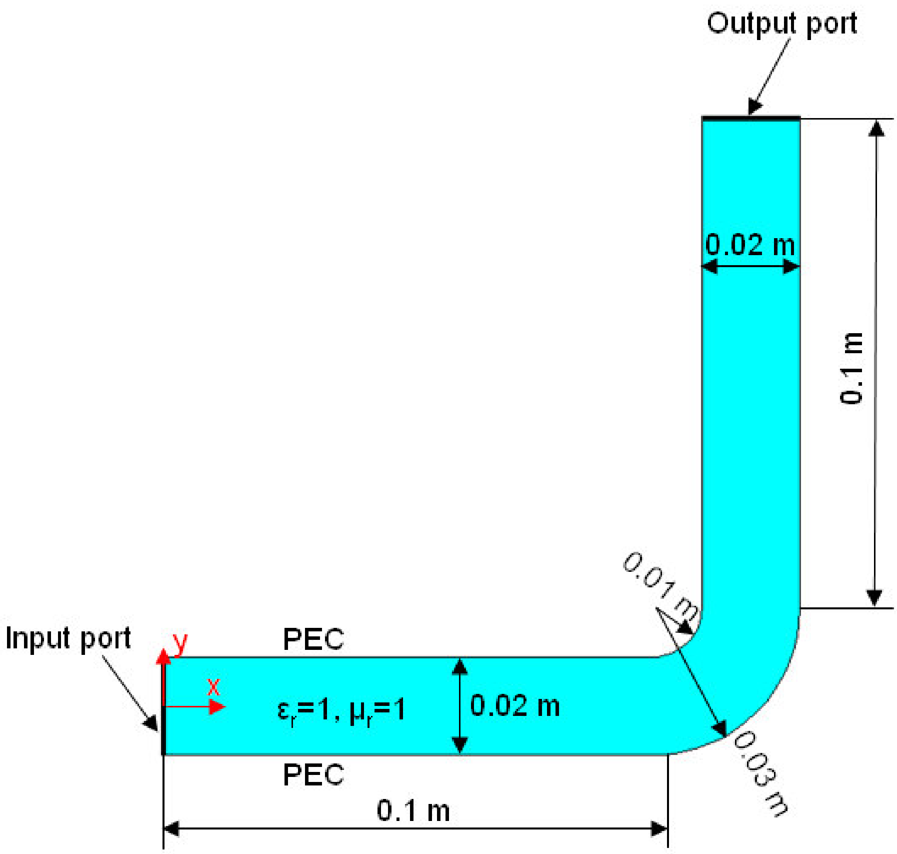
2D FEM Analysis of a 90º Waveguide Bend



x – propagation direction (before the bend); y – propagation direction (after the bend);

TE modes: $\vec{E} = E_z \cdot \vec{e}_z$

BVP :

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu_r}\frac{\partial E_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu_r}\frac{\partial E_z}{\partial y}\right)+\omega^2\mu_0\varepsilon_0\varepsilon_r E_z = 0 \text{, in the computational domain}$$

$$\frac{\partial E_z}{\partial n}(x,y)+jk_x E_z(x,y) = 2jk_x C_1(y)\cdot e^{-jk_x x} \text{, over the input port}$$

$$\frac{\partial E_z}{\partial n}(x,y)+jk_x E_z(x,y) = 0 \text{, over the output port}$$

$$E_z(x,y) = 0 \text{, PEC (waveguide wall)}$$

FEM discretization:

$$\sum_{j=1}^{N_n} E_{zj} \left\{ \sum_{\substack{e \\ e\in\sup(N_j)}} \iint_{(\Omega^e)} \frac{1}{\mu_r^{\,e}} \nabla N_i \cdot \nabla N_j \, d\Omega \right\} - \sum_{j=1}^{N_n} E_{zj} \left\{ \sum_{\substack{e \\ e\in\sup(N_j)}} \iint_{(\Omega^e)} \omega^2 \mu_0 \varepsilon_0 \varepsilon_r \, N_i N_j \cdot d\Omega \right\} +$$

$$+\sum_{j=1}^{N_n} E_{zj} \left\{ \sum_{\substack{e \\ e\in\sup(N_j)}} \int_{(\partial\Omega^e)} jk_x N_i N_j \cdot dl \right\} = \sum_{\substack{e \\ e\in\sup(N_j)}} \int_{(\partial_2\Omega^e)} 2\,jk_x C_1(y) N_i \, dl$$

Input signal normalization:

$$E_{z\,input} = E_0 \cos\left(\frac{\pi}{d} y\right) \cdot e^{-jk_x x}, \; H_y = \frac{1}{j\omega\mu} \frac{\partial E_z}{\partial x} = -\frac{k_x}{\omega\mu} E_0 \cos\left(\frac{\pi}{d} y\right) \cdot e^{-jk_x x}, \; Z = \frac{\omega\mu}{k_x}$$

$$P_{input}^{ex} = 1 \quad \Rightarrow \quad E_0 = \sqrt{2} \cdot \sqrt{\frac{2\omega\mu}{k_x d}} \text{ (Normalization)}$$

Magnetic field computation:

$$H_x = -\frac{1}{j\omega\mu} \frac{\partial E_z}{\partial y}, \; H_y = \frac{1}{j\omega\mu} \frac{\partial E_z}{\partial x}$$

$$H_x^e = -\frac{1}{j\omega\mu} \frac{1}{2A} \left[\left(x_3^e - x_2^e\right) E_{z1}^e + \left(x_1^e - x_3^e\right) E_{z2}^e + \left(x_2^e - x_1^e\right) E_{z3}^e \right]$$

$$H_y^e = \frac{1}{j\omega\mu} \frac{1}{2A} \left[\left(y_2^e - y_3^e\right) E_{z1}^e + \left(y_3^e - y_1^e\right) E_{z2}^e + \left(y_1^e - y_2^e\right) E_{z3}^e \right]$$

S – parameters:

$$S_{11} = \frac{\int_{\partial_2\Omega} \left(E_z - E_{1z}\right) \cdot E_{1z} \, dl}{\int_{\partial_2\Omega} E_{1z} \cdot E_{1z} \, dl}$$, E_z is the computed field, E_{1z} is the modal field at the input port

$$S_{12} = \frac{\int_{\partial_3\Omega} E_z \cdot E_{2z} \, dl}{\int_{\partial_3\Omega} E_{2z} \cdot E_{2z} \, dl}$$, E_{2z} is the modal field at the output port.