4.3

(Alternives geordnetes Paar) Seien A, B, C, D vier beliebige Objekte. Zeigen Sie dass

$$\Big\{\{\{A\},\emptyset\},\{\{B\}\}\Big\} = \Big\{\{\{C\},\emptyset\},\{\{D\}\}\Big\}$$

genau dann wenn $A = C$ und $B = D$.

$\underline{\Big\{\{\{A\},\emptyset\},\{\{B\}\}\Big\} = \Big\{\{\{C\},\emptyset\},\{\{D\}\}\Big\} \implies A = C, B = D:}$

$$\begin{aligned}
&\Big\{\{\{A\},\emptyset\},\{\{B\}\}\Big\} = \Big\{\{\{C\},\emptyset\},\{\{D\}\}\Big\} && |\text{Kuratowskis geordnetes Paar}\\
&\implies (\{\{A\},\emptyset\},B) = (\{\{C\},\emptyset\},D)\\
&\implies \{\{A\},\emptyset\} = \{\{C\},\emptyset\} \text{ und } B = D && |\text{Kuratowskis geordnetes Paar}\\
&\implies (\{A\},\emptyset) = (\{C\},\emptyset) \text{ und } B = D\\
&\implies \{A\} = \{C\} \text{ und } B = D\\
&\implies A = C \text{ und } B = D
\end{aligned}$$

$\underline{A = C, B = D \implies \Big\{\{\{A\},\emptyset\},\{\{B\}\}\Big\} = \Big\{\{\{C\},\emptyset\},\{\{D\}\}\Big\}:}$

$$\begin{aligned}
&A = C, B = D\\
\implies &\{A\} = \{C\} \text{ und } B = D\\
\implies &(\{A\},\emptyset) = (\{C\},\emptyset) \text{ und } B = D && |\text{Kuratowskis geordnetes Paar}\\
\implies &\{\{A\},\emptyset\} = \{\{C\},\emptyset\} \text{ und } B = D\\
\implies &(\{\{A\},\emptyset\},B) = (\{\{C\},\emptyset\},D) && |\text{Kuratowskis geordnetes Paar}\\
\implies &\Big\{\{\{A\},\emptyset\},\{\{B\}\}\Big\} = \Big\{\{\{C\},\emptyset\},\{\{D\}\}\Big\}
\end{aligned}$$

$\square$