Berechenbarkeit

Vorlesung 3: Mächtigkeit Turingmaschine

18. April 2024

Termine — Modul Berechenbarkeit

Übungen	Vorlesung
16.4. Übung 1 B-Woche	18.4. Turingmaschine II
23.4. Übung 1 A-Woche	25.4. Loop-Programme (Übungsblatt 2)
30.4. Übung 2 B-Woche (Mittwoch Feiertag)	2.5. While-Programme
7.5. Übung 2 A-Woche	9.5. (Übungsblatt 3)
14.5. Übung 3 B-Woche (Montag Feiertag)	16.5. Rekursion I
21.5. Übung 3 A-Woche	23.5. Rekursion II (Übungsblatt 4)

Übungen	Vorlesung
28.5. Übung 4 B-Woche	30.5. Entscheidbarkeit
4.6. Übung 4 A-Woche	6.6. Unentscheidbarkeit (Übungsblatt 5)
11.6. Übung 5 B-Woche	13.6. Spez. Probleme
18.6. Übung 5 A-Woche	20.6. Klasse P (Übungsblatt 6)
25.6. Übung 6 B-Woche	27.6. NP-Vollständigkeit
2.7. Übung 6 A-Woche	4.7. Komplexitätsklassen

Definition (§2.4 Turingmaschine; engl. *Turing machine*)

Turingmaschine ist Tupel $M = (Q, \Sigma, \Gamma, \Delta, \square, q_0, q_+, q_-)$

- endl. Menge Q von Zuständen (engl. *states*) mit $Q \cap \Gamma = \emptyset$
- endl. Menge Σ von Eingabesymbolen (engl. *input symbols*)
- endl. Menge Γ von Arbeitssymbolen (engl. *work symbols*) mit $\Sigma \subseteq \Gamma$
- Übergangsrelation (engl. *transition relation*)
  $\Delta \subseteq \Big(\big(Q \setminus \{q_+, q_-\}\big) \times \Gamma\Big) \times \Big(Q \times \Gamma \times \{\triangleleft, \triangleright, \diamond\}\Big)$
- Leersymbol (engl. *blank*) $\square \in \Gamma \setminus \Sigma$ $\qquad (\Gamma_M = \Gamma \setminus \{\square\})$
- Startzustand (engl. *initial state*) $q_0 \in Q$
- Akzeptierender Zustand (engl. *accepting state*) $q_+ \in Q$
- Ablehnender Zustand (engl. *rejecting state*) $q_- \in Q$

$\triangleleft$ = gehe nach links; $\triangleright$ = gehe nach rechts; $\diamond$ = keine Bewegung

Notizen

- Transformationssemantik für Berechnung Funktionen & Modularität
- Eingabe übersetzt in Bandinhalt bei Akzeptanz
  - Band vor Kopf leer
  - Ausgabe beginnend unter Kopf bis zum ersten $\square$
  - Band dahinter leer

Turingmaschine

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- Transformationssemantik für Berechnung Funktionen & Modularität
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- Beispiel §2.5 aus VL 2 berechnet

$$\{(ww^R, \varepsilon) \mid w \in \{a, b\}^*\}$$

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$$\{(ww^R, \varepsilon) \mid w \in \{a, b\}^*\}$$

§3.1 Definition (Transformationssemantik; engl. *input-output relation*)

Sei $M = (Q, \Sigma, \Gamma, \Delta, \square, q_0, q_+, q_-)$ TM und $\Gamma_M = \Gamma \setminus \{\square\}$

$$T(M) = \{(w, u) \in \Sigma^* \times \Gamma_M^* \mid \exists x, y \in \{\square\}^* \colon \varepsilon\ q_0\ w\square \vdash_M^* x\ q_+\ uy\}$$

§3.2 Theorem (Vereinigung)

Seien $M_1 = (Q, \Sigma, \Gamma, \Delta, \square, q_0, q_+, q_-)$ und $M_2 = (P, \Sigma, \Gamma, \nabla, \square, p_0, p_+, p_-)$ 2 Turingmaschinen. Dann existiert TM M mit

$$L(M) = L(M_1) \cup L(M_2) \qquad \text{und} \qquad T(M) = T(M_1) \cup T(M_2)$$

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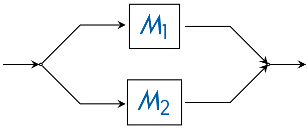
Beweisansatz

1. Nutze neuen Startzustand r_0
2. Neue Übergänge ohne Bandänderung zu alten Startzuständen q_0 und p_0



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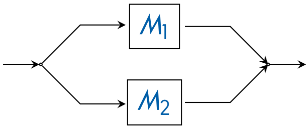
<u>Beweisansatz</u>

1. Nutze neuen Startzustand r_0
2. Neue Übergänge ohne Bandänderung zu alten Startzuständen q_0 und p_0
3. M_1 und M_2 laufen normal, wobei alle Übergänge in p_+ oder p_- gehen stattdessen in q_+ bzw. q_-



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Beweis.

OBdA sei $Q \cap P = \emptyset$ und $r_0 \notin Q \cup P$. Konstruiere TM

$$
\begin{aligned}
M &= \big(Q \cup P \cup \{r_0\}, \Sigma, \Gamma, \Delta \cup \nabla \cup R, \square, r_0, q_+, q_-\big) \\
R &= \big\{(r_0, \gamma) \to (q_0, \gamma, \diamond) \mid \gamma \in \Gamma\big\} \cup \\
&\quad \big\{(r_0, \gamma) \to (p_0, \gamma, \diamond) \mid \gamma \in \Gamma\big\} \cup \\
&\quad \big\{(p, \gamma) \to (q_+, \gamma', d) \mid (p, \gamma) \to (p_+, \gamma', d) \in \nabla\big\} \cup \\
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Dann $L(M) = L(M_1) \cup L(M_2)$ und $T(M) = T(M_1) \cup T(M_2)$ □

$\Gamma_M = \Gamma \setminus \{\Box\}$

§3.3 Definition (normierte TM; engl. *standardized TM*)

TM $M = (Q, \Sigma, \Gamma, \Delta, \Box, q_0, q_+, q_-)$ normiert (engl. *standardized*), falls $u \in \{\Box\}^*$ und $v \in \Gamma_M^*\{\Box\}^*$ für alle $w \in \Sigma^*$, $u, v \in \Gamma^*$ mit $\varepsilon\, q_0\, w\Box \vdash_M^* u\, q_+\, v$

Operationen auf Turingmaschinen

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<u>Notizen</u>

- Normierte TM kann nur akzeptieren, falls Band links des Kopfes aus $\{\Box\}^*$ und Band unter und rechts des Kopfes aus $\Gamma_M^*\{\Box\}^*$
- Wir konstruieren meist normierte TM
- Vereinigung normierter TM gemäß Theorem §3.2 ist normiert

§3.4 Definition (Verkettung; engl. *composition*)

Verkettung (oder Komposition; engl. *composition*) $R_1 \mathbin{;} R_2$
zweier Relationen $R_1 \subseteq A \times B$ und $R_2 \subseteq B \times C$ ist

$$R_1 \mathbin{;} R_2 = \big\{(a, c) \in A \times C \mid \exists b \in B \colon (a, b) \in R_1,\ (b, c) \in R_2\big\}$$

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Notizen

- Reihenschaltung (Hintereinanderschaltung)
- Erhalten für $\text{verdoppeln} = \big\{(n, 2n) \mid n \in \mathbb{N}\big\} \subseteq \mathbb{N} \times \mathbb{N}$

$$\text{verdoppeln} \,;\, \text{verdoppeln} = \big\{(n, 4n) \mid n \in \mathbb{N}\big\}$$

$\Gamma_{M_1} = \Gamma \setminus \{\Box\}$

§3.5 Theorem (Verkettung)

Seien $M_1 = (Q, \Sigma, \Gamma, \Delta, \Box, q_0, q_+, q_-)$ und $M_2 = (P, \Gamma_{M_1}, \Psi, \nabla, \Box, p_0, p_+, p_-)$ zwei TM mit M_1 normiert. Dann existiert TM M mit $T(M) = T(M_1) \,;\, T(M_2)$. Falls M_2 normiert ist, dann ist M normiert.

Operationen auf Turingmaschinen

$\Gamma_{M_1} = \Gamma \setminus \{\square\}$

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<u>Beweisansatz</u>

1. Starte M_1

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Beweisansatz

1. Starte M_1
2. Starte M_2 bei Akzeptanz von M_1
   (Normierung erzeugt Ausgangssituation)

Operationen auf Turingmaschinen

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<u>Beweisansatz</u>

1. Starte M_1
2. Starte M_2 bei Akzeptanz von M_1
   (Normierung erzeugt Ausgangssituation)
3. M_2 läuft normal

$$\longrightarrow \boxed{M_1} \longrightarrow \boxed{M_2} \longrightarrow$$

§3.5 Theorem (Verkettung)

Seien $M_1 = (Q, \Sigma, \Gamma, \Delta, \square, q_0, q_+, q_-)$ und $M_2 = (P, \Gamma_M, \Psi, \nabla, \square, p_0, p_+, p_-)$ zwei TM mit M_1 normiert. Dann existiert TM M mit $T(M) = T(M_1)\,;\,T(M_2)$. Falls M_2 normiert ist, dann ist M normiert.

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Beweis.

OBdA. sei $Q \cap P = \emptyset$. Wir konstruieren TM

$$M = (Q \cup P, \Sigma, \Psi, \Delta \cup \nabla \cup R, \square, q_0, p_+, p_-)$$
$$R = \{(q_+, \gamma) \to (p_0, \gamma, \diamond) \mid \gamma \in \Gamma\}$$

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Dann $T(M) = T(M_1) \,;\, T(M_2)$ $\Box$

§3.6 Definition (Iteration)

Iteration R^* (reflexive, transitive Hülle; engl. *iteration*) der Relation $R \subseteq A \times A$

$$R^* = \bigcup_{n \in \mathbb{N}} R^n \qquad \text{mit} \qquad R^0 = \mathrm{id}_A \qquad \text{und} \qquad R^{n+1} = R^n \,;\, R$$

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<u>Notizen</u>

- Beliebig häufige Wiederholung der Relation
- Erhalten für $\text{verdoppeln} = \{(n, 2n) \mid n \in \mathbb{N}\} \subseteq \mathbb{N} \times \mathbb{N}$

$$\text{verdoppeln}^* = \{(n, 2^m \cdot n) \mid m, n \in \mathbb{N}\}$$

§3.7 Theorem (Iteration)

Sei $M = (Q, \Gamma_M, \Gamma, \Delta, \square, q_0, q_+, q_-)$ normierte TM.
Dann existiert normierte TM N mit $T(N) = T(M)^*$

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Beweisansatz

1. Nutze neuen Startzustand p_0
   und neuen Akzeptanzzustand p_+
2. Übergang von p_0 zu p_+ (Abbruch)

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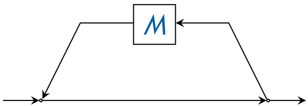
Beweisansatz

1. Nutze neuen Startzustand p_0 und neuen Akzeptanzzustand p_+
2. Übergang von p_0 zu p_+ (Abbruch)
3. Übergang von p_0 zu q_0 (Iteration)
4. M läuft normal; bei Erreichen von q_+ zurück in Startzustand p_0



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Beweis.

Seien $p_0 \notin Q$ und $p_+ \notin Q$ mit $p_0 \neq p_+$. Wir konstruieren TM

$$
\begin{aligned}
N &= \big(Q \cup \{p_0, p_+\}, \Gamma_M, \Gamma, \Delta \cup R, \Box, p_0, p_+, q_-\big) \\
R &= \big\{(p_0, \gamma) \to (p_+, \gamma, \diamond) \mid \gamma \in \Gamma\big\} \cup \\
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Operationen auf Turingmaschinen

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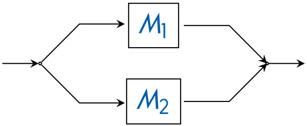
Seien $p_0 \notin Q$ und $p_+ \notin Q$ mit $p_0 \neq p_+$. Wir konstruieren TM

$$N = \big(Q \cup \{p_0, p_+\}, \Gamma_M, \Gamma, \Delta \cup R, \square, p_0, p_+, q_-\big)$$
$$\begin{aligned} R = {} & \big\{(p_0, \gamma) \to (p_+, \gamma, \diamond) \mid \gamma \in \Gamma\big\} \cup \\ & \big\{(p_0, \gamma) \to (q_0, \gamma, \diamond) \mid \gamma \in \Gamma\big\} \cup \\ & \big\{(q_+, \gamma) \to (p_0, \gamma, \diamond) \mid \gamma \in \Gamma\big\} \end{aligned}$$

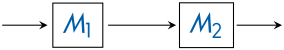
Dann $T(N) = T(M)^*$ $\square$

Operationen

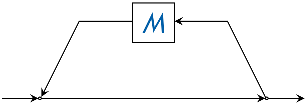
- Vereinigung



- Verkettung



- Iteration

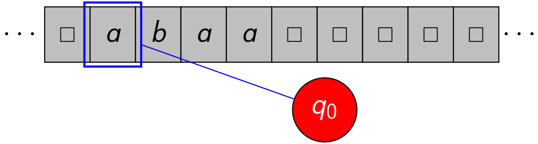


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

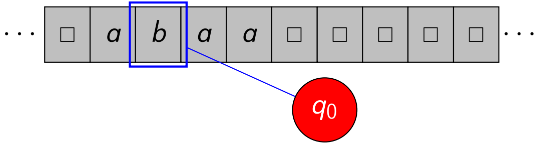
$$\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

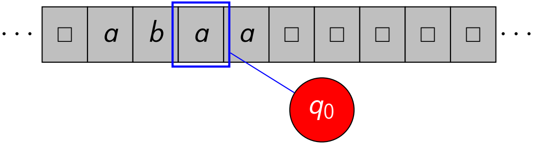
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \Box) \to (q_1, \Box, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \Box) \to (q_2, \Box, \triangleright)$	$(q_a, \Box) \to (q_*, a, \triangleleft)$	$(q_b, \Box) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \Box, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

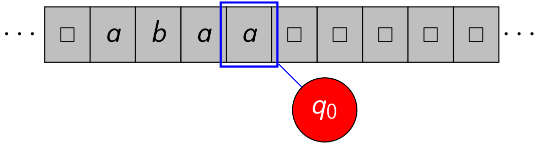
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \square) \to (q_1, \square, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \square) \to (q_2, \square, \triangleright)$	$(q_a, \square) \to (q_*, a, \triangleleft)$	$(q_b, \square) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \square, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

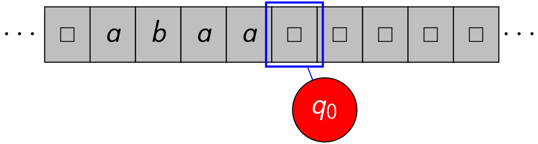
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \Box) \to (q_1, \Box, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \Box) \to (q_2, \Box, \triangleright)$	$(q_a, \Box) \to (q_*, a, \triangleleft)$	$(q_b, \Box) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \Box, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

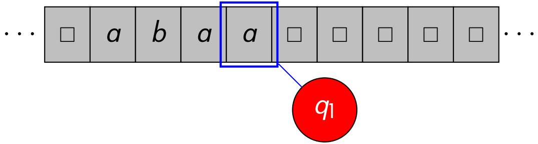
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

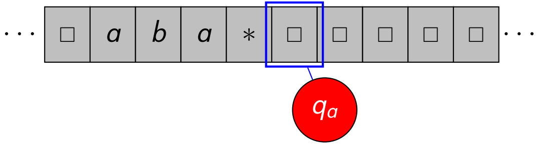
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

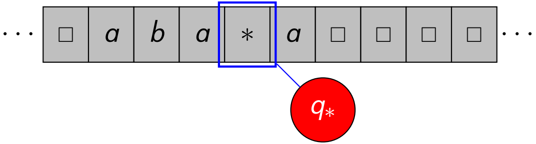
$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

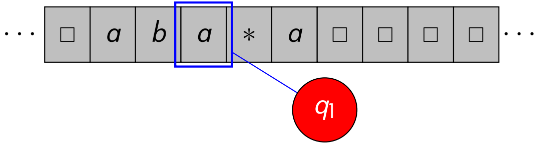
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \square) \to (q_1, \square, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \square) \to (q_2, \square, \triangleright)$	$(q_a, \square) \to (q_*, a, \triangleleft)$	$(q_b, \square) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \square, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

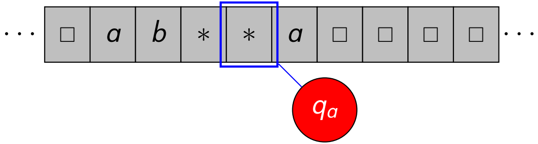
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \Box) \to (q_1, \Box, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \Box) \to (q_2, \Box, \triangleright)$	$(q_a, \Box) \to (q_*, a, \triangleleft)$	$(q_b, \Box) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \Box, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

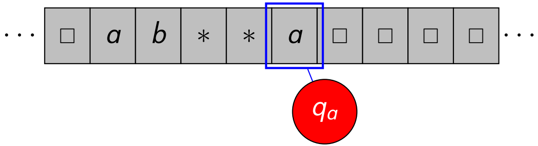
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

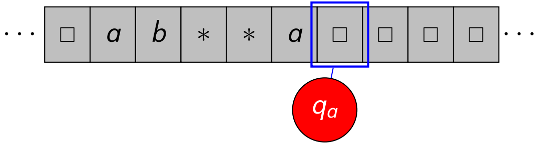
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

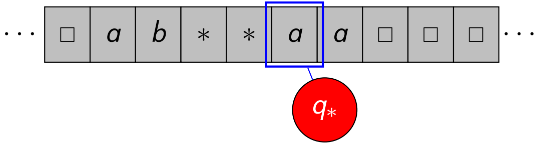
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \square) \to (q_1, \square, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \square) \to (q_2, \square, \triangleright)$	$(q_a, \square) \to (q_*, a, \triangleleft)$	$(q_b, \square) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \square, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

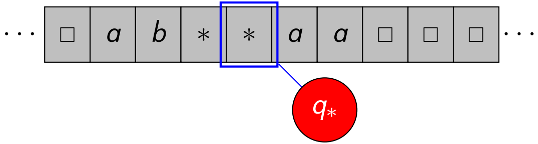
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

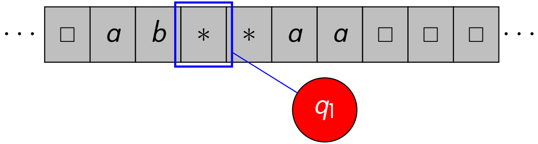
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

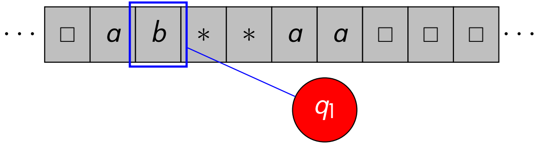
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \Box) \to (q_1, \Box, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \Box) \to (q_2, \Box, \triangleright)$	$(q_a, \Box) \to (q_*, a, \triangleleft)$	$(q_b, \Box) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \Box, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \Box) \to (q_1, \Box, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \Box) \to (q_2, \Box, \triangleright)$	$(q_a, \Box) \to (q_*, a, \triangleleft)$	$(q_b, \Box) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \Box, \triangleright)$

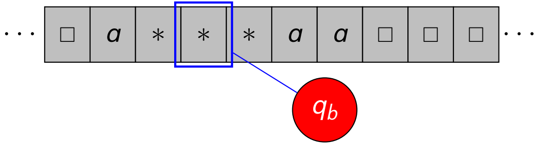


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$

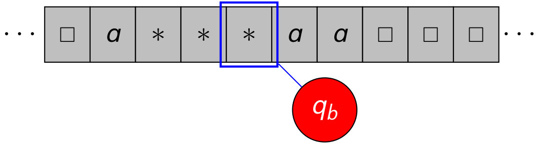


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

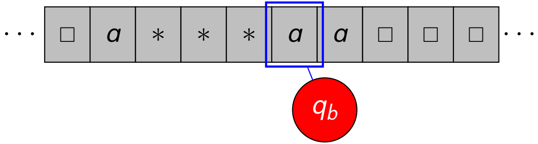
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

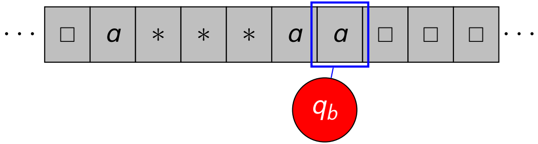
$$\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

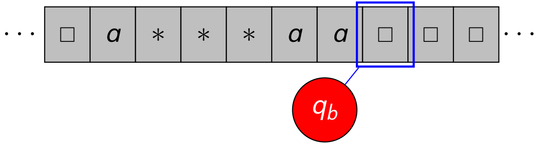
$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

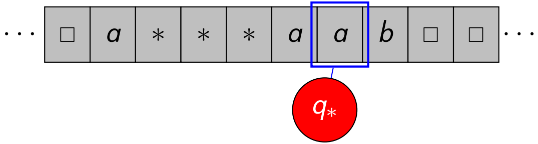
$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

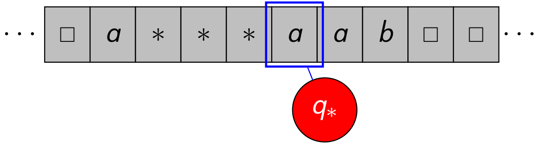
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \Box) \to (q_1, \Box, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \Box) \to (q_2, \Box, \triangleright)$	$(q_a, \Box) \to (q_*, a, \triangleleft)$	$(q_b, \Box) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \Box, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

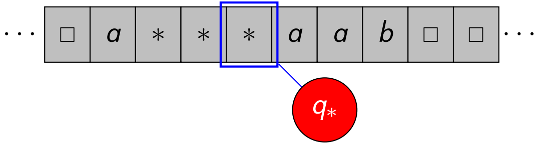
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

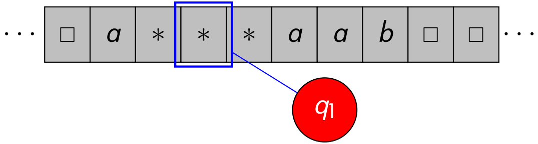
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

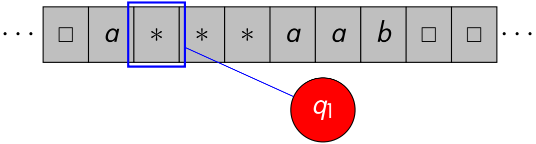
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

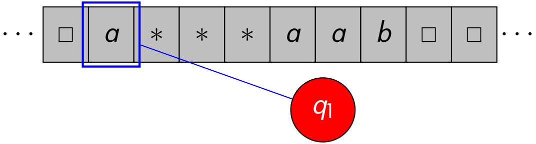
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

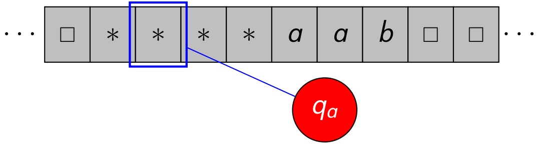
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

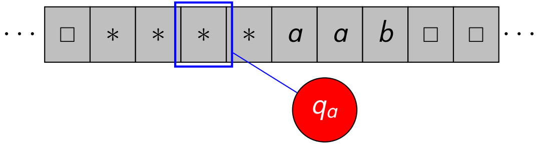
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

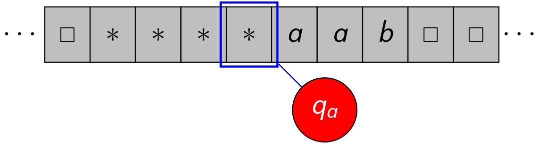
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \square) \to (q_1, \square, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \square) \to (q_2, \square, \triangleright)$	$(q_a, \square) \to (q_*, a, \triangleleft)$	$(q_b, \square) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \square, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

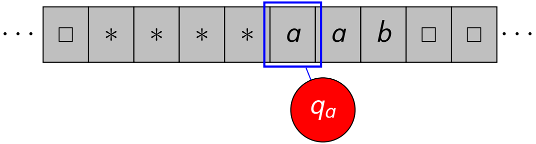
$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \square) \to (q_1, \square, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \square) \to (q_2, \square, \triangleright)$	$(q_a, \square) \to (q_*, a, \triangleleft)$	$(q_b, \square) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \square, \triangleright)$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$

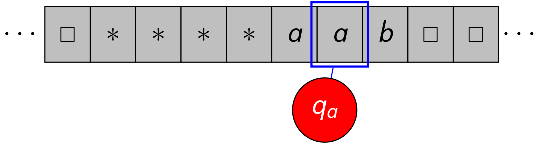


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

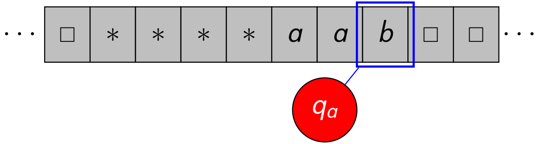
$$\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

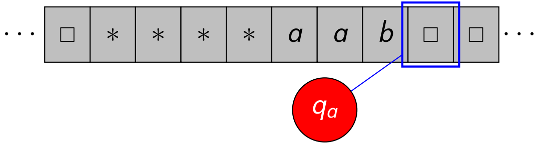
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

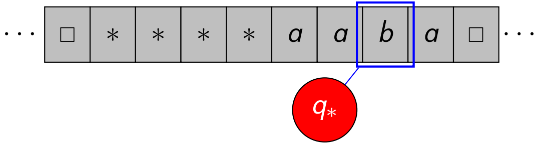
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$

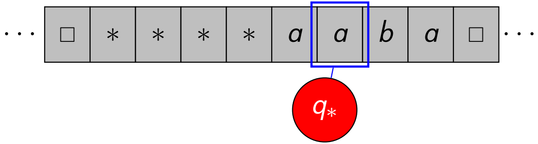


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$

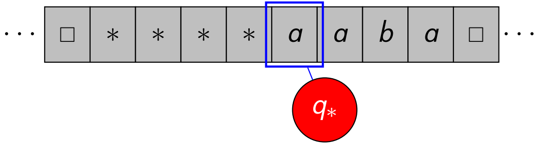


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

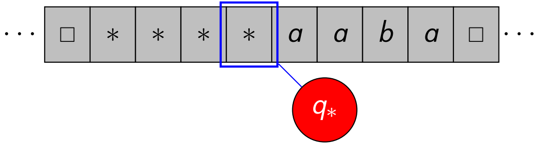
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
\color{red}{(q_*, a) \to (q_*, a, \triangleleft)} & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

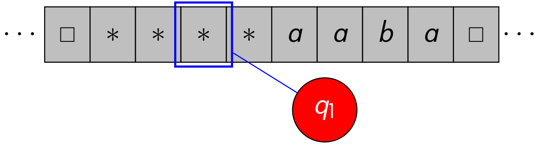
$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

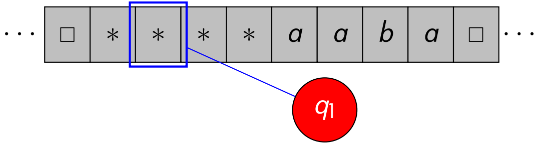
$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

$(q_0, a) \to (q_0, a, \triangleright)$	$(q_0, b) \to (q_0, b, \triangleright)$	$(q_0, \square) \to (q_1, \square, \triangleleft)$
$(q_1, a) \to (q_a, *, \triangleright)$	$(q_1, b) \to (q_b, *, \triangleright)$	$(q_1, *) \to (q_1, *, \triangleleft)$
$(q_1, \square) \to (q_2, \square, \triangleright)$	$(q_a, \square) \to (q_*, a, \triangleleft)$	$(q_b, \square) \to (q_*, b, \triangleleft)$
$(q_a, a) \to (q_a, a, \triangleright)$	$(q_a, b) \to (q_a, b, \triangleright)$	$(q_a, *) \to (q_a, *, \triangleright)$
$(q_b, a) \to (q_b, a, \triangleright)$	$(q_b, b) \to (q_b, b, \triangleright)$	$(q_b, *) \to (q_b, *, \triangleright)$
$(q_*, a) \to (q_*, a, \triangleleft)$	$(q_*, b) \to (q_*, b, \triangleleft)$	$(q_*, *) \to (q_1, *, \triangleleft)$
$(q_2, a) \to (q_+, a, \diamond)$	$(q_2, b) \to (q_+, b, \diamond)$	$(q_2, *) \to (q_2, \square, \triangleright)$

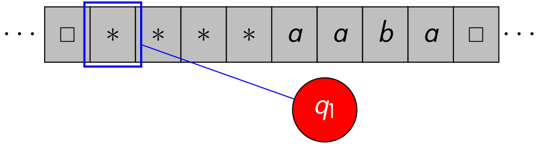


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

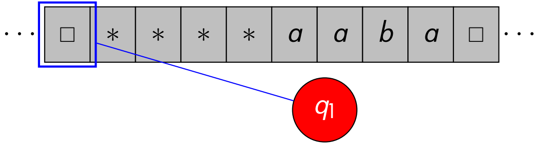
$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

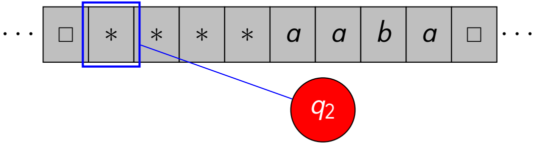
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
\color{red}{(q_1, \square) \to (q_2, \square, \triangleright)} & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$

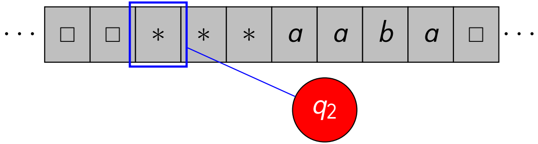


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

$$
\begin{array}{ccc}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$

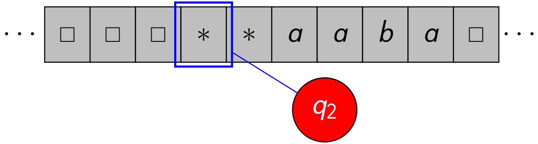


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \rhd) & (q_0, b) \to (q_0, b, \rhd) & (q_0, \square) \to (q_1, \square, \lhd) \\
(q_1, a) \to (q_a, *, \rhd) & (q_1, b) \to (q_b, *, \rhd) & (q_1, *) \to (q_1, *, \lhd) \\
(q_1, \square) \to (q_2, \square, \rhd) & (q_a, \square) \to (q_*, a, \lhd) & (q_b, \square) \to (q_*, b, \lhd) \\
(q_a, a) \to (q_a, a, \rhd) & (q_a, b) \to (q_a, b, \rhd) & (q_a, *) \to (q_a, *, \rhd) \\
(q_b, a) \to (q_b, a, \rhd) & (q_b, b) \to (q_b, b, \rhd) & (q_b, *) \to (q_b, *, \rhd) \\
(q_*, a) \to (q_*, a, \lhd) & (q_*, b) \to (q_*, b, \lhd) & (q_*, *) \to (q_1, *, \lhd) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \rhd)
\end{array}
$$

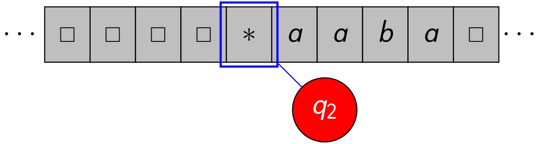


Mehrband-Turingmaschinen

§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

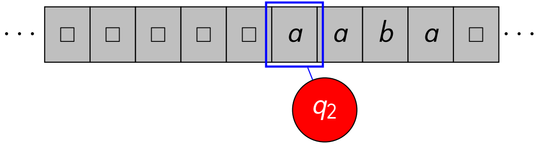
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

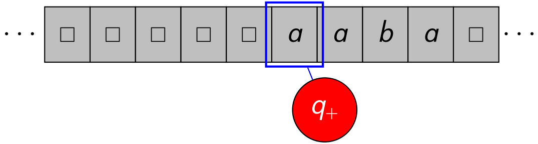
$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \Box) \to (q_1, \Box, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \Box) \to (q_2, \Box, \triangleright) & (q_a, \Box) \to (q_*, a, \triangleleft) & (q_b, \Box) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \Box, \triangleright)
\end{array}
$$



§3.8 Beispiel (Reversal-Turingmaschine)

TM $M = (\{q_0, q_1, q_a, q_b, q_*, q_2, q_+, q_-\}, \{a, b\}, \{a, b, *, \square\}, \Delta, \square, q_0, q_+, q_-)$

$$
\begin{array}{lll}
(q_0, a) \to (q_0, a, \triangleright) & (q_0, b) \to (q_0, b, \triangleright) & (q_0, \square) \to (q_1, \square, \triangleleft) \\
(q_1, a) \to (q_a, *, \triangleright) & (q_1, b) \to (q_b, *, \triangleright) & (q_1, *) \to (q_1, *, \triangleleft) \\
(q_1, \square) \to (q_2, \square, \triangleright) & (q_a, \square) \to (q_*, a, \triangleleft) & (q_b, \square) \to (q_*, b, \triangleleft) \\
(q_a, a) \to (q_a, a, \triangleright) & (q_a, b) \to (q_a, b, \triangleright) & (q_a, *) \to (q_a, *, \triangleright) \\
(q_b, a) \to (q_b, a, \triangleright) & (q_b, b) \to (q_b, b, \triangleright) & (q_b, *) \to (q_b, *, \triangleright) \\
(q_*, a) \to (q_*, a, \triangleleft) & (q_*, b) \to (q_*, b, \triangleleft) & (q_*, *) \to (q_1, *, \triangleleft) \\
(q_2, a) \to (q_+, a, \diamond) & (q_2, b) \to (q_+, b, \diamond) & (q_2, *) \to (q_2, \square, \triangleright)
\end{array}
$$



Notizen

- Viele Operationen nötig für Navigation
- Oft viele Läufe zwischen Ein- & Ausgabe nötig

Notizen

- Viele Operationen nötig für Navigation
- Oft viele Läufe zwischen Ein- & Ausgabe nötig
- Erhöhter Komfort durch mehrere Bänder (und intuitiver)

§3.9 Definition (k-Band-Turingmaschine; engl. *k-tape Turing machine*)

k-Band-Turingmaschine ist Tupel $M = (Q, \Sigma, \Gamma, \Delta, \square, q_0, q_+, q_-)$

- endl. Menge Q von Zuständen mit $Q \cap \Gamma = \emptyset$
- endl. Menge Σ von Eingabesymbolen
- endl. Menge Γ von Arbeitssymbolen mit $\Sigma \subseteq \Gamma$
- Übergangsrelation $\Delta \subseteq \Big((Q \setminus \{q_+, q_-\}) \times \Gamma^k\Big) \times \Big(Q \times (\Gamma \times \{\triangleleft, \triangleright, \diamond\})^k\Big)$
- Leersymbol $\square \in \Gamma \setminus \Sigma$ $\qquad$ $(\Gamma_M = \Gamma \setminus \{\square\})$
- Startzustand $q_0 \in Q$
- Akzeptierender Zustand $q_+ \in Q$
- Ablehnender Zustand $q_- \in Q$

$\triangleleft$ = gehe nach links; $\triangleright$ = gehe nach rechts; $\diamond$ = keine Bewegung

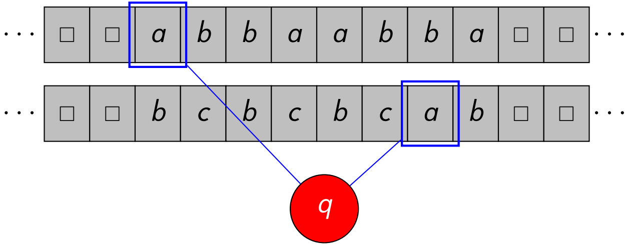
Notizen

- k Arbeitsbänder (gleiches Arbeitsalphabet)
- k unabhängige Lese- & Schreibköpfe (unabhängig beweglich)

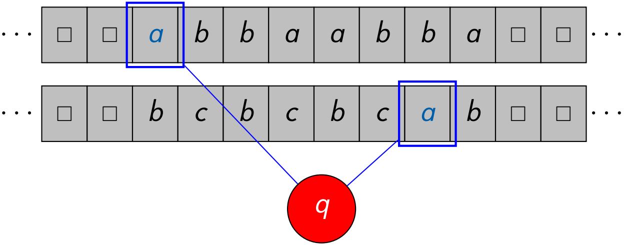
Mehrband-Turingmaschinen

<u>Notizen</u>

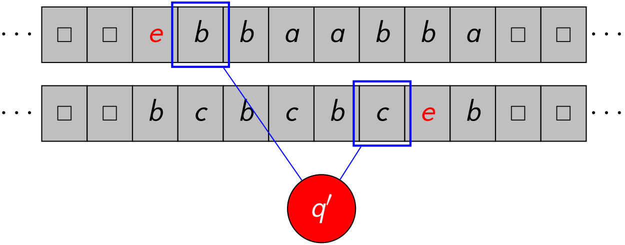
- k Arbeitsbänder (gleiches Arbeitsalphabet)
- k unabhängige Lese- & Schreibköpfe (unabhängig beweglich)
- Übergänge $\tau \in \Big(\big(Q \setminus \{q_+, q_-\}\big) \times \Gamma^k\Big) \times \Big(Q \times \big(\Gamma \times \{\triangleleft, \triangleright, \diamond\}\big)^k\Big)$
  - Aktueller globaler Zustand
  - Inhalt aktuellen Zellen auf allen k Bändern
  - Globaler Zielzustand
  - Neuer Inhalt aller k Zellen
  - k Bewegungsrichtungen für k Köpfe



Mehrband-Turingmaschinen



vom Übergang $(q, \langle a, a \rangle) \to (q', \langle (e, \triangleright), (e, \triangleleft) \rangle)$ überführt in



Mehrband-Turingmaschinen

1. Ausgangssituation
   - Eingabe auf erstem Band; andere Zellen & Bänder enthalten $\square$
   - TM in Startzustand q_0
   - Kopf erstes Band auf erstem Symbol der Eingabe

Mehrband-Turingmaschinen

1. Ausgangssituation
   - Eingabe auf erstem Band; andere Zellen & Bänder enthalten $\square$
   - TM in Startzustand q_0
   - Kopf erstes Band auf erstem Symbol der Eingabe
2. Übergänge gemäß Δ

1. Ausgangssituation
   - Eingabe auf erstem Band; andere Zellen & Bänder enthalten $\Box$
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   - Kopf erstes Band auf erstem Symbol der Eingabe
2. Übergänge gemäß Δ
3. Haltebedingung
   - Aktueller Zustand final; akzeptierend q_+ oder ablehnend q_-
   - Kein passender Übergang $\to$ TM hält nicht ordnungsgemäß

Mehrband-Turingmaschinen

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Akzeptanz Eingabe

Existenz Übergänge von Ausgangssituation in akzeptierenden Zustand

Mehrband-Turingmaschinen

1. Ausgangssituation
   - Eingabe auf erstem Band; andere Zellen & Bänder enthalten $\square$
   - TM in Startzustand q_0
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Akzeptanz Eingabe

Existenz Übergänge von Ausgangssituation in akzeptierenden Zustand

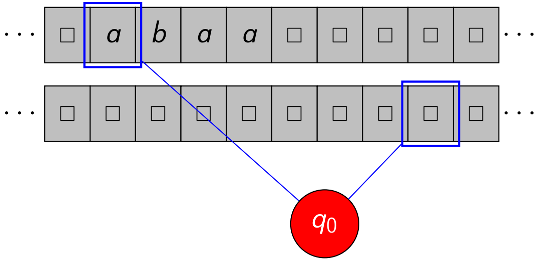
Ausgabe: auf letztem Band (Band k) (normiert mind. auf letztem Band)

§3.10 Beispiel (2-Band-Turingmaschine)

2-Band-TM $M = (\{q_0, q_+, q_-\}, \{a, b\}, \{a, b, \square\}, \Delta, \square, q_0, q_+, q_-)$

$(q_0, \langle a, \square\rangle) \to (q_0, \langle(\square, \triangleright), (a, \triangleleft)\rangle)$ $\quad (q_0, \langle b, \square\rangle) \to (q_0, \langle(\square, \triangleright), (b, \triangleleft)\rangle)$

$(q_0, \langle\square, \square\rangle) \to (q_+, \langle(\square, \diamond), (\square, \triangleright)\rangle)$



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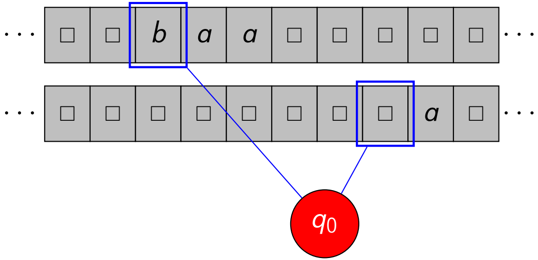

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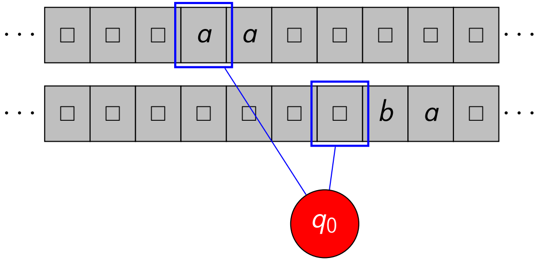

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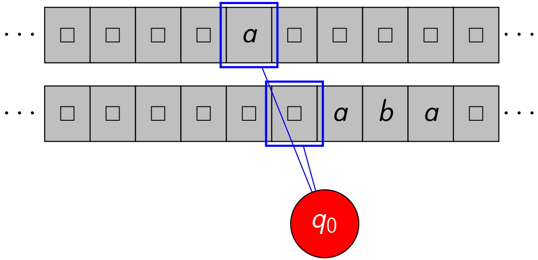

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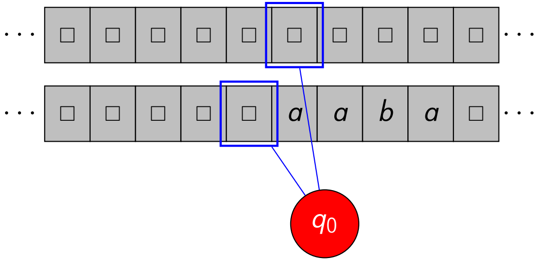
$(q_0, \langle \square, \square \rangle) \to (q_+, \langle (\square, \diamond), (\square, \triangleright) \rangle)$



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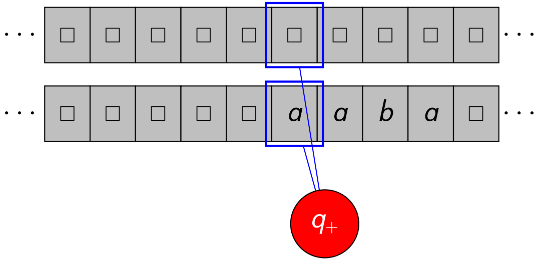
2-Band-TM $M = (\{q_0, q_+, q_-\}, \{a, b\}, \{a, b, \Box\}, \Delta, \Box, q_0, q_+, q_-)$

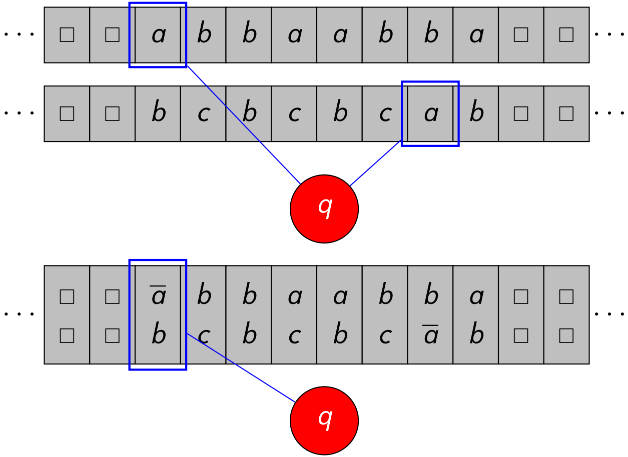
$$(q_0, \langle a, \Box\rangle) \to (q_0, \langle(\Box, \triangleright), (a, \triangleleft)\rangle) \quad (q_0, \langle b, \Box\rangle) \to (q_0, \langle(\Box, \triangleright), (b, \triangleleft)\rangle)$$
$$(q_0, \langle \Box, \Box\rangle) \to (q_+, \langle(\Box, \diamond), (\Box, \triangleright)\rangle)$$



Mehrband-Turingmaschinen

Simulation der k-Band-TM $(Q, \Sigma, \Gamma, \Delta, \square, q_0, q_+, q_-)$ durch TM

- Kodiere k Bänder durch 1 Band $\Gamma' = \Gamma \cup (\Gamma \cup \overline{\Gamma})^k$ (Tupelsymbole)
- Kodierung Position k Köpfe (Überstrich)



Simulation Ableitungsschritt k-Band-TM durch TM

1. Merken aktueller Zustand in Zuständen $(q, p, \dots)$
   1. Zustand q k-Band-TM
   2. Phase p in Bearbeitung mit weiteren Informationen

Mehrband-Turingmaschinen

Simulation Ableitungsschritt k-Band-TM durch TM

1. Merken aktueller Zustand in Zuständen $(q, p, \ldots)$
   1. Zustand q k-Band-TM
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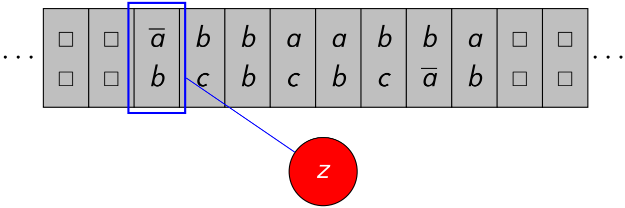
2. Aufsammeln Symbole unter Köpfen durch Ablaufen Band



$z = (q, \text{lese}, \langle \star, \star \rangle)$

Simulation Ableitungsschritt k-Band-TM durch TM

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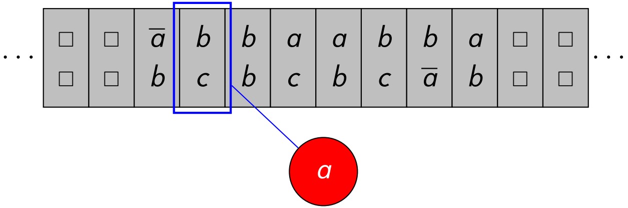

$a = (q, \text{lese}, \langle a, \star \rangle)$

Mehrband-Turingmaschinen

Simulation Ableitungsschritt k-Band-TM durch TM

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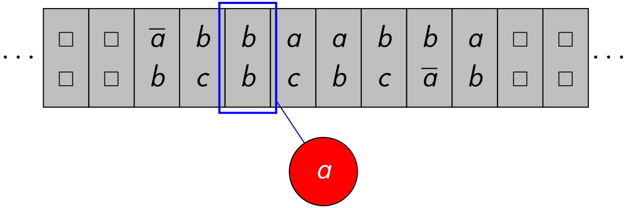

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Simulation Ableitungsschritt k-Band-TM durch TM

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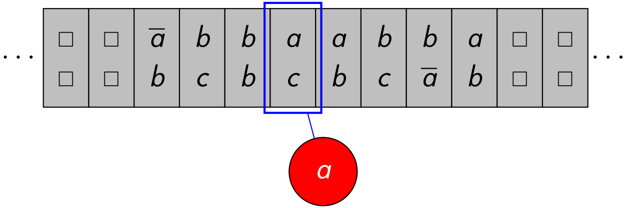

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Mehrband-Turingmaschinen

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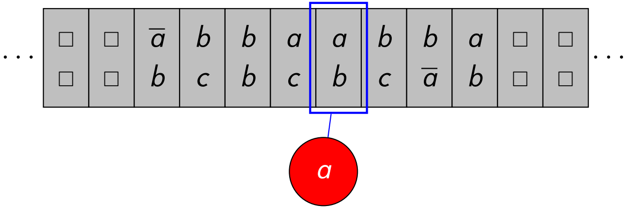

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Mehrband-Turingmaschinen

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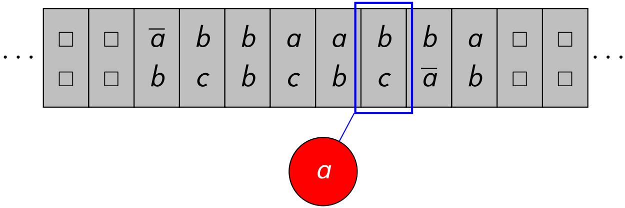

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Mehrband-Turingmaschinen

Simulation Ableitungsschritt k-Band-TM durch TM

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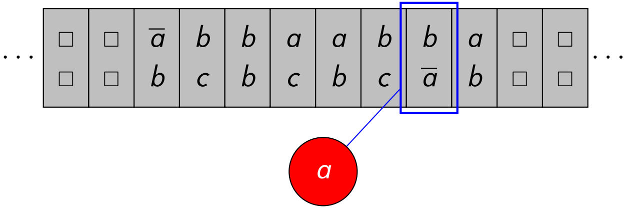

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Mehrband-Turingmaschinen

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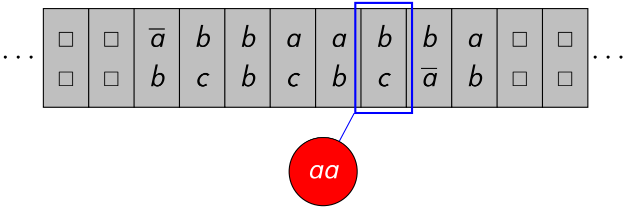

$aa = (q, \text{zurück}, \langle a, a \rangle)$

Mehrband-Turingmaschinen

Simulation Ableitungsschritt k-Band-TM durch TM

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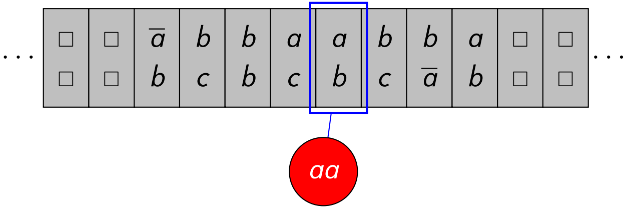

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Mehrband-Turingmaschinen

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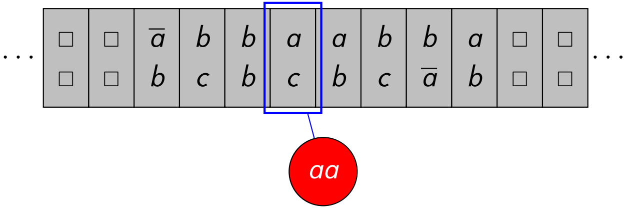

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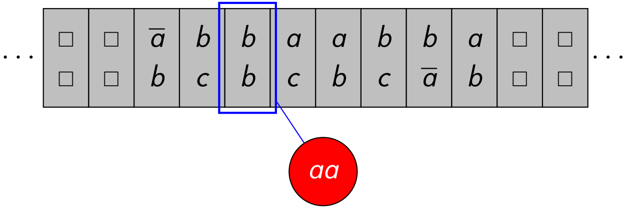

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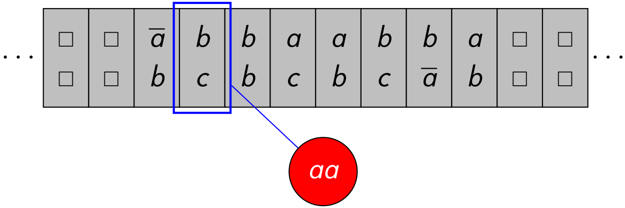

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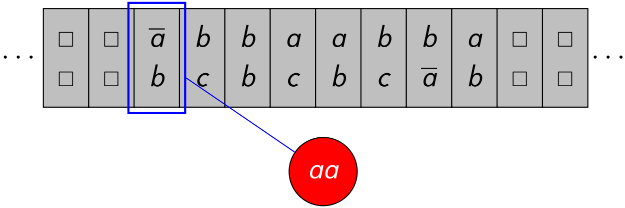

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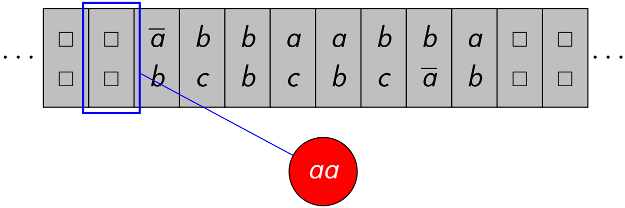

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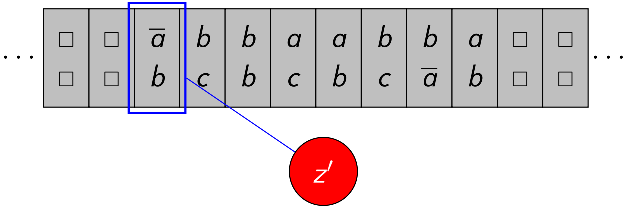


$$z' = (q, \text{wähle}, \langle a, a \rangle)$$

Simulation Ableitungsschritt k-Band-TM durch TM

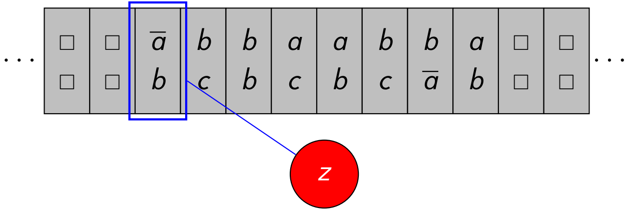
1. ...
2. ...
3. Nichtdeterministische Auswahl passender Übergang

$$\boxed{((q, \text{wähle}, \langle s_1, \ldots, s_k \rangle), \vec{a}) \to ((q', \text{schreibe}, \vec{r}), \vec{a}, \diamond) \in \Delta}$$

für alle Übergänge $(q, \langle s_1, \ldots, s_k \rangle) \to (q', \vec{r})$ der k-Band-TM

Simulation Ableitungsschritt k-Band-TM durch TM

1. ...
2. ...
3. ...
4. Anpassen Arbeitsband (Schreibvorgänge & Bewegungen)



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Simulation Ableitungsschritt k-Band-TM durch TM

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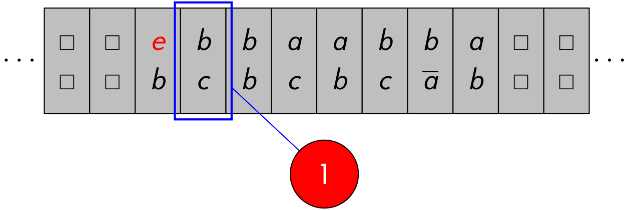

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Mehrband-Turingmaschinen

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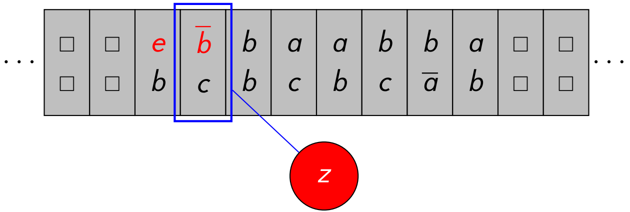

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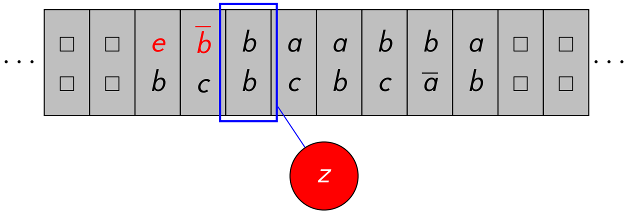

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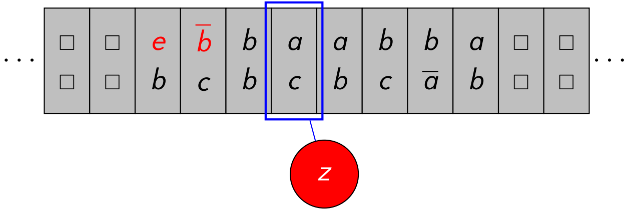

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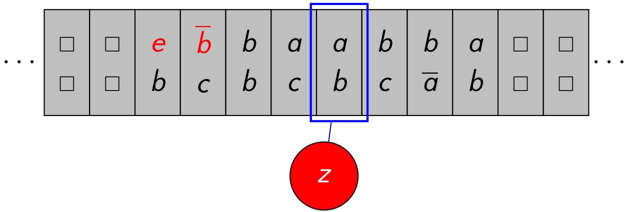

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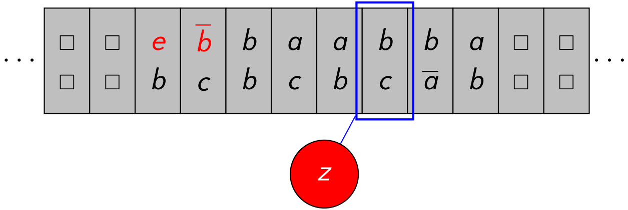

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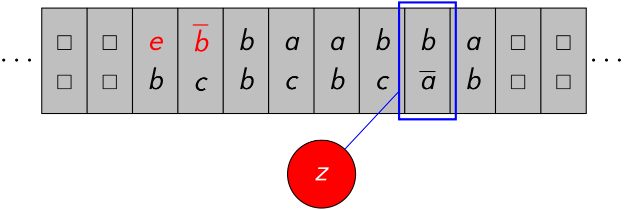

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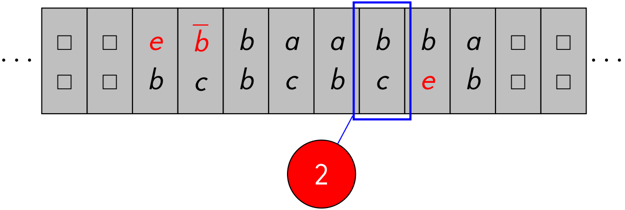

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Mehrband-Turingmaschinen

Simulation Ableitungsschritt k-Band-TM durch TM

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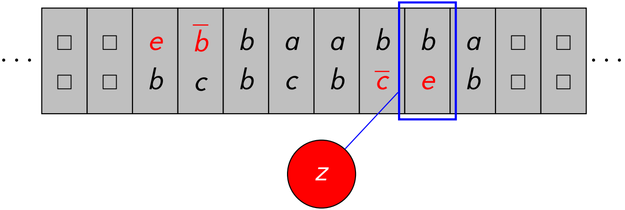

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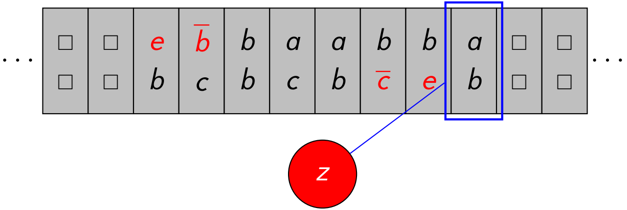

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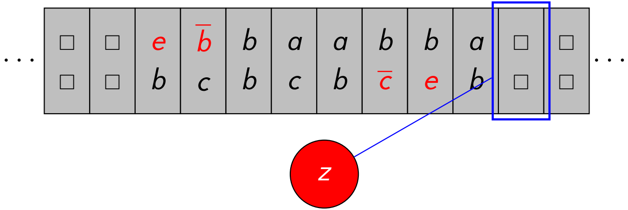

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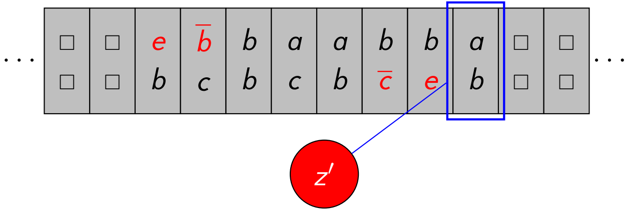

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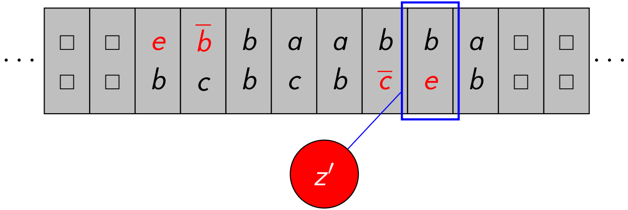

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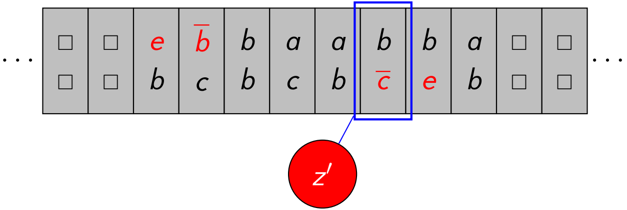

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Mehrband-Turingmaschinen

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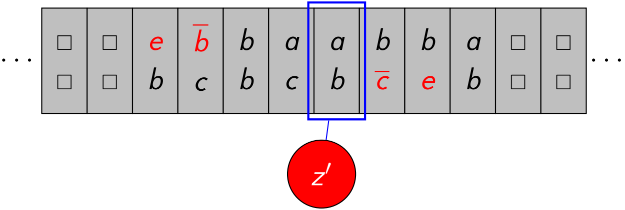

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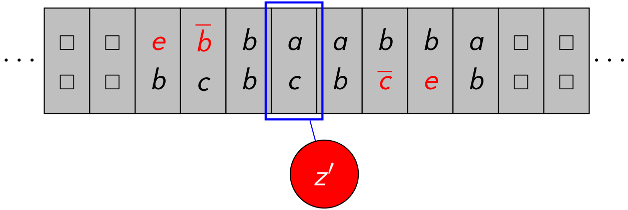

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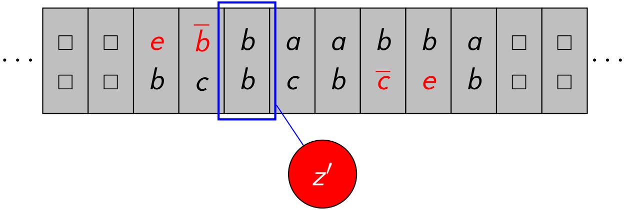

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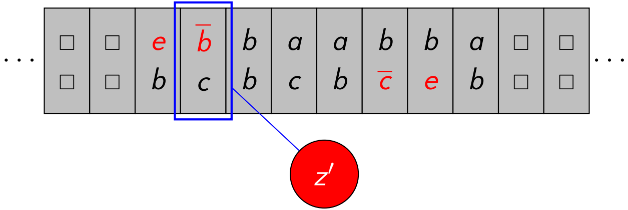

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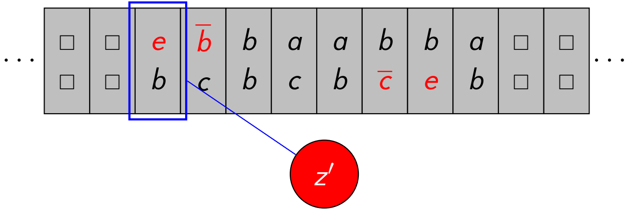

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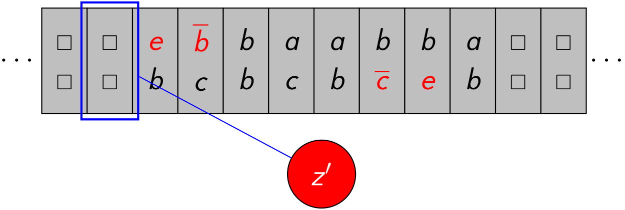

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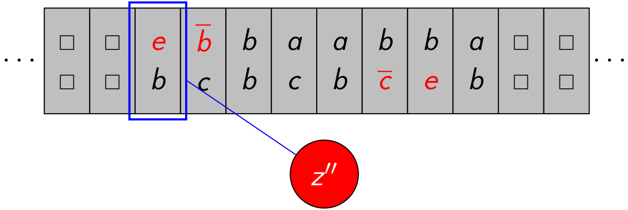

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Mehrband-Turingmaschinen

§3.11 Theorem

Für jede (normierte) k-Band-TM M existiert (norm.) TM N mit $T(N) = T(M)$

$$\longrightarrow \boxed{M_{\text{start}}} \longrightarrow \boxed{M_{\text{simul}}} \longrightarrow \boxed{M_{\text{ausgabe}}} \longrightarrow$$

Beweisskizze.

1. M_{start}: Einrichten Ausgangssituation (Erweitern Eingabe auf Tupel)
2. M_{simul}: Simulation Ableitungsschritte (wie gerade illustriert)
3. M_{ausgabe}: Ausgabe letztes Band (Löschen Bänder, Reduktion Tupel) □

Mehrband-Turingmaschinen

Standard-Operationen

- Band auf anderes Band kopieren

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- TM M auf Band i laufen lassen (($M(i)$ ist diese k-Band-TM)

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Konsequenzen

- Verwende Bänder wie Variablen

Mehrband-Turingmaschinen

Standard-Operationen

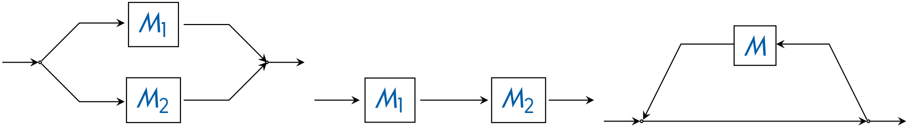
- Band auf anderes Band kopieren
- TM M auf Band i laufen lassen (${M(i)}$ ist diese k-Band-TM)

Konsequenzen

- Verwende Bänder wie Variablen
- Verwende k-Band-TM statt TM (äquivalente TM existiert)

Zusammenfassung

- Operationen auf Turingmaschinen
- k-Band-Turingmaschinen



Erste Übungsserie bereits im Moodle