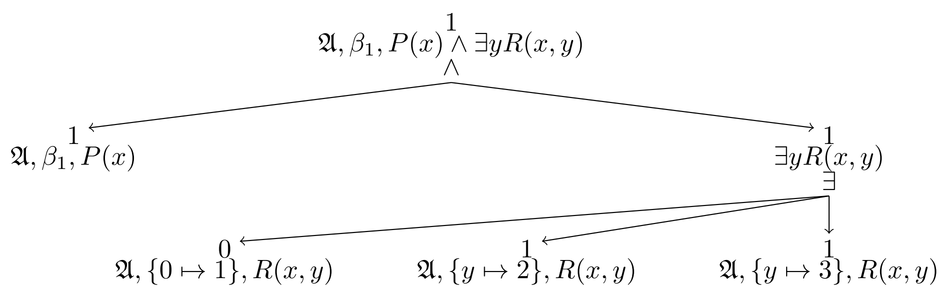
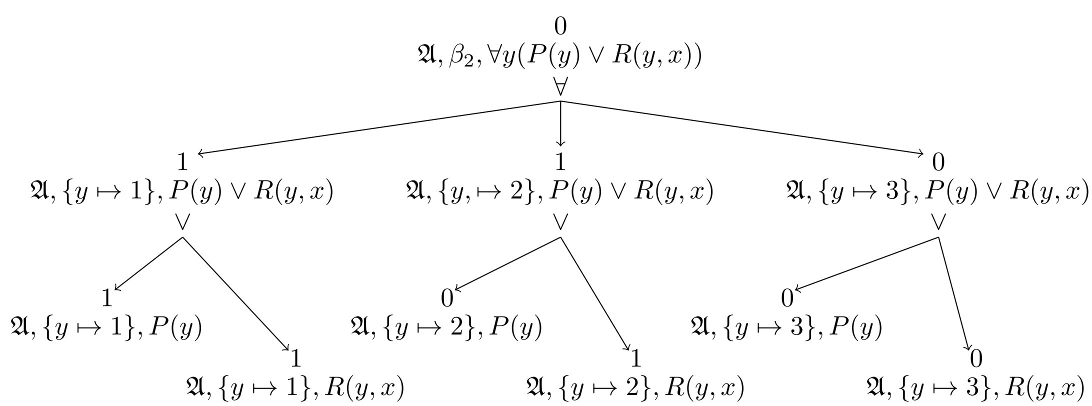
5.

a) test



b) test



6.

a)

$$\begin{aligned} \mathfrak{A} = (\{1,2\}, P^{\mathfrak{A}}, Q^{\mathfrak{A}})\ \textcircled{1}^{P}\ \textcircled{2}^{Q} \\ \varphi = P(x)\ \psi = Q(x) \\ \forall x(P(x) \wedge Q(x))\text{:}\ & x = 1 : P(1) \wedge Q(1) = 1 \wedge 0 = 0 \\ & x = 2 : P(2) \wedge Q(2) = 0 \wedge 1 = 0 \\ = 0 \\ \exists x P(x) \wedge \exists x Q(x) : x = 1; x = 2 \\ P(1) \wedge P(2) = 1 \wedge 1 = 1 \\ = 1 \lightning \end{aligned}$$

b)

$$\begin{aligned} (\forall x\varphi) \wedge (\exists x\varphi) &\equiv \forall x\varphi \\ (\forall x\varphi) \wedge (\exists x\varphi) &\equiv \forall x(\varphi \wedge \exists x\varphi) && \text{T 2.7} \\ &\equiv \forall x \exists x(\varphi \wedge \varphi) && \text{T 2.7} \\ &\equiv \forall x \exists x\varphi \\ &\equiv \forall x\varphi && \text{trivial} \end{aligned}$$

c)

$$\neg\exists x\varphi \equiv \forall x\neg\varphi \models \exists x\neg\varphi$$

d)

$$\begin{aligned} &\beta = (\{1,2\}, P^{\beta}) && \textcircled{1}^{P} && \textcircled{2} \\ &\beta \models \neg\forall x P(x) && P(1) = 1 \\ & && P(2) = 0\checkmark \\ &\beta \models \forall x\neg P(x) && \neg P(1) = 0 \\ & && \neg P(2) = 1\lightning \\ \Longrightarrow\ & \neg\forall x\varphi \# \forall x\neg\varphi \end{aligned}$$

7.

$$\varphi(x_4) = \exists x_1((\neg\forall x_4 S(x_1,x_4)) \vee \forall x_3(\neg\exists x_2 R(x_2,x_3) \vee S(x_4,x_3))) \equiv$$

a)

$$\begin{aligned}
&\equiv \exists x_1((\neg\forall x_4 S(x_1,x_4)) \vee \forall x_3(\neg\exists x_2(R(x_2,x_3) \vee S(x_4,x_3)))) \equiv\\
&\equiv \exists x_1\forall x_3((\neg\forall x_4 S(x_1,x_4)) \vee (\neg\exists x_2(Rx_2,x_3) \vee S(x_4,x_3))) \equiv\\
&\equiv \exists x_1\forall x_3(\neg\exists x_2((\neg\forall x_4 S(x_1,x_4)) \vee (R(x_2,x_3) \vee S(x_4,x_3))))\\
&\equiv \exists x_1\forall x_3\neg\exists x_2(\neg\forall x_4((S(x_1,x_4)) \vee (R(x_2,x_3) \vee S(x_4,x_3)))) \equiv\\
&\equiv \exists x_1\forall x_3\neg\exists x_2\neg\forall x_4(S(x_1,x_4) \vee R(x_2,x_3) \vee S(x_4,x_3)) \qquad = \varphi_P
\end{aligned}$$

b)

$$\begin{aligned}
\varphi_P =& \exists x_1\forall x_3\neg\exists x_2\neg\forall x_4(S(x_1,x_4) \vee R(x_2,x_3) \vee S(x_4,x_3)) \equiv\\
\equiv& \exists x_1\forall x_3\forall x_2\forall x_4(S(x_1,x_4) \vee R(x_2,x_3) \vee S(x_4,x_3))\\
\text{zu } & \forall x_3\forall x_2\forall x_4(S(f_1,x_4) \vee R(x_2,x_3) \vee S(x_4,x_3)) \qquad = \varphi_S
\end{aligned}$$

8.

φ_1 : Unerfüllbar. c und d sind als Konstanten Teil des Universums, da über x und y all-quantifiziert wird, muss folgender Fall eintreten: $c = x$ und $d = y$. In diesem Fall gilt: $R(c, d) \wedge \neg R(x, y) \equiv \psi \wedge \neg \psi$. Das ist eine Kontradiktion und somit ist φ_1 unerfüllbar.

φ_2 : $\mathfrak{A}_2 = (A, R^{\mathfrak{A}}, c^{\mathfrak{A}}, d^{\mathfrak{A}})$
$A = \{c, d\}, R^{\mathfrak{A}} = \{(c, c), (d, d), (c, d), (d, c)\}, c^{\mathfrak{A}} = c, d^{\mathfrak{A}} = d$

φ_3 : $\mathfrak{A}_2 = (A, R^{\mathfrak{A}}, c^{\mathfrak{A}}, d^{\mathfrak{A}}, f^{\mathfrak{A}})$
$A = \{c, d\}, R^{\mathfrak{A}} = \{(c, c), (d, d), (c, d)\}, c^{\mathfrak{A}} = c, d^{\mathfrak{A}} = d, f^{\mathfrak{A}} = \mathrm{id}_A$