If-Then Conditional Reformulation of Frequency Nadir Constraints

The frequency nadir τ^* may occur in the intervals $(0, t_{\mathrm{G}}^{\mathrm{DB}}]$, $(t_{\mathrm{G}}^{\mathrm{DB}}, T_{\mathrm{B}}^{\mathrm{D}}]$, $(T_{\mathrm{B}}^{\mathrm{D}}, T_{\mathrm{W}}^{\mathrm{D}}]$, $(T_{\mathrm{W}}^{\mathrm{D}}, T_{\mathrm{G}}^{\mathrm{D}}]$, $(T_{\mathrm{G}}^{\mathrm{D}}, T_{\mathrm{H}}^{\mathrm{D}}]$. The frequency nadir constraint (9) in the manuscript can be separated as (1a)-(1e), respectively.

$$I_{s,t}^{\mathrm{sys}}\left(\frac{R_{s,t}^{\mathrm{B}}}{T_{\mathrm{B}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{W}}}{T_{\mathrm{W}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{H}}}{T_{\mathrm{H}}^{\mathrm{D}}}\right) \geq \frac{\left(\Delta P_{s,t}^{\mathrm{D}}\right)^2}{4\Delta f^{\max}}, \tag{1a}$$

$$\left[I_{s,t}^{\mathrm{sys}}+\frac{R_{s,t}^{\mathrm{G}}(t_{\mathrm{G}}^{\mathrm{DB}})^2/T_{\mathrm{G}}^{\mathrm{D}}}{4\Delta f^{\max}}\right]\left(\frac{R_{s,t}^{\mathrm{B}}}{T_{\mathrm{B}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{W}}}{T_{\mathrm{W}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{G}}}{T_{\mathrm{G}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{H}}}{T_{\mathrm{H}}^{\mathrm{D}}}\right)$$
$$\geq \frac{\left(\Delta P_{s,t}^{\mathrm{D}}+R_{s,t}^{\mathrm{G}}t_{\mathrm{G}}^{\mathrm{DB}}/T_{\mathrm{G}}^{\mathrm{D}}\right)^2}{4\Delta f^{\max}}, \tag{1b}$$

$$\left[I_{s,t}^{\mathrm{sys}}-\frac{R_{s,t}^{\mathrm{B}}T_{\mathrm{B}}^{\mathrm{D}}}{4\Delta f^{\max}}+\frac{R_{s,t}^{\mathrm{G}}(t_{\mathrm{G}}^{\mathrm{DB}})^2/T_{\mathrm{G}}^{\mathrm{D}}}{4\Delta f^{\max}}\right]\left(\frac{R_{s,t}^{\mathrm{W}}}{T_{\mathrm{W}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{G}}}{T_{\mathrm{G}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{H}}}{T_{\mathrm{H}}^{\mathrm{D}}}\right)$$
$$\geq \frac{\left(\Delta P_{s,t}^{\mathrm{D}}-R_{s,t}^{\mathrm{B}}+R_{s,t}^{\mathrm{G}}t_{\mathrm{G}}^{\mathrm{DB}}/T_{\mathrm{G}}^{\mathrm{D}}\right)^2}{4\Delta f^{\max}}, \tag{1c}$$

$$\left[I_{s,t}^{\mathrm{sys}}-\frac{R_{s,t}^{\mathrm{B}}T_{\mathrm{B}}^{\mathrm{D}}+R_{s,t}^{\mathrm{W}}T_{\mathrm{W}}^{\mathrm{D}}}{4\Delta f^{\max}}+\frac{R_{s,t}^{\mathrm{G}}(t_{\mathrm{G}}^{\mathrm{DB}})^2/T_{\mathrm{G}}^{\mathrm{D}}}{4\Delta f^{\max}}\right]\left(\frac{R_{s,t}^{\mathrm{G}}}{T_{\mathrm{G}}^{\mathrm{D}}}+\frac{R_{s,t}^{\mathrm{H}}}{T_{\mathrm{H}}^{\mathrm{D}}}\right)$$
$$\geq \frac{\left(\Delta P_{s,t}^{\mathrm{D}}-R_{s,t}^{\mathrm{B}}-R_{s,t}^{\mathrm{W}}+R_{s,t}^{\mathrm{G}}t_{\mathrm{G}}^{\mathrm{DB}}/T_{\mathrm{G}}^{\mathrm{D}}\right)^2}{4\Delta f^{\max}}, \tag{1d}$$

$$\left[I_{s,t}^{\mathrm{sys}}-\frac{R_{s,t}^{\mathrm{B}}T_{\mathrm{B}}^{\mathrm{D}}+R_{s,t}^{\mathrm{W}}T_{\mathrm{W}}^{\mathrm{D}}+R_{s,t}^{\mathrm{G}}T_{\mathrm{G}}^{\mathrm{D}}}{4\Delta f^{\max}}\right]\frac{R_{s,t}^{\mathrm{H}}}{T_{\mathrm{H}}^{\mathrm{D}}}$$
$$\geq \frac{\left(\Delta P_{s,t}^{\mathrm{D}}-R_{s,t}^{\mathrm{B}}-R_{s,t}^{\mathrm{W}}-R_{s,t}^{\mathrm{G}}\right)^2}{4\Delta f^{\max}}. \tag{1e}$$

The constraints on frequency nadir for the five intervals in (9) and (1a)-(1e) are the *if-then* conditional second-order cone constraints. We first introduce five groups of binary variables $z_{s,t}^1, z_{s,t}^2, z_{s,t}^3, z_{s,t}^4$, and $z_{s,t}^5$ to indicate each interval whether the frequency nadir occurs. Subsequently, we denote the two terms on the left side of inequality sign and the one term on the right side of inequality in (1a)-(1e) by $A_{s,t}^d(\boldsymbol{x})$, $B_{s,t}^d(\boldsymbol{x})$, and $C_{s,t}^d(\boldsymbol{x})$ for clarity, where $d \in \mathcal{D} = \{1, 2, ..., 5\}$ is the index for intervals and $\boldsymbol{x}$ represents the vector of optimization variables. By using the big-M method and variable transformation, the if-then conditional constraints, i.e., (1a)-(1e), can be replaced by a set of following constraints:

$$D_{s,t}^2 \leq K_{s,t}L_{s,t} \tag{2a}$$

$$z_{s,t}^1+z_{s,t}^2+z_{s,t}^3+z_{s,t}^4+z_{s,t}^5=1 \tag{2b}$$

$$-M^F\left(1-z_{s,t}^d\right) \leq D_{s,t}-A_{s,t}^d(\boldsymbol{x}) \leq M^F\left(1-z_{s,t}^d\right), d \in \mathcal{D} \tag{2c}$$

$$-M^F\left(1-z_{s,t}^d\right) \leq K_{s,t}-B_{s,t}^d(\boldsymbol{x}) \leq M^F\left(1-z_{s,t}^d\right), d \in \mathcal{D} \tag{2d}$$

$$-M^F\left(1-z_{s,t}^d\right) \leq L_{s,t}-C_{s,t}^d(\boldsymbol{x}) \leq M^F\left(1-z_{s,t}^d\right), d \in \mathcal{D}, \tag{2e}$$

where M^F is a predefined big positive number. $D_{s,t}$, $K_{s,t}$, and $L_{s,t}$ are auxiliary variables in scenario s at time t.